RDST 2020

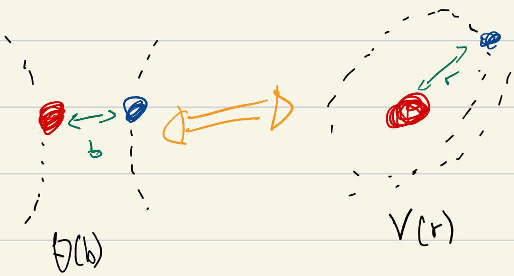
From Spinning Particles to Classical Gravitational Radiation

A. Guevara
(Perimeter / Harvard)

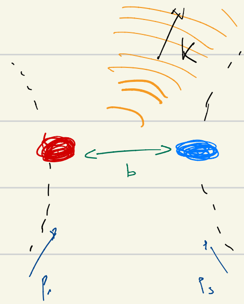
1812.06895 w/ Ochirov & Vines
1903.12419 w/ Bautista
1908.11349
20XX. w/ Bautista, Kavanagh & Vines
20XX. w/ " , N. Siemonsen

3 problems in Classical GR.

i) 2-body conservative dynamics



ii) Radiation from 2-body system.

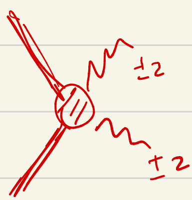


(see Chia Hsien's talk)

(see Sen's talk)

Generalized unitarity + EFT

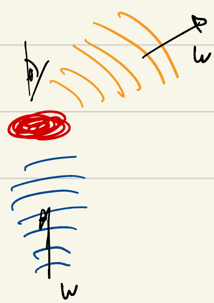
residues (radiation modes $k^2=0$)



: Gravitational Compton

$\hbar \to 0$

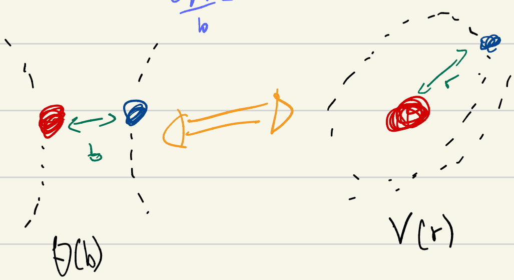
iii) Scattering GW's off a BH



3 problems in Classical GR.

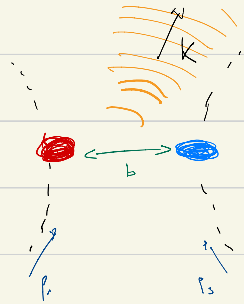
i) 2-body conservative dynamics

$\frac{Gm}{b} \ll 1$



ii) Radiation from 2-body system.

$\frac{Gm}{r} \ll 1$



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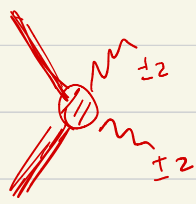

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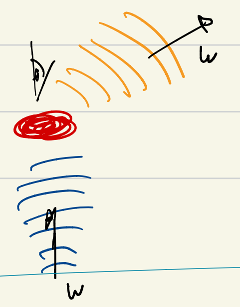
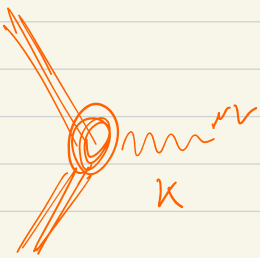


$GM\omega \ll 1$

What about Spinning Black holes?

Kerr BH from minimal coupling amplitudes.

A.G. '17
+ Ochirov, Vines '18
Chung, Huang, Kim & Lee '18



$\Rightarrow h^{\mu\nu}$ (linearized Kerr metric) @ $k^2 = 0$

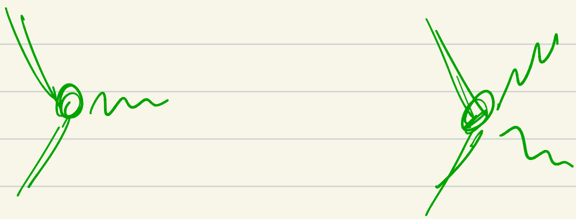
$s \to \infty$

$\longrightarrow \dfrac{d\sigma^{GW}}{d\Omega}$ (gravitational wave cross-section)

$s \to \infty$

Outline

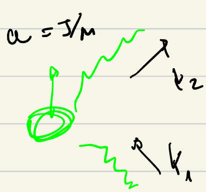
I) Construction of



for spinning particles.

II) Applications:

Gravitational Radiation

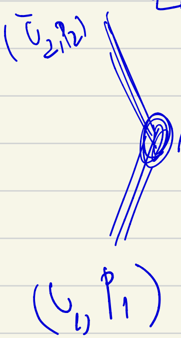


Gravitational Wave
-Scattering-

OBS: A spin-s amplitude involves 2s+1 (mass and current) multipole moments.

↳ intrinsic multipoles of the Kerr Black Hole ($s \to \infty$)

Ex: $s = 1/2$,



$$M_k^{\mu\nu} = \alpha\, p_1^{(\mu}\, \bar{u}_2 \big(\underbrace{p_1^{\nu)} \mathbb{I}}_{\text{scalar}} + g \underbrace{\gamma^{\nu)\alpha} k_\alpha}_{\text{dipole}} \big) u_1$$

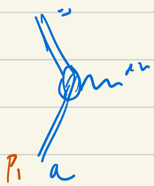
scalar ↓ Weinberg's soft theorem

$\alpha = \sqrt{8\pi G}$

dipole ↓ Cachazo-Strominger soft theorem

$g = 2$

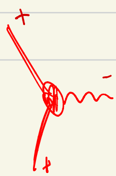
...



$$M^{\mu\nu}\bar{\varepsilon}_{\mu\nu} = \frac{\kappa}{2} \Bigg(\overbrace{\frac{[k|p_1|\mu\rangle}{\langle k\mu\rangle}}^{A_0} \Bigg)^2 (\varepsilon\cdot p)^2 \langle \varepsilon_2 | \, \mathbb{I} + \frac{F_{\mu\nu} J^{\mu\nu}}{\varepsilon\cdot p_{1k}} \, | \varepsilon_1 \rangle$$

$\bar{\varepsilon}_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu$

Key observation: The multipole structure is preserved in the $m \to 0$ limit, e.g.

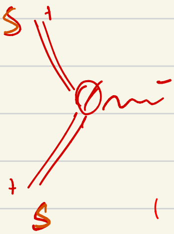


$$= \frac{\kappa}{2} A_0 \left(\langle \varepsilon_2 | \, 1 + \frac{F_{\mu\nu} J^{\mu\nu}}{\varepsilon\cdot p_1} \, | \varepsilon_1 \rangle \right)$$

Minimal Coupling criteria

We can use the $m \to 0$ limit to obtain the multipole structure @ any spin s.

fixed by little group scaling $\mathbb{R}$ in $D=4$



$$= \frac{\kappa}{2} A_0 \left(\langle \varepsilon_2^{(1/2)} | \, 1 + \frac{F_{\mu\nu} J^{\mu\nu}}{\varepsilon \cdot p} | \varepsilon_1^{(1/2)} \rangle \right)^{S} \qquad |\varepsilon^{(s)}\rangle = |\varepsilon^{(1/2)}\rangle^{\otimes s}$$

$$= \frac{\kappa}{2} A_0 \langle \varepsilon_2^{(s)} | \left(1 + \frac{F_{\mu\nu} J^{(s)}_{\mu\nu}}{s\, \varepsilon \cdot p} \right)^{\otimes s} | \varepsilon_1^{(s)} \rangle \qquad J^{(s)}_{\mu\nu} = J_{\mu\nu} \otimes \mathbb{I}^{2s-1} + \mathbb{I} \otimes J_{\mu\nu} \otimes \mathbb{I}^{2s-2} + \dots$$

$$\boxed{s \to \infty} = \frac{\kappa}{2} A_0 \langle \varepsilon_2^{(s)} | \, e^{\frac{F_{\mu\nu} J^{\mu\nu}}{\varepsilon \cdot p}} \, | \varepsilon_1^{(s)} \rangle$$

To obtain the 3pt amplitude for a Kerr BH $(m \neq 0)$ we demand consistency with the above as $m \to 0$ ("no-hair theorem")

$$= \kappa/2 \, \boxed{A_0} \langle \varepsilon_2 | \, e^{F \cdot J^{(s)} / \varepsilon \cdot p} \, | \varepsilon_1 \rangle \, // \qquad a^\mu = \frac{1}{m^2} \epsilon^{\mu\nu\rho\sigma} p_\nu S_{\rho\sigma}$$

$$= \frac{\kappa}{2} A_0 \langle 2^b |^{\otimes s} \, e^{2 a \cdot k} \, | 1^a \rangle^{\otimes s} \quad \to \text{massive spinor helicity}$$

Classical limit $a = \hat{a}/\hbar$, $k = \hat{k}\hbar$, $\hbar \to 0$

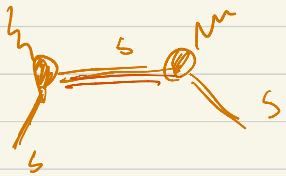
$\Rightarrow a \cdot k \sim \hbar^0$.

(CD " $h^{\mu\nu}_{Kerr} = \frac{1}{\Box}$

$T^{\mu\nu}(k^2 \neq 0)$ " $\leadsto$ Newman - Janis boost.

There's no consistent $\overbrace{\text{higher-spin}}^{\text{massless}}$ amplitude @ $\underline{\underline{n=4}}$

Unitarity demands



$\rightarrow \dfrac{1}{m^{\alpha}}$

(For GW scattering $r_s/\underline{\lambda} \ll 1$)

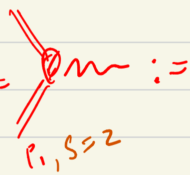
i) should be fine as long as $s \ll m^2$ (effective description)

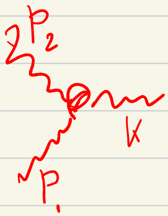
ii) As $m \to 0$ more higher-spin states are needed

We can still obtain consistent information @ $s \leq 2$.

In this case the minimal coupling criteria can be implemented via dimensional reduction

$\boxed{S=2}$

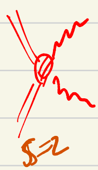
$A_3 =$ 

$:=$ 

$P_i^2 = 0$

$P_i = (p_i, m) \qquad K = (k, 0)$

$\mathcal{E}_i = (\epsilon_i, 0)$

$A_4 =$ 

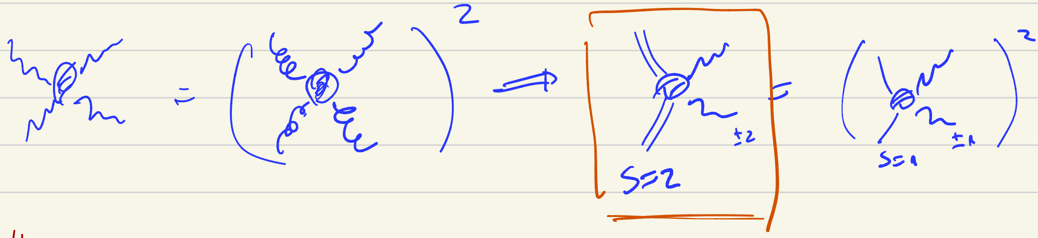
$:=$ 

$P_i = (p_i, m) \qquad K = (k, 0)$

$\mathcal{E}_i = (\epsilon_i, 0)$

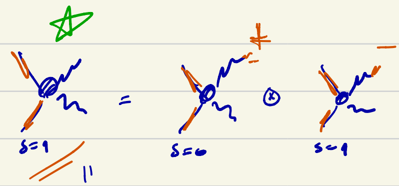
These have the correct $m \to 0$ limit by construction!

Digression: minimal coupling $\Longrightarrow$ double copy

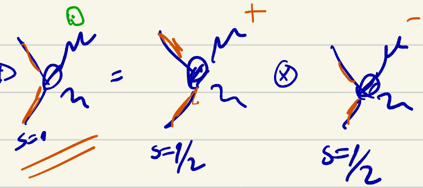


other examples:

$\mathcal{N}=4$ SUGRA = YMS $\otimes$ YM $\Longrightarrow$
(bosonic)



$\mathcal{N}=8$ SUGRA = $\mathcal{N}=4$ SYM $\otimes$ $\mathcal{N}=4$ SYM $\Longrightarrow$



Gravitational amplitudes are universal

↪ $\star = \odot$

↪ $0 \otimes 1 = 1/2 \otimes 1/2$

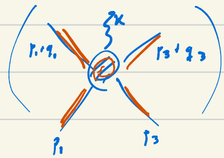
Dilaton-axion amplitudes are not!

(but they decouple)

Applications I: Classical Radiation @ tree level

→ Kosower - Maybee - O'Connell : Radiation Kernel

$$"T^{\mu\nu}(k)" = \int \frac{d^Dq_1\, d^Dq_3}{(2\pi)^{D-2}}\, \delta(q_1+q_3-k)\, \delta(2q_1 \cdot p_1)\, \delta(2q_3 \cdot p_3)\; e^{iq_1 b_1}\, e^{iq_3 b_3}$$



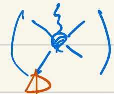
$k = \hbar \hat{k}$

$q_i = \hbar \hat{q}_i$

$\hbar \to 0$

$\boxed{q = \frac{q_1 - q_3}{2}}$

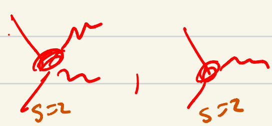
$$= \int \frac{d^Dq}{(2\pi)^{D-2}}\, e^{iq\cdot b}\, e^{ik\cdot\tilde{b}}\, \delta(\;)\,\delta(\;)$$



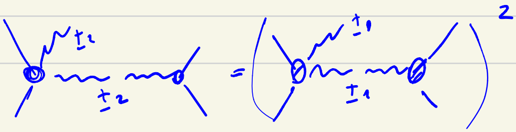
$\boxed{\begin{array}{l} q = \hbar \hat{q} \\ k = \hbar \hat{k} \end{array}}$

But we have

∴ $O(G)$ classical radiation is completely constrained by

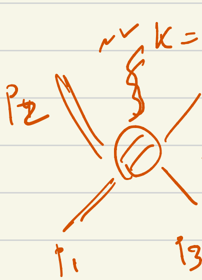


Each term is a double copy!



⟹ recovers Goldberger, Li, Prabhu (Spin dipole) classical double copy.

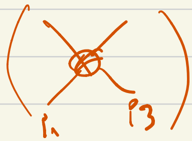
(Leading) Soft theorem



$$= \kappa A \underbrace{\sum_{i=1,3} \frac{p_i^\mu p_i^\nu}{k\cdot p_i}}_{S^{(0)}}$$

$$+ O(\omega^0)$$

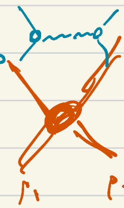
$$= \left(\frac{p_1^\mu p_1^\nu}{k\cdot p_1} - \frac{p_2^\mu p_2^\nu}{k\cdot p_2} + \frac{p_3^\mu p_3^\nu}{k\cdot p_3} - \frac{p_4^\mu p_4^\nu}{k\cdot p_4} \right)$$



$$\boxed{p_2 = p_1 + q_{\perp}}$$

Classically. ($\kappa \to 0$)

$$\longrightarrow \Delta p^\mu = \int \frac{d^D q}{(2\pi)^{D-2}}\, q^\mu e^{iq\cdot b}$$

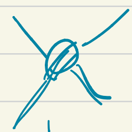


$$\Big|\, \delta(2q\cdot p_1)\, \delta(2q\cdot p_3)$$

$q = \hbar \hat{q}$, $\hbar \to 0$, $q\cdot b$, $e^{ik\cdot b}$

$$\epsilon_{\mu\nu} T^{\mu\nu} = \int \frac{d^D q}{(2\pi)^{D-2}}\, \delta\!\left(q\cdot p_1 + \cancel{\frac{k\cdot p_1}{2}}\right) \delta(q\cdot p_3)\, e \quad e$$

$$\left[S^{(0)}_\mu q^\mu \right]$$



$$+ O(\omega)$$

$$= S^{(0)}_\mu \underbrace{\int \frac{d^D q}{(2\pi)^{D-2}}\, q^\mu \; e^{iq\cdot b}\, \delta(q\cdot p_1)\, \delta(q\cdot p_3)}_{}$$

$$+ \text{other} \sim O(G)$$

$$= S^{(0)}_\mu \Delta p^\mu = \Delta \sum_{i=1,3} \frac{p_i^\mu p_i^\nu}{k\cdot p_i}\, \epsilon_{\mu\nu}$$

See also Laddha et al. log terms

Outlook: S-matrix program for classical GR.

First principles
(unitarity, locality + minimal coupling etc)

$\Longrightarrow$ Amplitudes for massive spinning particle
(universality)

$\Longrightarrow$ Classical observables in G.R.

- $\hookrightarrow$ leading order radiation
- $\hookrightarrow$ GW scattering
  (leading order in a)
  $GM\omega \to 0$
- $\hookrightarrow$ Binary conservative dynamics.
  1PM all orders.
  2PM $\underline{\underline{a^2}}$
  $\downarrow$