# A hybrid geometric-random template placement algorithm for gravitational wave searches from compact binary coalescences

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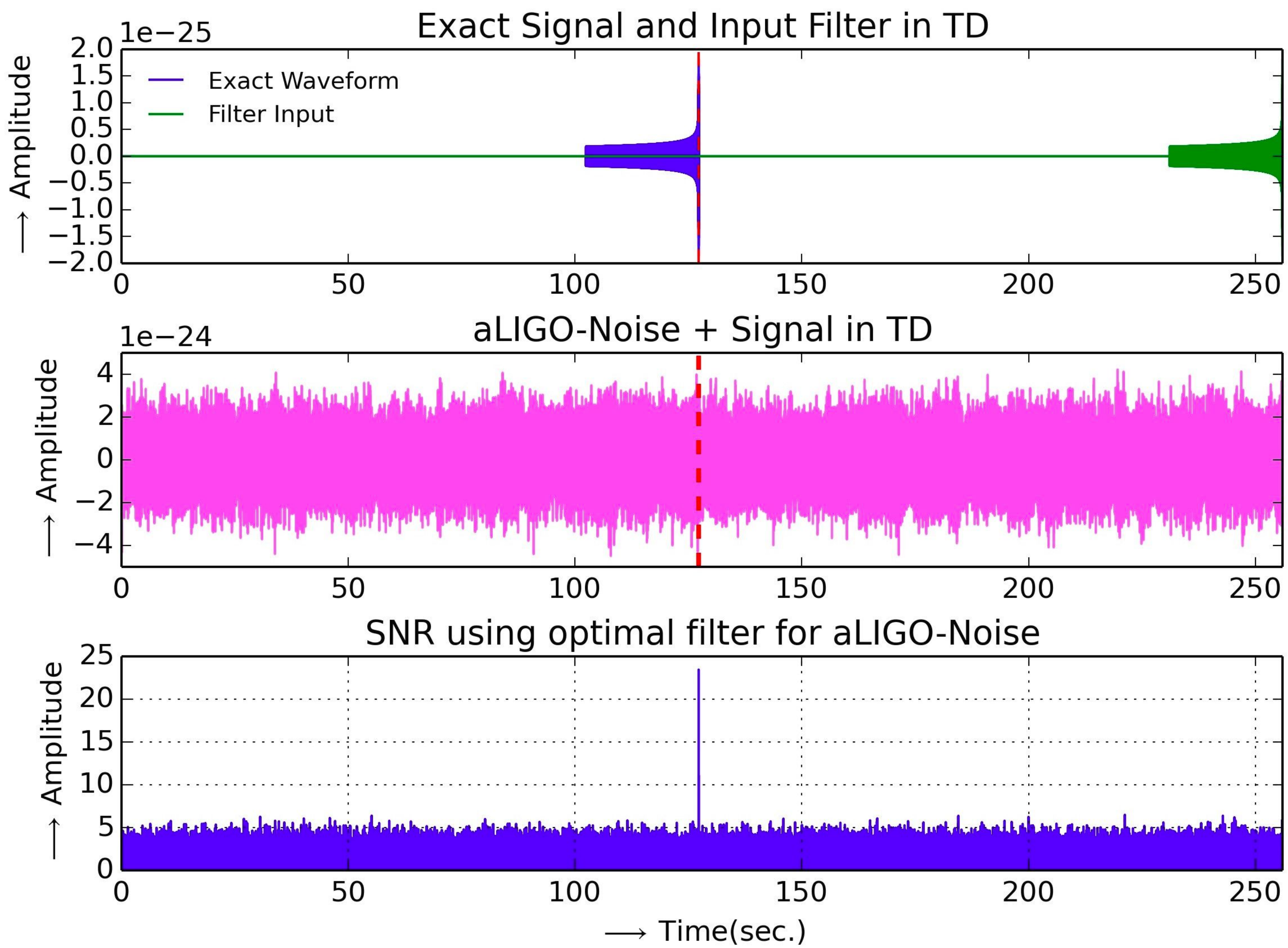
# Templated matched filtering

$$\langle s | h(\vec{\lambda}) \rangle_{\Delta t} = 4\,\mathrm{Re} \int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} \frac{\tilde{s}^*(f)\,\tilde{h}(f;\vec{\lambda})}{S_n(f)} e^{-2\pi i f \Delta t}\, df$$

$$\langle h(\vec{\lambda}) | h(\vec{\lambda} + \Delta\vec{\lambda}) \rangle = 1 - g_{\mu\nu} \Delta\lambda^{\mu} \Delta\lambda^{\nu}$$




$$g_{\mu\nu} = \frac{1}{2} \frac{\partial^2 \langle h(\vec{\lambda}) | h(\vec{\lambda} + \Delta\vec{\lambda}) \rangle}{\partial\Delta\lambda^{\mu}\, \partial\Delta\lambda^{\nu}} \Bigg|_{\Delta\vec{\lambda}=0}$$

# Art of template placement

Maximise inter-template separation without violating the constraint on minimal match of the bank.

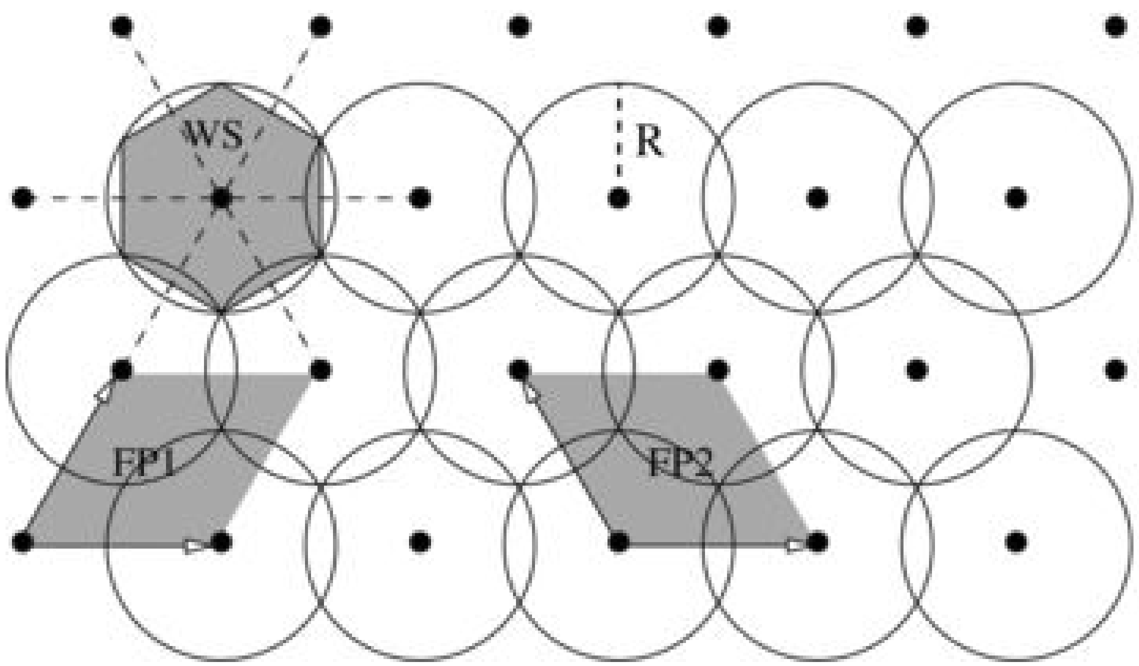
Geometrical template placement can be mapped to a sphere-packing problem with overlap using cells of radius $\sqrt{1 - \mathcal{M}_{\rm min}}$

$$g_{\mu\nu}\Delta\lambda^{\mu}\Delta\lambda^{\nu} = 1 - \mathcal{M}_{\rm min}$$

"Spherical" cell at some point in $\lambda$

# Two ways of template placement

Geometric:



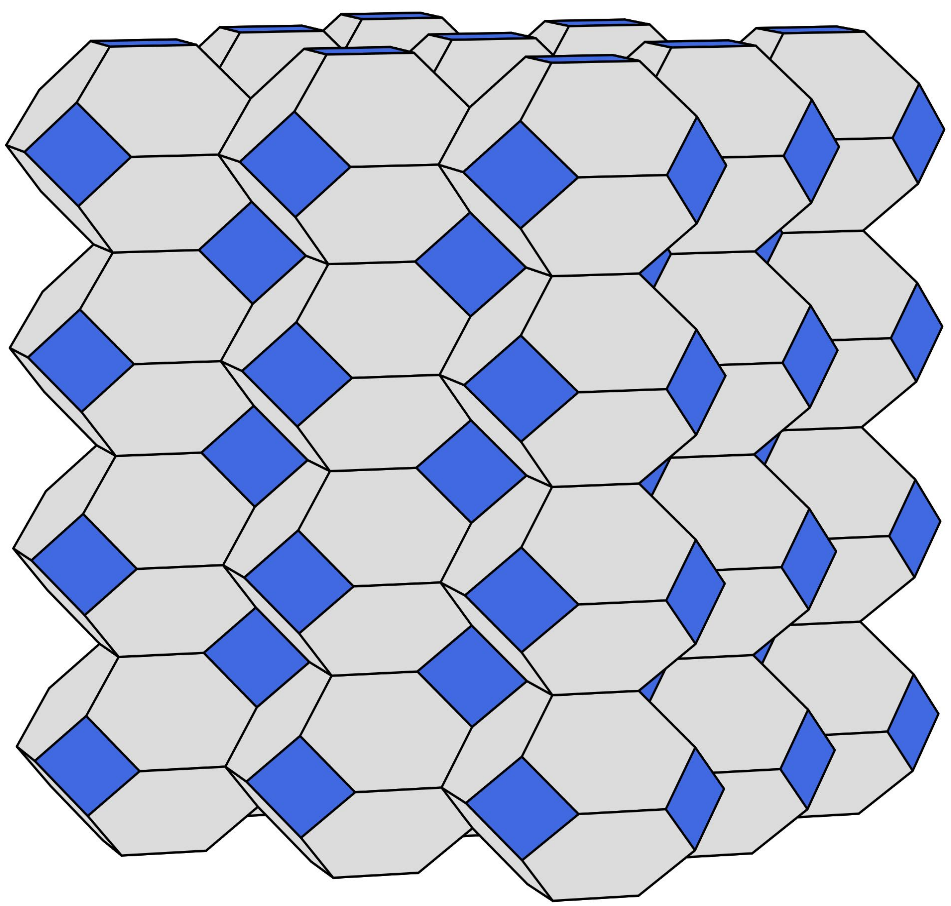

**Stochastic:**

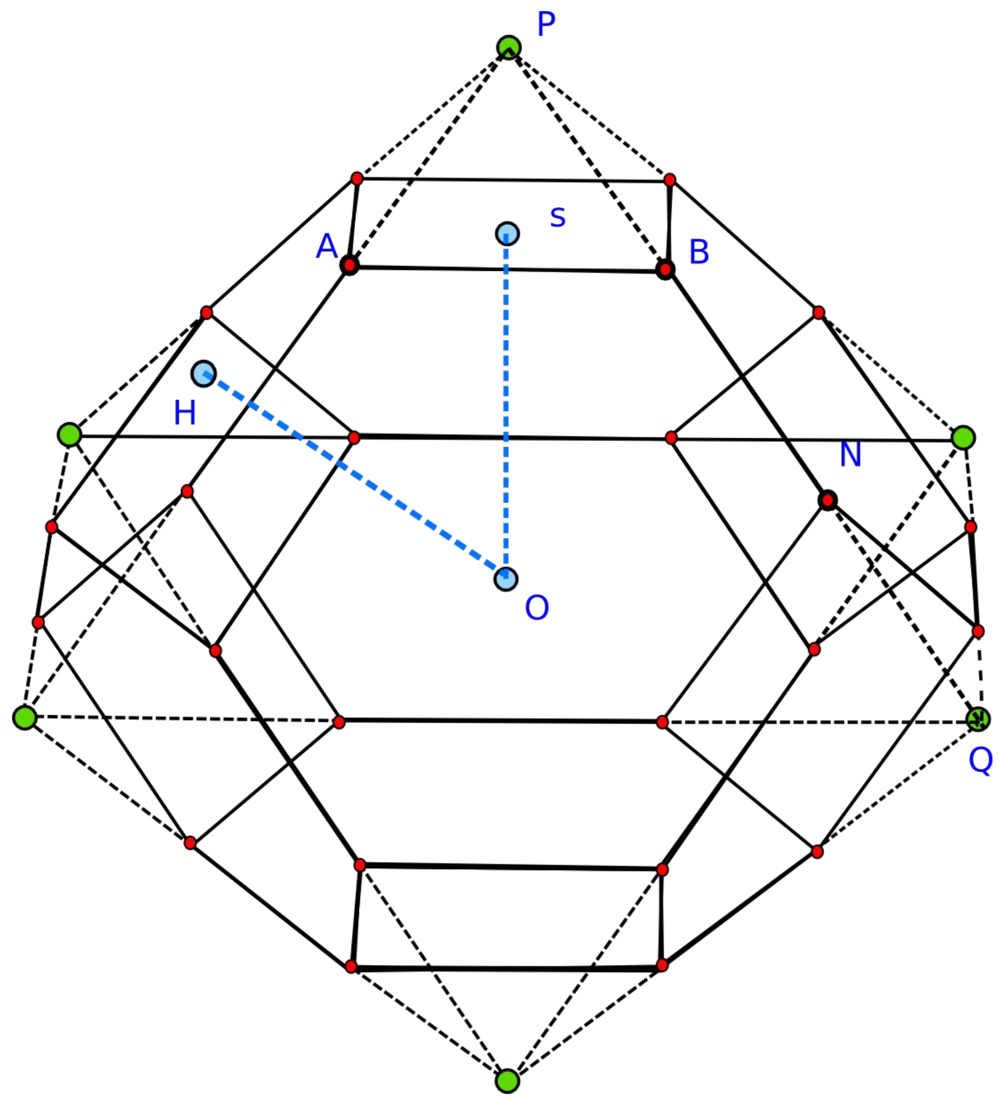
Start with empty bank and build it up from random proposals, for each proposal calculate fitting factor

$$\mathrm{FF} = \max_{\lambda \in B_0} \mathcal{M}(\lambda, \lambda - \lambda^{\mathrm{prop}})$$

# The three dimensional sphere packing

What is the optimal way to fill a three dimensional space with cells of equal volume to achieve the minimum interfacial area?
--1887 Lord Kelvin



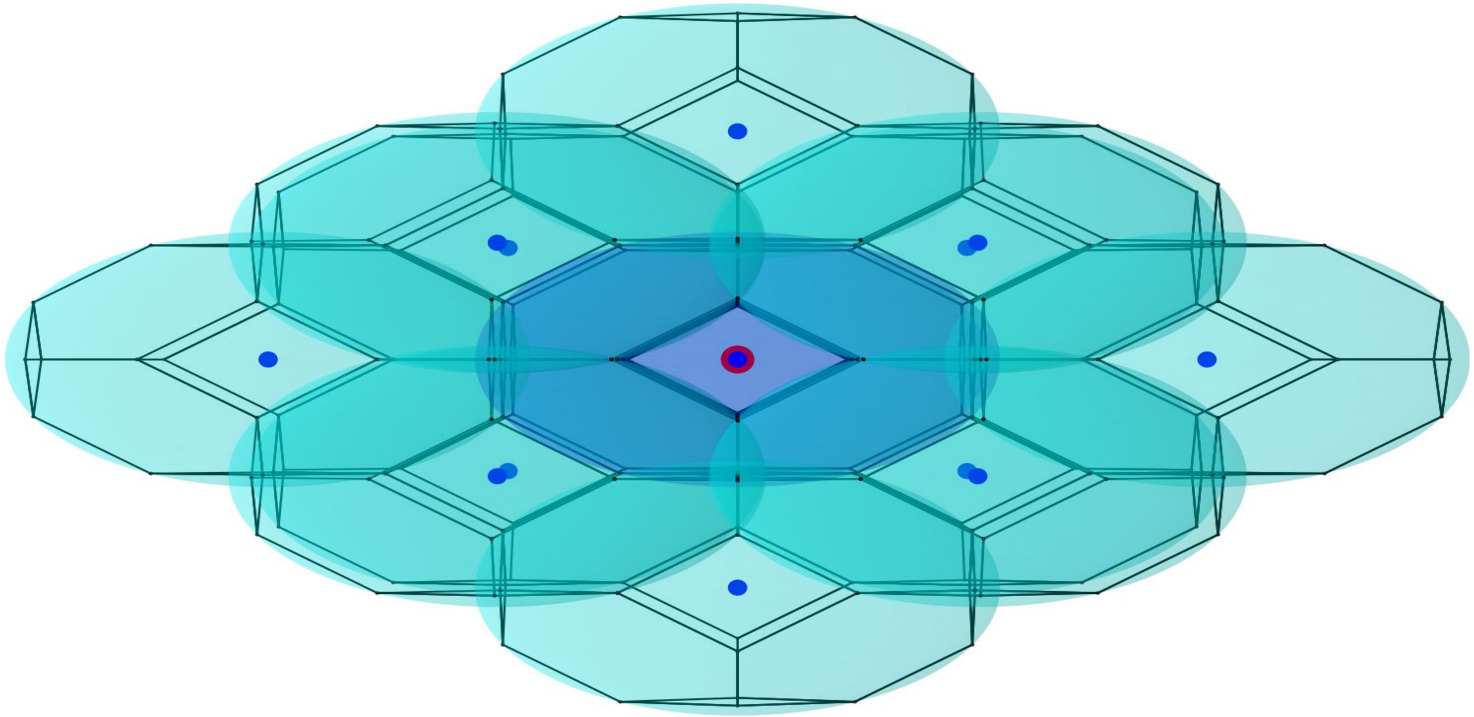


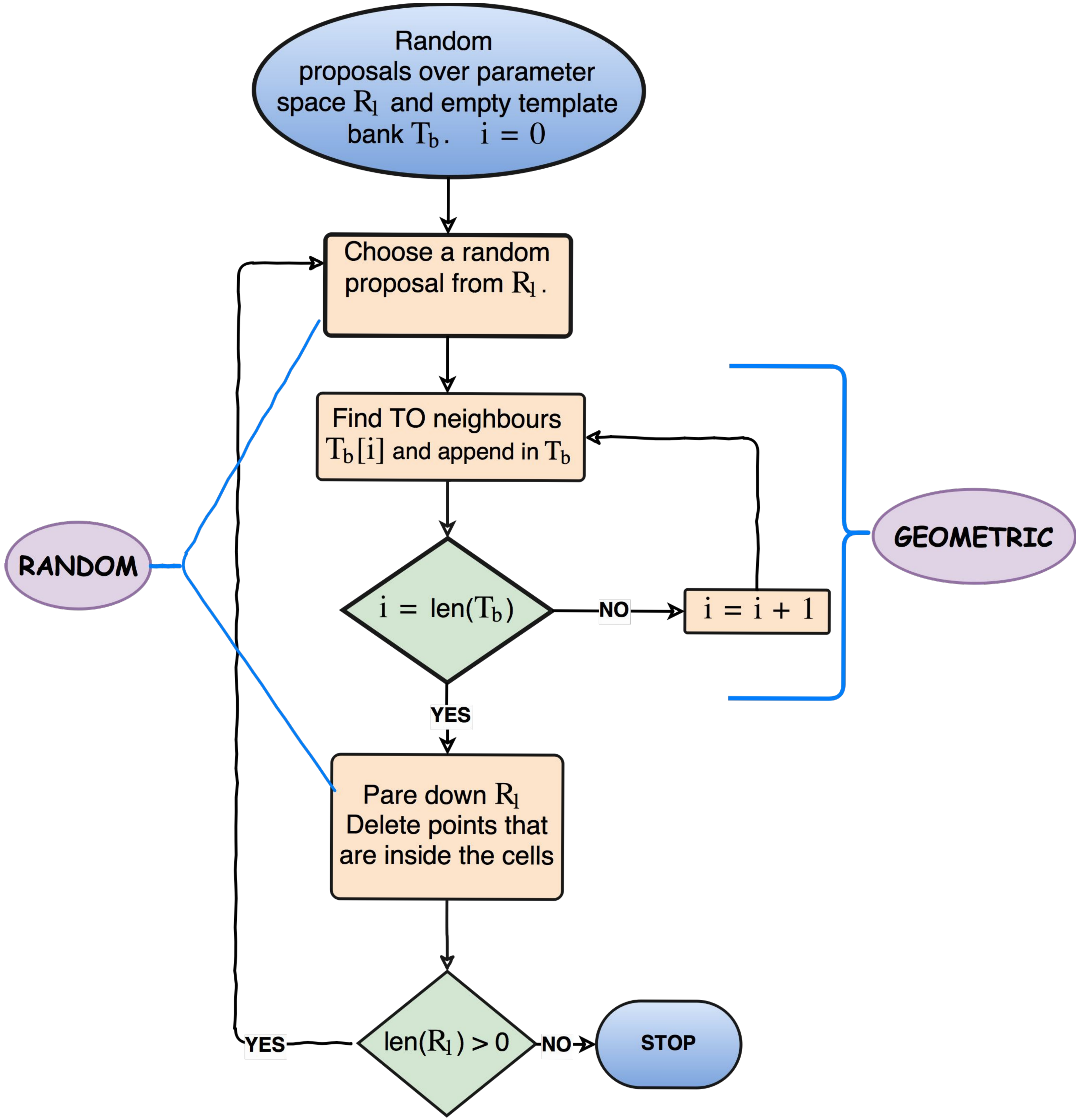

Truncated octahedrons:
Dirichlet-Voronoi polytopes of the body-centred $A_3^*$ lattice

# Packing in curved space

Calculate ellipsoid neighborhood position using rotation and scaling matrix from the metric

$$\bar{N}_i^p = \sum_{j,k=1}^{3} \mathcal{R}_{ij}^T \mathcal{S}_{jk} N_k^p$$



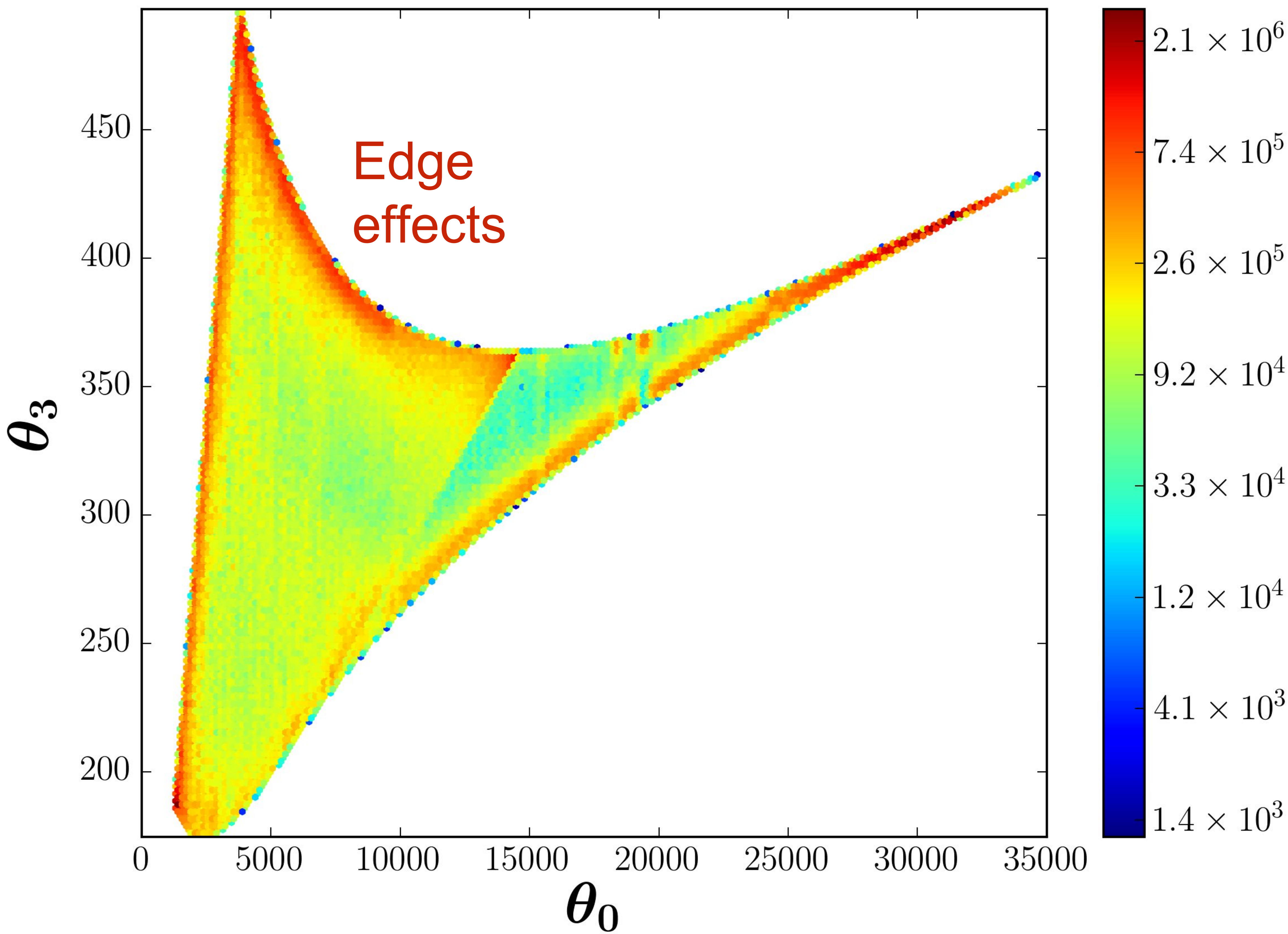


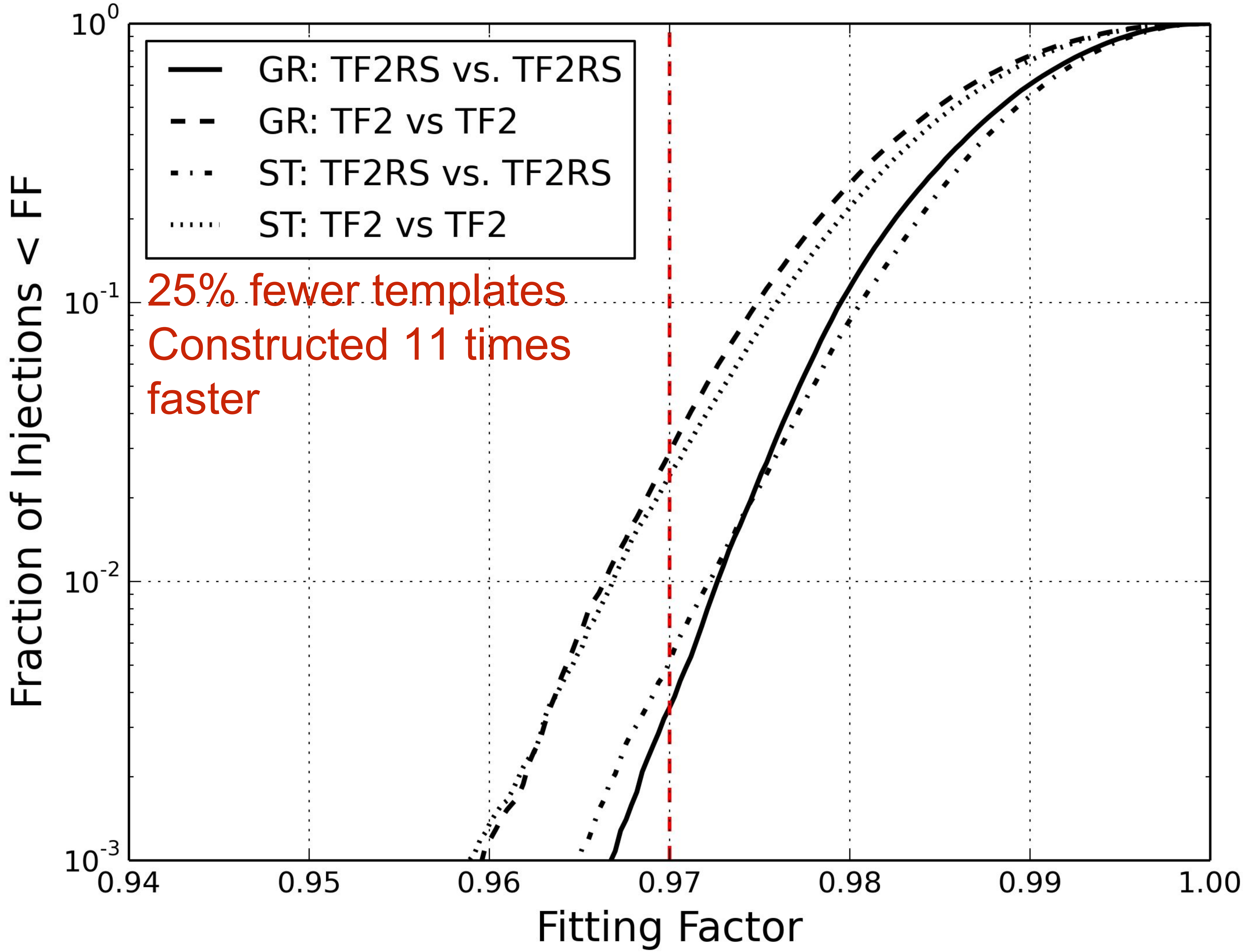
Random proposals over parameter space $R_l$ and empty template bank $T_b$. $i = 0$
Choose a random proposal from $R_l$.
Find TO neighbours $T_b[i]$ and append in $T_b$
$i =$ len($T_b$)
NO
$i = i + 1$
YES
Pare down $R_l$ Delete points that are inside the cells
len($R_l$) > 0
YES
NO
STOP
RANDOM
GEOMETRIC

# Parameter Space

| Bank Parameter | Set-I | Set-II |
|---|---|---|
| Waveform model | TaylorF2RedSpin | TaylorF2RedSpin |
| Noise model | aLIGOZeroDetHighPower | aLIGOZeroDetHighPower |
| Lower cut-off frequency $f_{low}$ | 20Hz | 30Hz |
| Higher cut-off frequency $f_{high}$ | 2048Hz | 1024Hz |
| Mass of first object $m_1$ | $[1, 20]\mathrm{M}_\odot$ | $[3, 15]\mathrm{M}_\odot$ |
| Mass of second object $m_2$ | $[1, 3]\mathrm{M}_\odot$ | $[1, 3]\mathrm{M}_\odot$ |
| Spin of first object $\chi_1$ | $[-0.98, 0.98]$ | $[-0.6, 0.6]$ |
| Spin of second object $\chi_2$ | $[-0.4, 0.4]$ | $[-0.05, 0.05]$ |
| Size of $\mathcal{R}_\ell$ | $[1 - 8] \times 10^7$ | $1 \times 10^7$ |
| Minimal Match $\mathcal{M}_{\min}$ | 0.97 | 0.97 |

# Template density normalized by $\sqrt{|g_{ij}|}$




GR: TF2RS vs. TF2RS
GR: TF2 vs TF2
ST: TF2RS vs. TF2RS
ST: TF2 vs TF2
25% fewer templates
Constructed 11 times
faster
Fraction of Injections < FF
Fitting Factor
10^0
10^-1
10^-2
10^-3
0.94
0.95
0.96
0.97
0.98
0.99
1.00

# Final results for various banks

| Bank Parameters | Placement Algorithm | Size of $\mathcal{R}_\ell$ | Bank Size | Execution Time (min) | Comments |
|---|---|---|---|---|---|
| Set-I of Table I | Geometric-Random | $1 \times 10^7$ | $694,422$ | 375 | 25% fewer templates $\times(8-10)$ faster |
| | | $2 \times 10^7$ | $749,705$ | 482 | |
| | | $3 \times 10^7$ | $777,113$ | 616 | |
| | | $4 \times 10^7$ | $798,269$ | 885 | |
| | | $5 \times 10^7$ | $812,570$ | 990 | |
| | | $6 \times 10^7$ | $824,541$ | 1191 | |
| | | $8 \times 10^7$ | $843,177$ | 1712 | |
| | Vanilla-Stochastic | $--$ | $939,787$ | 3666 | |
| Set-II of Table I | Geometric-Random | $1 \times 10^7$ | $107,547$ | 69 | 25% fewer templates $\times 11$ faster |
| | Vanilla-Stochastic | $--$ | $134,563$ | 762 | |
| | $A_3^*$ seeded Stochastic | $--$ | $128,185$ | $--$ | |

# Future Research

- Implementation in higher dimension.
- Construction of template bank using IMR signal model.