Quantum mechanics and the geometry of spacetime

Juan Maldacena

Institute for Advanced Study

Geometry of space → Euclidean

Non-Euclidean geometry → e.g. Hyperbolic space.

Euclidean+ time → Minkowski space

Metric of our spacetime

General relativity or Geometrodynamics

Two stunning predictions

- Black holes
- Expanding universe

"Your math is great, but your physics is dismal" (Einstein to LeMaitre)

Both involve drastic stretching of space and/or time

Quantum Mechanics and spacetime

- General relativity → is a classical field theory
- We should quantize it
- It is hard to change the shape of spacetime
- For most situations → quantum fields in a fixed geometry is a good approximation
- Even this simple perturbative quantization has very interesting consequences.
- Big Problem at the Big Bang : need a full theory of quantum gravity.

Predictions of a Perturbative quantization

Two surprising predictions

- Black holes have a temperature

$$T = \hbar / r_H$$

- An accelerating universe also has a temperature

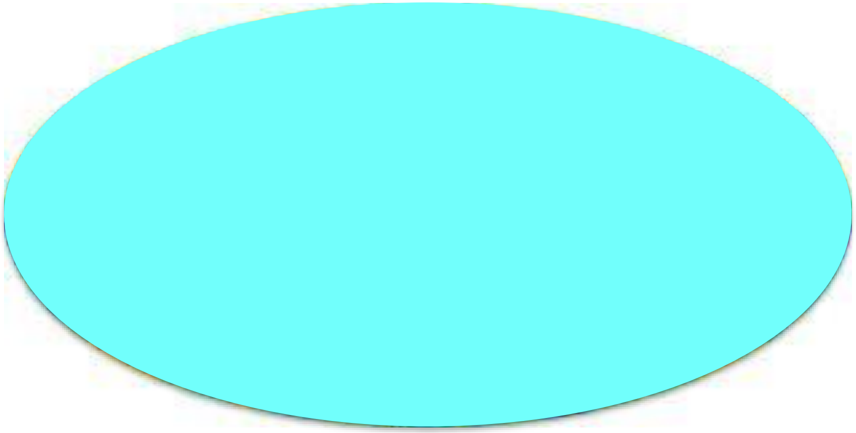
$$T = \hbar H = \frac{\hbar}{R_H}$$

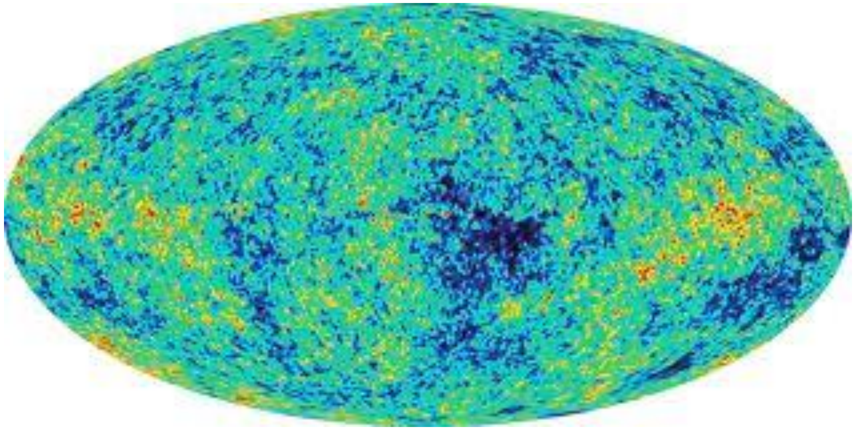
Very relevant for us!

Inflation

Starobinski, Mukhanov, …
Guth, Linde,
Albrecht, Steinhardt, …

- Period of expansion with almost constant acceleration.
- Produces a large homogeneous universe





Quantum mechanics is crucial for understanding the large scale geometry of the universe.

This was the first instance of a connection between quantum mechanics and the geometry of spacetime.

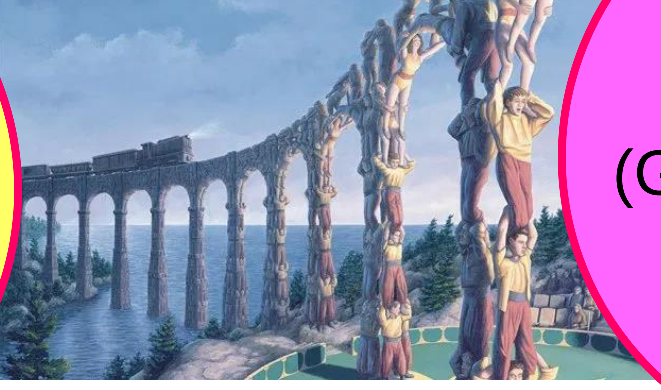
We will now discuss theoretical developments suggesting a stronger connection

Gauge/Gravity Duality

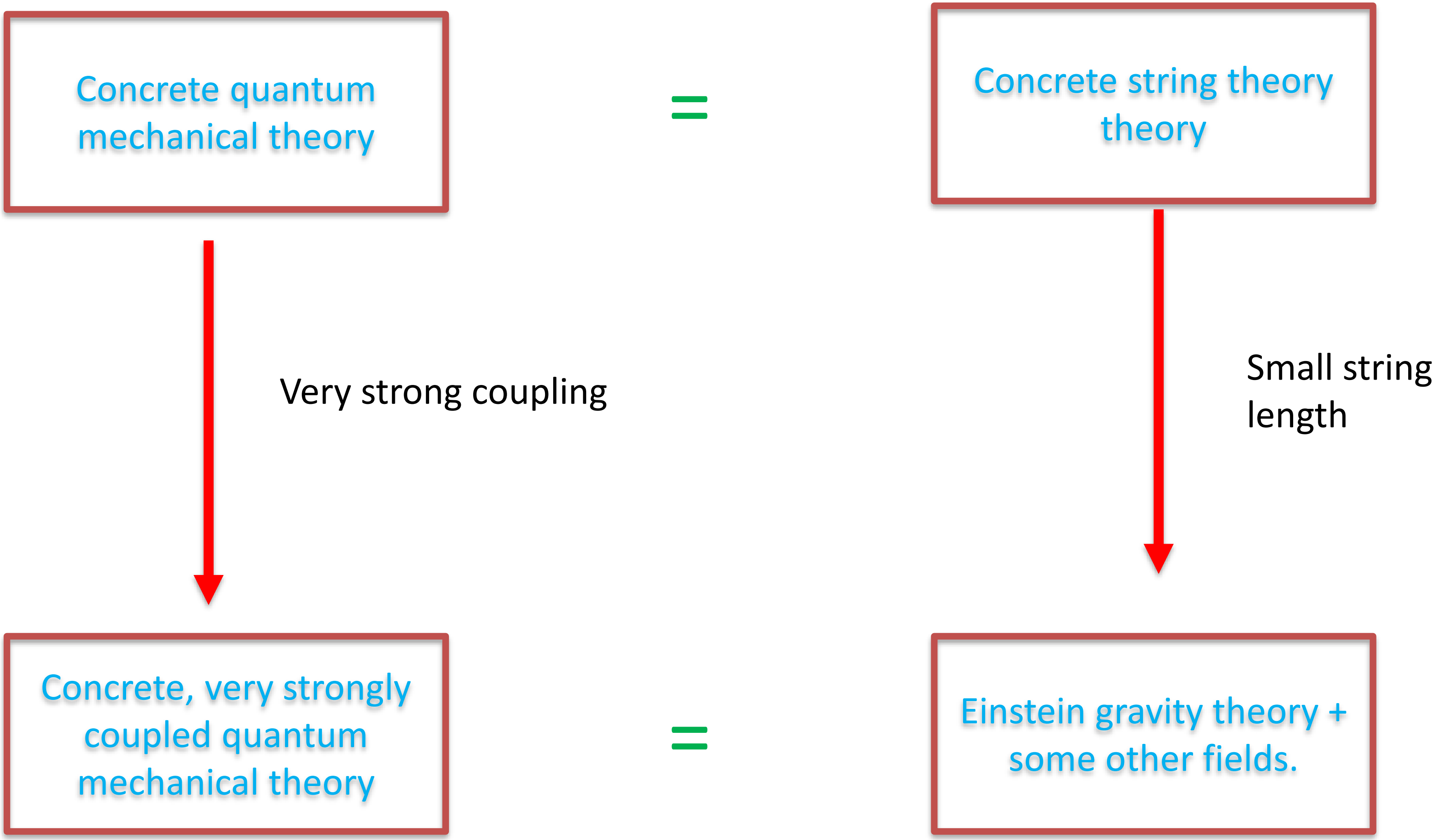
(or gauge/string duality, AdS/CFT, holography)

Theories of quantum
interacting particles

Quantum Field
Theory



Dynamical
Space-time
(General relativity)
string theory


Concrete quantum mechanical theory
=
Concrete string theory theory
Very strong coupling
Small string length
Concrete, very strongly coupled quantum mechanical theory
=
Einstein gravity theory + some other fields.

Example: Anharmonic oscillators

Start with harmonic oscillators and Majorana fermions

$$L = \int dt \dot{X}^2 - \omega^2 X^2 + i\psi\partial_t\psi + i\omega\psi\psi$$

Add interactions

$$L = \int dt \dot{X}^2 - \omega^2 X^2 + i\psi\partial_t\psi + i\omega\psi\psi + g\psi X\psi + g^2 X^4 + g\omega X^3$$

de Wit, Hoppe, Nicolai

$$X \to X_a^I$$

I = 1, ..., 9. SO(9) symmetry

Adjoint of U(N) , or N^2 components

Used by
Banks, Fischler,
Shenker, Susskind

All interactions fixed by U(N) symmetry and supersymmetry.

Effective coupling constant:

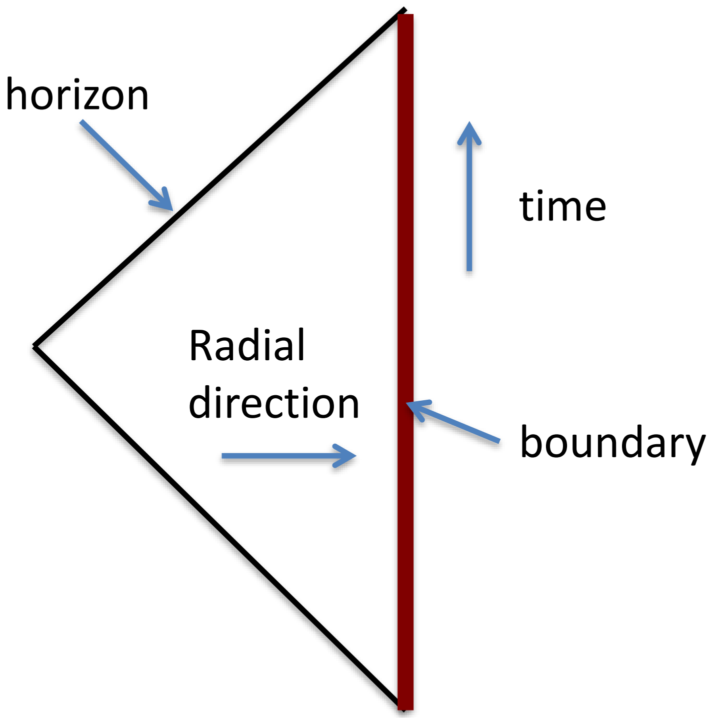
$$\frac{g^2 N}{\omega^3}$$

Consider very strong coupling and finite temperature.

$$\frac{g^2 N}{\omega^3} \gg 1 \ , \qquad \underbrace{\frac{g^2 N}{T^3} \gg 1 \ ,}_{\text{System is strongly coupled at the thermal scale}} \qquad T \gg \omega$$

The Geometry

Horowitz, Strominger
Itzhaki, J.M., Sonnenschein, Yankielowicz
Lin & JM
Costa, Greenspan, Penedones, Santos



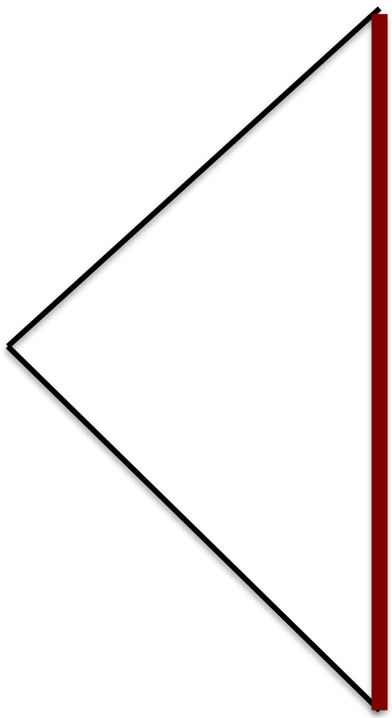
Finite temperature.

Is ten dimensional. Solution of 10d supergravity

Two dimensions x 8-sphere (SO(9) symmetry).

$$\frac{\text{(Curvature radius)}}{\text{(String length)}} \sim \text{Effective interaction strength}$$

The Geometry



Using the Hawking Bekenstein black hole entropy formula,

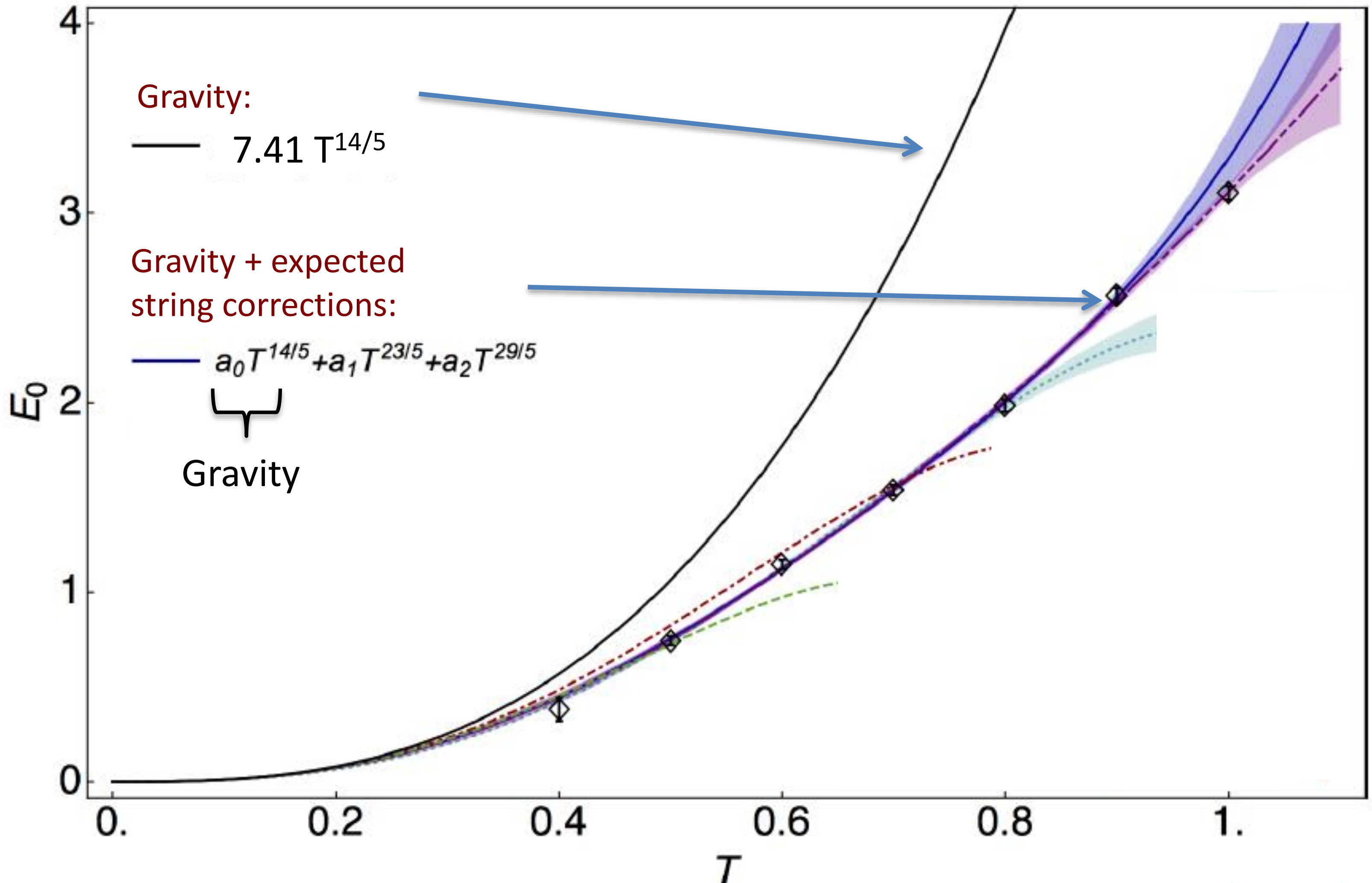
$$S = \frac{(\mathrm{Area})}{4G_N}$$

we can calculate the entropy and the free energy as a function of the temperature

- We can compare to a direct numerical computation in the quantum mechanical model…

Computation of the free energy in the quantum mechanical model

Berkowitz, Rinaldi, Hanada, Ishiki, Shimasaki, Vranas. 2016



Strong coupling

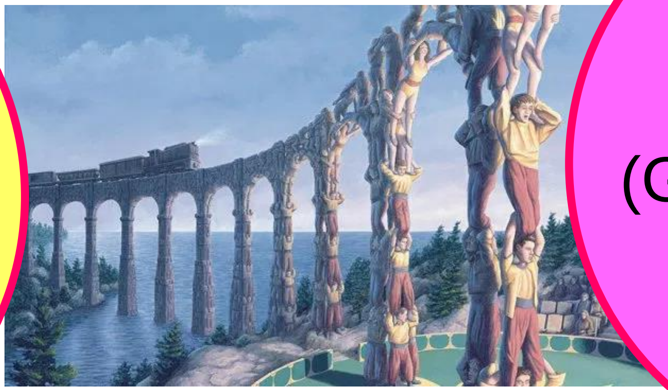
Weak coupling

(a_0 in agreement with gravity within the numerical error bars of about 7%)

Why do we have such relations ?

Theories of quantum interacting particles

Quantum Field Theory



Dynamical Space-time (General relativity) string theory

A very brief history

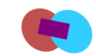
Strings in nature

Experimental observation:

Strings are produced in hadron collisions

(they decay to other hadrons)

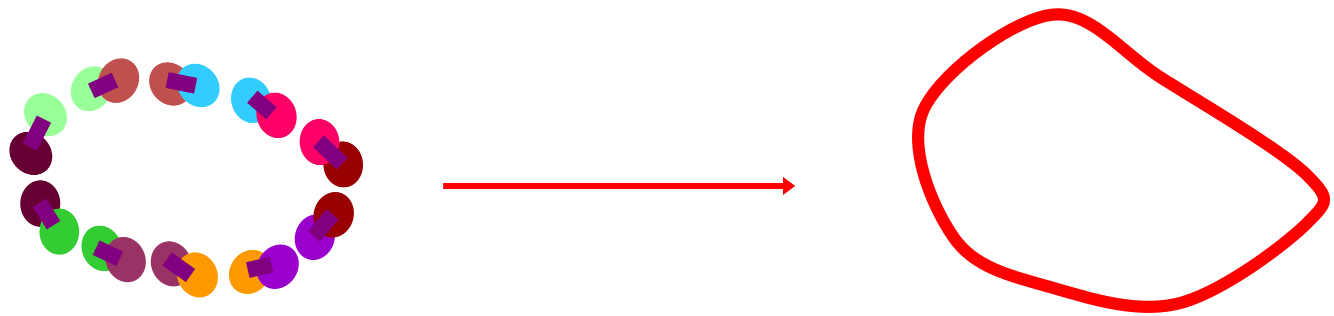
Large N gauge theories and strings

Quantum Chromodynamics: Gluon: color and anti-color 

Take N colors instead of 3, SU(3) → SU(N)

t' Hooft '74

Large N limit



g^2N = effective interaction strength.
Keep this fixed when N → infinity

String coupling ~ 1/N

Closed strings → glueballs

- We have experimental evidence for these strings.

- It was initially thought that these strings propagate in four dimensions.

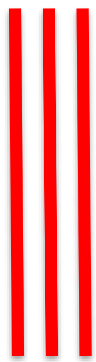
- They should move in (at least) one more dimension. Polyakov

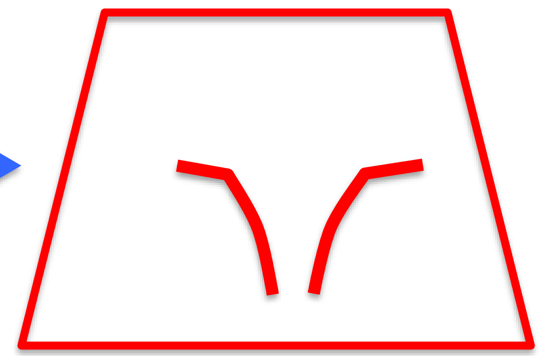
- We do not know the precise string background for large N QCD, ☹

- But we do know it for its maximally supersymmetric cousin, ☺

D-branes in 10d superstring theory

Polchinski





Horowitz
Strominger

Collection of N branes
with 3+1 dimensional world-volumes

Geometry of a black brane

Low energies

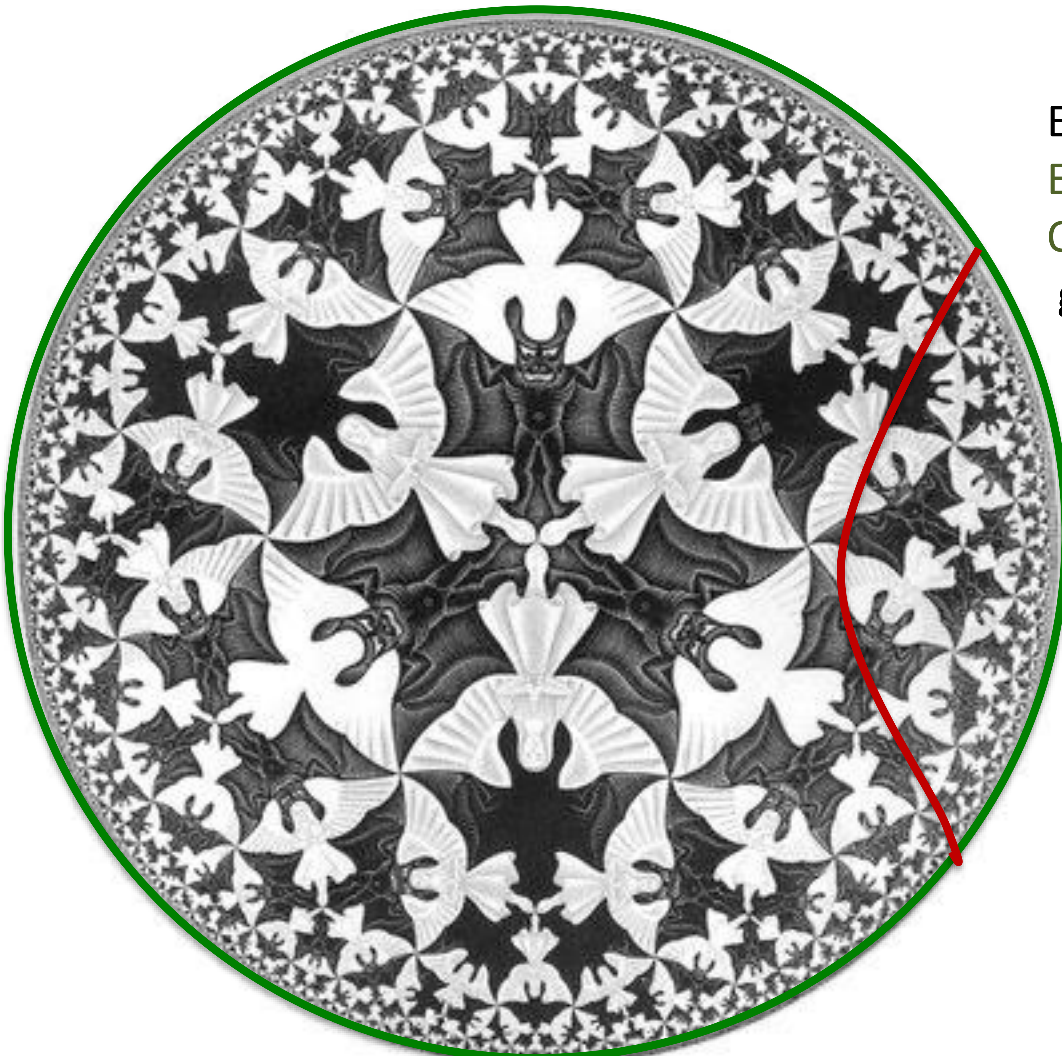
Gubser Klebanov

SU(N) Super Quantum Chromodynamics = string theory on $AdS_5 \times S^5$

JM 1997

- Probabilities in the quantum particle theory = computed in terms of propagation of particles in Anti-de-Sitter space.

Gubser, Klebanov, Polyakov, Witten



Example:
Energy fluctuations in the boundary QFT = quantum fluctuations of gravity waves in the interior

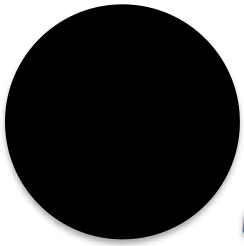
Duality

- Many examples.
- Many checks that are very impressive (specially at large N).

- Not proven.
- No general method to go from the QFT to gravity or viceversa.

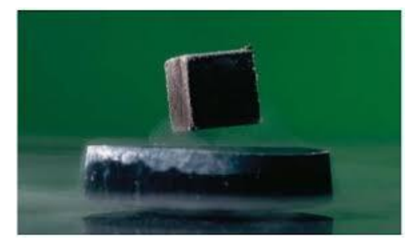
Holography
QFT = gravity

Black holes



Quark gluon plasma

Non trivial phases of
condensed matter physics



Quantum information theory,
Quantum error correction

“Strength in diversity”

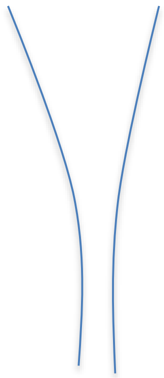
Discuss one example

Universal bound on quantum chaos

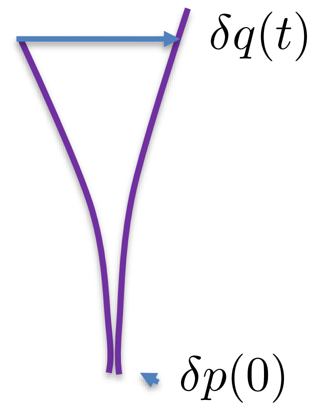
inspired by black holes

Shenker-Stanford
Kitaev

Signature of classical chaos $\rightarrow$ divergence of nearby trajectories



Classical



Liapunov exponent

$$\delta q(t) \propto e^{\lambda t}$$

$$\frac{\delta q(t)}{\delta p(0)} = \{q(0), q(t)\} \ , \qquad \{q(0), q(t)\}^2$$

Quantum

$$[Q(t), Q(0)]^2 \qquad [Q_i(t), Q_j(0)]^2$$

General quantum many body system:

$$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{N} e^{\lambda t}$$

Quantum General: $$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{N} e^{\lambda t}$$

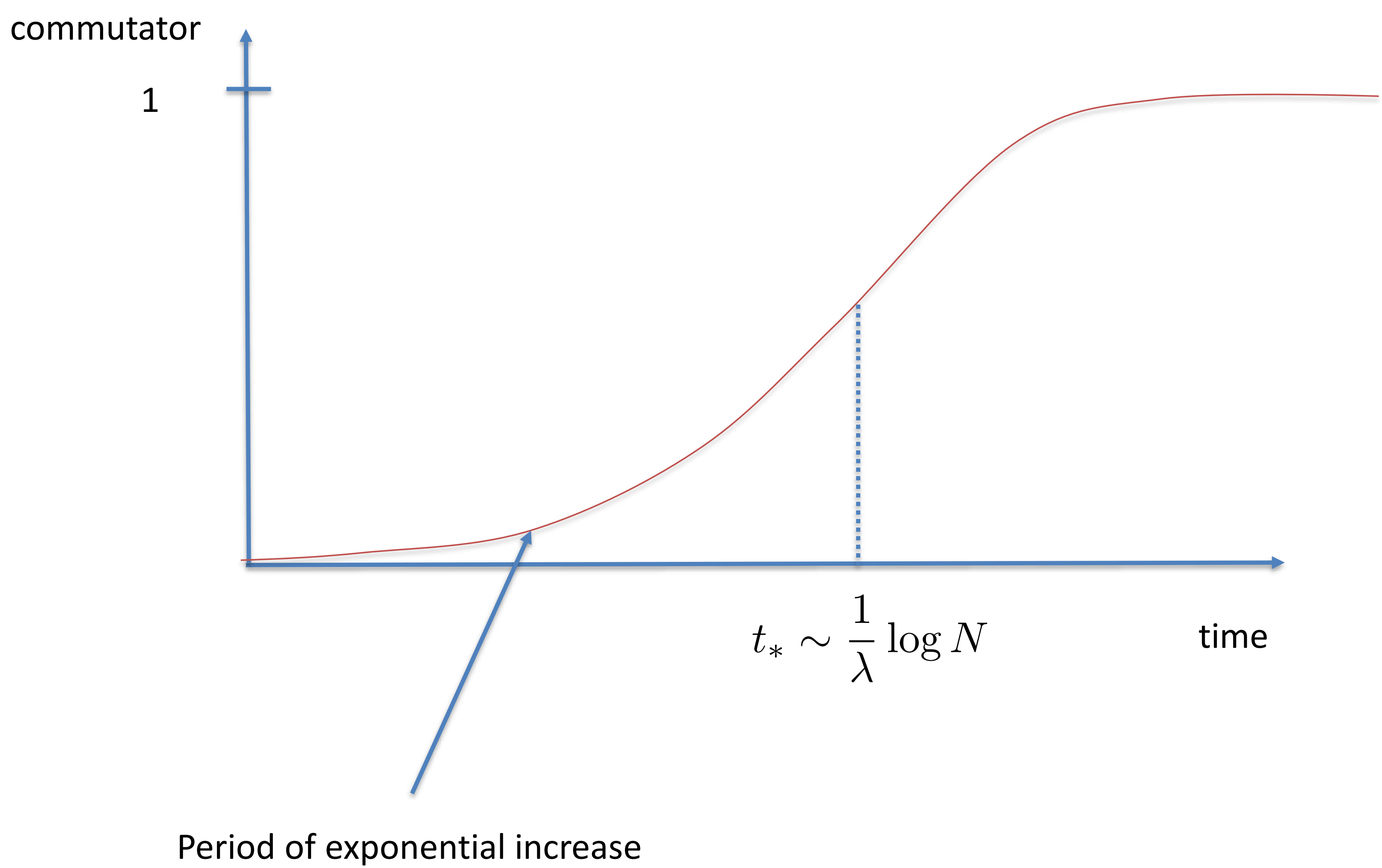
W, V are two ``simple'' (initially commuting) observables of a large many body system, with N components.

Imagine we have large entropy system, N>>1.

This is the definition of the quantum Liapunov exponent

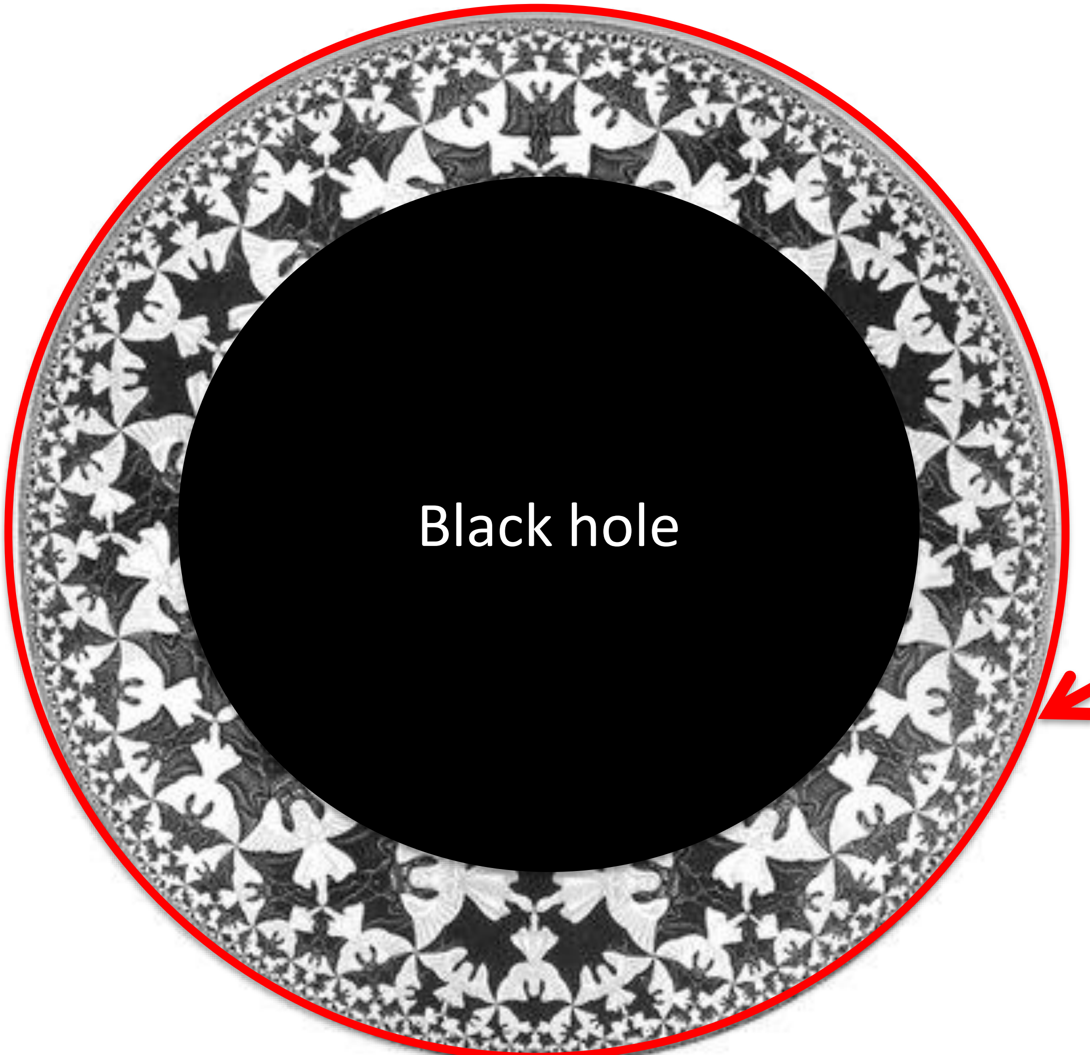
It is defined by its initial increase.

At very long times it saturates to a quantity of order one.


commutator
1
$t_* \sim \frac{1}{\lambda} \log N$
time
Period of exponential increase

Use now holography and black holes

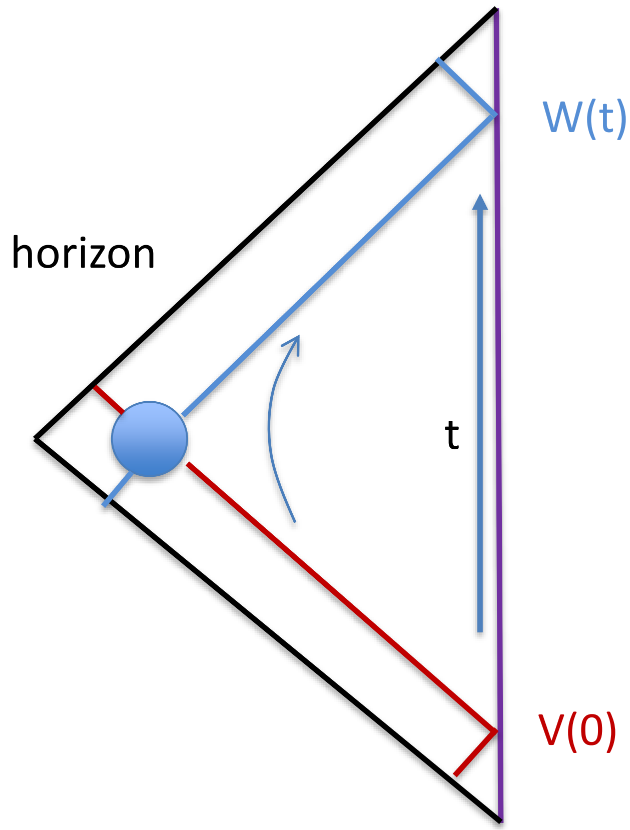
Black holes in a gravity box



Hot fluid made out of
very strongly
Interacting particles.

For quantum systems that have a gravity dual

Shenker Stanford
Kitaev



Commutator → involves the scattering amplitude between these two excitations.

Large t → large boost between the two particles.

Leading order → graviton exchange

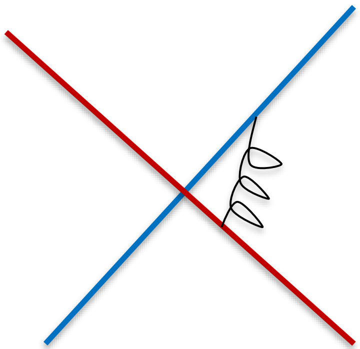
Gravitational interaction has spin 2, Shapiro time delay proportional to energy.

Energy goes as e^t

$$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{N} e^{\lambda t}$$

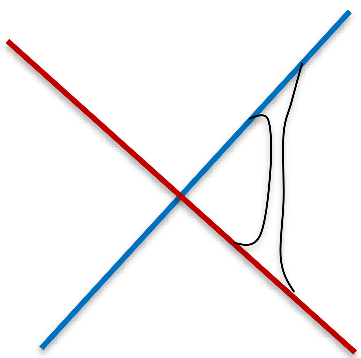
$$\lambda = \frac{2\pi}{\beta} = 2\pi T$$

Can it be different ?



Graviton → phase shift : $\delta(s) \sim G_N s \longrightarrow \lambda = \frac{2\pi}{\beta}$

High energy limit , s>>t,   s >> 1



String → phase shift

Typical size of string
(of graviton in string theory)

$$\delta(s) \sim G_N s^{1+\alpha' t} \longrightarrow \lambda = \frac{2\pi}{\beta}(1 - \frac{l_s^2}{R^2})$$

s, t = Mandelstam invariants

Radius of curvature of black hole

Shenker Stanford

It can be less…

More ?

In flat space, a phase shift has to scale with a power of s less than one in order to have a causal theory

Maybe there is a bound…

Universal upper bound on chaos

$$\lambda \leq \frac{2\pi}{\beta} = 2\pi T = \frac{2\pi T}{\hbar}$$

Sekino Susskind

JM, Shenker, Stanford

Proof: Uses analyticity in Euclidean time, unitarity, and that simple observables thermalize.

For <u>any</u> large N quantum many body system

Let us go back to the

quantum mechanics / geometry relation

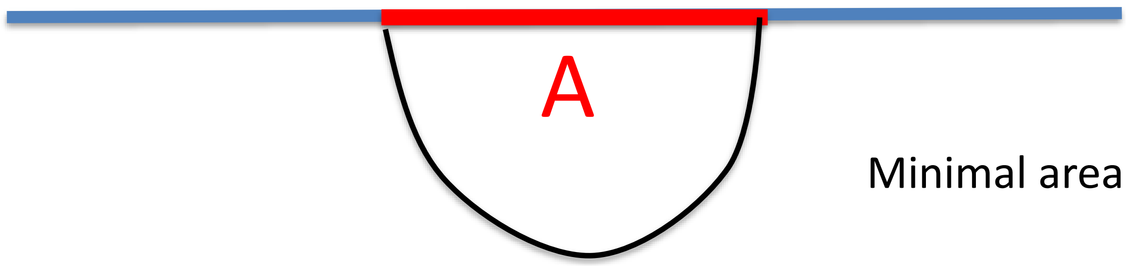
$$\underbrace{\text{Entanglement}}_{\substack{\text{Fundamental quantum} \\ \text{mechanical property}}} \text{ and geometry}$$

Entanglement and geometry in AdS/CFT

- The entanglement pattern present in the state of the boundary theory can translate into geometrical features of the interior.

- Spacetime is closely connected to the entanglement properties of the fundamental degrees of freedom.

Entropy of subregions
(entanglement entropy)



Minimal area surface in the bulk

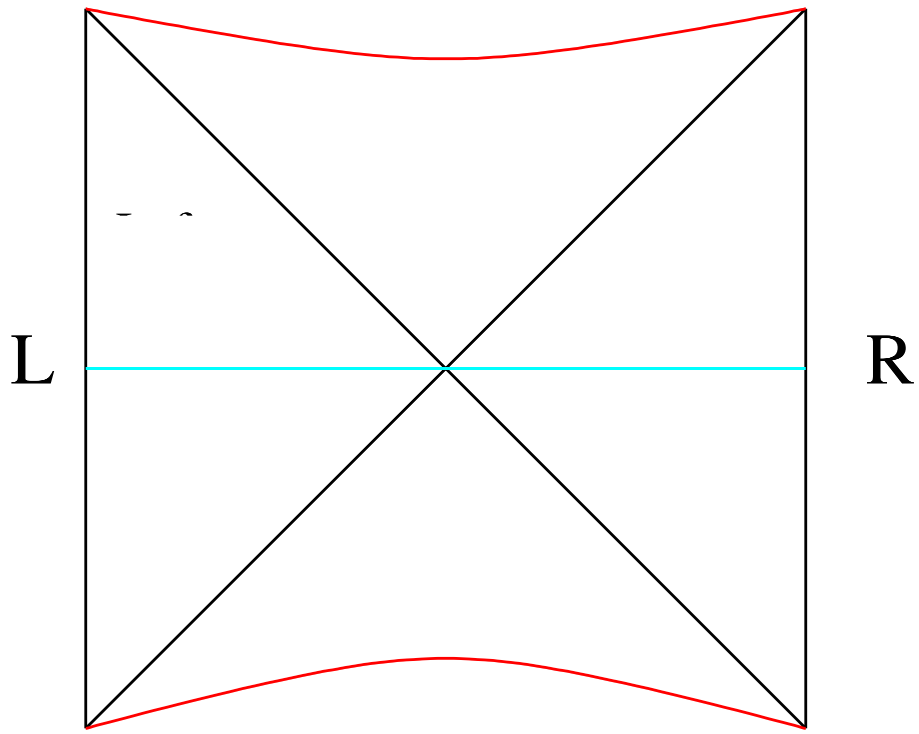
$$S_A = \frac{A_{\mathrm{minimal}}}{4G_N}$$

Ryu-Takayanagi
(Hubeny, Rangamani ..)

(Leading order in G_N expansion)

This a generalization of the Bekenstein-Hawking formula for black hole entropy

Two sided AdS black hole (wormhole)

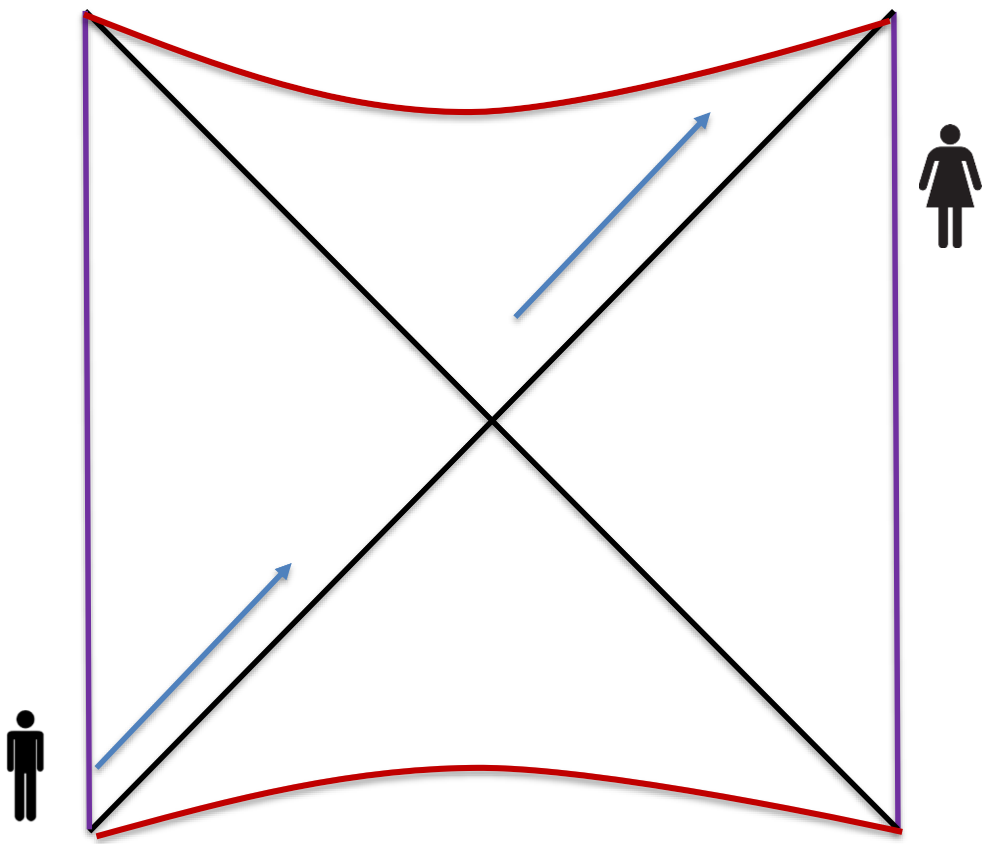


Entangled state in two <u>non-interacting</u> CFT's.

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

Israel
Balasubramanian, Kraus
Lawrence, Trivedi
JM , Susskind

True causal separation



If Romeo sends a signal , then
Juliet cannot receive it.

These wormholes are not traversable,
due to the integrated null energy condition

$$0 \leq \int dx^- T_{--}$$

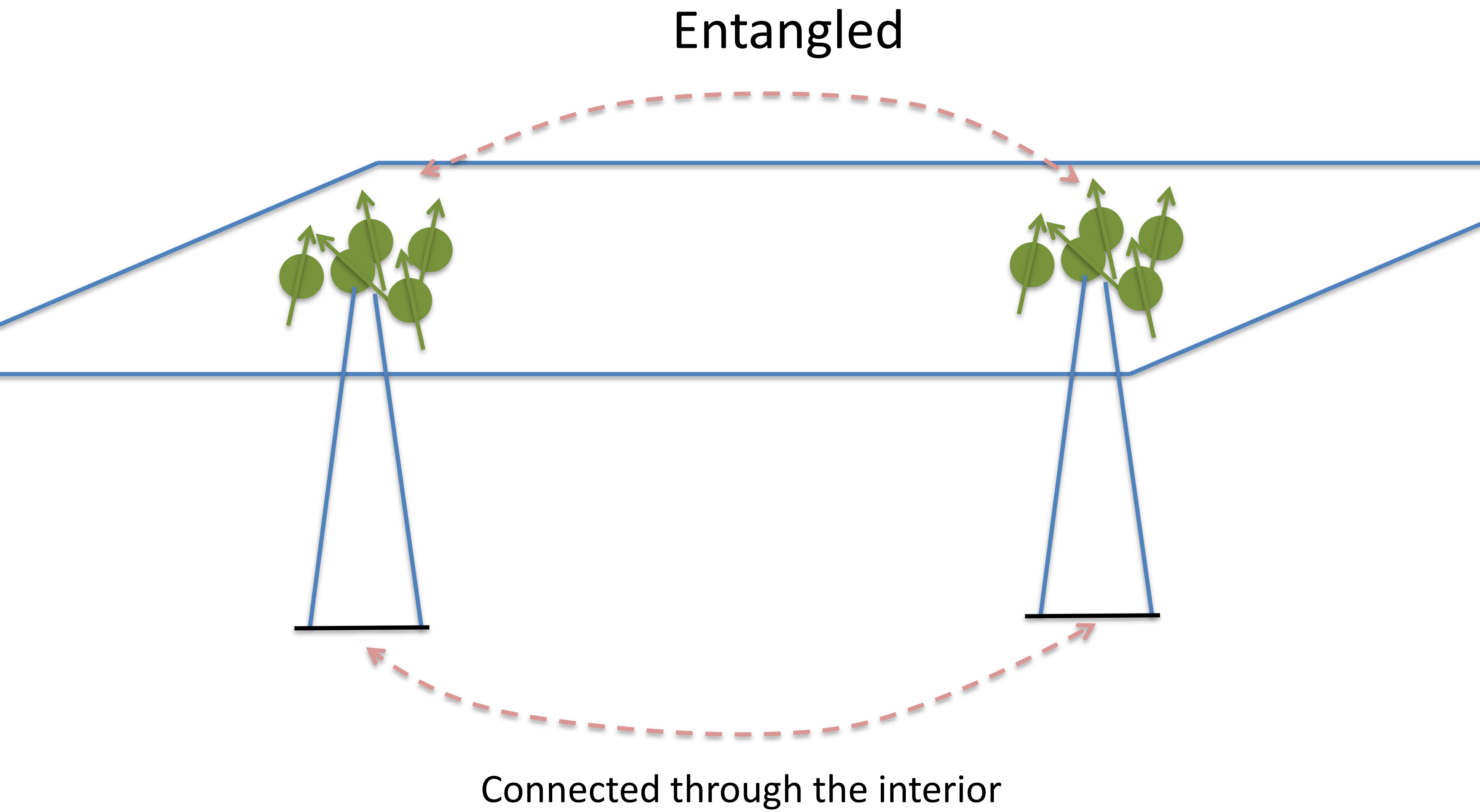
Balakrishnan, Faulkner, Khandker, Wang

Not good for science fiction.

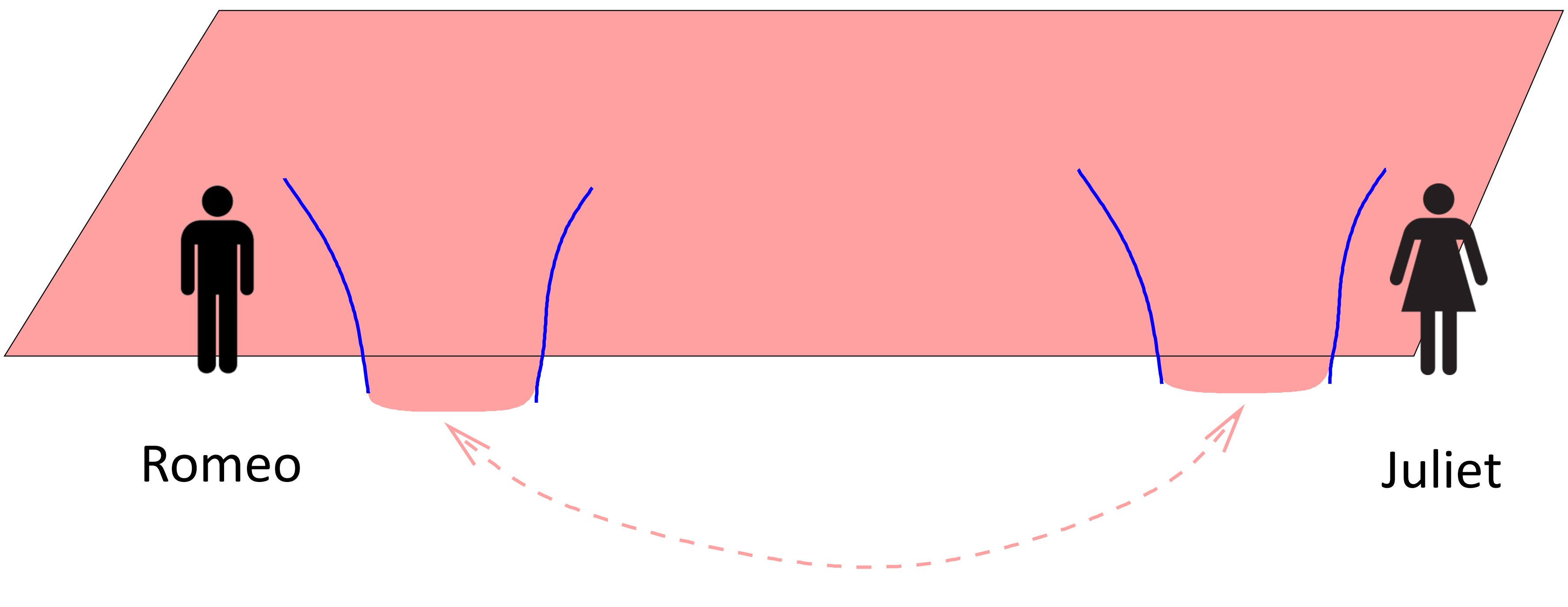
Good for science!

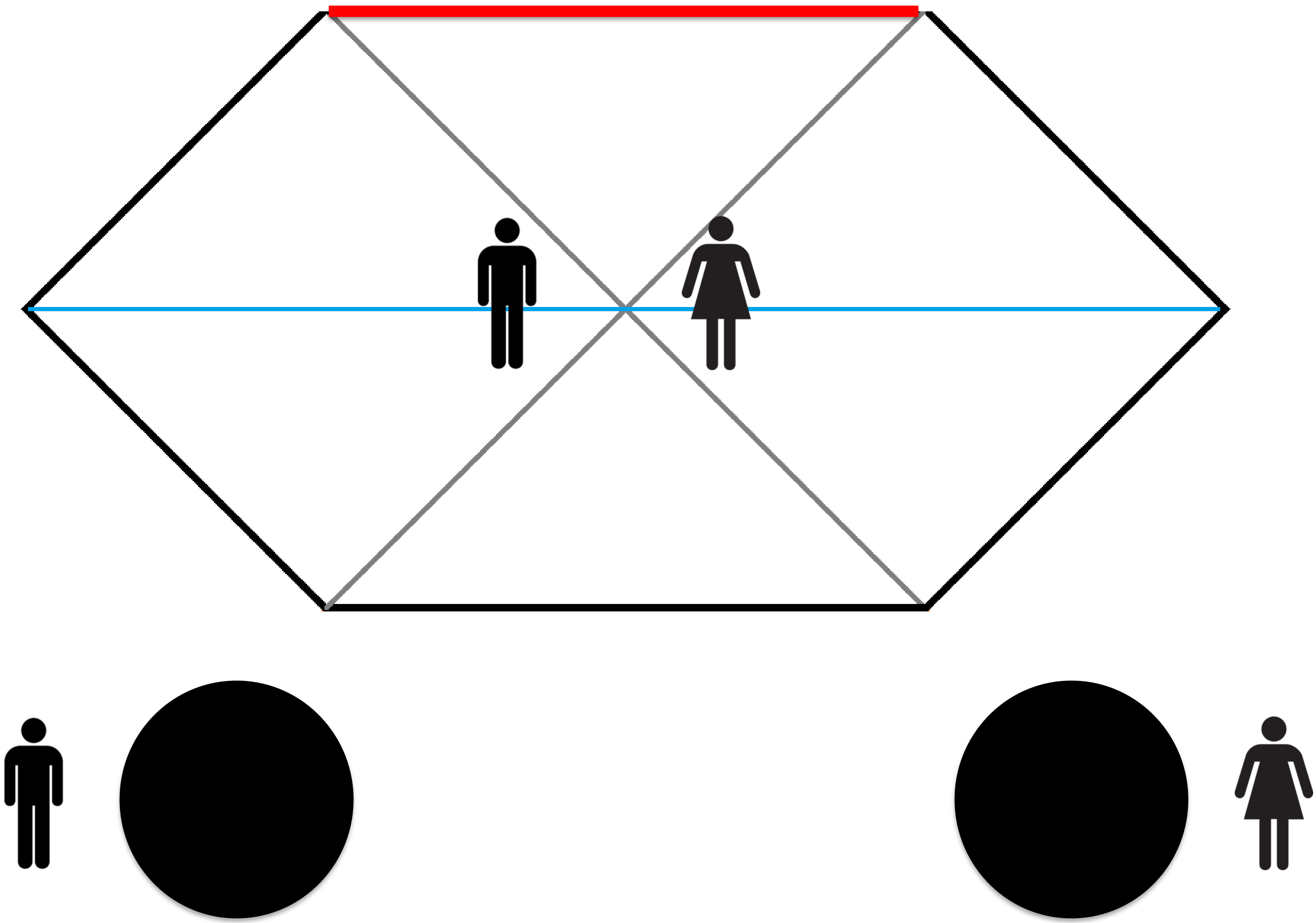
$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Lab systems



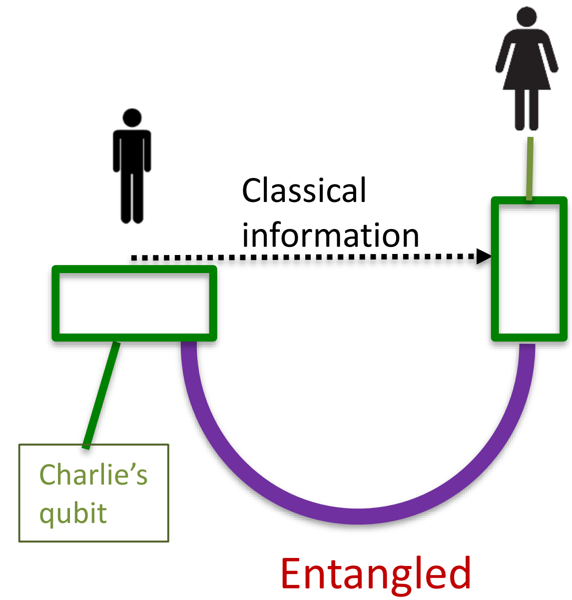
A forbidden meeting





Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters

Quantum teleportation



Romeo and Juliet share an entagled pair of qubits.

Charlie gives Romeo a qubit and he wants to send it to Alice.

Romeo does a joint measurement of Charlie's qubit and his qubit.

Sends the the result to Juliet as classical information

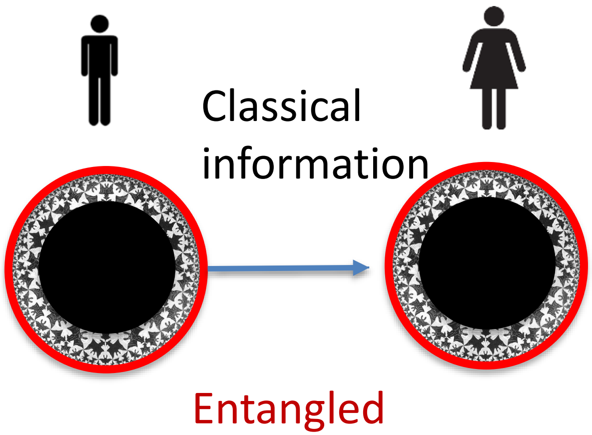
Juliet does an operation on his qubit that depends on Romeo's result.

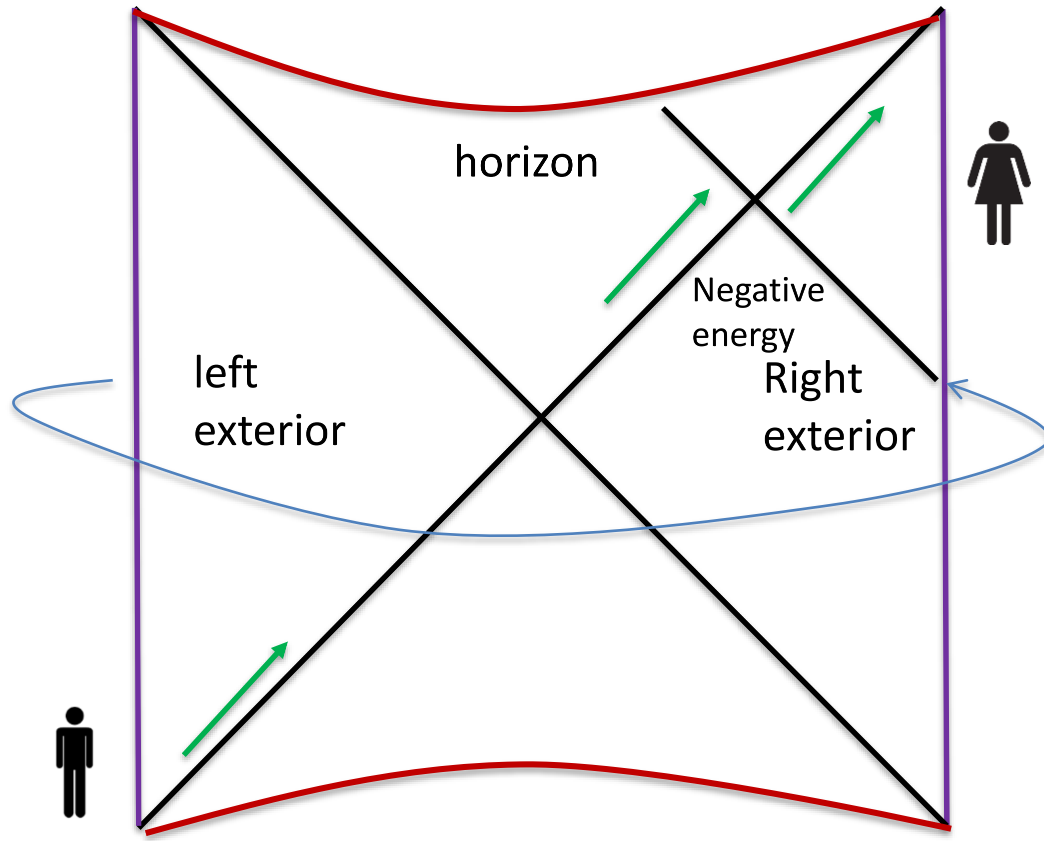
Juliet gets the qubit.

Resources needed to send a qubit: One entangled qubit and 2 bits of classical information.

How does the qubit travel ? Would you like to be teleported ?

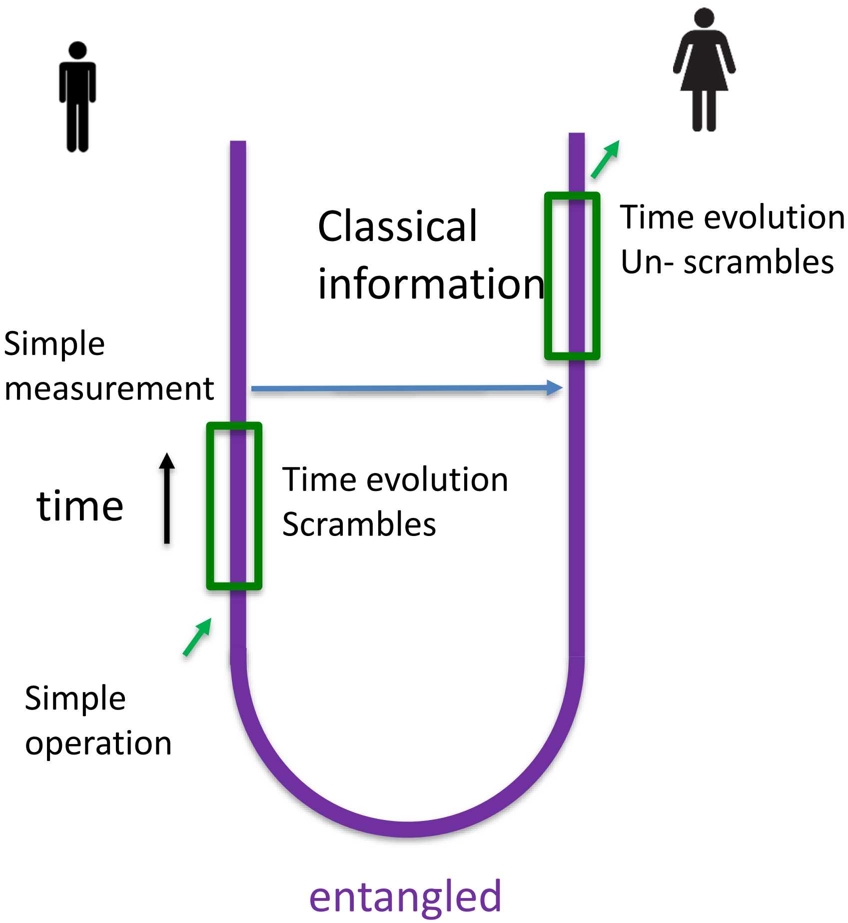
Teleportation through the wormhole

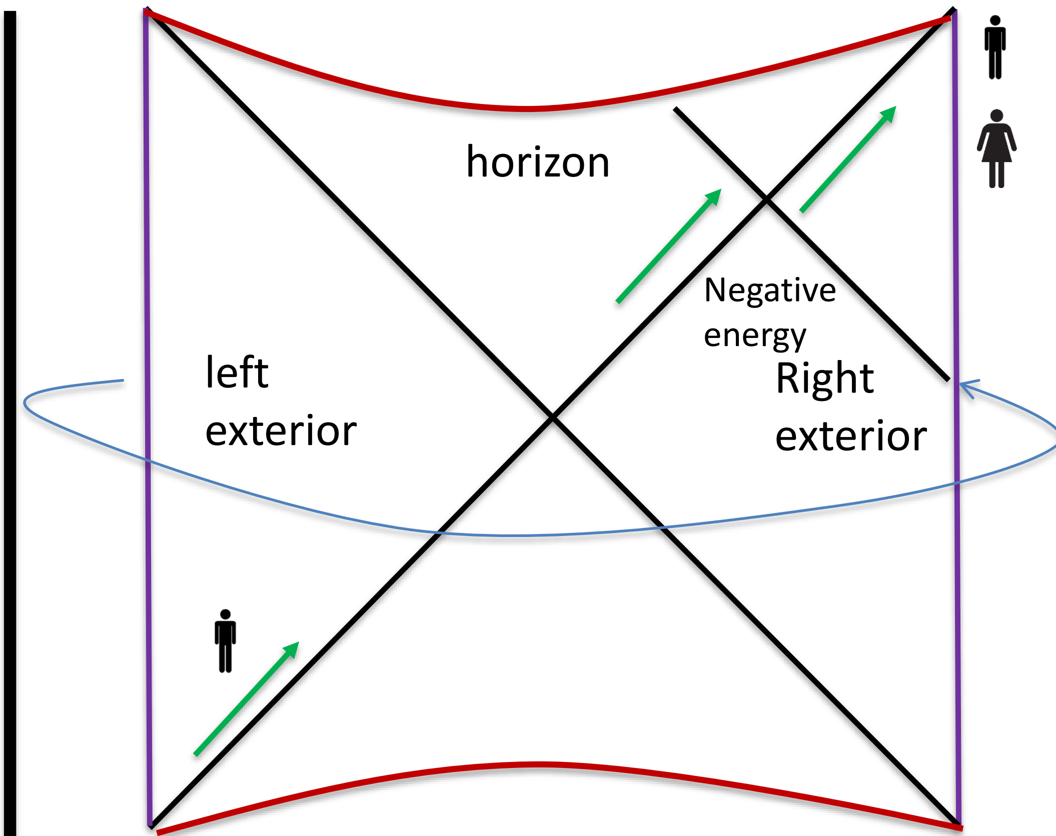




Gao, Jafferis, Wall

Teleportation through the wormhole





In two dimensional black holes.

Or if the measurement involves an s-wave (independent of the extra dimensions)

You do not feel anything as you are teleported !

You feel as if you are travelling through empty space.

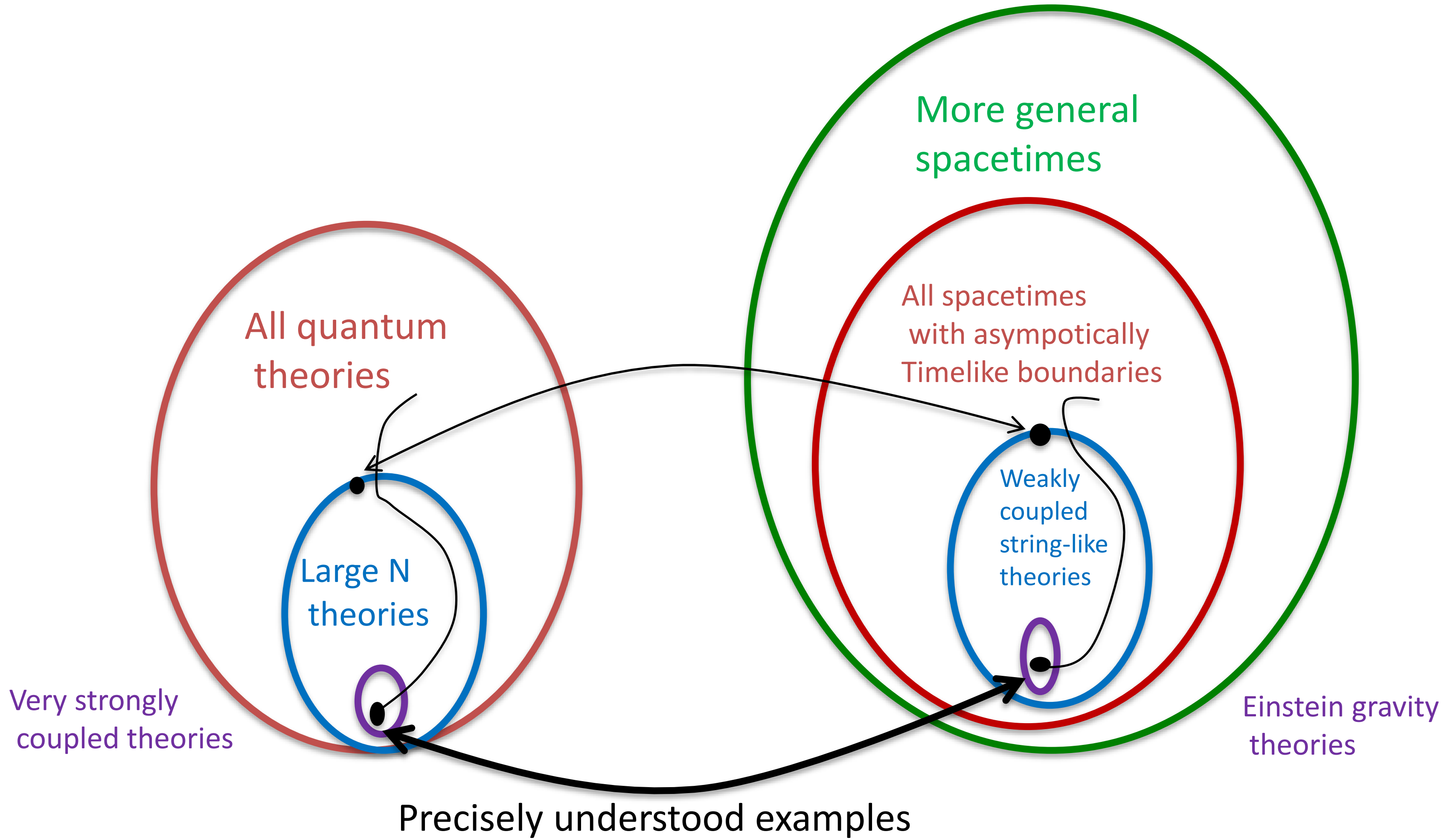
You are as comfortable as possible !

What is interesting is HOW the qubit is being moved from one side to the other.

Conclusions

- Quantum mechanics determines the large scale structure of the universe.
- Certain strongly interacting quantum systems have concrete gravity duals.
- Near horizon dynamics of black holes is related to chaotic dynamics of the quantum system.
- The structure of spacetime is intimately connected to patterns of entanglement.
- Entanglement is related to wormholes.
- Teleportation is related to travel through wormholes.

Challenges



More general spacetimes
All spacetimes with asympotically Timelike boundaries
All quantum theories
Weakly coupled string-like theories
Large N theories
Very strongly coupled theories
Einstein gravity theories
Precisely understood examples

Duality vs Emergence ?

- Is gravity just an emergent description at large N ?
- Or is it independently defined as a theory of quantum gravity ?

Singularities

- What ``happens'' at a black hole singularity or Big Bang singularity ?
- How do we describe the black hole interior ?