Critical dynamics of an exclusion process with hole-dependent rates

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Critical dynamics of systems with nonequilibrium steady state

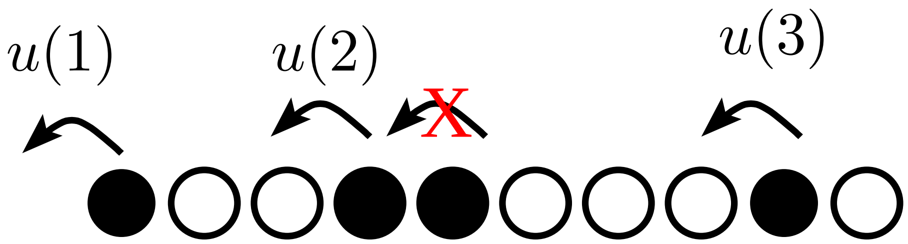
- one dimension
- nonequilibrium phase transition: jamming (or condensation) transition
- steady state is known exactly

Model

- System of $N = L\rho$ particles on a ring with L sites
- Each site can be either empty or occupied by one particle
- Particle attempts to hop to the left neighbor, if it is empty

(totally asymmetric)

- Hop rate depends on vacancies in front of it



Jamming transition in the steady state (review by Evans & Hanney, 2005)

Hop rates that decrease with increasing number of vacancies

$$u(n) = 1 + \frac{b}{n}\ ,\ b \geq 0, n \geq 1$$

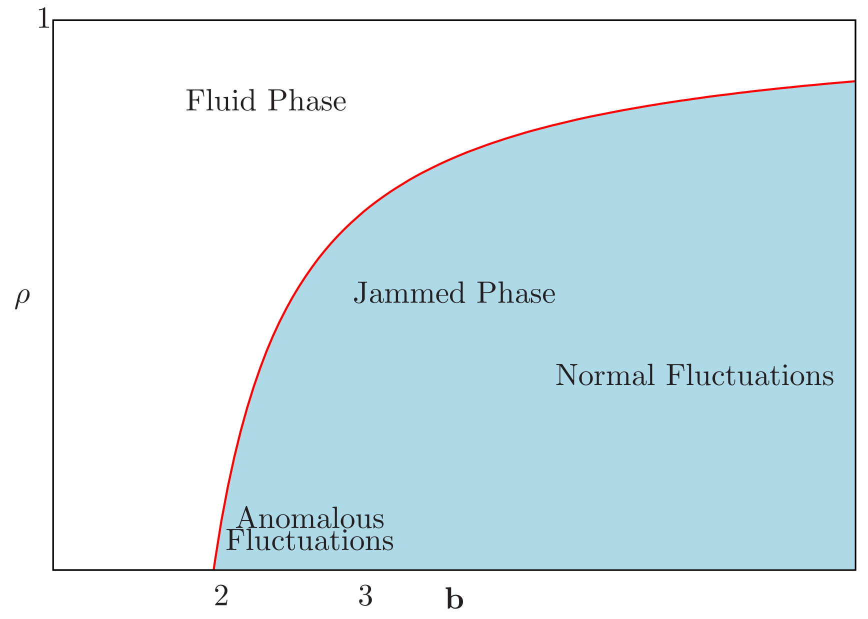
$b = 0$: Obtain TASEP

$b > 2: \quad \rho_c = \frac{b-2}{b-1}$

Fluid phase: interparticle spacing is of order unity

Jammed phase: fluid phase + hole cluster of macroscopic length

Critical density: hole cluster length is sublinear

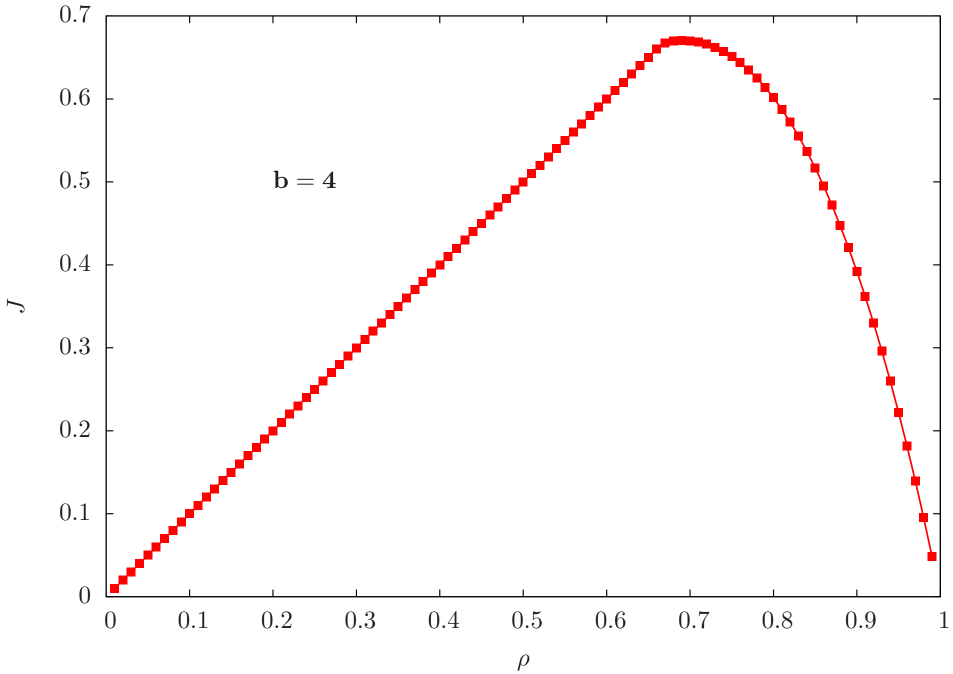


Density-Density correlation function for $\rho \geq \rho_c$

$$S(i,t) = \langle n_0(0) n_{i+vt}(t) \rangle - \rho^2$$

Speed of density fluctuations, $v = \partial J / \partial \rho$

(Recall: $\partial_t \delta\rho(x,t) + v \partial_x \delta\rho(x,t) = 0$)



$$\begin{aligned} \text{Current, } J &= \rho\, z \\ \frac{1}{\rho} - 1 &= z \frac{\partial \ln g(z)}{\partial z} \\ g(z) &= {}_2F_1(1,1;b+1;z) \end{aligned}$$

At the critical point,

$$v^{(-)} = \frac{\partial J^{(-)}}{\partial \rho}\bigg|_{\rho_c} = 1,\ b > 2$$

$$v^{(+)} = \frac{\partial J^{(+)}}{\partial \rho}\bigg|_{\rho_c} = \begin{cases} 1 & ,\ 2 < b < 3 \\ 1 - \frac{(b-2)(b-3)}{b-1} & ,\ b > 3 \end{cases}$$

We set $v = v^{(+)}$ since

- consistent with Monte Carlo measurement of speed

- hole cluster is not macroscopic at the critical density

Height-Height correlation function for KPZ equation (Prähofer+Spohn, 2004)

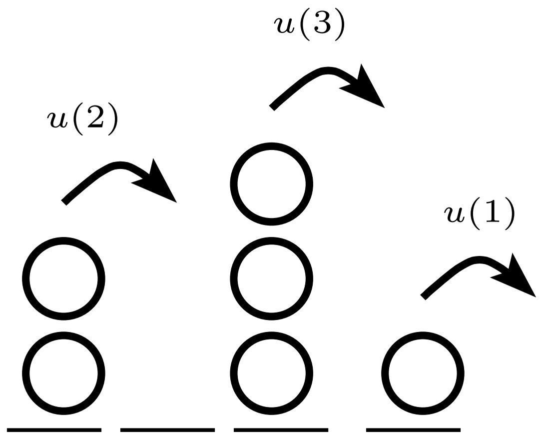
Hypothesis: Dynamics here are determined by the KPZ equation

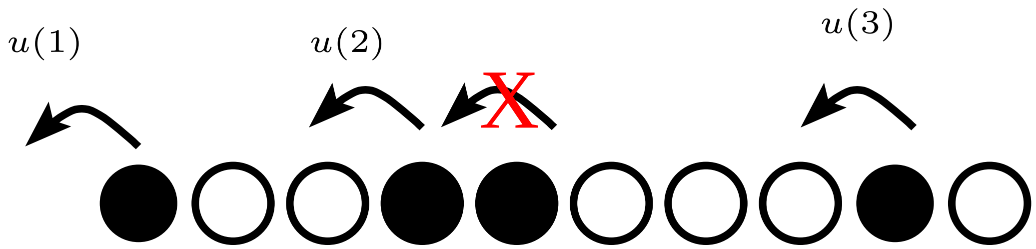
$$
\begin{aligned}
2n_{i+1}(t) - 1 &= h(i+1,t) - h(i,t) \\
8S(i,t) &= C(i+1,t) - 2C(i,t) + C(i-1,t)
\end{aligned}
$$

$$
\begin{aligned}
C(x,t) &= \langle (h(x,t) - h(0,0))^2 \rangle \\
&= \left(\frac{\lambda A^2 t}{2}\right)^{2/3} \mathcal{C}\left(\frac{x}{(2\lambda^2 A t^2)^{1/3}}\right)
\end{aligned}
$$

where, $C(x,0) = A|x|, \lambda = J''(\rho), \mathcal{C}''(y) = 4e^{-0.295|y|^3}$

Equal time correlation function (Priyanka, Ayyer & Jain, 2014)





$$\langle n_0 n_r \rangle = \rho \sum_{s=1}^{r} \text{Prob}(s \text{ sites have } r-s \text{ particles in ZRP})$$

$$= \rho \sum_{s=1}^{r} \sum_{\{m_j\}} \frac{\prod_i f(m_i) \delta_{\sum_{j=1}^{s} m_j - (r-s)}}{Z_{s,r-s}}$$

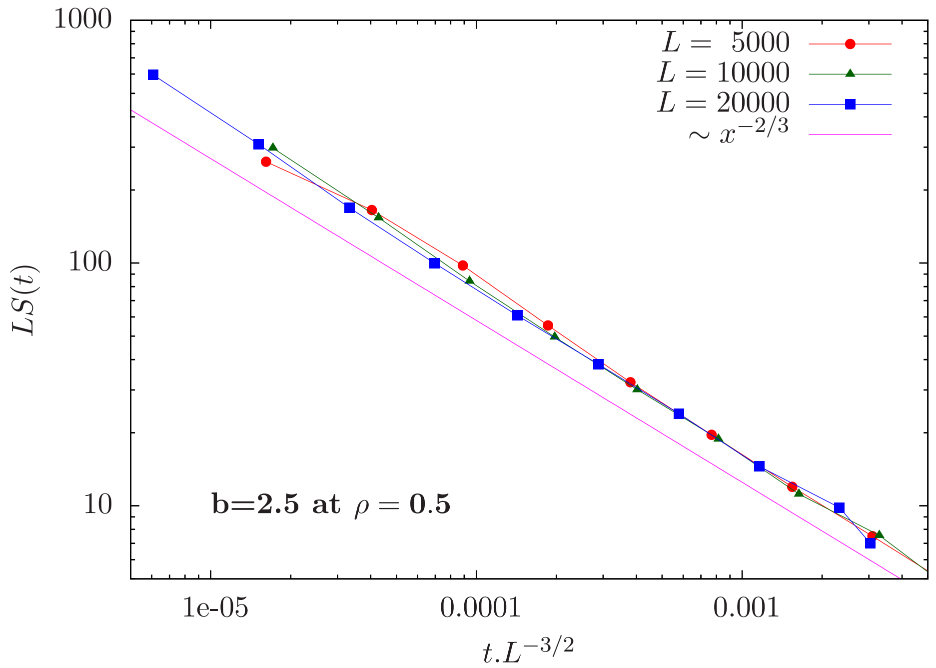
The generating function of the density-density correlation function is

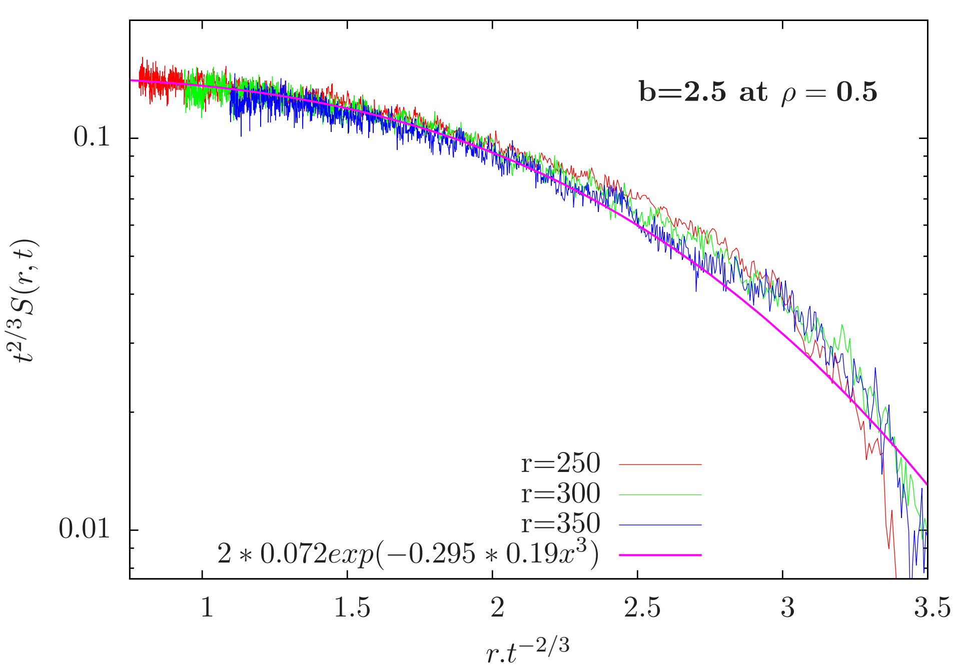
$$\begin{aligned} G(y) &= \sum_{r=0}^{\infty} y^r S(r,0) \\ &= \frac{\rho}{1 - y\,\frac{g(yz)}{g(z)}} - \frac{\rho^2}{1-y} \end{aligned}$$

[where $g(z) = {}_2F_1(1,1;b+1;z)$]

$$\begin{aligned} S(x,0) &= \frac{d^2 C(x,0)}{dx^2} \\ C(x,0) &= A|x| \end{aligned}$$

Correlation function in the fluid phase





KPZ theory holds (as expected)

Density-density correlation function at the critical point

Static correlation function:

$$C(x,0) = a_1 x + a_2 x^{4-b} \sim x^{2\alpha}$$

- For $b > 3$: The coefficient A is defined and roughness exponent $\alpha = 1/2$
- For $2 < b < 3$: the roughness exponent $\alpha = (4-b)/2$ is anomalous

Steady state current near critical point

$$J(\rho) = \rho z(\rho) = \begin{cases} \rho_c + v\delta\rho + \mathcal{O}(\delta\rho^{\frac{1}{b-2}}) + ..., 2 < b < 3 \\ \rho_c + v\delta\rho + \mathcal{O}(\delta\rho^{b-2}) + ..., 3 < b < 4 \\ \rho_c + v\delta\rho + \mathcal{O}(\delta\rho^{2}) + ..., b > 4 \end{cases}$$

$$\frac{\partial^2 J}{\partial \rho^2} = \begin{cases} \delta\rho^{\frac{5-2b}{b-2}} & , \ 2 < b < 3 \\ \delta\rho^{b-4} & , \ 3 < b < 4 \\ \text{finite} & , \ b > 4 \end{cases}$$

$$\lambda = \left.\frac{\partial^2 J}{\partial \rho^2}\right|_{\rho_c} = \begin{cases} 0 & , \ 2 < b < 5/2 \\ \text{finite} & , \ b = 5/2 \\ \text{infinite} & , \ 5/2 < b < 4 \\ \text{finite} & , \ b > 4 \end{cases}$$

Hydrodynamic equations

$$\frac{\partial \delta\rho(x,t)}{\partial t} = -\frac{\partial J(\rho(x,t))}{\partial x} + \text{diffusive+noise term}$$

Stationary state current has the following expansion:

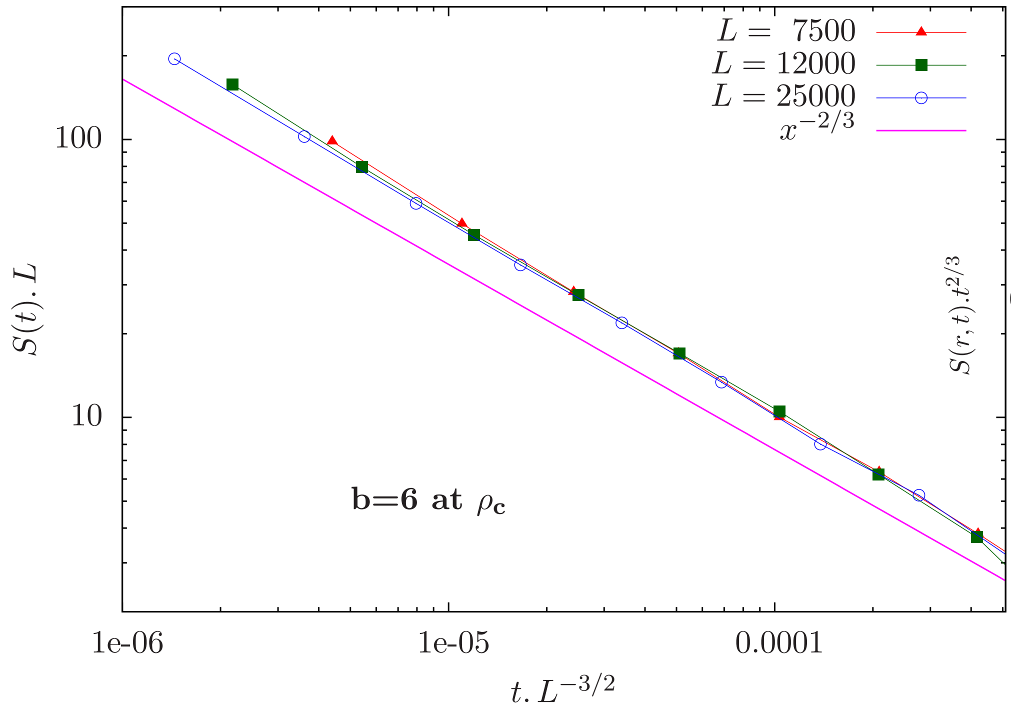
$$J(\rho) = \rho_c + v\delta\rho + (\delta\rho)^{\mu}$$

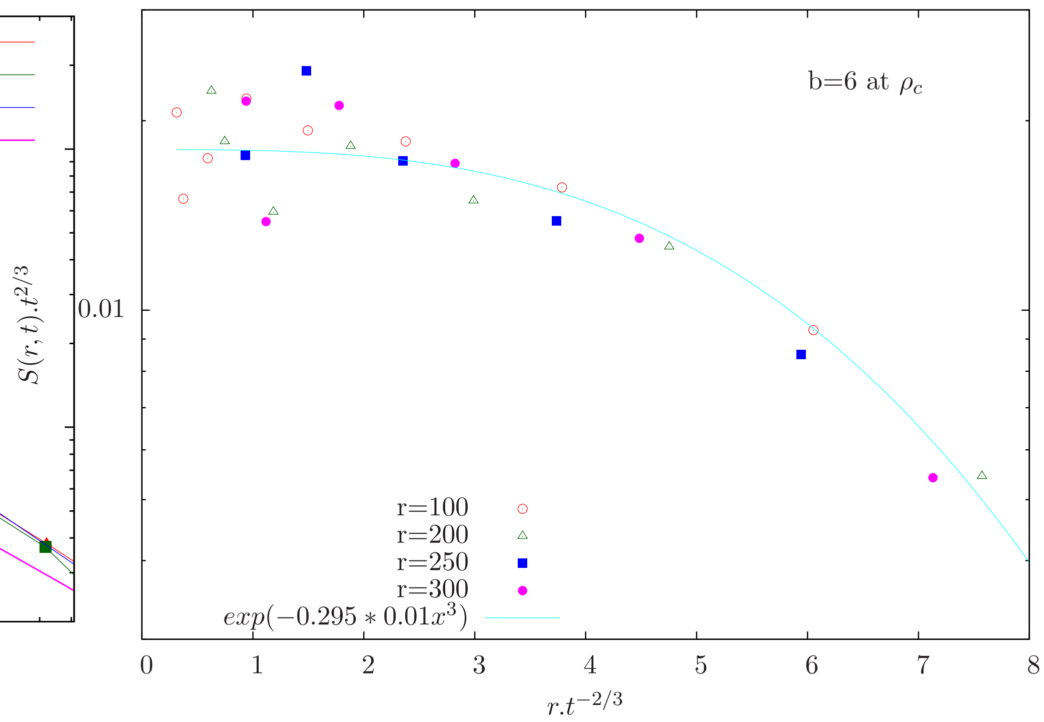
where

$$\mu = \begin{cases} \frac{1}{b-2} & ,\ 2 < b < 3 \\ b-2 & ,\ 3 < b < 4 \\ 2 & ,\ b > 4 \end{cases}$$

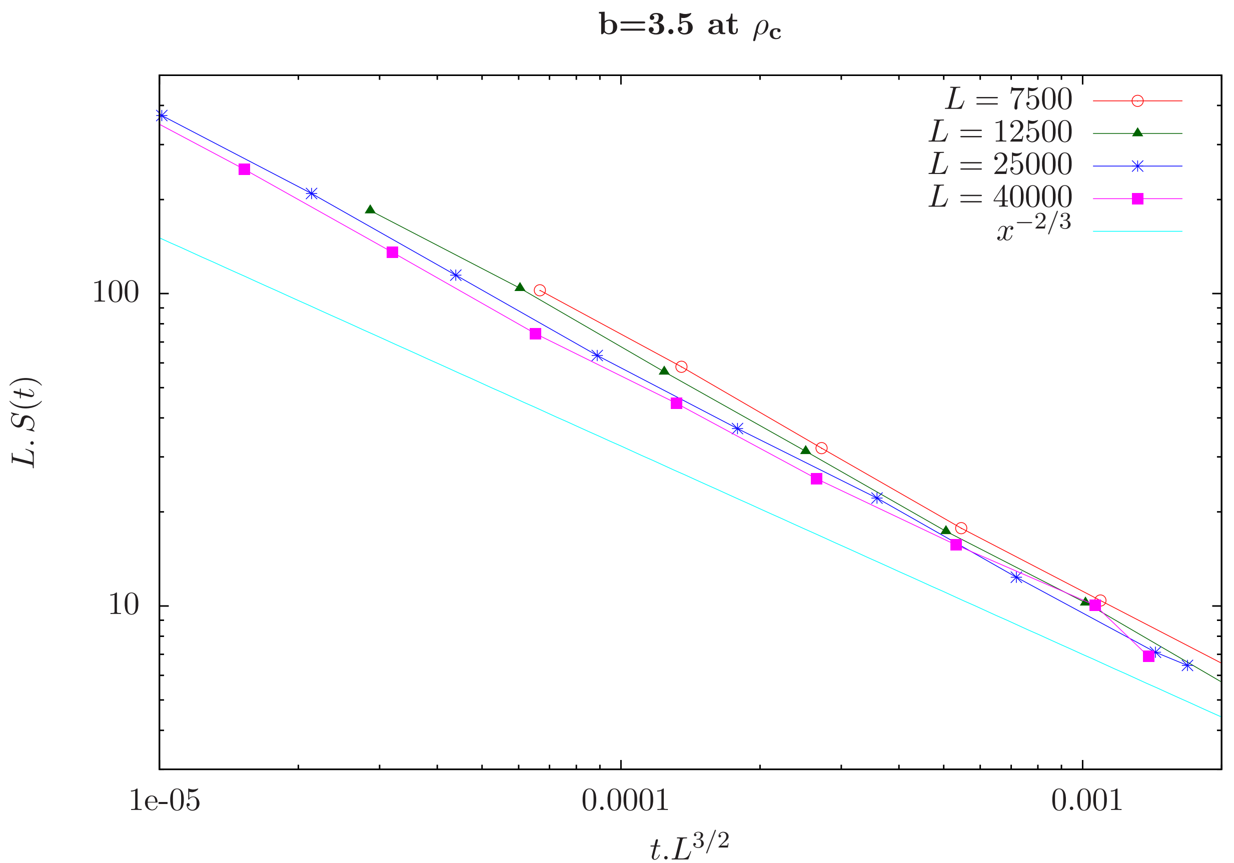
(Krug & Spohn, 1988; Amar & Family, 1993)

When $b > 4$: KPZ universality



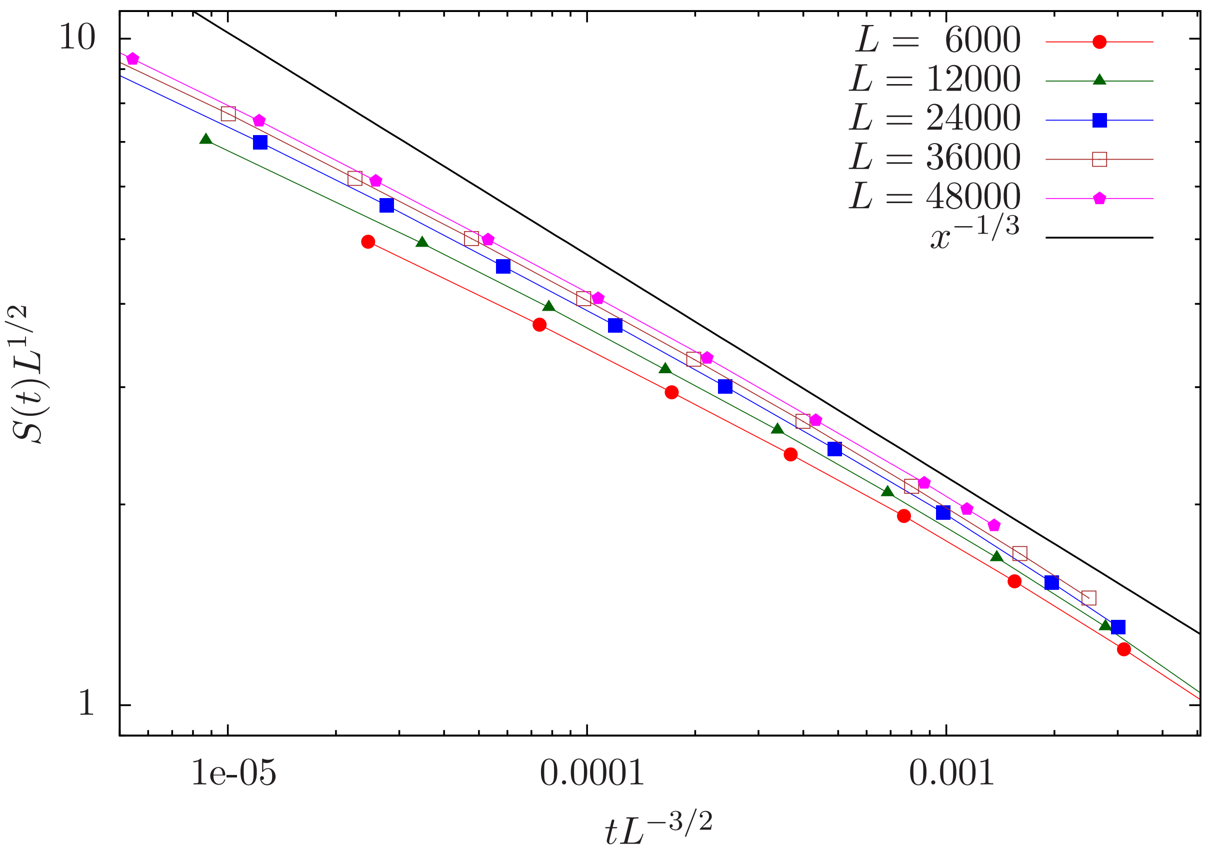


When $3 < b < 4$: KPZ universality?



Data collapse for auto-correlation function with KPZ exponents

When $2 < b < 3$



Data collapse for auto-correlation function using $\alpha = (4 - b)/2, z = 3/2$

Anomalous roughness exponent ($\alpha \neq 1/2$) for KPZ equation

- Noise is delta-correlated in space and time, but non-Gaussian (Zhang, 1990)

$$P(\eta) \sim \eta^{-1-a}, a > 2$$

An argument gives $\alpha \approx 3/(a+1)\,,\ a < a_c$

- Spatial correlations in noise (Medina et al., 1989)

$$\langle \eta(k,\omega)\eta(k',\omega')\rangle \sim k^{-2\rho}\delta(k+k')\delta(\omega+\omega')\,,\ 0 \leq \rho < 1$$

An R-G analysis gives $\alpha \approx (1+2\rho)/3, \rho > 1/4$ and $1/2$ otherwise

But in these scenarios, Galilean invariance continues to hold $(\alpha + z = 2)$

Here, even for $b = 5/2$ where deterministic part has quadratic nonlinearity,

$\alpha + z \neq 2$

- Temporal correlations in noise (Medina et al., 1989)

$$\langle \eta(k,\omega)\eta(k',\omega')\rangle \sim \omega^{-2\theta}\delta(k+k')\delta(\omega+\omega')$$

Using

$$\alpha = 1.69\theta + 0.22 = \frac{4-b}{2}$$

$$z = \frac{2\alpha + 1}{1 + 2\theta} = 1.5 - 1.56$$

Summary

- Several other models also show jamming transition

- hop rate depends on vacancies in front and back

(related to misanthrope process)

- hop rate depends on the particle (particlewise disorder)

- We find the roughness exponent to be anomalous in a parameter regime for the above models also

- Similar current-density diagram.

Critical dynamics of condensation transition?

Interface model