REAL TOPOLOGICAL STRING AMPLITUDES

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1) MOTIVATION AND STATEMENT OF THE QUESTION

2) Results

3) Summary of the Method.

4) Conclusions & Open questions.

1) MOTIVATION & STATEMENT OF THE PROBLEM

→ Closed String (IIA/B) TOPOLOGICAL STRING PARTITION Fn.

(BCOV '93)

II / CY

CFT DESCRIPTION OF CY is $N=2$

SCFT. ALGEBRA $(T, G^{\pm}, J)$ with dimensions $T \to 2$, $G^{\pm} \to 3/2$, $J \to 1$

→ TWISTING

$$\hat{T} = T + \partial J$$

MODIFIED DIMENSIONS $G^{+} \longrightarrow 1$, $G^{-} \longrightarrow 2$

$$Q_{Top} = \int G^{+}$$

$$Q\, G^{-} = \hat{T}$$

Physical States are in Q_{Top} Cohomology

$$Q_{Top} |\psi\rangle = 0$$

$$|\psi\rangle \sim |\psi\rangle + Q_{Top} |\;\rangle$$

RECALL USUAL BOSONIC STRING

Q_{BRST} String BRST

b, c ghost system

$$b \to 2, \quad c \to -1$$

$$Q_{BRST}\, b = T$$

This allows one to define Amplitudes in Riemann Surfaces of genus g

$$\int_{\mathcal{M}_g} \mathcal{D}\Phi \prod^{3g-3} (\mu b)\, e^{-S} \underbrace{V\, V\, V \ldots}_{\text{Vertex operators}} \qquad g \geq 2$$

If we insert BRST exact operators

$$\Updownarrow$$

Q_{BRST}

Then ~~using~~ Deforming the contours.

$$\mu\, Q_{BRST}\, b = \mu T \longrightarrow \text{Total derivative}$$

$\Rightarrow$ Decoupling of BRST-Exact operators. (ALMOST)

Topological PARTITION FN.

$$F_g = \int_{\mathcal{M}_g} \mathcal{D}\phi \prod^{3g-3} (\mu G^-)\, e^{-S}$$

Q_{Top} EXACT OPERATORS (Anti holomorphic Derivatives)

$\longrightarrow$ $(\mu \hat{T})$

$\longrightarrow$ Total Derivative in the moduli Space $\mathcal{M}_g$

$\longrightarrow$ Boundary terms

$\longrightarrow$ Holomorphic anomaly which is <u>RECURSIVE</u>

<u>QUESTION</u>: Is There a PHYSICAL STRING AMPLITUDE THAT REPRODUCES F_g ?

or Equivalently

DOES F_g COMPUTE AN EFFECTIVE ACTION TERM IN SUGRA ?

<u>ANSWER</u> BCOV, AGNT ('93)

$$\int F_g \, (W^2)^g$$

where $W^2 = W_{\mu\nu} W^{\mu\nu}$ → CHIRAL WEYL SUPERFIELD

$$W_{\mu\nu} = T_{\mu\nu} + \cdots \theta \sigma_{\lambda\sigma} \theta \, R_{\mu\nu}{}^{\lambda\sigma} + \cdots$$

$$\longrightarrow \int F_g \, R^2 \, (T^2)^{g-1}$$

→ HETEROTIC DUAL (AGNT '95)

USED TO STUDY ~~Si~~ CONIFOLD SINGULARITY STRUCTURE

(SCHWINGER LIKE FORMULA)

A KEY INGREDIENT IN GV

→ BLACK HOLE PHYSICS

→ NEKRASOV PARTITION FN.

(FOR $\epsilon_+ = 0$)

→ TOPOLOGICAL STRING FOR $\mathcal{N}=1$

(4 SUPERCHARGES)

→ BCOV '93 (OPEN STRING)

→ AGNT '96 (HETEROTIC STRING)
(Semi topological)

$$\int \hat{F}_g \, (W^2)^g$$

$$W = \lambda + \theta F$$

Chiral gauge Superfield.

→ Holomorphic anomaly equations do not close on F_g's

Walcher 2009 Topological String for Type I like theories (Surfaces with boundaries & Cross Caps)

→ Twisted CY CFT with D-brane & orientifold plane wrapped on a Lagrangian Submanifold.

→ Holomorphic ANOMALY Equation closes on $\hat{F}_g$.

MAIN QUESTION

~~W~~ DOES Walcher Top. String Partition Function ~~Gom~~ $\hat{F}_g$ compute some Effective action term in a suitable Type I theory ?

or equivalently a IIA orientifold Theory. ?

II RESULT

→ IIA on $R^4 \times CY$

$\downarrow$

admits an anti-analytic involution g with the fixed point submanifold being Lagrangian

$IIA / \Omega \hat{g}$ where Ω is world sheet parity

and $\hat{g} = g \cdot h$

$$R^2 \times \underset{\substack{\downarrow \\ h}}{\underbrace{R^2}} \times \underset{\substack{\\ g}}{CY}$$

Z_2 orbifold.

This results in O4 − D4 plane wrapped on first R^2 and g-fixed Lagrangian submanifold.

Equivalently one can think of this as

Type I on $R^2 \times R^2 \times CY$

and do 5 T-dualities

on second R^2

& 3-directions Transversal

to the g-fixed Lagrangian

Submanifold.

which $(O9-D9) \longrightarrow (O4-D4)$

$\boxed{\text{Claim}}$

Nontrivial part of Walcher Topological String Partition fn. (ie removing the purely closed oriented part) Computes

$$\int d^2\theta \; F_\chi \; W_{11}^{g'} \qquad \text{where } \chi = 1-g'$$

$\downarrow$ unbroken chiral superspace — Euler character where $\chi = 1 - g'$ is the Euler number of the Rieman surface ~~ed~~ with boundaries & cross caps. $\Sigma_{(g,h,c)}$

and g' is the genus of the closed oriented double cover of the relevant surface.

$$W_{11} = T_{11} + R_{11}\,\theta^2 + \cdots$$

T_{11} & R_{11} are the orientifold invariant graviphoton & Rieman Tensors

$$\Rightarrow \langle R_{11}\, T_{11}^{\,g'-1} \rangle$$

III SUMMARY OF THE METHOD

→ $V_{T_{11}} = V_L \, V_R$

$$V_L = \cdots e^{i \frac{\sqrt{3}}{2} H}$$

$$V_R = \cdots e^{-i \frac{\sqrt{3}}{2} \tilde{H}}$$

H, $\tilde{H}$ are left & Right Bosonisation of $U(1)$ currents

→ Double Cover $\Sigma_{g'}$

so that $\Sigma_{g'} / Z_2 = \Sigma_{(g,h,c)}$

→ Image Trick

$$V_{T_{11}}(p) = V_L(p) \, \hat{V}_R(\bar{p})$$

where $\bar{p}$ is the image of p in $\Sigma_{g'}$

and $\hat{V}$ includes the action of Z_2 reflections on CY & second R^2

→ In particular the internal part of the $V_{T_{11}}(p)$ becomes

$$e^{i\frac{\sqrt{3}}{2}H}(p) \quad e^{i\frac{\sqrt{3}}{2}H}(\bar{p})$$

as under Z_2 Involution $\tilde{H} \to -H$

$\Rightarrow$ there are $2g'-2$ $\quad e^{i\frac{\sqrt{3}}{2}H}$ on $\Sigma_{g'}$

→ Thus the problem reduces to the type II calculation (Just the left moving sector)
And one can repeat the steps of (AGNT '93)

→ This results in the twisted internal theory

→ The space time part gives now $\det(\mathrm{Im}\,\tau)$ instead of $(\det(\mathrm{Im}\,\tau))^2$

→ This cancels with the determinant contribution from

$$\begin{array}{ccc} R^2 & \times & R^2 \\ N & & D \\ \downarrow & & \downarrow \\ \det \Delta_+ & & \det \Delta_- \end{array}$$

$$\underbrace{\det \Delta_+ \quad \det \Delta_-}_{\det \Delta_+ \Delta_- = \det \Delta}$$

instead of $(\det \Delta)^2$ ~~as~~ in closed string case

→ non-zero modes of Bosonic & fermionic determinants of R^4 cancel.

→ (b, c) & $(\beta\ \gamma)$ ghost determinants cancel.

Conclusions & Open questions

1) We identified Walcher Top. string partition fn. with the effective action term

$$\int \hat{F}_\chi \, W_{11}^{g'}$$

2) Walcher et al have a conjecture regarding the conifold singularity structure of $\hat{F}_\chi$.

This is more complicated than closed string F_g case. It involves Bernoulli numbers & Euler numbers

Question

→ Can we get a dual string description where would-be massless states are perturbatively accessible

(as in AGNT '95 for usual F_g)

Under Investigation