The hadron spectrum on the lattice

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from the 2010 Review of Particle Physics.
Please use this **CITATION**: K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

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LIGHT UNFLAVORED MESONS ($S = C = B = 0$)

For I=1 (π, b, ρ, a): $u\bar{d}$, $(u\bar{u}-d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for I=0 (η, η', h, h', ω, φ, f, f'): $c_1(u\bar{u}+d\bar{d}) + c_2(s\bar{s})$

	$\pi^\pm$	$1^-(0^-)$
	π^0	$1^-(0^{-+})$
	η	$0^+(0^{-+})$
	$f_0(600)$ or σ	$0^+(0^{++})$
	$\rho(770)$	$1^+(1^{--})$
	$\omega(782)$	$0^-(1^{--})$
	$\eta'(958)$	$0^+(0^{-+})$
	$f_0(980)$	$0^+(0^{++})$
	$a_0(980)$	$1^-(0^{++})$
	$\varphi(1020)$	$0^-(1^{--})$
	$h_1(1170)$	$0^-(1^{+-})$
	$b_1(1235)$	$1^+(1^{+-})$
	$a_1(1260)$	$1^-(1^{++})$
	$f_2(1270)$	$0^+(2^{++})$
	$f_1(1285)$	$0^+(1^{++})$
	$\eta(1295)$	$0^+(0^{-+})$
	$\pi(1300)$	$1^-(0^{-+})$
	$a_2(1320)$	$1^-(2^{++})$
	$f_0(1370)$	$0^+(0^{++})$
•	$h_1(1380)$	$?^-(1^{+-})$
	$\pi_1(1400)$	$1^-(1^{-+})$
	$\eta(1405)$	$0^+(0^{-+})$
	$f_1(1420)$	$0^+(1^{++})$
	$\omega(1420)$	$0^-(1^{--})$
•	$f_2(1430)$	$0^+(2^{++})$
	$a_0(1450)$	$1^-(0^{++})$
	$\rho(1450)$	$1^+(1^{--})$

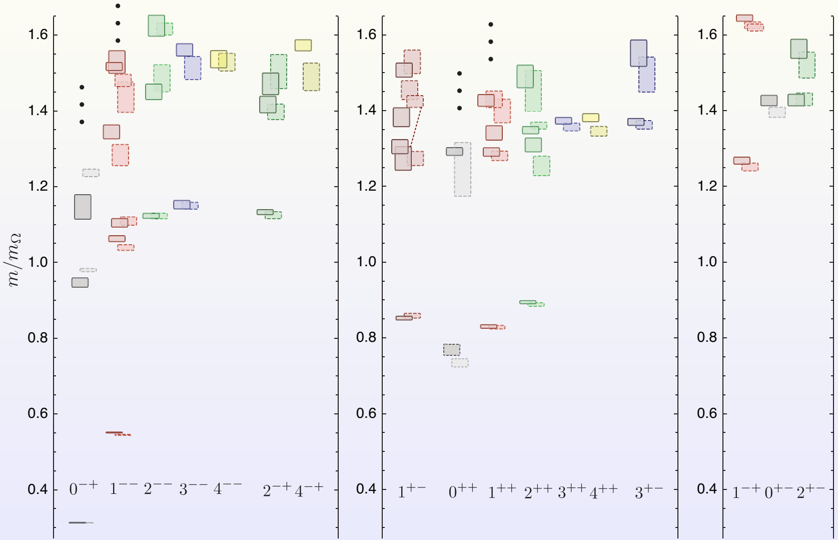
	$\eta(1475)$	$0^+(0^{-+})$
	$f_0(1500)$	$0^+(0^{++})$
•	$f_1(1510)$	$0^+(1^{++})$
	$f_2'(1525)$	$0^+(2^{++})$
•	$f_2(1565)$	$0^+(2^{++})$
•	$\rho(1570)$	$1^+(1^{--})$
•	$h_1(1595)$	$0^-(1^{+-})$
	$\pi_1(1600)$	$1^-(1^{-+})$
•	$a_1(1640)$	$1^-(1^{++})$
•	$f_2(1640)$	$0^+(2^{++})$
	$\eta_2(1645)$	$0^+(2^{-+})$
	$\omega(1650)$	$0^-(1^{--})$
	$\omega_3(1670)$	$0^-(3^{--})$
	$\pi_2(1670)$	$1^-(2^{-+})$
	$\varphi(1680)$	$0^-(1^{--})$
	$\rho_3(1690)$	$1^+(3^{--})$
	$\rho(1700)$	$1^+(1^{--})$
•	$a_2(1700)$	$1^-(2^{++})$
	$f_0(1710)$	$0^+(0^{++})$
•	$\eta(1760)$	$0^+(0^{-+})$
	$\pi(1800)$	$1^-(0^{-+})$
•	$f_2(1810)$	$0^+(2^{++})$
•	$X(1835)$	$?^?(?^{-+})$
	$\varphi_3(1850)$	$0^-(3^{--})$
•	$\eta_2(1870)$	$0^+(2^{-+})$
	$\pi_2(1880)$	$1^-(2^{-+})$
•	$\rho(1900)$	$1^+(1^{--})$

•	$f_2(1910)$	$0^+(2^{++})$
	$f_2(1950)$	$0^+(2^{++})$
•	$\rho_3(1990)$	$1^+(3^{--})$
	$f_2(2010)$	$0^+(2^{++})$
•	$f_0(2020)$	$0^+(0^{++})$
	$a_4(2040)$	$1^-(4^{++})$
	$f_4(2050)$	$0^+(4^{++})$
•	$\pi_2(2100)$	$1^-(2^{-+})$
•	$f_0(2100)$	$0^+(0^{++})$
•	$f_2(2150)$	$0^+(2^{++})$
•	$\rho(2150)$	$1^+(1^{--})$
	$\varphi(2170)$	$0^-(1^{--})$
•	$f_0(2200)$	$0^+(0^{++})$
•	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
•	$\eta(2225)$	$0^+(0^{-+})$
•	$\rho_3(2250)$	$1^+(3^{--})$
	$f_2(2300)$	$0^+(2^{++})$
•	$f_4(2300)$	$0^+(4^{++})$
•	$f_0(2330)$	$0^+(0^{++})$
	$f_2(2340)$	$0^+(2^{++})$
•	$\rho_5(2350)$	$1^+(5^{--})$
•	$a_6(2450)$	$1^-(6^{++})$
•	$f_6(2510)$	$0^+(6^{++})$

• — OMITTED FROM SUMMARY TABLE

What are these states? $\bar{q}q$ mesons?

Isoscalar meson spectrum



- $16^3, 20^3$ lattice (about 2-2.5 fm), $m_\pi \approx 440$ MeV

[arXiv:0909.0200, arXiv:1004.4930]

Particle(s) in a box

- Spatial lattice of extent L with periodic boundary conditions
- Allowed momenta are quantized: $p = \frac{2\pi}{L}(n_x, n_y, n_z)$ with $n_i \in \{0, 1, 2, \ldots L-1\}$
- Energy spectrum is a set of **discrete** levels, classified by p: Allowed energies of a particle of mass m

$$E = \sqrt{m^2 + \left(\frac{2\pi}{L}\right)^2 N^2} \quad \text{with } N^2 = n_x^2 + n_y^2 + n_z^2$$

- Can make states with **zero total momentum** from pairs of hadrons with momenta $p, -p$.
- "Density of states" **increases** with energy since there are more ways to make a particular value of N^2 e.g. $\{3, 0, 0\}$ and $\{2, 2, 1\} \rightarrow N^2 = 9$

Avoided level crossings

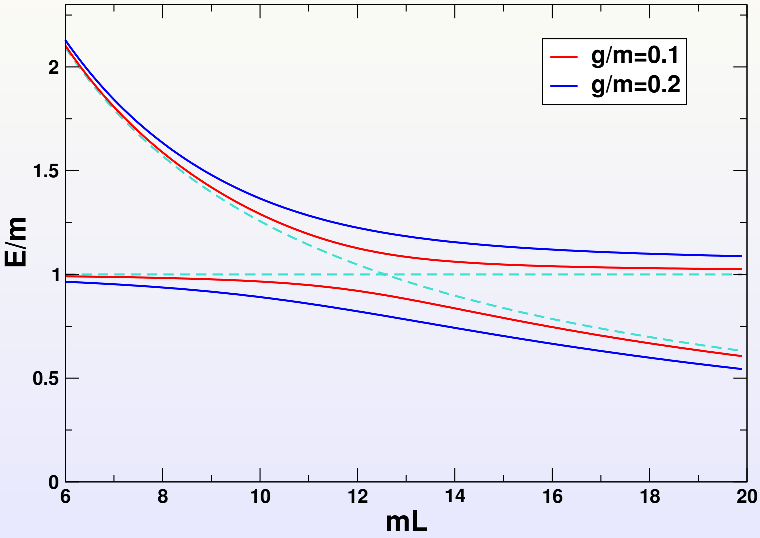
- Consider a toy model with two states (a resonance and a two-particle decay mode) in a box of side-length L
- Write a mixing hamiltonian:

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$

- Now the energy eigenvalues of this hamiltonian are given by

$$E_{\pm} = \frac{(m + \frac{4\pi}{L}) \pm \sqrt{(m - \frac{4\pi}{L})^2 + 4g^2}}{2}$$

Avoided level crossings



Avoided level crossings

- **Ground-state** smoothly changes from resonance to two-particle state
- Need a large box. This example, levels cross at $mL = 4\pi \approx 12.6$
- Example: $m = 1$ GeV state, decaying to two massless pions - avoided level crossing is at $L = 2.5\text{fm}$.
- If the decay product pions have $m_\pi = 300$ MeV, this increases to $L = 3.1\text{fm}$

Lüscher's method

- Relates the spectrum in a finite box to the scattering phase shift (and so resonance properties)

Lüscher's formula

$$\delta(p) = -\phi(\kappa) + \pi n$$

$$\tan \phi(\kappa) = \frac{\pi^{3/2}\kappa}{Z_{00}(1;\kappa^2)}$$

$$\kappa = \frac{pL}{2\pi}$$

- p_n is defined for level n with energy E_n from the dispersion relation:

$$E_n = 2\sqrt{m^2 + p_n^2}$$

- Z_{00} is a generalised Zeta function:

$$Z_{js}(1,q^2) = \sum_{n \in Z^3} \frac{r^j Y_{js}(\theta,\phi)}{(n^2 - q^2)^s}$$

[Commun.Math.Phys.105:153-188,1986.]

- With the phase shift, and for a well-defined resonance, can fit a Breit-Wigner to extract the **resonance width** and **mass**.

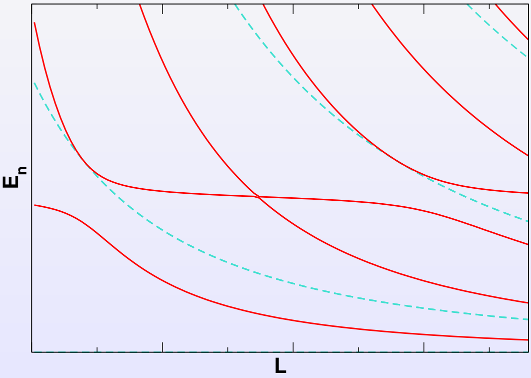
$$\delta(p) \approx \tan^{-1}\left(\frac{4p^2 + 4m_\pi^2 - m_\sigma^2}{m_\sigma \Gamma \sigma}\right)$$

Schrödinger equation

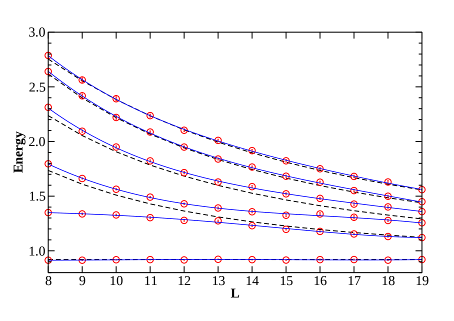
Exercise: find the phase shift for a 1-d potential

$$V(x) = V_0\delta(x-a) + V_0\delta(x+a)$$

- Now compute the spectrum in a finite box and use Lüscher's method to compare the two

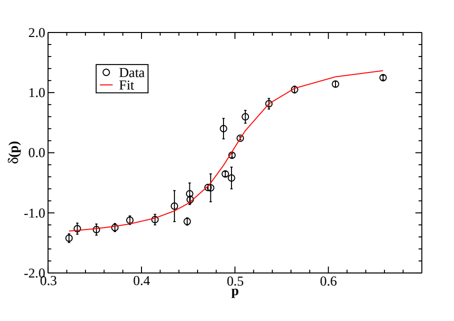


Test: O(4) Sigma model

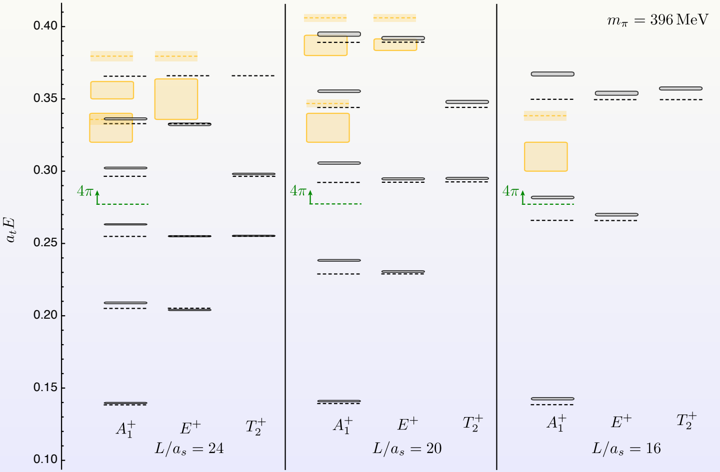


Spectrum of $O(4)$ model in broken phase

Phase shift inferred from Lüscher's method

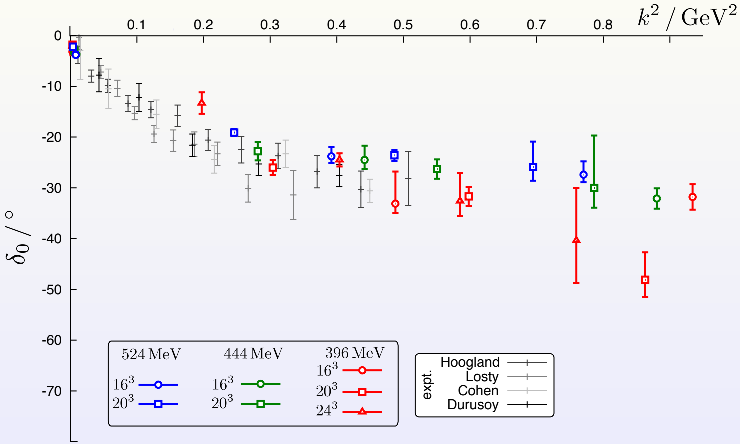


I=2 $\pi\pi$ scattering



Resolve shifts in masses away from non-interacting values

I=2 $\pi\pi$ scattering



- Non-resonant scattering in S-wave - compares well with experimental data

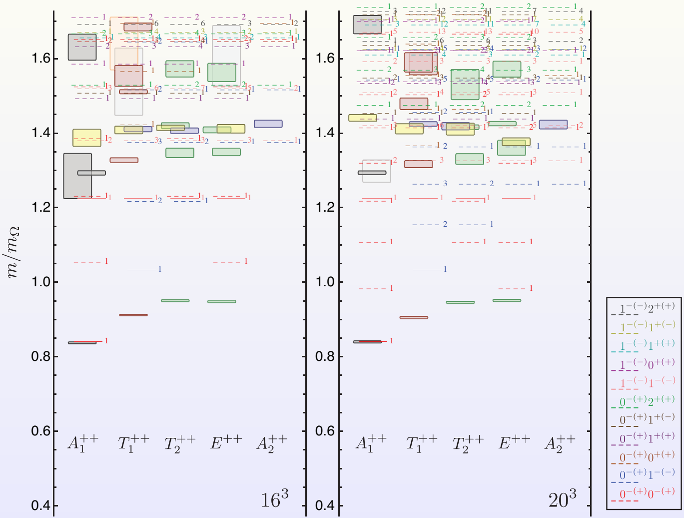
Group theory of two particles in a box

- Consider two identical particles, with momentum p and $-p$ (so zero total momentum).
- $\Omega(p)$, set of all momentum directions related by rotations in O_h
- Can make a set of operators, $\{\phi(p)\}$ from Ω and these form a (reducible) representation of O_h.
- Example: $\Phi = \{\phi(1,0,0), \phi(0,1,0), \phi(0,0,1)\}$ contains the A_1 and E irreps
- Different particles: $+p$ and $-p$ are not equivalent

p	irreducible content
(0,0,0)	A_1^g
(1,0,0)	$A_1^g \oplus E^g$
(1,1,0)	$A_1^g \oplus E^g \oplus T_2^g$
(1,1,1)	$A_1^g \oplus T_2^g$

- More complicated if mesons have internal spin

Multi-meson states in QCD



- Multi-hadron states not seen in this calculation

The inelastic threshold

- Lüscher's method is based on **elastic** scattering.
- Since m_π is small, most resonances are above this threshold
- Not clear how to proceed - perhaps a histogram approach will help us gain some expertise
- It will be crucial to ensure we have a comprehensive **basis of operators that create multi-hadron states**.

Summary

- The lattice provides a robust framework for investigating the hadron spectrum. New methods to improve precision continue to develop.
- Computations of the hadron masses is an important check for many aspects of lattice QCD. Universality seems to work!
- Are we near the $m_q \to 0, V \to \infty, a \to 0$ limit?
- Many open questions in hadron spectroscopy:
    - Are there **intrinsic excitations** of gluons inside light hadrons?
    - What are the states above the **open-charm threshold**?
    - Can precision spectroscopy help us to understand **what degrees of freedom are important** in confined systems?
- We need a lot more expertise in **scattering measurements**