Efficiency of a Brownian Heat Engine

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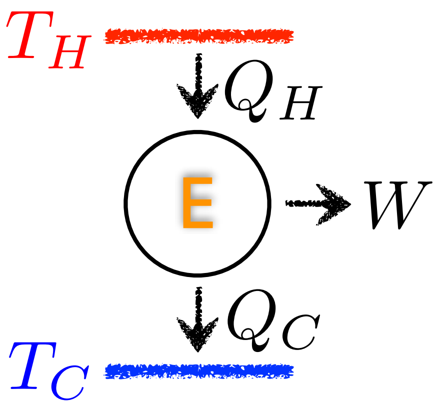
Heat engine



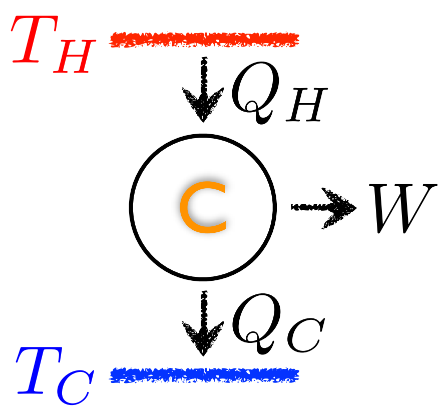
efficiency $\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$

Carnot efficiency

reversible engine

no entropy production

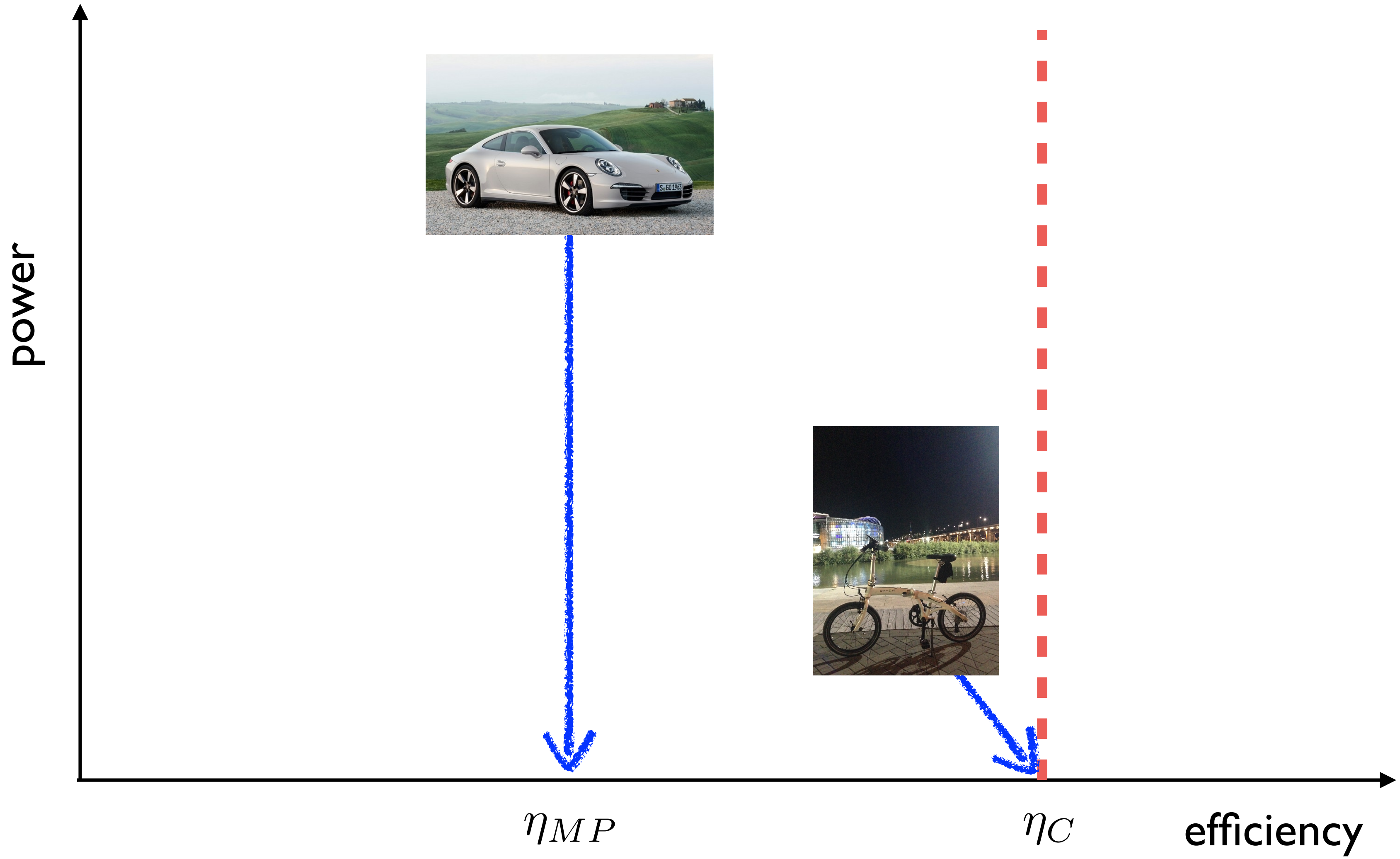
$$\Delta S_{tot} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} + \Delta S_E = 0$$



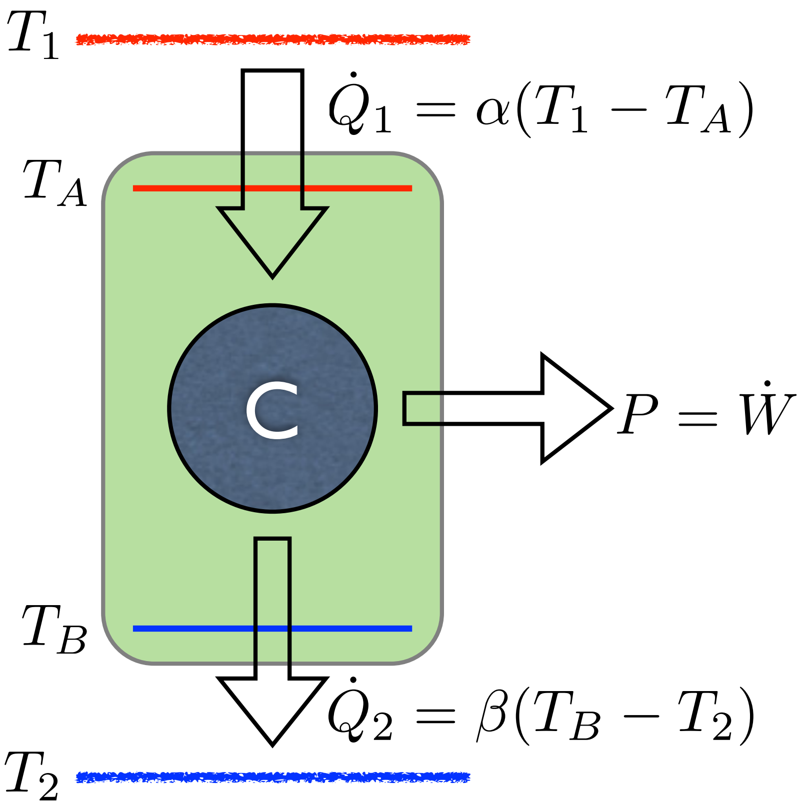
Carnot efficiency

$$\eta_C = 1 - \frac{T_C}{T_H} \geq \eta$$

Efficiency at Maximum Power



Endoreversible engine



Endoreversibility condition

$$\frac{\alpha(T_1 - T_A)}{T_A} = \frac{\beta(T_B - T_2)}{T_B}$$

Maximize P with respect to T_A and T_B

Efficiency at the Maximum Power

$$\eta_{MP} = \eta^* \equiv 1 - \sqrt{\frac{T_2}{T_1}}$$

Curzon and Ahlborn, Am. J. Phys. 43, 22 (1975)

Novikov (1958), Chambadal (1957)
Yvon (1955)
Henri B. Reitlinger (1929)

Near equilibrium

$T_1 \simeq T_2$ or $\eta_C = 1 - T_2/T_1 \ll 1$

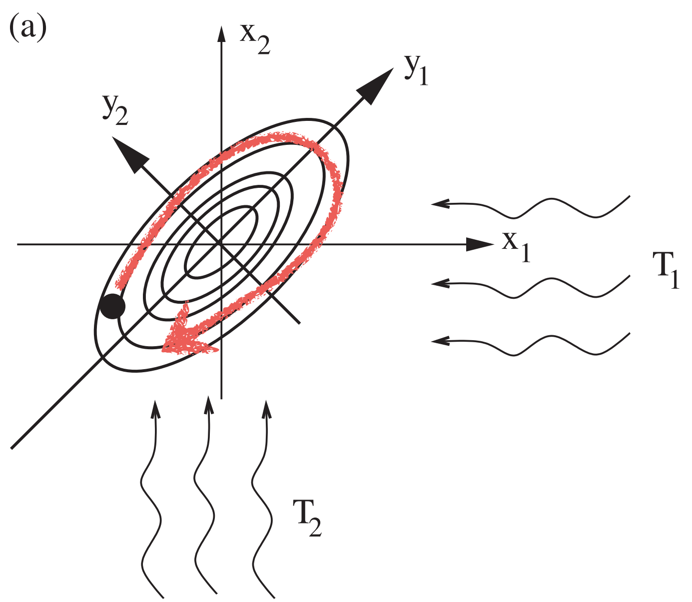
$$\eta * = 1 - \sqrt{\frac{T_2}{T_1}} = \frac{1}{2}\eta_C + \frac{1}{8}\eta_C^2 + \cdots$$

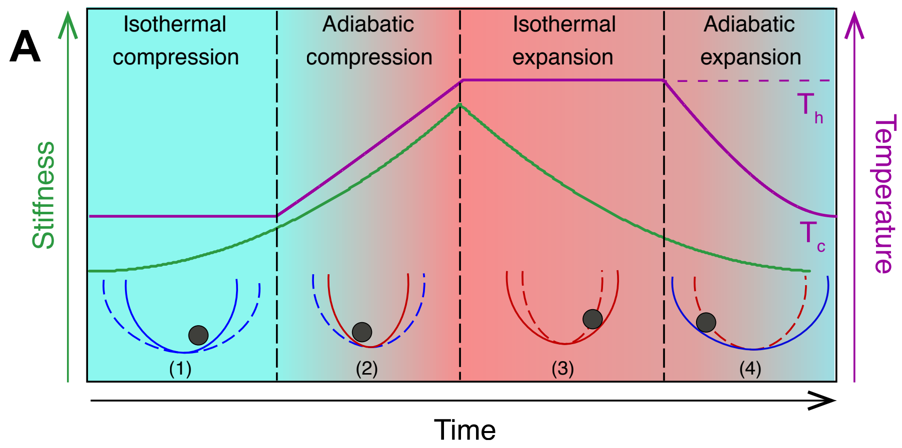
Van den Broeck (2005) : strong coupling

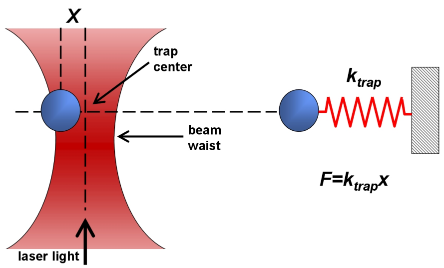
Esposito, Lindenberg, Van den Broeck (2009) : strong coupling + left-right symmetry

Stochastic efficiency

Brownian particle heat engine







Felliger and Reimann, PRL 2007

Martinez et al, arXiv:1412.1282

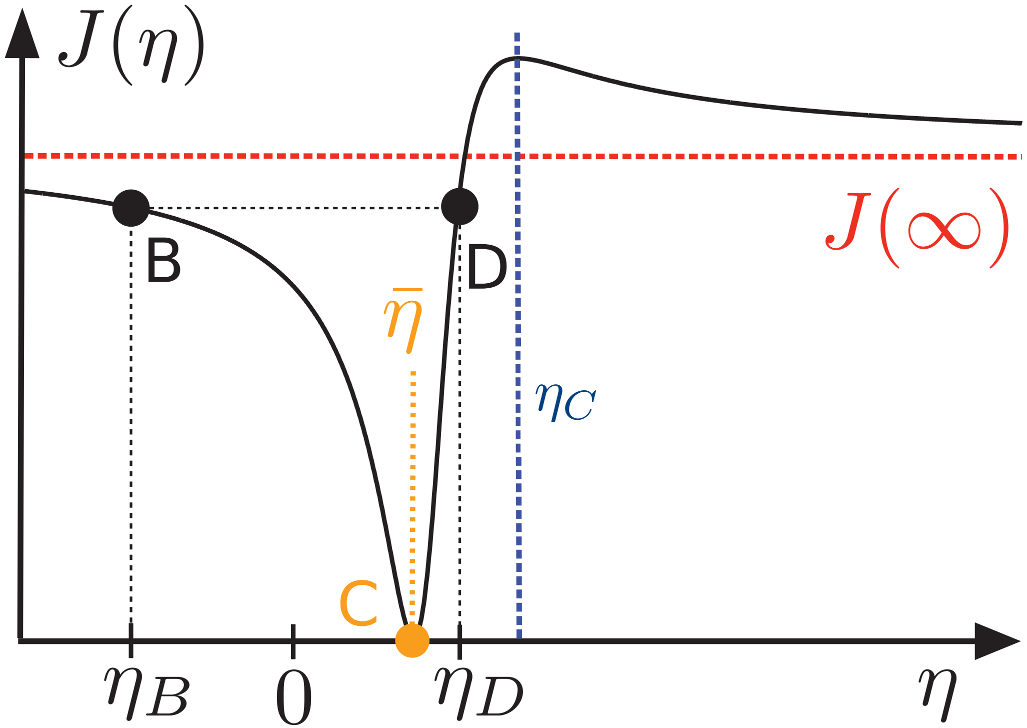
Q and W are stochastic, so is the efficiency.

Least Likeliness of Carnot efficiency

Verley et al., Nat. Commun. (2014)
Verley et al., PRE (2014)

Large deviation function for the efficiency $$J(\eta) = -\lim_{t\to\infty} \frac{1}{t} \ln P(\eta)$$

$$\left(P(\eta) \sim e^{-tJ(\eta)}\right)$$



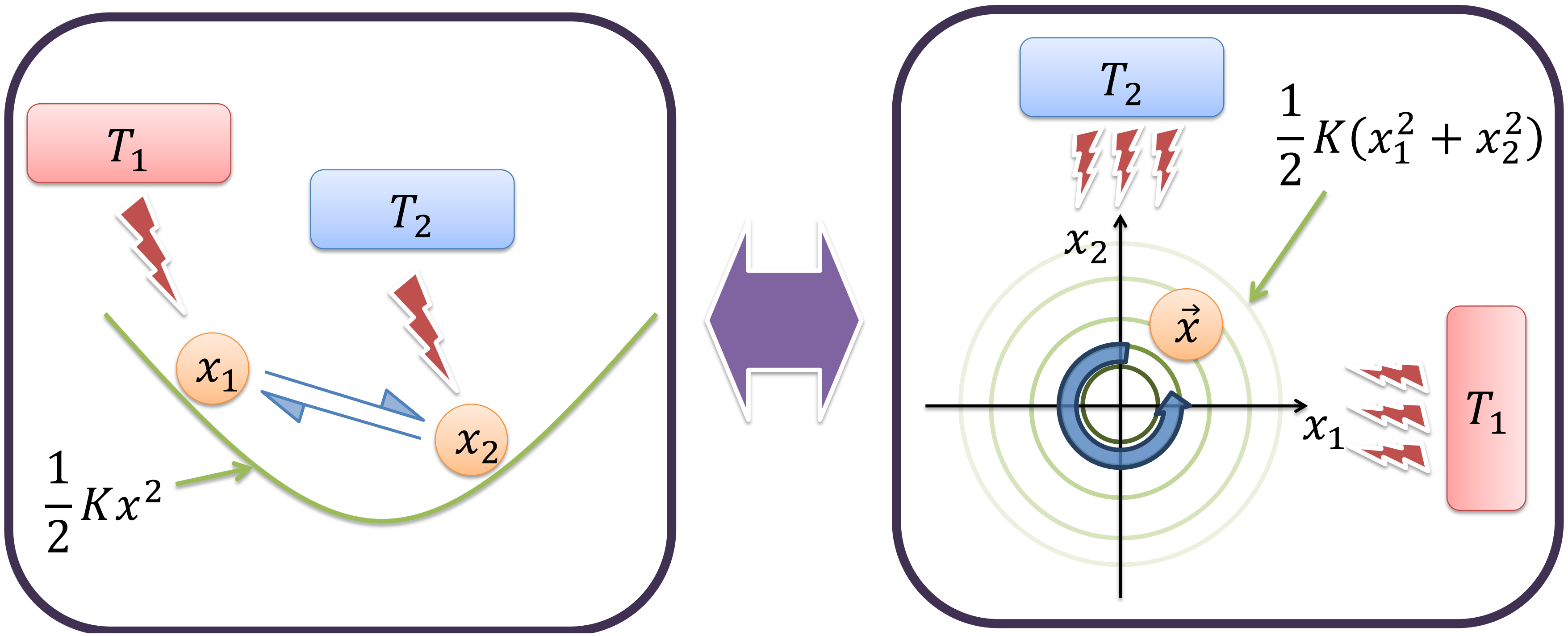
a finite number of states

time periodic protocol

Outline

- Linear Brownian heat engine model
- Efficiency at maximum power
- Efficiency fluctuations

Linear Brownian Heat Engine



$$
\begin{aligned}
\dot{x}_1 &= v_1, \\
\dot{x}_2 &= v_2, \\
m\dot{v}_1 &= -\gamma v_1 - Kx_1 + \epsilon x_2 + \xi_1(t) \\
m\dot{v}_2 &= -\gamma v_2 - Kx_2 + \delta x_1 + \xi_2(t)
\end{aligned}
$$

Ornstein-Uhlenbeck process

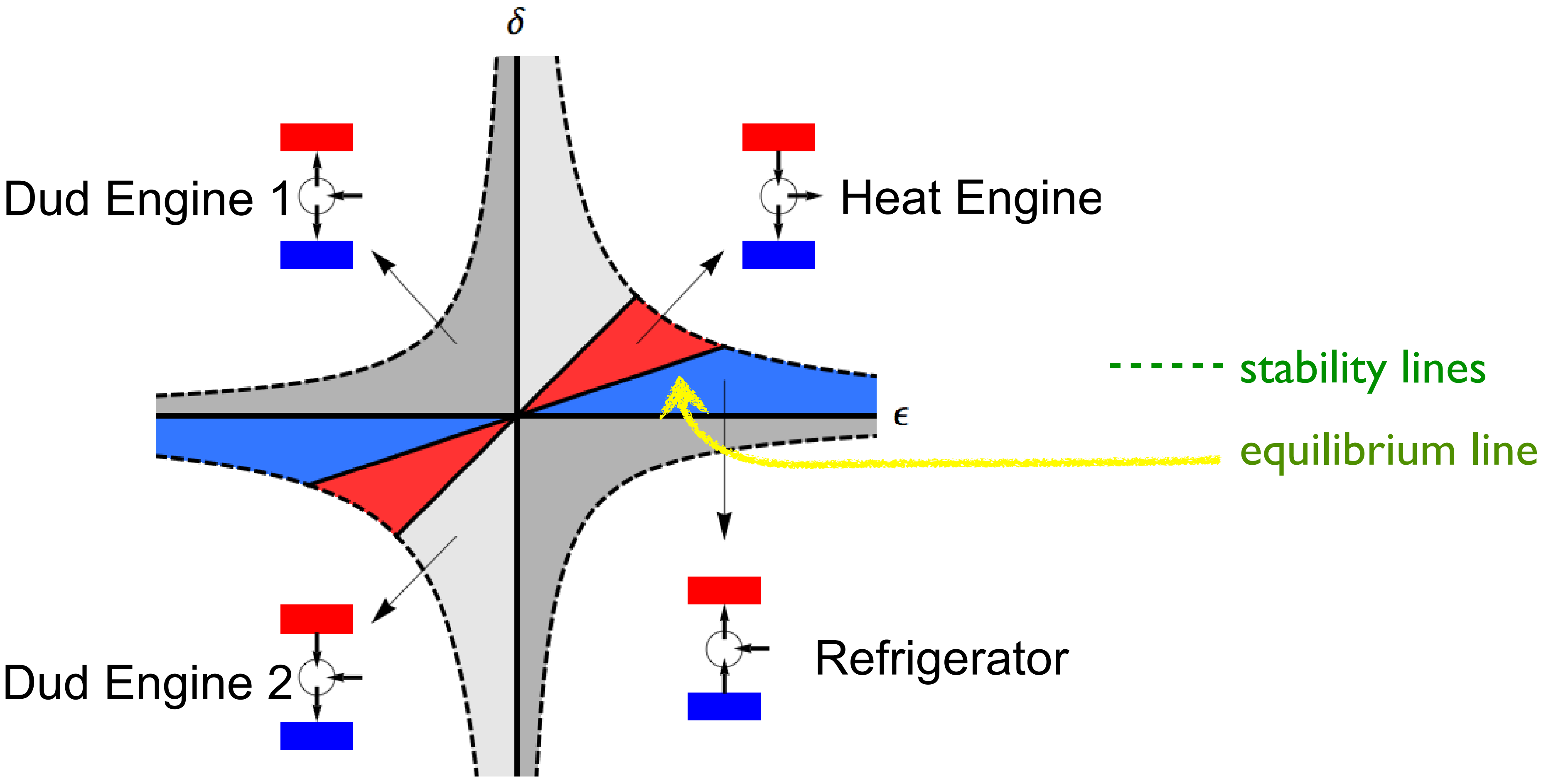
conserved force non-conserved force

Engine diagram

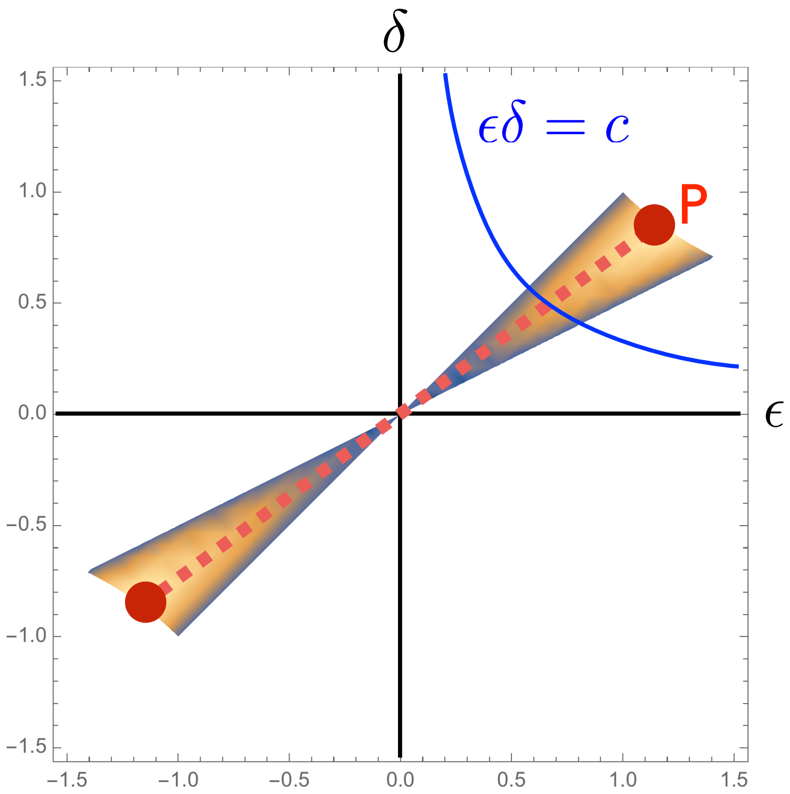
heat flow rate $q \equiv \langle \dot{x}_1 \circ (-\gamma \dot{x}_1 - \xi_1) \rangle = \dfrac{\gamma\epsilon(\delta T_1 - \epsilon T_2)}{2(\gamma^2 K + m\epsilon\delta)}$

power $w \equiv -\langle \dot{\boldsymbol{x}} \circ \boldsymbol{f}_{nc} \rangle = \dfrac{\gamma(\epsilon - \delta)(\delta T_1 - \epsilon T_2)}{2(\gamma^2 K + m\epsilon\delta)}$

$\Rightarrow \quad \eta = 1 - \dfrac{\delta}{\epsilon}$



Efficiency at Maximum Power



global maximum at P

$$\eta_{MP} = 1 - \sqrt{\frac{T_2}{T_1}} = \eta^*$$

local maximum along constant $\epsilon\delta$ curves

$$\eta_{MP} = 1 - \sqrt{\frac{T_2}{T_1}} = \eta^*$$

The model exhibits the Curzon-Ahlborn EMP without endoreversibility.

Condition for CA

entropy loss of the hot reservoir $y = q/T_1$

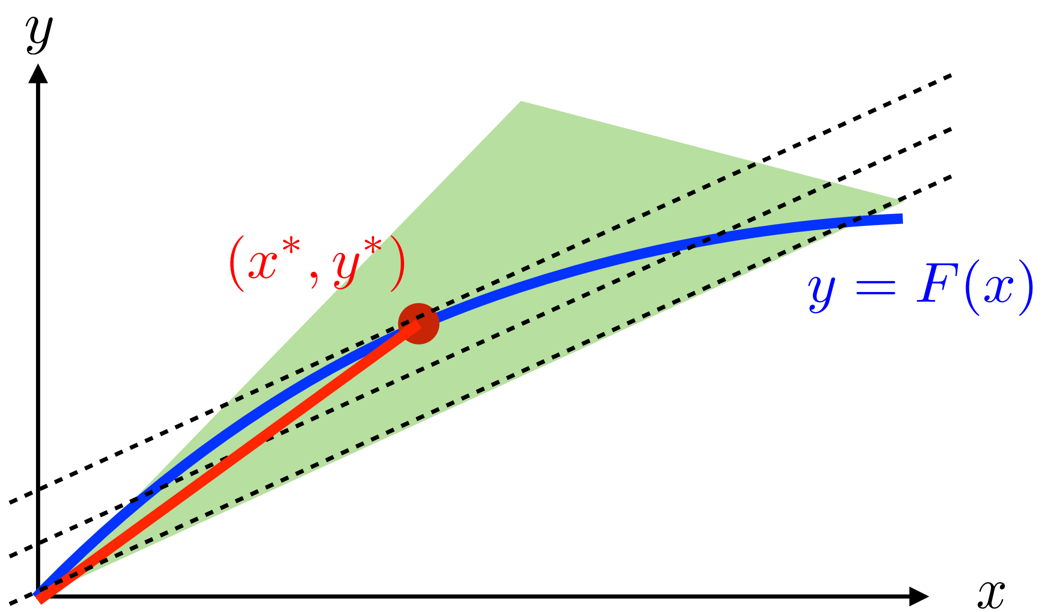
entropy gain of the cold reservoir $x = q_2/T_2$

$\Rightarrow$ parametric equation $y = F(x)$

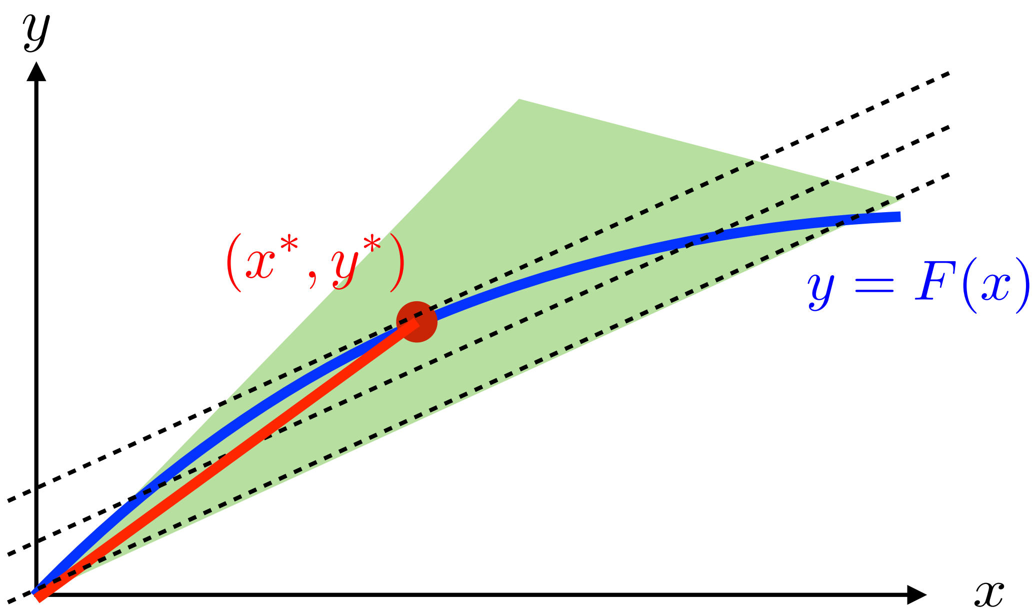
power gain $w = q - q_2 = T_1 y - T_2 x$

$\Rightarrow$ constant-w line : $y = (T_2/T_1)x + w/T_1$

efficiency $\eta = (q - q_2)/q = 1 - \left(\frac{T_1}{T_2}\right)\left(\frac{x}{y}\right)$



Condition for CA



MP condition $\quad F'(x^*) = \dfrac{T_2}{T_1}$

$$\otimes$$

CA EMP $\quad \dfrac{F(x^*)}{x^*} = \sqrt{\dfrac{T_2}{T_1}}$

$$\Downarrow$$

$$F'(x) = (F(x)/x)^2$$

$$\Rightarrow F(x) = \frac{x}{1+cx}$$

$c = \dfrac{1}{\alpha} + \dfrac{1}{\beta}$ for the endoreversible engine

$c = \dfrac{2m}{\gamma}\left(1 + \dfrac{\gamma^2 K}{m\epsilon\delta}\right)$ for the linear engine model

Efficiency fluctuations

probability distribution for W in linear systems [Kwon, Noh, Park (2011,2013)]

joint distribution for Q and W in linear systems [Noh (2014)]

over-damped limit

$$\dot{x}_1 = -Kx_1 + \epsilon x_2 + \xi_1(t) \qquad \frac{\partial P_{12}}{\partial t} = \mathcal{L}_{12} P_{12}$$

$$\dot{x}_2 = -Kx_2 + \delta x_1 + \xi_2(t) \qquad \mathcal{L}_{12} = \mathcal{L}_{12}(x_1, x_2, \partial_{x_1}, \partial_{x_2})$$

$$\oplus\ W \text{ and } Q \qquad \mathcal{L}_{12WQ}(x_1, x_2, \partial_{x_1}, \partial_{x_2}, \partial_Q, \partial_W)$$

$$G_t(x_1, x_2, \lambda_Q, \lambda_W) = \left\langle e^{-\lambda_Q Q - \lambda_W W} \right\rangle_{P_t} \qquad \widetilde{\mathcal{L}}(x_1, x_2, \partial_{x_1}, \partial_{x_2}, \lambda_Q, \lambda_W)$$

quadratic in x and ∂_x

$\simeq H_{\text{2D SHO}}$

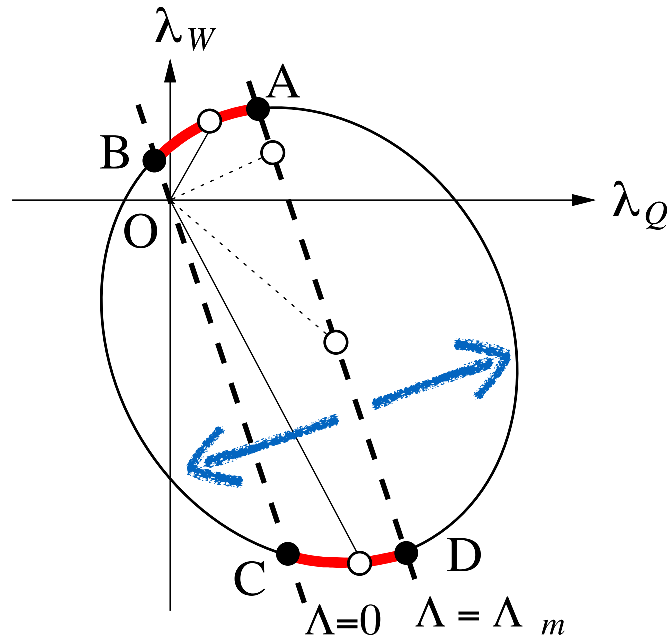
Efficiency fluctuation

$$\tilde{G}(\lambda_Q, \lambda_W) \sim e^{-t\phi(\lambda_Q,\lambda_W)}$$

cumulant generating function $\phi(\lambda_Q, \lambda_W)$

$:$ function of a single parameter $\Lambda = \lambda_Q + \bar{\eta}\lambda_W$

$:$ bounded by the branch cut



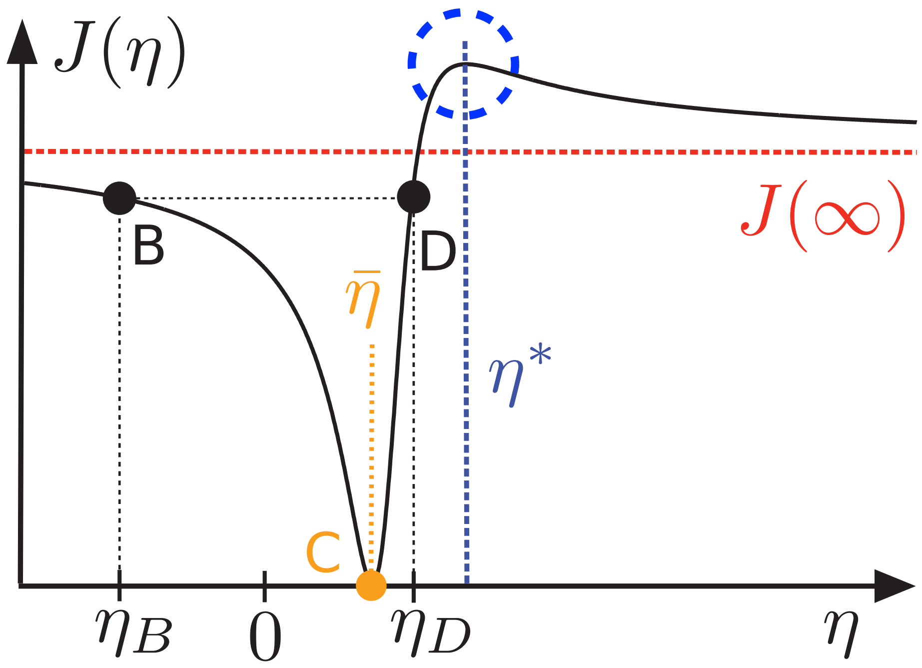
large deviation function for the efficiency $J(\eta) = -\lim_{t\to\infty} \frac{1}{t} \ln P(\eta)$

$$\left(P(\eta) \sim e^{-tJ(\eta)}\right)$$

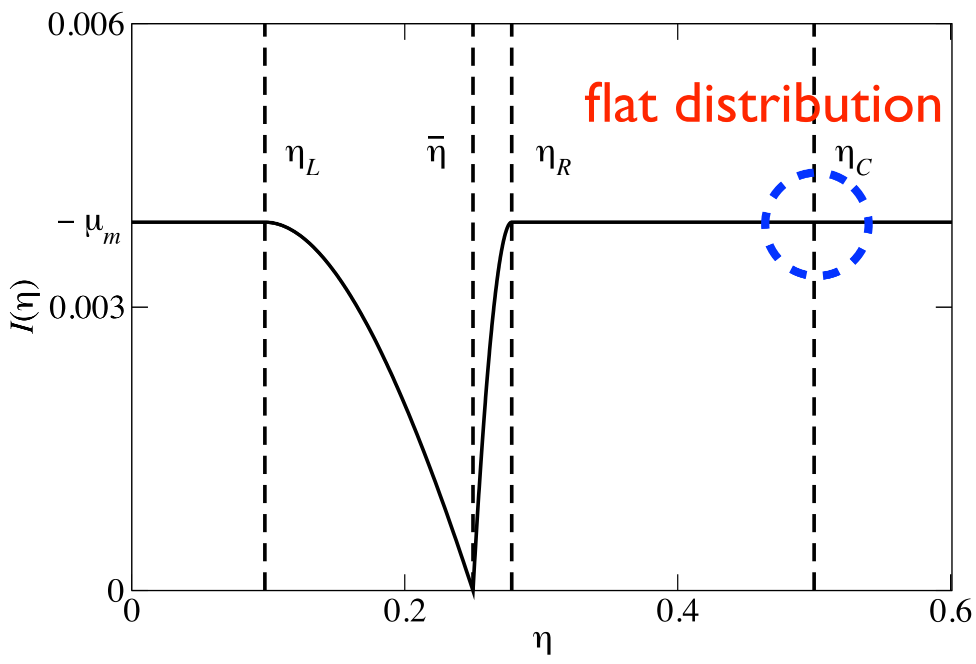
$$J(\eta) = -\min_{\lambda} \phi(-\eta\lambda, \lambda)$$

Verley et al PRE(2014)

Large deviation function



Verley et al



Energy fluctuations

entropy production $$\Delta S_{tot} = -\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \Delta S_e$$

energy conservation $$\Delta E = Q_1 - Q_2 - W$$

finite state space
bounded E&S fluctuations

$$\Delta S_{tot} = \left(\frac{1}{T_2} - \frac{1}{T_1}\right) Q_1 - \frac{W}{T_2} - \frac{\Delta E}{T_2} + \Delta S_e$$

fluctuation theorem by Garcia-Garcia (2010)

$$\frac{P(Q_1, W)}{P(-Q_1, -W)} = e^{-(1/T_2 - 1/T_1)Q_1 - W/T_2}$$

systems with unbounded phase space?

counter example by Noh and Park (PRL 2013) :

for systems with $Q = W + \Delta E$

Q does not obey the FT, while W obeys the FT.

Summary

- From the analytically solvable linear heat engine model,
  - the Curzon-Ahlborn efficiency is achieved without endoreversibility.
  - the least likeliness of the Carnot efficiency is not the case for systems with unbounded energy spectrum.