Anisotropic Geometries, Viscosity, And Cold Atoms

Sandip Trivedi, TIFR

Outline

1) Introduction

2) Anisotropic Geometries

3) Viscosity: Some General Results $\frac{\eta_{xz}}{s} \ll \frac{1}{4\pi}$

4) Testing the result using Cold Atoms

5) Conclusions

Introduction

In strongly coupled field theories dual to a smooth geometry

Eg: N=4 SYM dual to AdS_5

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

(Kovtun, Son, Starinets (KSS), …,
Das, Gibbons, Mathur, … Damour, …)

Introduction

KSS suggested that this might be a bound.

$$\frac{\eta}{s} \sim \frac{\lambda_{mfp}}{\lambda_{dB}}$$

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

Introduction

We now know that this is not true.

Adding higher derivative corrections can reduce the ratio.

But in these instances the correction is small.

No examples with say parametrically large violations of this bound are known.

Nor are any examples which violate the bound known in nature.

(So far isotropic case).

Introduction

Anisotropic Systems which admit smooth gravity duals often lead to parametric violations

$$\frac{\eta}{s} \sim \left(\frac{T}{\rho}\right)^p$$

$$\ll \frac{1}{4\pi} \qquad \text{for } T << \rho$$

ρ : Anisotropy Parameter

Anisotropic Geometries

No hair theorems might lead one to expect that very few black hole/brane solutions should exist

On the other hand there are many –many phases seen in nature, and expected to also arise in strongly coupled systems. AdS/CFT suggests then that perhaps the naïve expectation above is not correct.

In fact that turns out to be true. Asymptotically AdS space is different.

Many different kinds of black brane solutions in asymptotically AdS space are now known to exist.

Anisotropic Geometries

Including several anisotropic ones, i.e., solutions which break rotational symmetry

E.g.: Bianchi Classified general spacetimes which can be anisotropic but are homogeneous.
Used extensively in Cosmology.

Turns out that with reasonable matter many of the Bianchi types can be realised in asymptotically AdS space!

E.g., Types II III, V, VII, IX realised with matter consisting of U(1) gauge fields and scalars.
(Iizuka, Kachru, Kundu, Narayan, Sircar, ST, Wang …)

Anisotropic Geometries

Simple Example:

$$S = \int d^5x \sqrt{-g}[R - \Lambda - \frac{1}{2}(\partial\phi)^2]$$

Jain, Kundu, Sen, Sinha, ST

$$AdS_5$$

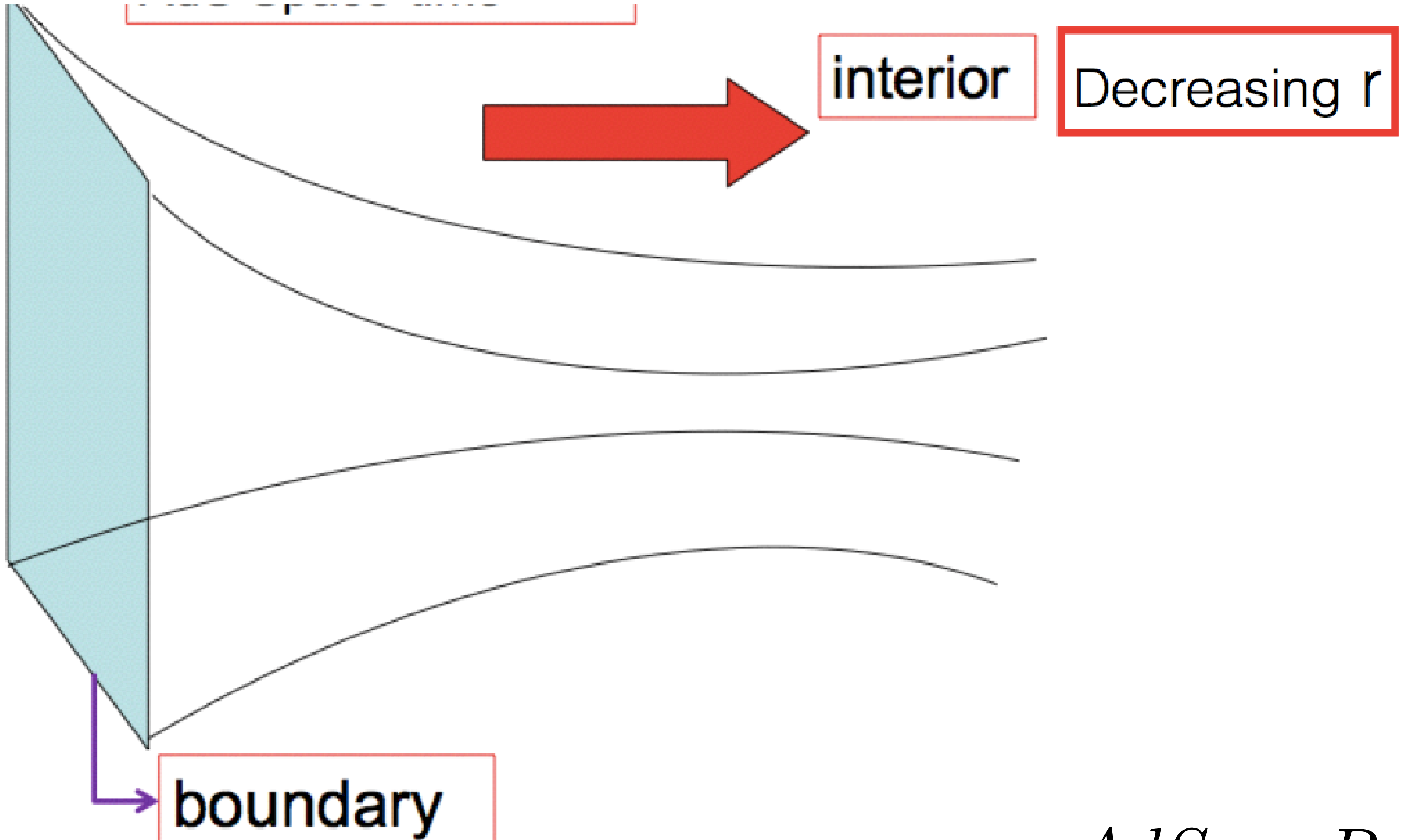
$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^2 + dy^2 + dz^2) + \frac{R^2}{r^2}dr^2$$

Turn On Dilaton to break isotropy:

$\phi = \rho z$ (Non-normalisable)

Resulting metric interpolates between

$$AdS_5 \quad \to AdS_4 \times R$$



AdS_5 $AdS_4 \times R$

$$AdS_4 \times R$$

$$ds^2 = \frac{r^2}{\tilde{R}^2}(-dt^2 + dx^2 + dy^2) + \frac{\tilde{R}^2}{r^2}dr^2 + L^2 dz^2$$

$$SO(3) \to SO(2)$$

Full interpolating metric preserves translational invariance.

Boundary theory subjected to a force which is constant.

$$< \partial_\mu T^{\mu\nu} >= \partial^\nu \phi < O >$$

Shear Viscosity

In general shear viscosity is a tensor with 21 components.

In this case it has three components: Spin 2, Spin 1, and Spin 0.

Spin 1: η_{xz}, η_{yz}

η_{xz} obtained from two point function of T_{xz} etc.

Spin 2: η_{xy}

Spin 1:

$$T \gg \rho : \frac{\eta_{xz}}{s} = 1 - c_1 \frac{\rho^2}{T^2} \qquad c_1 = \frac{\log 2}{16\pi^3}$$

$$T \ll \rho : \frac{\eta_{xz}}{s} = c_2 \frac{T^2}{\rho^2}$$

parametrically small!

Spin 2:

$$\frac{\eta_{xy}}{s} = \frac{1}{4\pi}$$

How general is this large reduction in the value of components of Viscosity?

Other examples, some known earlier: Rehban and Steineder, …, Polchinski and Silverstein, …

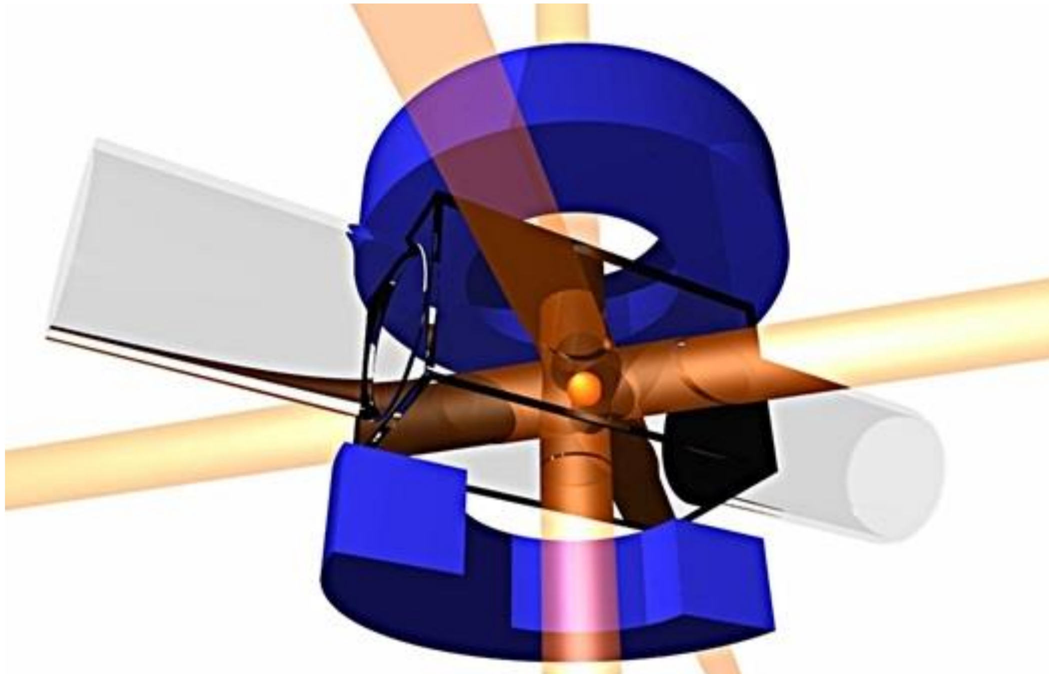
In example above the breaking was due to an externally applied force which preserved translational invariance (spatially constant)

One can show in general that for such situations:

$$\frac{\eta_{xz}}{s} = \frac{1}{4\pi} \frac{g_{xx}}{g_{zz}}\Big|_h$$ (Jain, Samanta, SPT)

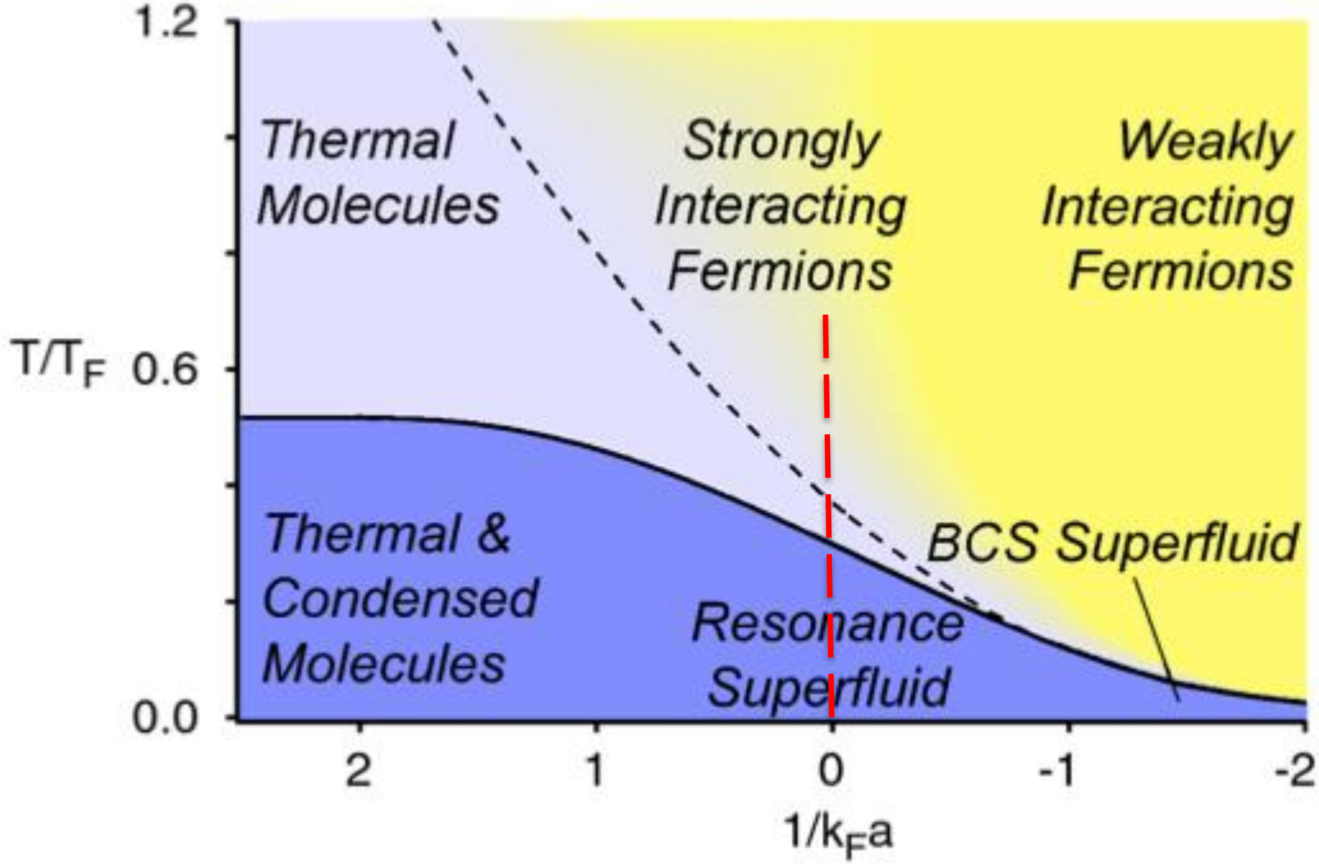
(includes gapped and conformal systems).

Cold Atom Systems



10^6 Atoms,
10^{-6} Meters
$10^{-5}K$ temperature

Lots of funky lasers!!



Phase diagram (Ketterley and Zwerlein arXiv: 0801.2500)

At unitary point we have a scale invariant theory.
Strongly coupled.

$$\frac{\eta}{s} \sim 6 \times \frac{1}{4\pi}$$

Question: Can we set up a situation which will probe if the viscosity becomes small in presence of anisotropy?

Rickmoy Samanta, Rishi Sharma, ST: arxiv: 1607.04799, arXiv: 1611.02720

Bottom line: Yes! Quite promising!

Caveats: I) Of course this system does not have a gravity dual. But if the generic behaviour seen in gravity systems also occurs here there is an experiment which seems quite feasible to do which will probe it.

II) Cannot go to the extreme anisotropic limit. Etc.

Basic Idea: Make the trap anisotropic

$$V = \frac{1}{2} m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

$$\omega_z \gg \omega_x = \omega_y$$

Identify a mode whose dissipation dominantly arises due the spin 1 component of the shear viscosity: Scissor Mode

Equations of superfluid mechanics are complicated. We obtain a solution in the linearised approximation (small amplitude).

$$\frac{\partial(\rho s)}{\partial t} + \nabla\cdot(\rho s \mathbf{v}_n) = 0,$$
$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{g} = 0\,,$$
$$\frac{\partial g_i}{\partial t} + \nabla_j \Pi_{ij} = -n\nabla\phi(\mathbf{r}),$$
$$\frac{\partial \mathbf{v}_s}{\partial t} = -\nabla\left(\frac{\mathbf{v}_s^2}{2} + \frac{\phi(\mathbf{r})}{m} + \frac{\mu(\mathbf{r})}{m}\right).$$

$$\mathbf{g} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s\,,$$

$$\Pi_{ij} = P\delta_{ij} + \rho_n \mathbf{v}_{n,i}\mathbf{v}_{n,j} + \rho_s \mathbf{v}_{s,i}\mathbf{v}_{s,j}\;.$$

$$\mathbf{v_n} = \mathbf{v_s} = \mathbf{v}$$

$$\mathbf{v} = \alpha e^{i\omega t}(z\hat{x} + x\hat{z})$$

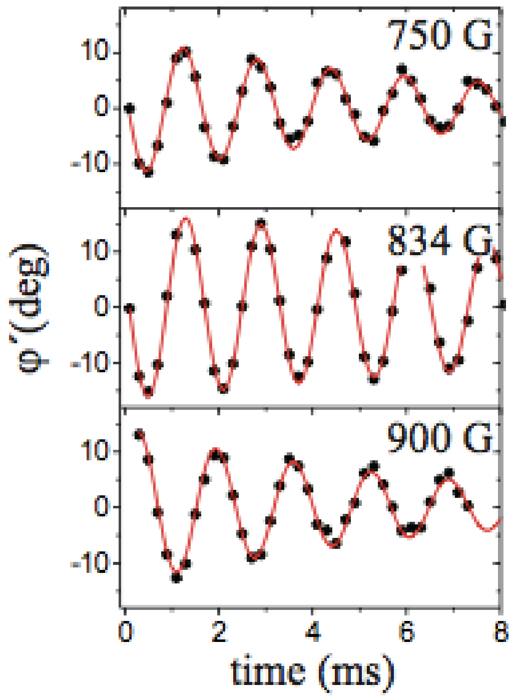
$$\omega = \sqrt{\omega_x^2 + \omega_z^2}$$

$$\dot{E} = -\int d^3r\ \eta_{xz}(\mathbf{r})\alpha^2$$

$$\tau^{-1} = \frac{\dot{E}}{E}$$

Viscosity neglected to leading order- selfconsistently.

In fact this scissor mode has already been excited and studied observationally.
Wright, Riedl, Altmeyer, Kohstall. Sanchej Guajardo, Hecker Denschlag and Grimm, arXiv: 0707.3593



Decay observed on milli second time scale. As anisotropy is increased this decay time should increase if viscosity is decreasing.

Fly in ointment!
Anisotropy cannot be made arbitrarily big.

The effective anisotropy parameter is not $\frac{\omega_z}{\omega_x}$

But more correctly: $k_{LDA} \equiv (\frac{\partial \mu}{\partial z})\frac{1}{\mu k_F}$

$$= \frac{\hbar \omega_z}{\mu} \leq 1$$

For 3 dimensional hydrodynamics to hold

Anisotropy can be made order unity

Still one can estimate:

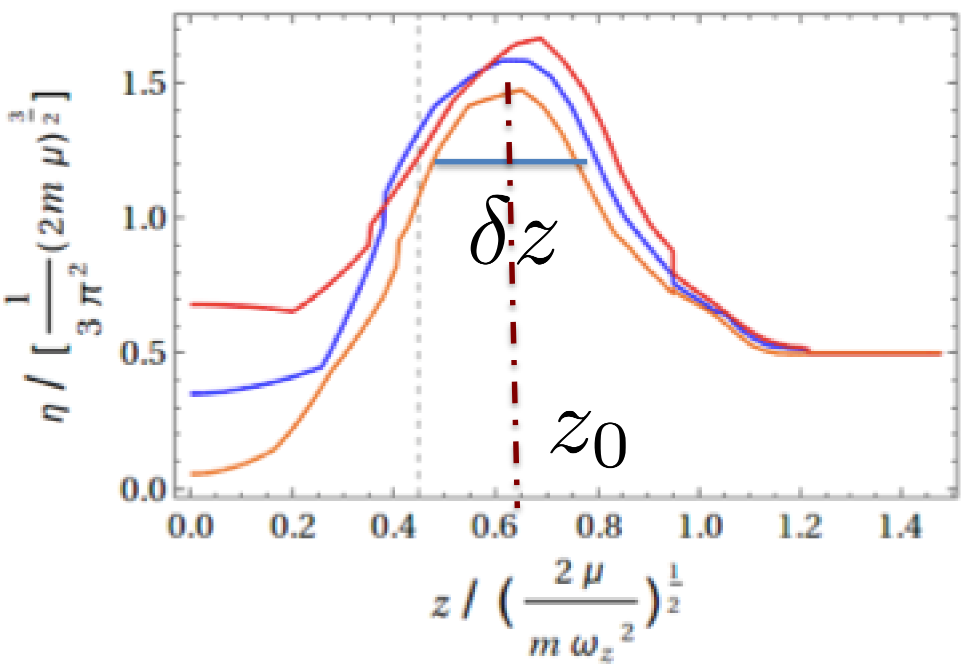
$$\delta\eta_{xz} \sim \kappa_{LDA}^2$$

So for

$$\kappa_{LDA} \sim O(1)$$

There should be an order one effect that can be observed (subject to the caveats mentioned earlier of course!) .

Another Important Point: Linearity Condition



$$\frac{\delta z}{z_0} \sim 0.3$$

Ideally for linear approximation to hold $\frac{\delta z}{z_0} \ll 1$

Linearlity Condition

$\frac{\delta z}{z_0}$ becomes smaller if one starts with lower temperatures at the centre of the trap.

But then the phonon contribution to viscosity becomes more significant.

This results in a tradeoff, and the best one can do is about

$$\frac{\delta z}{z_0} \sim 0.2$$

Bottom Line

We can have substantial but not arbitrarily large anisotropy

The dominant contribution to the viscosity can come from a somewhat but not very localised region of trap. So linear approximation of potential is not very good.

Still if nature is kind, the tendency for viscosity to decrease with increasing anisotropy could be seen experimentally.

And since the scissor mode has already been excited and required time resolution etc seem well at hand it is worth looking for this effect!

Conclusions

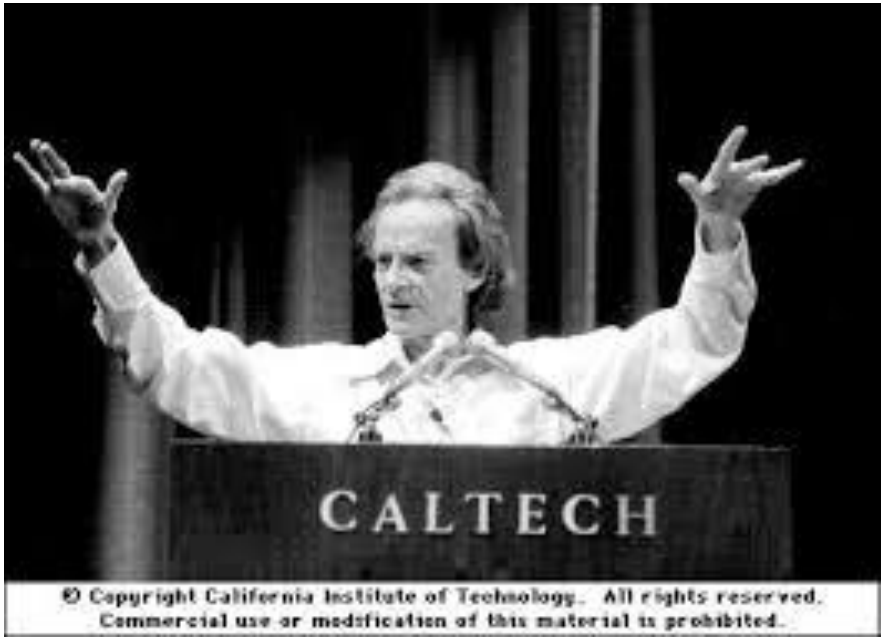
- There are a rich set of anisotropic black brane solutions in systems consisting of gravity coupled to reasonable matter fields.

- These could shed light on the behaviour of strongly coupled anisotropic systems, e.g. in quasi crystals, etc.

Conclusions

In the relatively simple case, where translational invariance is preserved and rotational invariance is broken, the gravity systems already reveal a fairly general and interesting property, namely the reduction of the spin one component of the viscosity.

This could be tested experimentally quite easily, in cold atom systems.
(If nature is kind and such systems have the good tasted to reproduce the gravity results!)



As Feynman Said:

`‘‘Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry.’’

Thank you And Congratulations!