KIAS-TIFR-ICTS Joint Advanced School on Algebraic Surfaces and Related Topics

21 – 30 November, 2015

Schedule and Abstracts of Talks

International Centre for Theoretical Sciences
Tata Institute of Fundamental Research

Title of Talks (Workshop)

Margarida Lopes *On the algebraic fundamental group of surfaces of general type*

Sheng-Li Tan *Chern numbers of families of algebraic curves and ordinary differential equations*

Miles Reid *Unprojection and applications to constructing varieties*

Masayoshi Miyanshi *Unipotent structures of algebraic varieties*

Marius Koras *The BMY inequality and its applications*

Jong-Hae Keum *To be Announced*

Rajendra Gurjar *A positively graded domain is determined at its vertex. An application to Invariant Theory; a new property about the quotient $\mathbf{A^n}//\mathbf{G}$.*

Rajendra Gurjar *On a Conjecture of V. Popov.*

Title of Talks (Conference)

Zbigniew Jelonek	*On the group of automorphisms of a quasi-affine variety*
Insong Choe	*Maximal isotropic subbundles of symplectic and orthogonal bundles over a curve*
Adrien Dubouloz	*Fake real euclidean planes*
Karol Palka	*Almost minimal log surfaces and the logMMP with half-integral coefficients*
Jinhyun Park	*On algebraic cobordism version of Grothendieck standard conjecture D*
Young-Hoon Kiem	*Moduli spaces of sheaves on curves and Calabi-Yau 3-folds*
S.M. Bhatwadekar	*On a quadratic plane*
Donghoon David Hyeon	*Generic semistability of representations of reductive groups*
Neena Gupta	*Projective modules over the kernel of a locally nilpotent derivation on a polynomial ring*
Junmyeong Jang	*A lifting of an automorphism of a K3 surface over odd characteristic*
Giancarlo Urzua	*Geography in positive characteristic and connections with Kawamata-Viehweg vanishing theorem*
Hideo Kojima	*Some results on open algebraic surfaces of non-negative logarithmic Kodaira dimension*
Dongseon Hwang	*Log del Pezzo surfaces of rank one*
Kyoung-Seog Lee	*To be Announced*
D. S. Nagraj	Tangent bundle of $\mathbb{P}^2$ and morphism to $Gr(2,4)$.

Abstracts (Workshop)

Saturday, 21 November 2015 (14:30-15:00) ; Sunday, 22 November 2015 (11:30–12:30)
Speaker : **Magarida Lopes**
Title : **On the algebraic fundamental group of surfaces of general type**

 The algebraic fundamental group of an algebraic variety is an important tool in Algebraic Geometry. In these two lectures, after an introduction to both the main properties of invariants of complex projective surfaces and to the algebraic fundamental group, some results will be discussed. The focus will be mainly on results concerning the algebraic fundamental group of minimal surfaces of general type with small Chern ratio c_1^2/c_2, illustrating the fact that for those surfaces the structure of the algebraic fundamental group is relatively simple

Saturday, 21 November 2015 (16:00–17:00) ; Sunday, 22 November 2015 (14:00–15:00)
Speaker : **Sheng-Li Tan**
Title : **Chern numbers of families of algebraic curves and ordinary differential equations**

The main purpose of these two talks is to introduce the Chern numbers of families of algebraic curves and ordinary differential equations of the following type

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)}.$$

The Chern numbers c_1^2 and c_2 provide us new topological invariants for polynomials $f(x,y)$ or rational polynomials $f(x,y)/g(x,y)$ by considering the family of algebraic curves C_t defined by $f(x,y) - tg(x,y) = 0$, where t is the parameter. The Chern numbers c_1^2 and c_2 of an ODE on an algebraic surface are invariants desired by some mathematicians of 19th century. Poincaré and Painlevé proposed some problems on the algebraic integrability of these ODEs. As an application of these invariants, we will give positive answers to these problems for ODEs with $c_1^2 < \frac{1}{4}c_2$. Finally, we will discuss the birational and biregular classification of ODEs by using their Chern numbers.

Sunday, 22 November 2015 (10:00–11:00) ; Monday, 23 November 2015 (16:00–17:00)
Speaker : Miles Reid
Title   : Unprojection and applications to constructing varieties

Some of the basic constructions of algebraic varieties are in terms of graded rings; this applies, for example, to weighted hypersurfaces: a hypersurface $X(d)$ in $\mathbb{P}(a_1,...,a_n)$ is the Proj of the graded algebra $k[x_1,...,x_n]/F_d$ where $F_d(x_1,...,x_n)$ is a weighted homogeneous polynomial in variables. Kustin-Miller unprojection (''constructing bigger Gorenstein ideals from smaller ones'') is a method in commutative algebra with strong links to birational geometry. I will give an entry level treatment of some of the theory behind this, and discuss some interesting applications.

Monday and Tuesday, 23 and 24 November 2015 (10:00–11:00)
Speaker : Masayoshi Miyanshi
Title   : Unipotent structures of algebraic varieties

A unipotent structure of an algebraic variety X is the existence of a Zariski open set U such that $U \cong \mathbb{A}^1 \times S$, where S is an algebraic variety of dimension one less than $\dim X$. The open set U is called an $\mathbb{A}^1$-cylinder over S. If a smooth affine variety X has a unipotent structure $U \cong \mathbb{A}^1 \times S$ then the projection $p_U : U \to S$ extends to an $\mathbb{A}^1$-fibration $f : X \to Y$. If Y is further affine, the morphism f is the quotient morphism of a certain G_a-action. On the other hand, if X is a smooth projective variety, we are interested in the question whether the property of X having a unipotent structure is preserved under a birational morphism $\varphi : X \to X'$. We discuss these topics as well as the related ones.

Monday and Tuesday, 23 and 24 November 2015 (11:30–12:30)
Speaker : Marius Koras
Title   : The BMY inequality and its applications

The BMY inequality was proved by F.Bogomolov,Y.Miyaoka and independently by 2 S.Yau. It states that if X is a smooth projective minimal surface then $K_X^2 \leq 3e_{top}(X)$. It was generalized to the case of smooth open surface by R.Kobayashi. We will introduce basic notions needed to formulate the BMY inequality in case of an open surface, in particular we will describe the process of minimalization of a pair (X, D) where X is a projective surface and D a divisor. Some applications of BMY inequality will be discussed i.e. a classification of $\mathbb{Z}$ and $\mathbb{Q}$-homology planes with quotient singularities, singular Z-homology planes of general type, a classification of smooth embeddings of $\mathbb{C}$ into $\mathbb{C}^2$.

Speaker : **Jong-Hae Keum**
Title : **To be Announced**

Speaker : **Rajendra Gurjar**
Title : **A positively graded domain is determined at its vertex. An application to Invariant Theory; a new property about the quotient $\mathbf{A^n}//\mathbf{G}$.**

Let k be an algebraically closed field of characteristic 0. For an affine positively graded domain R with $R_0 = k$, let M be the irrelevent maximal ideal of M. Then M is determined by the isomorphism class of th completion of R_M w.r.t. its maximal ideal. This result was conjectured by me twenty five years ago. Using crucially an idea of O. Mathieu it is now proved.

Corollary. Let G be a reductive algebraic group acting linearly on an affine space $\mathbf{A^n}$. Let V be the quotient $\mathbf{A^n}//\mathbf{G}$ corresponding to the ring of invariants of G. Then the logarithmic Kodaia dimension of the smooth locus of V is $-\infty$. This was also conjectured by me around 1990.

Speaker : **Rajendra Gurjar**
Title : **On a Conjecture of V. Popov**

Let $f : W \to V$ be a finite surjective morphism from a complex manifold W to a normal complex space. Then the order of the local fundamental group at points in V is an upper continuous function of the point in V.

Corollary. An application to a conjecture of V. Popov. Let G be a connected algebraic group acting linearly on an affine space $\mathbf{A^n}$. If the quotient map $\mathbf{A^n} \to \mathbf{A^n}//\mathbf{G}$ has all the fibers of the same dimension then Popov conjectured in 1976 that $\mathbf{A^n}//\mathbf{G}$ is smooth. We will prove that the singular locus of $\mathbf{A^n}//\mathbf{G}$ has codimension at least 3

Abstracts (Conference)

Friday, 27 November 2015 (10:00–10:50)
Speaker : **Zbigniew Jelonek**
Title : **On the group of automorphisms of a quasi-affine variety**

Let $\mathbb{K}$ be an algebraically closed field of characteristic zero. We show that if the automorphisms group of a quasi-affine variety X over $\mathbb{K}$ is infinite, then X is uniruled

Friday, 27 November 2015 (11:10–12:00)
Speaker : **Insong Choe**
Title : **Maximal isotropic subbundles of symplectic and orthogonal bundles over a curve**

Segre-Nagata's bound computes the largest degree among line subbundles of a general stable rank 2 vector bundle over a curve. Also, similar bounds are known for vector bundles of higher rank. We consider the same question for symplectic and orthogonal bundles and give a sharp bound. We also provide the details of the related stratifications on the moduli spaces. This is based on a series of joint works with George H. Hitching.

Friday, 27 November 2015 (13:30–14:20)
Speaker : **Adrien Dubouloz**
Title : **Fake real euclidean planes**

A fake real euclidean plane is a complexification of the euclidean plane $\mathbb{R}^2$ into a smooth $\mathbb{Q}$-acyclic complex algebraic surface S different from $\mathbb{C}^2$. I will present families of examples of such fake planes in every logarithmic Kodaira dimension, and give some partial results concerning their classification up to real-biregular and real-birational equivalence.(Joint work with F. Mangolte)

Speaker : **Karol Palka**
Title : **Almost minimal log surfaces and the logMMP with half-integral coefficients**

An important tool in studying open algebraic surfaces is the logarithmic Minimal Model Program. Although in dimension two its general mechanism is well understood, in concrete problems we need an analysis which goes far beyond the general framework. For log surfaces with reduced boundaries the usage of almost minimal models (due to Miyanishi, Fujita and others), which allows to avoid singularities, turned out to be very successful. Recently we generalized this approach to boundaries with coefficient 1/2 and used it in particular in the proof of the Coolidge-Nagata conjecture (2015, coauthored with M. Koras) concerning cuspidal planar curves. We will discuss a more general form of this approach showing how to apply it to other problems concerning surfaces of log general type.

Saturday, 28 November 2015 (10:00–10:50)
Speaker : **Jinhyun Park**
Title : **On algebraic cobordism version of Grothendieck standard conjecture D**

Algebraic cobordism can be seen as a common generalization of Grothendieck groups and Chow groups. Various natural questions on these groups can thus be lifted and asked on algebraic cobordism. I will discuss some of these lifts, including the Grothendieck standard conjecture D on algebraic cobordism.

Saturday, 28 November 2015 (11:10–12:00)
Speaker : **Young-Hoon Kiem**
Title : **Moduli spaces of sheaves on curves and Calabi-Yau 3-folds**

Let C be a smooth projective curve of genus $g>1$ and let Y be the total space of a general rank 2 stable vector bundle on C whose determinant is the canonical line bundle of C, so that Y is a Calabi-Yau 3-fold. Let N be the moduli space of stable rank 2 vector bundles of odd degree d on C. Let M be the moduli space of stable coherent sheaves on Y with Hilbert polynomial 2m+d-2g+2. In this talk I will discuss the problem of comparing N with M. Away from a divisor D, N is isomorphic to M but the scheme structure is different along D. This problem arose

from a mathematical theory of the Gopakumar-Vafa invariant. I will provide a quick introduction to the history of the Gopakumar-Vafa invariant and its relation with the categorification of the Donaldson-Thomas invariant.

Sunday, 29 November 2015 (10:00–10:50)

Speaker : **S. M. Bhatwadekar**
Title : **On a quadratic plane**

Let k be an algebraically closed field of characteristic zero and let F k[X, Y, Z] be a prime element such that k[X, Y, Z]/(F) = k[U, V]. In this set up Abhyankar-Sathaye Epimorphism question asks whether F is a variable in k[X, Y, Z]. In my talk I will address this question when $F = a(X, Y)Z^2 + b(X, Y)Z + c(X, Y)$. This talk is based on a thesis of Garcia (former student of Peter Russell) and my joint work with Neena Gupta.

Sunday, 29 November 2015 (11:10–12:00)

Speaker : **Donghoon Hyeon**
Title : **Generic semistability of representations of reductive groups**

Given an algebraic group G and a rational representation $\rho : G \to GL(V)$, a point $v \in V$ is said to be semistable if the closure of the G-orbit of v does not contain the origin. For reductive groups, the Hilbert-Mumford theorem states that v is unstable if and only if there is a co-character λ of G such that $\lim_{t \to 0} \lambda(t).v = 0$. It has long been observed by researchers that, to find a destabilizing co-character, one has to choose a maximal torus carefully. In this talk, we shall present a framework in which this observation can be made precise in the semisimple case, and how it can be generalized to the case of reductive groups. This is joint work with J. Park and D. Ph. Bac.

Sunday, 29 November 2015 (13:30–14:20)

Speaker : **Neena Gupta**
Title : **Projective modules over the kernel of a locally nilpotent derivation on a polynomial ring**

Let k be an algebraically closed field of characteristic zero, B the polynomial algebra in n variables over k, D a locally nilpotent derivation on B and A the kernel of D. A question of M. Miyanishi asks whether finitely generated projective modules over A are free. It is well known that for n=2, i.e., when B=k[X,Y], the

kernel A is a polynomial algebra in one variable over k. Again, when n=3, i.e., when B= k[X,Y,Z], a theorem of Miyanishi shows that the kernel A is a polynomial algebra in two variables over k. Thus, by a theorem of C.S. Seshadri, Miyanishi's question has an affirmative answer when $n<4$.

Sunday, 29 November 2015 (14:40–15:30)

Speaker : **Junmyeong Jang**
Title : **A lifting of an automorphism of a K3 surface over odd characteristic**

For a K3 surface over an algebraically closed field of odd characteristic, all the liftings over the Witt vector ring are classified by the Hodge filtration on the second crystalline cohomology. Based on this fact, we can prove that for a K3 surface of finite height, under a mild condition, there exists a lifting over the Witt vector ring to which whole automorphism group can be extended. We can also prove that a purely non-symplectic automoprhism can be lifted over the Witt vector ring. Then, from the uniqueness of a purely non-symplectic automorphism of high order on complex K3 surfaces, we can prove the uniqueness of a non-symplectic automorphism of the same kind over odd characteristic.

Sunday, 29 November 2015 (15:50–16:40)

Speaker : **Giancarlo Urzua**
Title : **Geography in positive characteristic and connections with Kawamata-Viehweg vanishing theorem**

I will explain recent results on density of Chern slopes for surfaces of general type in positive characteristic (arXiv:1509.05260), which extend my joint results (in collaboration with X. Roulleau) on density of Chern slopes in characteristic zero (arXiv:1402.5801). The result in positive characteristic includes density of Chern slopes in [2,infinity[of etale-simply connected surfaces of general type with Picard scheme equal to a reduced point. It turns out that its proof gives a possible strategy to prove the Kawamata-Viehweg vanishing theorem for rational surfaces. I will discuss details around a specific missing ingredient, and how it effectively works in the proof of the above mentioned reduced density.

Speaker : **Hideo Kojima**

Title : **Some results on open algebraic surfaces of non-negative logarithmic Kodaira dimension**

We discuss open algebraic surfaces of non-negative logarithmic Kodaira dimension over an algebraically closed field of arbitrary characteristic. After recalling structure theorems for open algebraic surfaces, we give some results on logarithmic plurigenera of open algebraic surfaces. We also give some results on open algebraic surfaces of logarithmic Kodaira dimension zero.

Monday, 30 November 2015 (11:10–12:00)

Speaker : **Dongseon Hwang**

Title : **Log del Pezzo surfaces of rank one**

De-Qi Zhang initiated the systematic study of the classification problem of log del Pezzo surfaces of rank one in his doctoral thesis. It has been developed further by many other mathematicians and has been used to investigate many properties of log del Pezzo surfaces.

In this talk, I will report my recent progress on his program and explain how it leads to the classification of log del Pezzo surfaces of rank one.

Monday, 30 November 2015 (13:30–14:20)

Speaker : **Kyoung-Seog Lee**

Title : **To be Announced**

Monday, 30 November 2015 (14:40–15:30)

Speaker : **D. S. Nagraj**

Title : **Tangent bundle of $\mathbb{P}^2$ and morphism to $\mathrm{Gr}(2,4)$.**

In this talk we discuss about morphisms from $\mathbb{P}^2$ to $\mathrm{Gr}(2,4)$. This is a joint work with A. El Mazouni of University of D' Artios.