QTCM – Model and Concepts

[Neelin and Zeng, Jounal of the Atmospheric Science, 2000]

Vijit Maithel

Divecha Center for Climate Change, IISc

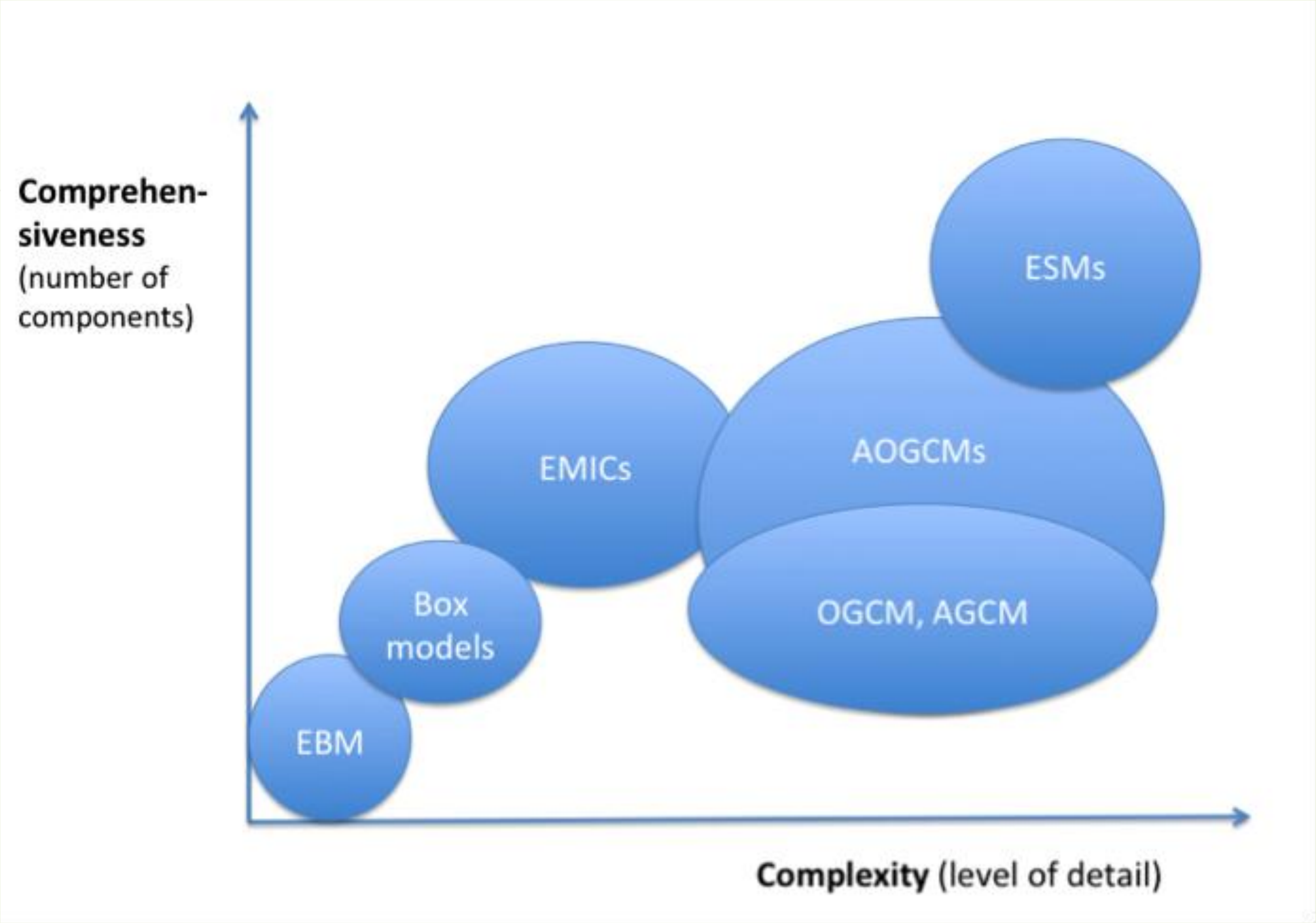


Figure taken from www.scisnak.com

Outline

- Primitive Equations
- Quasi-equilibrium – concept and constraints on vertical profiles
- Galerkin expansion – reference profiles and basis functions
- Precipitation parameterisation/convective heating (Betts Miller scheme)
- Final form of QTCM equations
- Simplified versions of QTCM
- Vertical stability terms and gross moist stability

Primitive Equations

- Thermodynamic Equation

$$(\partial_t + v.\nabla)T + \omega\partial_P s$$
$$= Q_c + g\partial_P R^{\uparrow} - g\partial_P R^{\downarrow} - g\partial_P S + g\partial_P F_T$$
$$+ K_H \nabla^2 T$$

- Moisture equation

$$(\partial_t + v.\nabla\ )q\ + \omega\partial_P q = Q_q + g\partial_P F_q + K_H \nabla^2 q$$

T & q are in energy units

- Energy constraint, $\widehat{Q_c} + \widehat{Q_q} = 0$

- Vertically integrating and adding the two equations gives the moist static energy equation

$$\partial_t(\hat{T} + \hat{q}) + \widehat{\mathcal{D}_T T} + \widehat{\mathcal{D}_q q} + \widehat{\omega\partial_P h}$$
$$= g/p_T(E + H + R)$$

Where,

dry static energy, $s = T + gz$

moist static energy,
$h = s + q = T + gz + q$

$$\mathcal{D}_T = \mathcal{D}_q = v.\nabla - K_H \nabla^2$$

Source – Neelin and Zeng 2000

- Hydrostatic balance is balance between the gravitational and pressure forces acting on a parcel.
- In general the vertical acceleration observed in the atmosphere is much smaller than what one would expect solely due to gravity. Hence, we assume that hydrostatic balance is maintained.
- Momentum equation (combined with hydrostatic balance)

$$(\partial_t + \mathcal{D}_V)v + fk \times v + g\partial_P\tau = -\nabla \int_p^{p_{rs}} \kappa T dlnp - \nabla\phi_s$$

Where, $\mathcal{D}_V = v.\nabla + \omega\partial_P - K_H\nabla^2$

- Mass conservation equation for an incompressible flow

$$\omega = \omega_s + \int_p^{p_s} \nabla. v\, dp$$

Where, ω_s is approximated to zero, as effects of topography are neglected

Source – Neelin and Zeng 2000

Quasi-equilibrium (QE) framework

- Interaction between large scale circulation, and moist convection.
- Moist convection responds at time scales much faster than large scale circulation, and dissipates the available energy for convection (APE) being produced by the large scale circulation. As a result, the rate of change of APE is nearly zero, and the system is in said to be in "quasi-equilibrium". (Arakawa and Schubert, J. Atmos. Sciences, 1974)
- One implication of QE is that it constrains the temperature profile. The result of moist convection is to adjust the atmosphere towards a equilibrium temperature profile, T^c

$$T^c = T_r^c(p) + A_1(p)T_1^c(x, y, t)$$

Temperature reference profile and basis function

$$T^c = T_r^c(p) + A_1(p)T_1^c$$

Moist static energy perturbation at the boundary layer

Temperature reference profile

- Dry adiabat – temperature lapse rate observed by a dry parcel rising adiabatically ~ 9.8K/km
- Moist adiabat – temperature lapse rate observed by a saturated parcel rising adiabatically ~ 4.5K/km

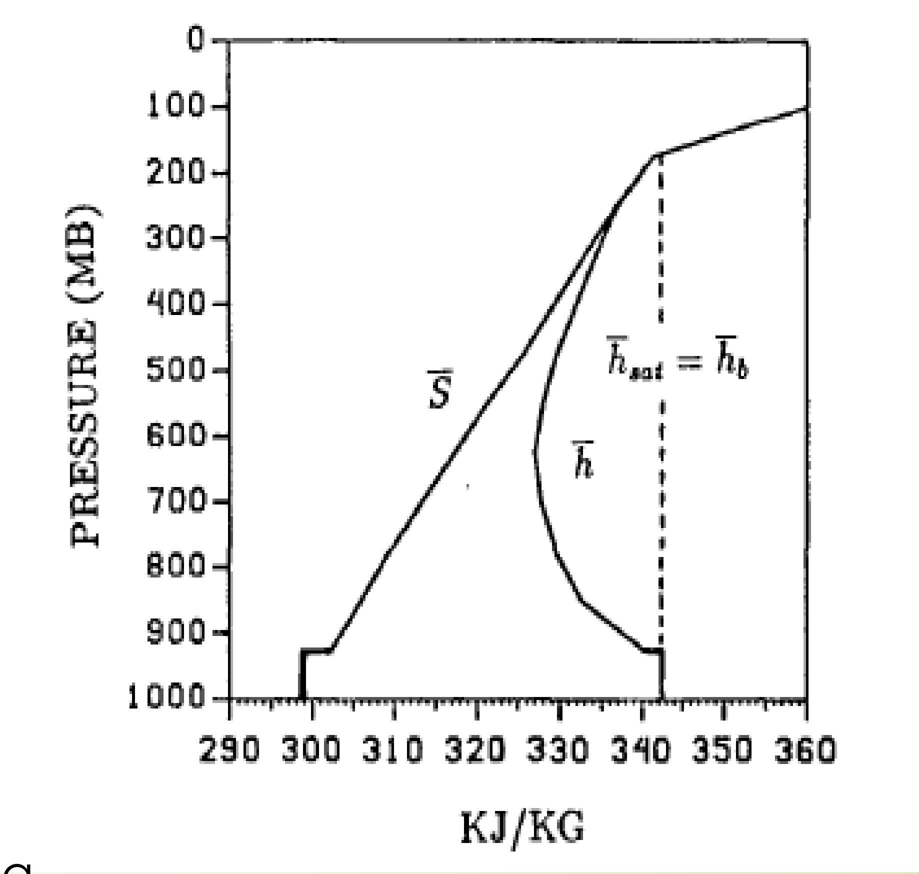
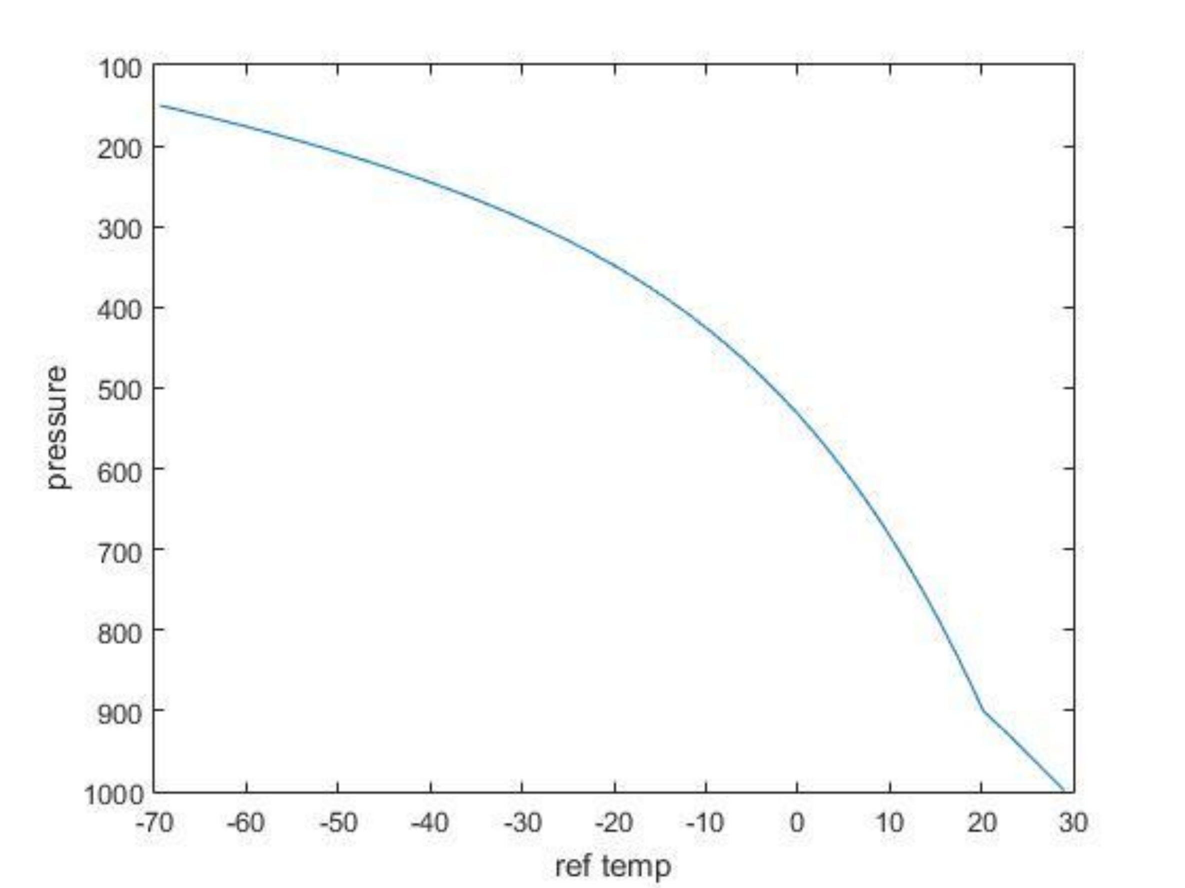


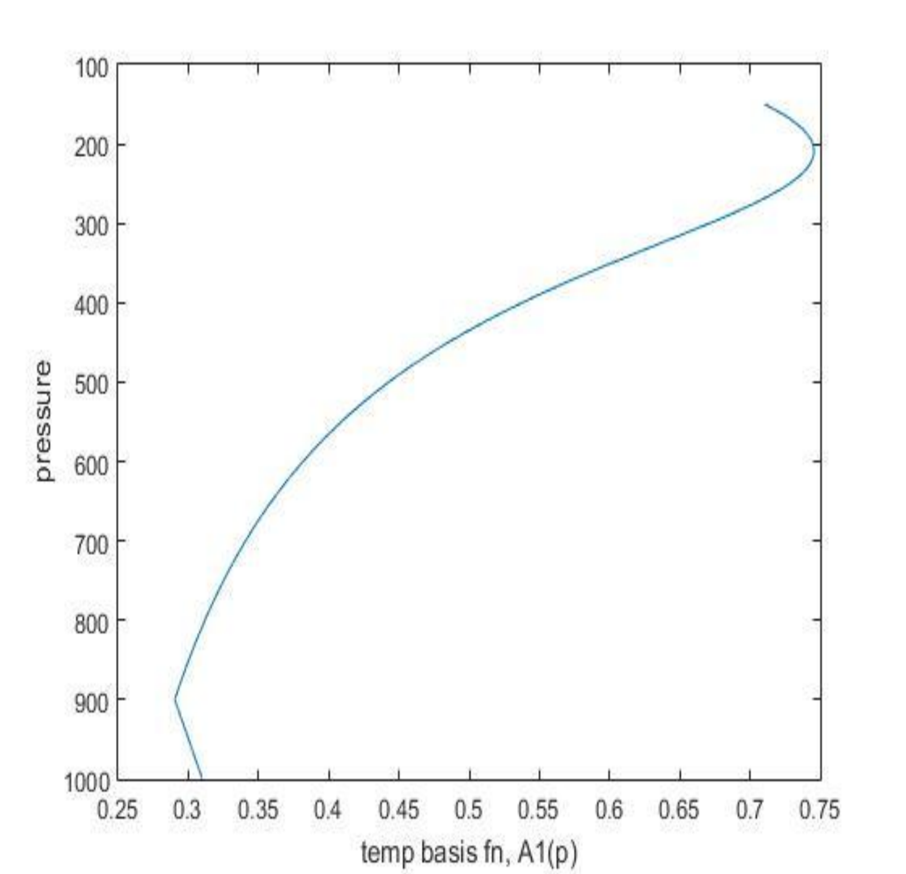
Figure from Neelin and Yu, J. Atmos. Sciences, 1994

- Reference temperature profile, $T_r^c(p)$



- Temperature basis function, $A_1(p)$

$$c_P T^{c\prime} = A_1(p) h'_b$$



Velocity components

$$(\partial_t + \widetilde{\mathcal{D}}_V)v + fk \times v + g\partial_P \tau = -\nabla \int_p^{p_{rs}} \kappa T^c dlnp - \nabla \phi_s$$

- Solution comprises of a barotropic and baroclinic term.

$$v = v_0(x, y, t) + V_1(p)v_1(x, y, t)$$

Barotropic
(height invariant)

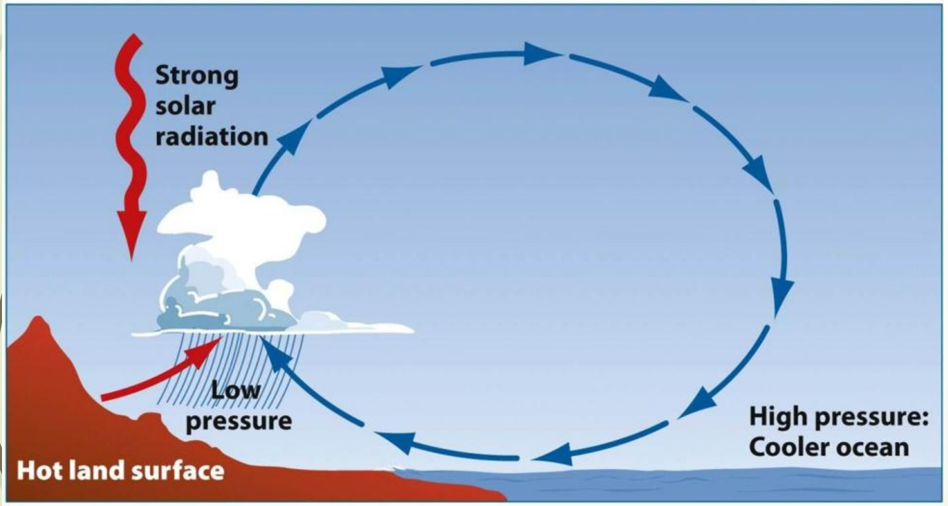
Baroclinic
(varying with height)

Source – Neelin and Zeng 2000

Baroclinic velocity basis function, $V_1(p)$

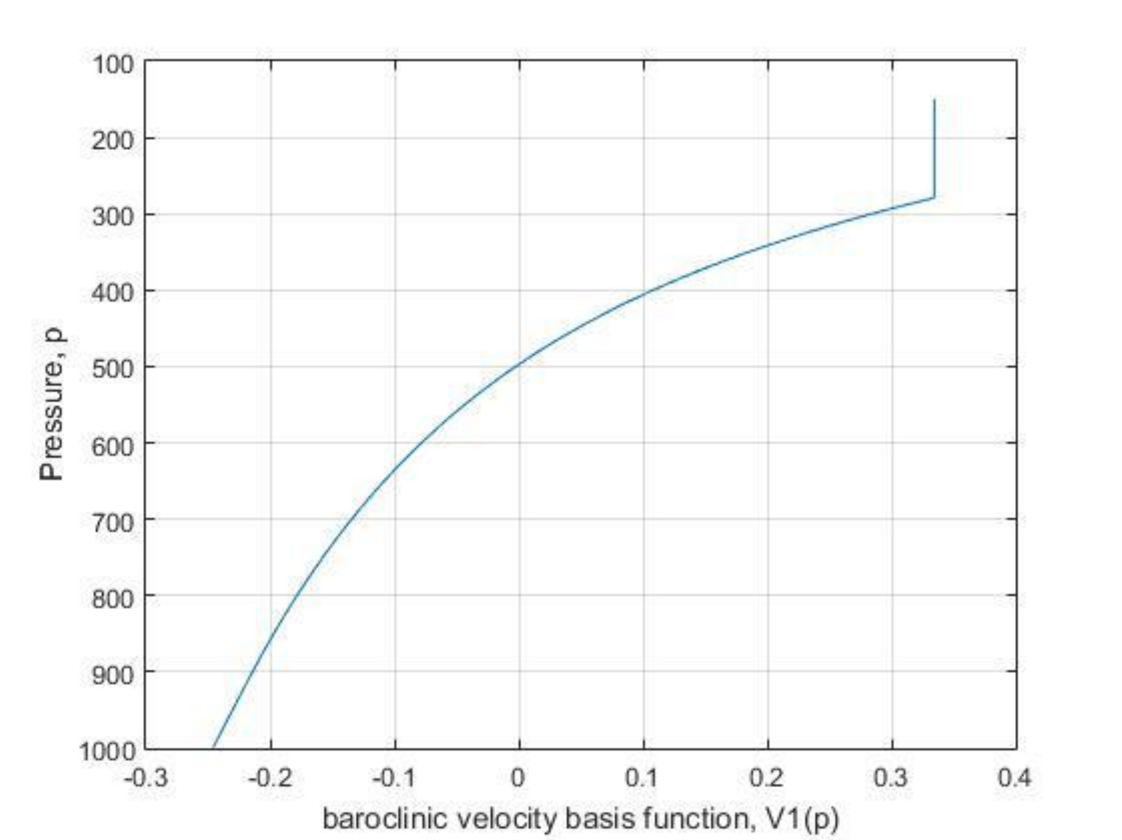
$$V_1(p) = A_1^+(p) - < A_1^+ >$$

Where, $A_1^+(p) = \int_p^{p_{rs}} A_1(p')dlnp'$



Summer monsoon

Figure 8-1
Earth's Climate: Past and Future, Second Edition

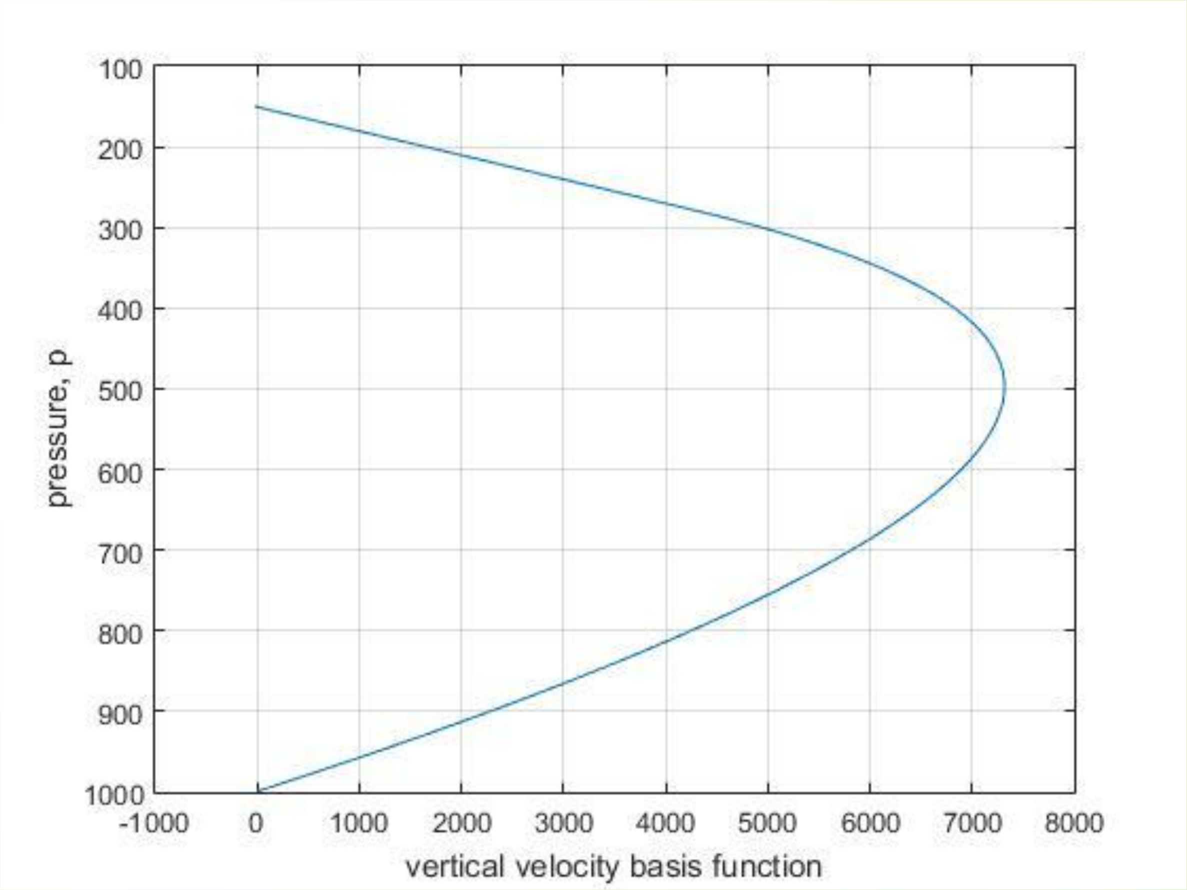


Source – Neelin and Zeng 2000

Vertical velocity basis function, $\Omega_1(p)$

$$\omega = \int_p^{p_s} \nabla . v_0 \, dp + \int_p^{p_s} \nabla . (V_1 v_1) dp$$

$$\Omega_1(p) = -\int_p^{p_s} V_1(p') dp'$$



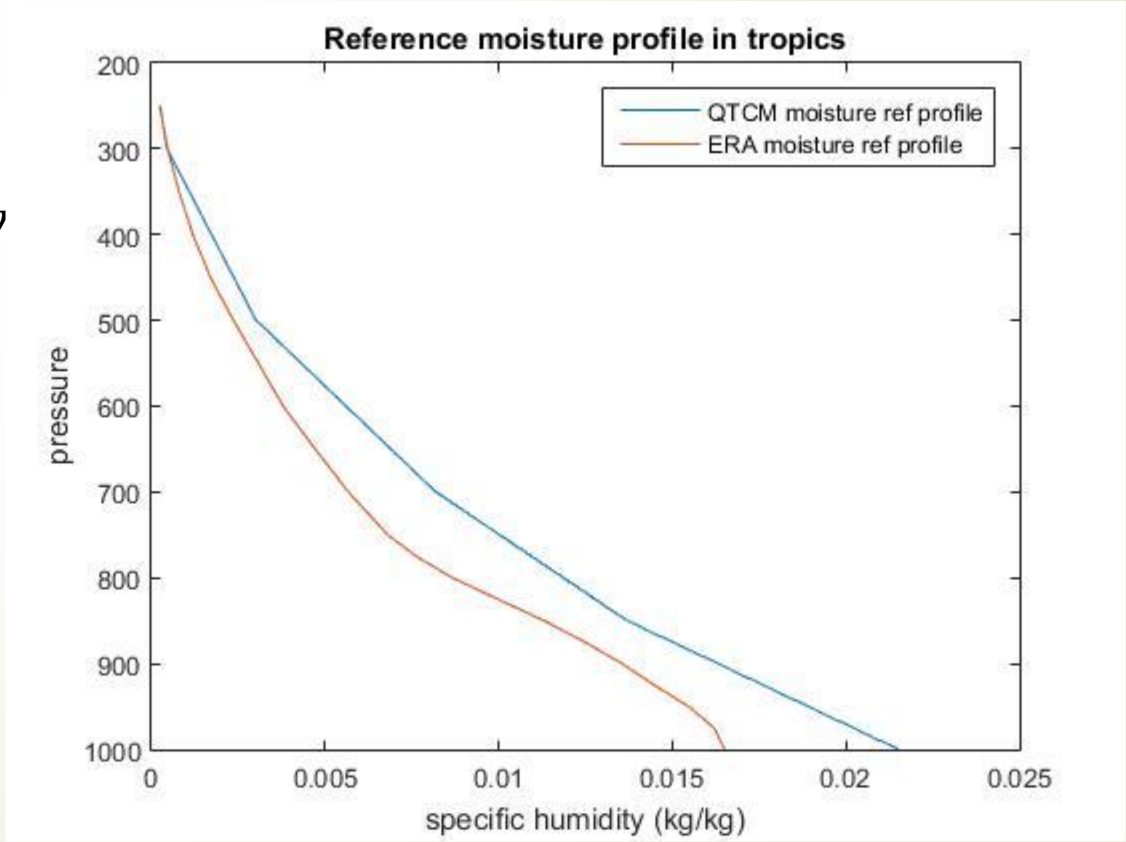
Source – Neelin and Zeng 2000

Moisture reference profile and basis function

- Similar to temperature, moisture is also adjusted towards a equilibrium profile due to convection.

$$q^c = q_r^c(p) + B_1(p) q_1^c(x, y$$

$$where, B_1 = q_r^c(p)/q_r^c(p_s)$$



Source – Neelin and Zeng 2000

Galerkin expansion of variables

$$T = T_r(p) + \sum a_k\,(p) T_k(x, y, t)$$

$$q = q_r(p) + \sum b_k\,(p) q_k(x, y, t)$$

$$v = \sum V_k\,(p) v_k(x, y, t)$$

- $T_r(p)$, $q_r(p)$ are the reference profiles whereas $a_k, b_k, and v_k$ act as the various basis functions.
- Using QE concepts, constraints are placed on the vertical reference temperature profile

Source – Neelin and Zeng 2000

- Also, based on QE, the series are truncated after the first basis function for temperature and moisture, and two basis function for winds.

- Quote from Neelin and Zeng, 2000 –

“In a more general case, a limited number of basis functions tailored to the dominant physical processes, in particular convective QE constraints, would be used. This approach is predicated on the assumption that convective QE constraints tend to reduce the number of vertical degrees of freedom that are crucial to the solution.”

$$T = T_r(p) + a_1 (p) T_1(x, y, t)$$

$$q = q_r(p) + b_1 (p) q_1(x, y, t)$$

$$v = v_0(x, y, t) + V_1(p) v_1(x, y, t)$$

Source – Neelin and Zeng 2000

Precipitation Parameterisation and adjustment time scale

$$Q_c = \frac{T^c - T}{\tau_c} \quad if \quad (T^c - T) > 0$$

$$0 \qquad \text{otherwise}$$

- τ_c is the convective adjustment time scale of the order of ~ 2h (Betts Miller 1986).
- Limiting case of τ_c --> 0 represents strict QE

Source – Neelin and Zeng 2000

Derivation of final QTCM equations

- The Galerkin series expansion is truncated after the first basis function for temperature and moisture, and the second basis function for momentum.

$$T = T_r(p) + A_1\,(p)T_1(x, y, t)$$

$$q = q_r(p) + B_1\,(p)q_1(x, y, t)$$

$$v = v_0(x, y, t) + V_1(p)v_1(x, y, t)$$

- The primitive equations are projected onto these basis functions and further simplified to yield the final form of model equations used in QTCM

Final form of equations

$$\partial_t v_1 + \mathcal{D}_{V1}(v_0, v_1) + fk \times v_1 = -\kappa \nabla T_1 - \epsilon_1 v_1 - \epsilon_{01} v_0$$

$$\partial_t \xi_0 + curl\big(\mathcal{D}_{V0}(v_0, v_1)\big) + \beta v_0 = -curl(\epsilon_0 v_0) - curl(\epsilon_{10} v_1)$$

$$\widehat{A_1}(\partial_t + \mathcal{D}_{T1})T_1 \ + M_{s1}\nabla . v_1 = \langle Q_c \rangle + \frac{g}{p_T} \times [R + H]$$

$$\widehat{B_1}(\partial_t + \mathcal{D}_{q1})q_1 \ - M_{q1}\nabla . v_1 = \langle Q_q \rangle + \frac{g}{p_T} \times E$$

$$\langle Q_c \rangle = -\langle Q_q \rangle = \frac{q_1 - T_1}{{\tau_c}^*}$$

Source – Neelin and Zeng 2000

Simplified versions of QTCM

- Zonal symmetry assumed.
- Mean background flow, can be neglected by setting it to zero. $(v_0 = 0)$
- The non-linear advection terms and diffusion terms, present in the momentum equation, can be neglected as a first approximation to linearize about a resting state. $(\mathcal{D}_{V1}(v_0, v_1) \sim 0)$
- Effect of rotation of Earth can be neglected for small scale systems.

Reduced form of QTCM

$$\partial_t v_1 + \varepsilon_1 v_1 = -\kappa \frac{dT_1}{dy}$$

$$\widehat{A_1} \partial_t T_1 + a_T\, v_1 \frac{\partial T_1}{\partial y} + M_{s1} \frac{dv_1}{dy} = P + \frac{g}{p_T} \times [R + H]$$

$$\widehat{B_1} \partial_t q_1 + \ a_q v_1 \frac{\partial q_1}{\partial y} - M_{q1} \frac{dv_1}{dy} = -P + \frac{g}{p_T} \times E$$

$$P = \frac{q_1 - T_1}{\tau_c} \mathcal{H}(q_1 - T_1)$$

Gross moist stability, GMS

$$M_{s1} = p_T^{-1} \int \Omega_1(-\partial_p s)dp$$ Dry static stability

$$M_{q1} = p_T^{-1} \int \Omega_1(\partial_p q)dp$$ Gross moisture stratification

$$\text{GMS} = M_{s1} - M_{q1}$$ (Neelin and Held, Monthly Weather Review, 1987)

$$= p_T^{-1} \int \Omega_1(-\partial_p h)dp$$

GMS > 0 implies that horizontal motion in monsoonal circulation exports energy out of the system

Conclusions

- QTCM is an intermediate complexity model based on quasi-equilibrium framework and Betts-Miller convective adjustment scheme.
- QE constraints allow to restrict the vertical degrees of freedom crucial to the model and analytically solve for part of the solution. This makes the model computationally cheaper.
- The constraints on the structure of the basis functions and the vertical stability parameters allow for analytical studies on the dynamics and stability in the model.
- Due to QE constraints, the model is expected to work reasonably in convective regions. Outside these regions, the model maybe highly truncated making it unable to capture the circulation effectively.
- QTCM can be easily used to develop a range of reduced models to study specific phenomena.

Supplementary

$$A_1(p) = \frac{1}{1+\gamma'_c} \exp[-\kappa \int_p^{p_s} \frac{1}{1+\gamma} dlnp]$$

$$\gamma = \frac{d\mathcal{L}q_{sat}}{dc_pT}$$