Higher Spin and Yangian

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AdS/CFT 20 ICTS, 2018/05/23

Reference

1. **Higher Spins and Yangian Symmetries**
   JHEP **1704**, 152 (2017), [arXiv:1702.05100]
   with *Matthias Gaberdiel, Rajesh Gopakumar, and Cheng Peng*

2. **Twisted sectors from plane partitions**
   JHEP **1609**, 138 (2016), [arXiv:1606.07070]
   with *Shouvik Datta, Matthias Gaberdiel, and Cheng Peng*

3. **The supersymmetric affine yangian**
   JHEP in press, [arXiv:1711.07449]
   with *Matthias Gaberdiel, Cheng Peng, and Hong Zhang*

4. **Twin plane partitions and $\mathcal{N}=2$ affine yangian**
   (to appear)
   with *Matthias Gaberdiel and Cheng Peng*

Motivations

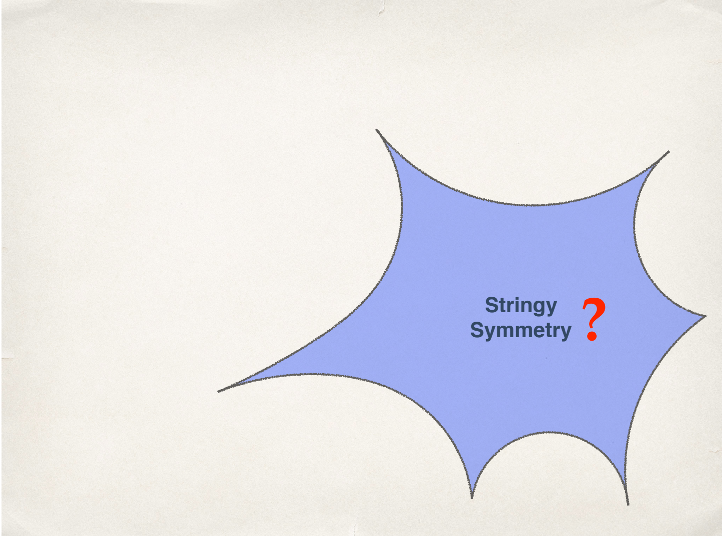
Motivation-1 (conceptual, vague)

Higher spin symmetry and integrability are both (large) symmetry structures of string theory. Relations? Unification?

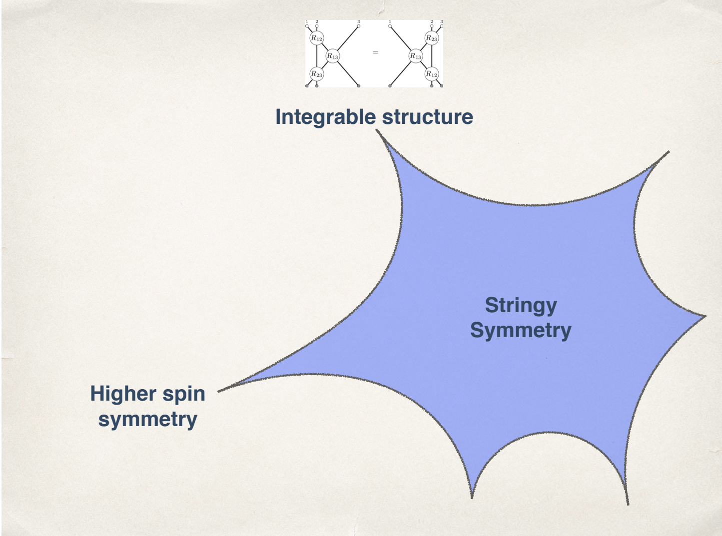
Motivation-2 (practical, concrete)

$\mathcal{W}_\infty$ is ubiquitous, but computationally unwieldy. Better formulation?

Motivation-1: What is the hidden stringy symmetry?



Different manifestation of stringy symmetry



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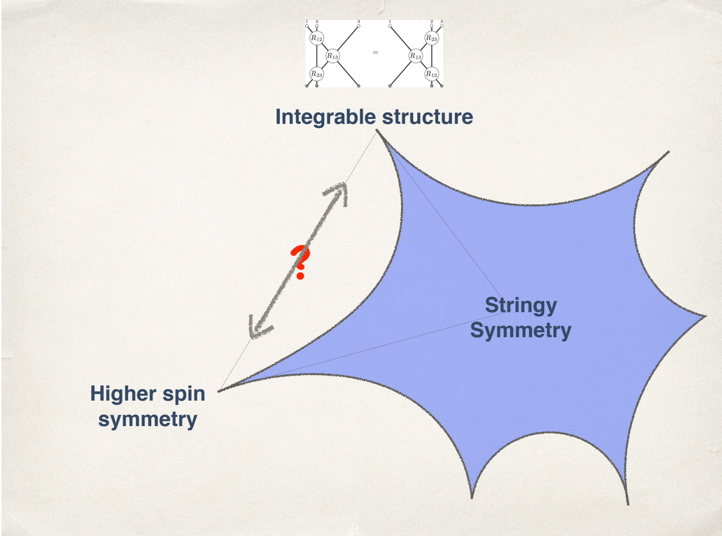

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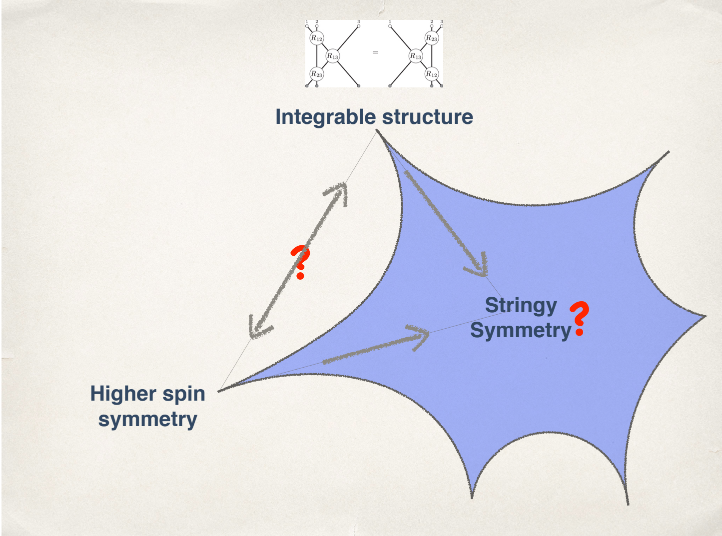
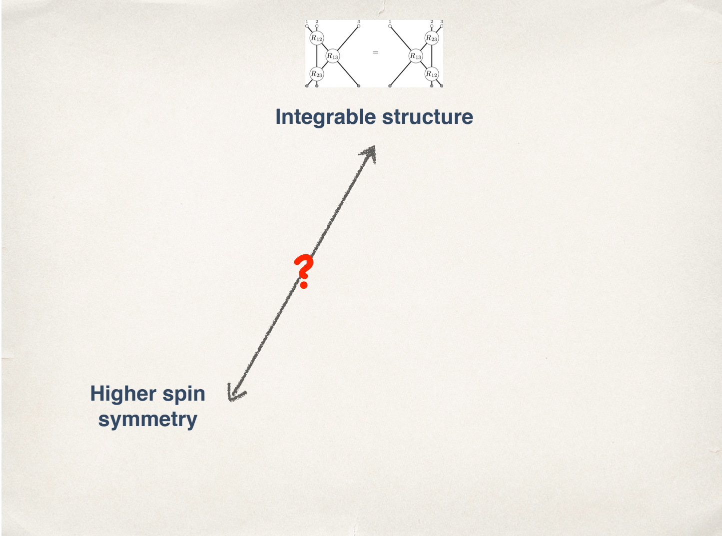
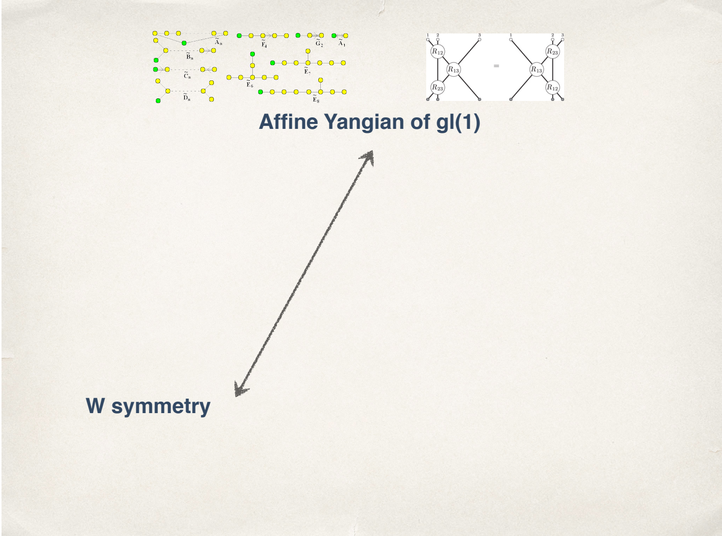


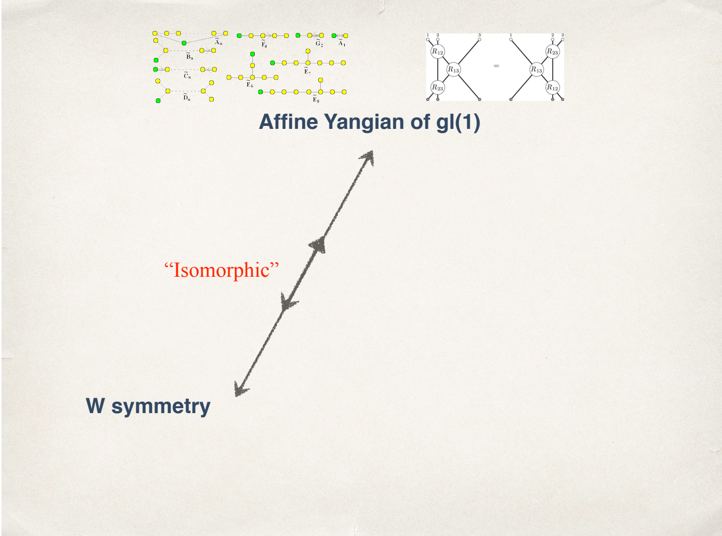
Today



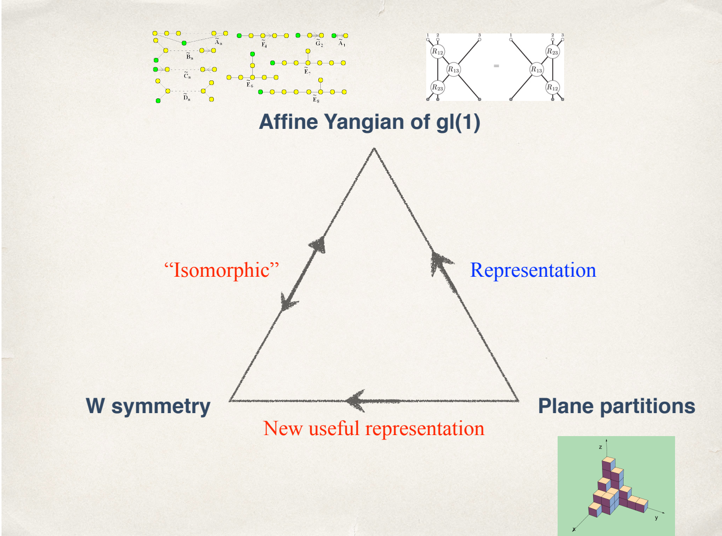
Today : higher spin AdS_3/CFT_2



A concrete relation between HS and integrability



Motivation-2: plane partition is useful for $\mathcal{W}_\infty$



Two questions

1. Supersymmetrize $\triangle$?

2. $\triangle$ as lego pieces for new VOA/affine Yangian?

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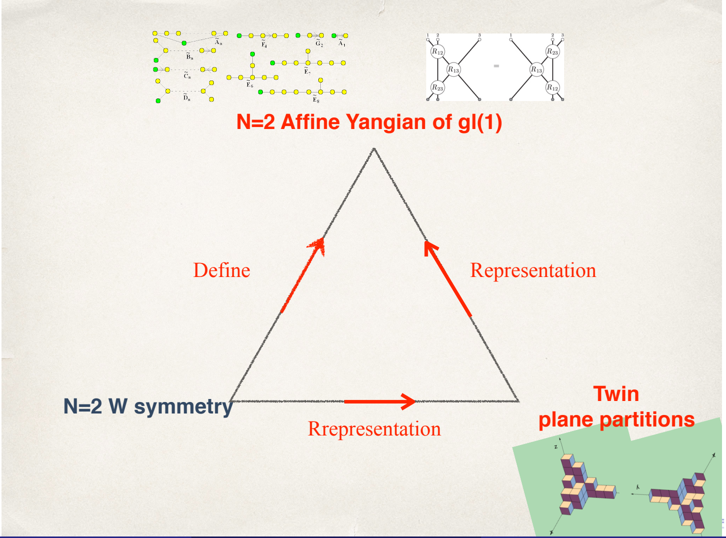
1. Supersymmetrize $\triangle$?
2. $\triangle$ as lego pieces for new VOA/affine Yangian?

A surprising (partial) answer

Glue two $\triangle$ to get $\mathcal{N}=2$ version of $\triangle$

New Yangian algebra from W algebra



Outline

Outline

Intro

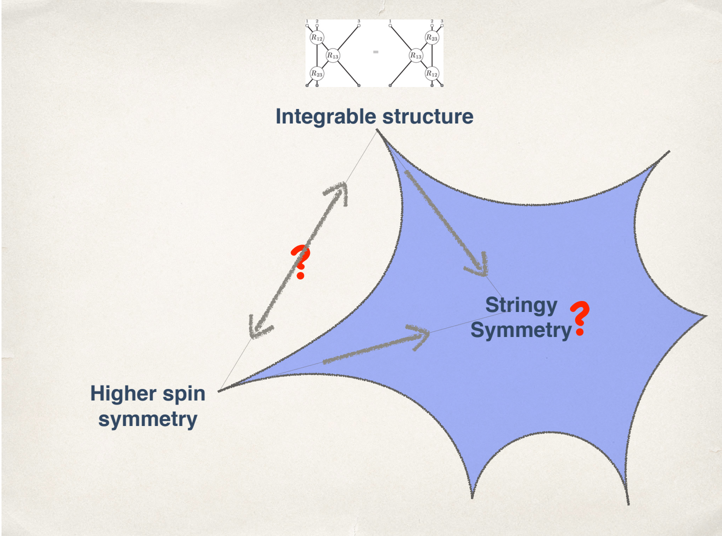
W and affine Yangian

Plane Partition

$\mathcal{N} = 2$ affine Yangian

Summary

Different manifestation of stringy symmetry



Higher spin symmetry and stringy symmetry

- String theory has infinite number of massive higher spin particles

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- Tensionless limit:
  massive higher spin particle $\Longrightarrow$ massless $\Longrightarrow$ stringy symmetry
  - subalgebra: Vasiliev higher spin symmetry (one per spin)
    (from Leading Regge trajectory)
    Vasiliev '91
    Sundborg '01, Witten '01, Mikhailov '02, Klebanov-Polyakov '02

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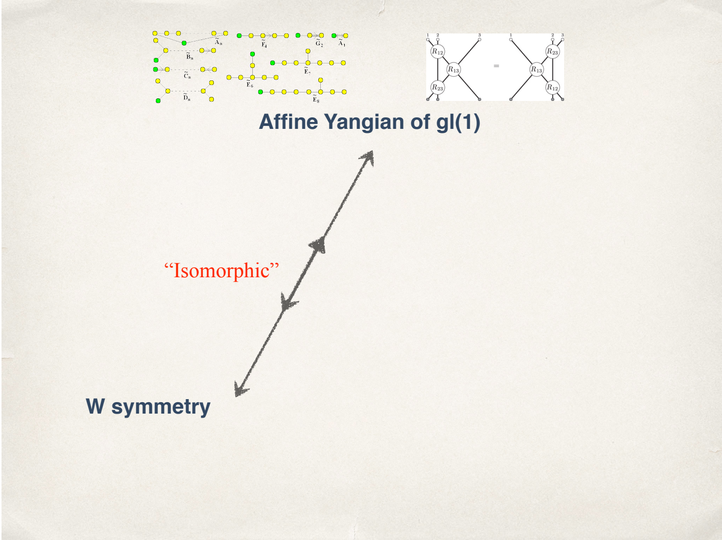
    Sundborg '01, Witten '01, Mikhailov '02, Klebanov-Polyakov '02
- Tensionless String in AdS_3 $\Longrightarrow$ maximal stringy symmetry?
  Gaberdiel Gopakumar '15
  - higher spin symmetry $\Longrightarrow$ $\mathcal{W}$ symmetry
    (Virasoro + higher spin currents)
    Campoleoni Fredenhagen Pfenninger Theisen '10, Henneaux Rey '10

Affine Yangian



Modes of $\mathcal{W}_{1+\infty}$

$$W^{(s)}(z) = \sum_{n\in\mathbb{Z}} \frac{W_n^{(s)}}{z^{n+s}} \qquad s = 1, 2, 3, \ldots$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
spin-5	$\ldots$	X_{-4}	X_{-3}	X_{-2}	X_{-1}	X_0	X_1	X_2	X_3	X_4	$\ldots$
spin-4	$\ldots$	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_0	U_1	U_2	U_3	U_4	$\ldots$
spin-3	$\ldots$	W_{-4}	W_{-3}	W_{-2}	W_{-1}	W_0	W_1	W_2	W_3	W_4	$\ldots$
spin-2	$\ldots$	L_{-4}	L_{-3}	L_{-2}	L_{-1}	L_0	L_1	L_2	L_3	L_4	$\ldots$
spin-1	$\ldots$	J_{-4}	J_{-3}	J_{-2}	J_{-1}	J_0	J_1	J_2	J_3	J_4	$\ldots$

Regrouping the modes

$$W^{(s)}(z)=\sum_{n\in\mathbb{Z}}\frac{W^{(s)}_n}{z^{n+s}}\qquad s=1,2,3,\dots$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
spin-5	$\dots$	X_{-3}	X_{-2}	$X_{-1}\sim e_4$	$X_0\sim\psi_5$	$X_1\sim f_4$	X_2	X_3	X_4
spin-4	$\dots$	U_{-3}	U_{-2}	$U_{-1}\sim e_3$	$U_0\sim\psi_4$	$U_1\sim f_3$	U_2	U_3	U_4
spin-3	$\dots$	W_{-3}	W_{-2}	$W_{-1}\sim e_2$	$W_0\sim\psi_3$	$W_1\sim f_2$	W_2	W_3	W_4
spin-2	$\dots$	L_{-3}	L_{-2}	$L_{-1}\sim e_1$	$L_0\sim\psi_2$	$L_1\sim f_1$	L_2	L_3	L_4
spin-1	$\dots$	J_{-3}	J_{-2}	$J_{-1}\sim e_0$	$J_0\sim\psi_1$	$J_1\sim f_0$	J_2	J_3	J_4

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Generators

$$e(z) = \sum_{j=0}^{\infty} \frac{e_j}{z^{j+1}} \qquad \psi(z) = 1 + \sigma_3 \sum_{j=0}^{\infty} \frac{\psi_j}{z^{j+1}} \qquad f(z) = \sum_{j=0}^{\infty} \frac{f_j}{z^{j+1}}$$

Affine Yangian of $\mathfrak{gl}_1$

Def: Associative algebra with generators e_j, f_j and ψ_j, $j = 0, 1, \ldots$

- Generators

$$\psi(z) = 1 + (h_1 h_2 h_3) \sum_{j=0}^{\infty} \frac{\psi_j}{z^{j+1}} \qquad e(z) = \sum_{j=0}^{\infty} \frac{e_j}{z^{j+1}} \qquad f(z) = \sum_{j=0}^{\infty} \frac{f_j}{z^{j+1}}$$

- Parameters (h_1, h_2, h_3) with $h_1 + h_2 + h_3 = 0$
- One $\mathcal{S}_3$ invariant function $\varphi(z) = \frac{(z+h_1)(z+h_2)(z+h_3)}{(z-h_1)(z-h_2)(z-h_3)}$
- Defining relations

$$[e(z)\,, f(w)] = -\frac{1}{h_1 h_2 h_3} \frac{\psi(z) - \psi(w)}{z - w}$$

$$\psi(z)\, e(w) \sim \varphi(z-w)\, e(w)\, \psi(z) \qquad \psi(z)\, f(w) \sim \varphi(w-z)\, f(w)\, \psi(z)$$

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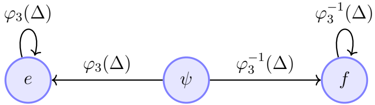

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- Initial conditions

$$[\psi_{0,1}, e_j] = 0 \quad [\psi_2, e_j] = 2e_j \quad [\psi_{0,1}, f_j] = 0 \quad [\psi_2, f_j] = -2f_j$$

- Serre relation

$$\mathrm{Sym}_{(j_1,j_2,j_3)}\, [e_{j_1}, [e_{j_2}, e_{j_3+1}]] = 0 \qquad \mathrm{Sym}_{(j_1,j_2,j_3)}\, [f_{j_1}, [f_{j_2}, f_{j_3+1}]] = 0$$

Affine Yangian of $\mathfrak{gl}_1$

In terms of modes e_j, f_j and ψ_j, $j = 0, 1, \ldots$

$$
\begin{aligned}
0 =& [\psi_j, \psi_k] \\
\psi_{j+k} =& [e_j, f_k] \\
\sigma_3\{\psi_j, e_k\} =& [\psi_{j+3}, e_k] - 3[\psi_{j+2}, e_{k+1}] + 3[\psi_{j+1}, e_{k+2}] - [\psi_j, e_{k+3}] \\
& + \sigma_2[\psi_{j+1}, e_k] - \sigma_2[\psi_j, e_{k+1}] \\
-\sigma_3\{\psi_j, f_k\} =& [\psi_{j+3}, f_k] - 3[\psi_{j+2}, f_{k+1}] + 3[\psi_{j+1}, f_{k+2}] - [\psi_j, f_{k+3}] \\
& + \sigma_2[\psi_{j+1}, f_k] - \sigma_2[\psi_j, f_{k+1}] \\
\sigma_3\{e_j, e_k\} =& [e_{j+3}, e_k] - 3[e_{j+2}, e_{k+1}] + 3[e_{j+1}, e_{k+2}] - [e_j, e_{k+3}] \\
& + \sigma_2[e_{j+1}, e_k] - \sigma_2[e_j, e_{k+1}] \\
-\sigma_3\{f_j, f_k\} =& [f_{j+3}, f_k] - 3[f_{j+2}, f_{k+1}] + 3[f_{j+1}, f_{k+2}] - [f_j, f_{k+3}] \\
& + \sigma_2[f_{j+1}, f_k] - \sigma_2[f_j, f_{k+1}]
\end{aligned}
$$

with

$$
h_1 + h_2 + h_3 = 0 \qquad \sigma_2 \equiv h_1 h_2 + h_2 h_3 + h_1 h_3 \qquad \sigma_3 \equiv h_1 h_2 h_3
$$

Schiffmann Vasserot '12 *Maulik Okounkov '12*

Feigin Jimbo Miwa Mukhin '10-11

Tsymbaliuk '14

W algebra and affine Yangian

$$\mathcal{Y}[\widehat{\mathfrak{gl}_1}] \cong \mathrm{UEA}[\mathcal{W}_{1+\infty}[\lambda]]$$

Procházka '15

Gaberdiel Gopakumar Li Peng '17

for q-version $\mathcal{U}[\widehat{\widehat{\mathfrak{gl}_1}}] \cong \mathrm{UEA}[\text{q-}\mathcal{W}_{1+\infty}[\lambda]]$

Miki '07

Feigin Jimbo Miwa Mukhin '10-11

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$\mathcal{N}=2$ affine Yangian

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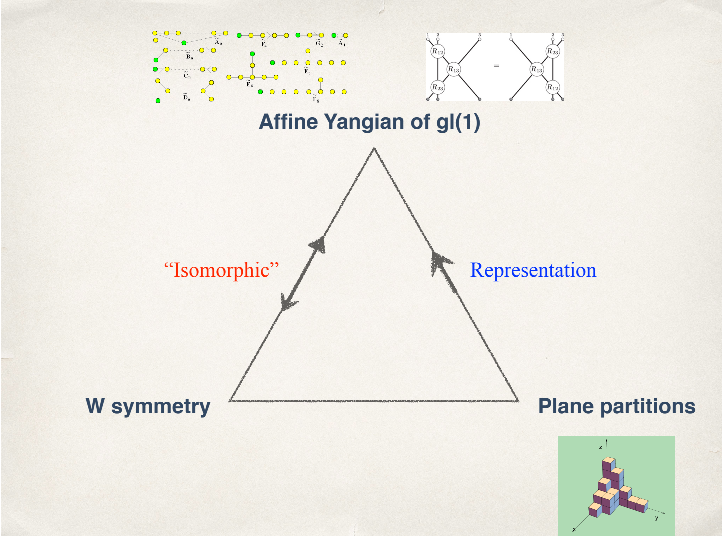
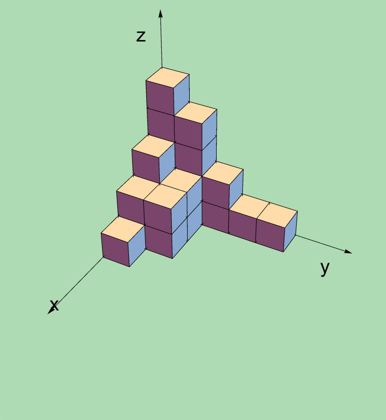
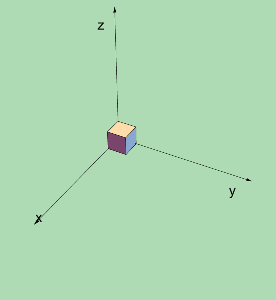
Plane partition as representations of affine Yangian



Plane partition via box stacking

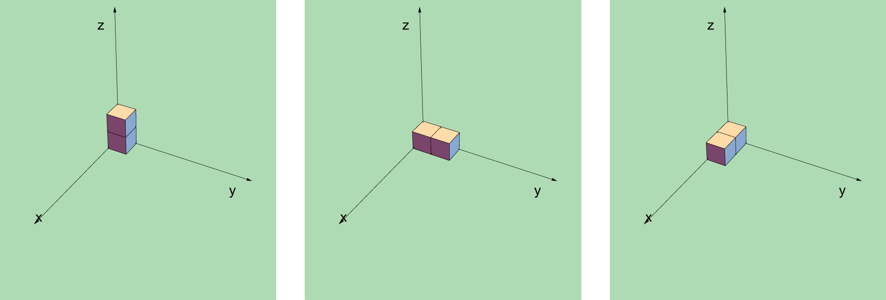


Stacking 1 box



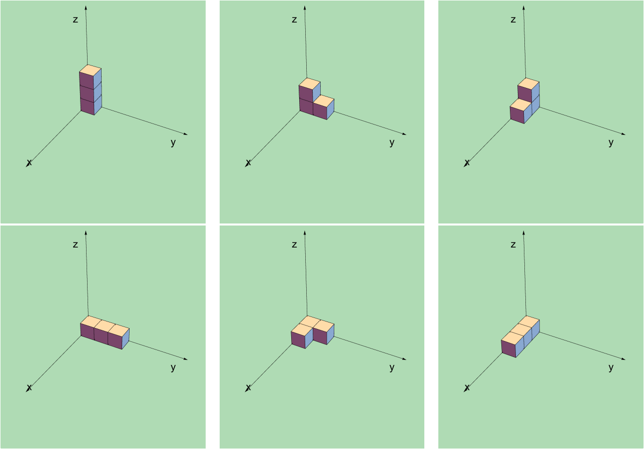
$$1+q+\cdots$$

Stacking 2 boxes



$$1 + q + 3\,q^2 \cdots$$

Stacking 3 boxes



$$1 + q + 3\,q^2 + {\color{blue}6}\,q^3 + \cdots$$

MacMahon function

Generating function of Plane partition

$$\sum_{n=0} M(n)q^n = \prod_{k=1}^{\infty} \frac{1}{(1-q^k)^k}$$
$$= 1 + q + 3\,q^2 + 6\,q^3 + 13\,q^4 + 24\,q^5 + 48\,q^6 + \cdots$$

$$M(n) \sim n^{-\frac{25}{36}} \cdot \exp\left(\frac{3\zeta(3)^{\frac{1}{3}}}{2^{\frac{2}{3}}} n^{\frac{2}{3}}\right)$$

Wright '31

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Wright '31

Generating function of partition

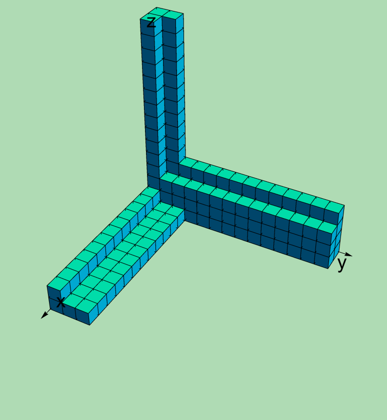
$$\sum_{n=0} p(n) q^n = \prod_{k=1}^{\infty} \frac{1}{1-q^k}$$
$$= 1 + q + 2\,q^2 + 3\,q^3 + 5\,q^4 + 7\,q^5 + 11\,q^6 + \cdots$$

$$p(n) \sim \frac{1}{n} \cdot \exp\left(\sqrt{\frac{2}{3}}\, \pi \sqrt{n} \right)$$

Hardy Ramanujan '18

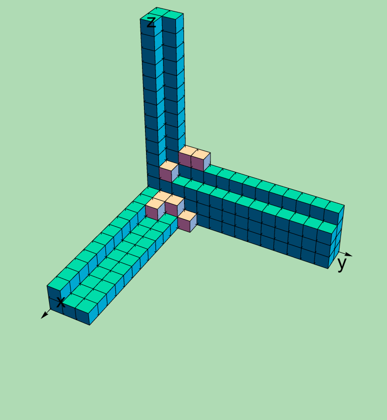
Plane partition with non-trivial asymptotics

Ground state of $(\Lambda_x, \Lambda_y, \Lambda_z)$

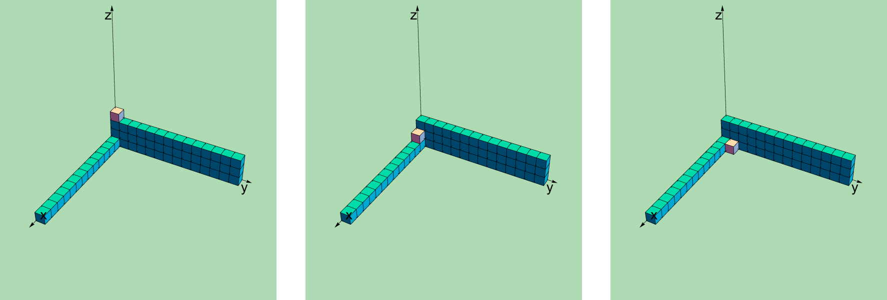


Plane partition with non-trivial asymptotics

a level-7 excited states of $(\Lambda_x, \Lambda_y, \Lambda_z)$

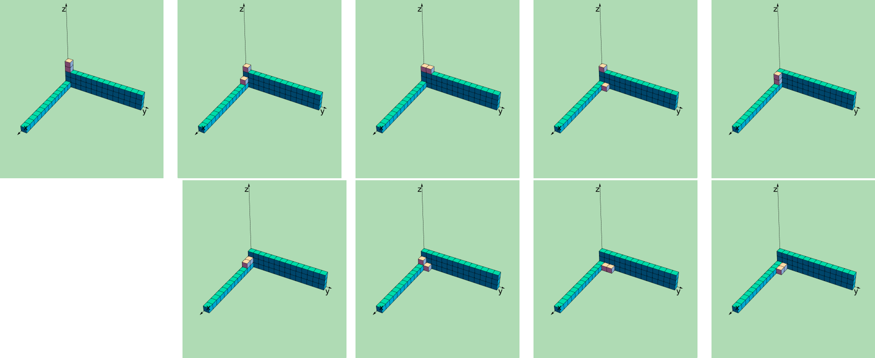


Stacking 1 boxes



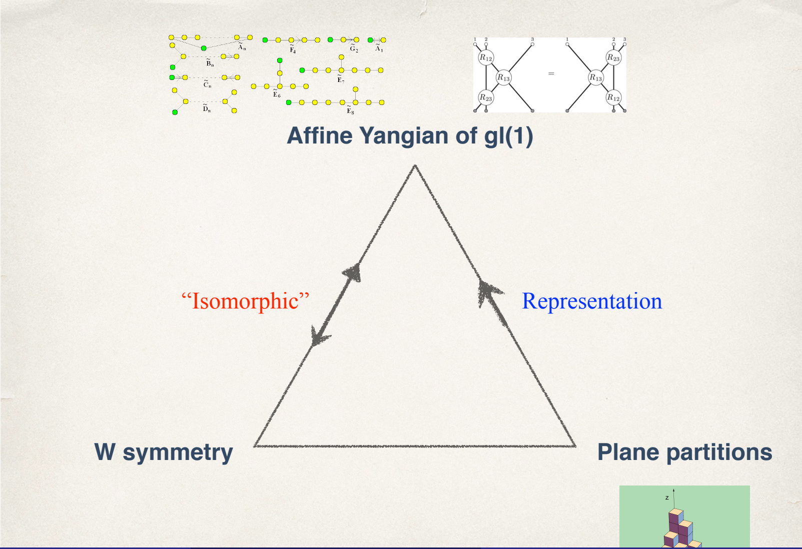
$$1 + 3\,q + \cdots$$

Stacking 2 boxes



$$1 + 3\,q + 9\,q^2 + \cdots$$

Plane partitions are faithful representations of $\hat{\mathcal{Y}}(\mathfrak{gl}_1)$



Action of $\hat{\mathcal{Y}}(\mathfrak{gl}_1)$ on a plane partition

- $\psi(z)$ acts diagonally *Tsymbaliuk '14, Prochazka '15*

$$\psi(z)|\Lambda\rangle = \psi_\Lambda(z)|\Lambda\rangle$$

$$\psi_\Lambda(z) \equiv \left(1+\frac{\psi_0\sigma_3}{z}\right) \prod_{\square\in(\Lambda)} \varphi(z-h(\square))$$

$$h(\square) = h_1 x(\square) + h_2 y(\square) + h_3 z(\square)$$

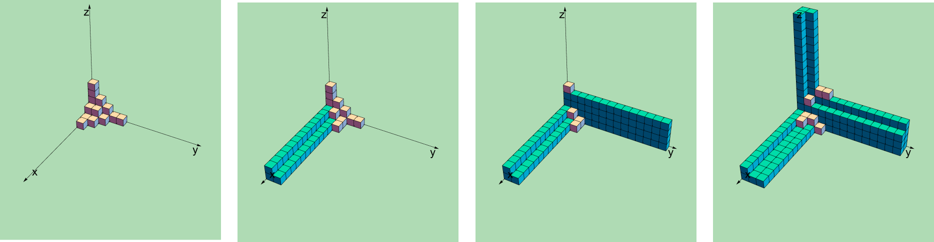
- $e(z)$ adds one box

$$e(z)|\Lambda\rangle = \sum_{\square\in \mathrm{Add}(\Lambda)} \frac{\left[-\frac{1}{\sigma_3}\mathrm{Res}_{w=h(\square)}\psi_\Lambda(w)\right]^{\frac{1}{2}}}{z-h(\square)}|\Lambda+\square\rangle$$

- $f(z)$ removes one box

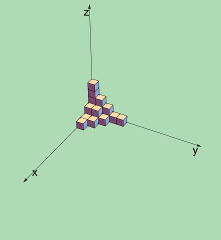
$$f(z)|\Lambda\rangle = \sum_{\square\in \mathrm{Rem}(\Lambda)} \frac{\left[-\frac{1}{\sigma_3}\mathrm{Res}_{w=h(\square)}\psi_\Lambda(w)\right]^{\frac{1}{2}}}{z-h(\square)}|\Lambda-\square\rangle$$

Plane partition with non-trivial boundary condition

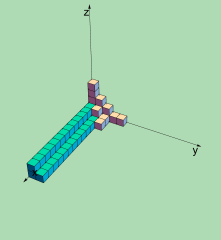


Trivial b.c. $(\Lambda_x;0)$ $(\Lambda_x;\Lambda_y)$ $(\Lambda_x;\Lambda_y;\Lambda_z)$

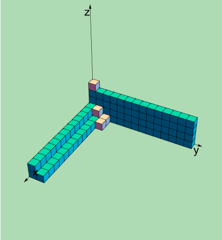
Plane partition with non-trivial boundary condition



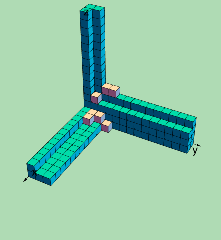
Trivial b.c.



$(\Lambda_x; 0)$



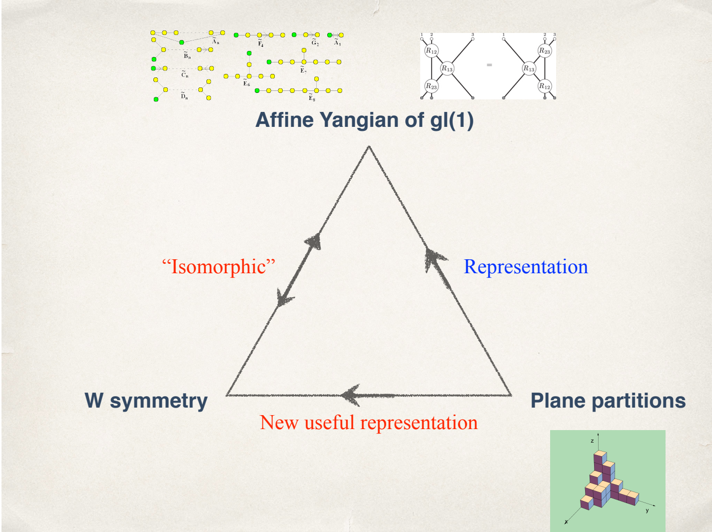
$(\Lambda_x; \Lambda_y)$



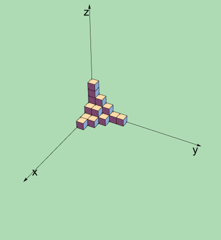
$(\Lambda_x; \Lambda_y; \Lambda_z)$

character of affine Yangian = generating function of plane partition

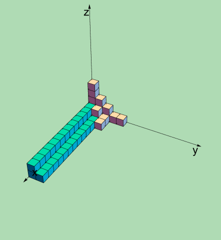
PP as representions of affine yangian



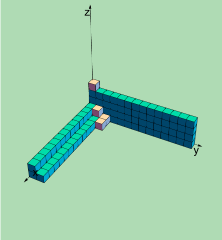
Plane partition as representations of W



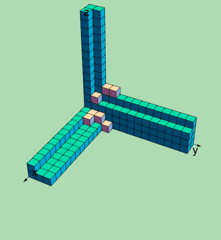
Trivial b.c.



$(\Lambda_x; 0) = (\Lambda; 0)$

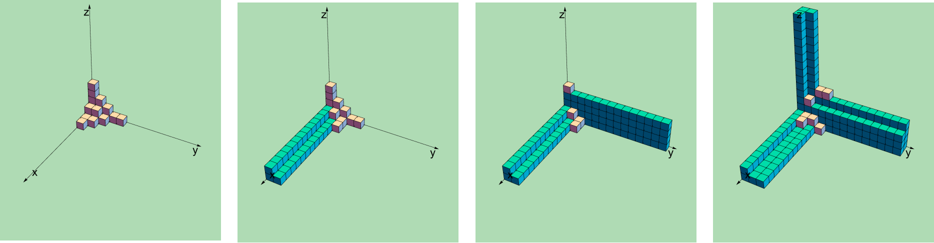


$(\Lambda_x; \Lambda_y) = (\Lambda_+; \Lambda_-)$



$(\Lambda_x; \Lambda_y; \Lambda_z)$

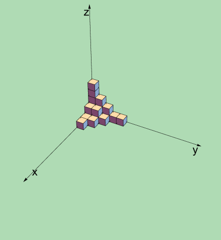
Plane partition as representations of W



Trivial b.c. $\quad (\Lambda_x;0)=(\Lambda;0) \quad (\Lambda_x;\Lambda_y)=(\Lambda_+;\Lambda_-) \quad (\Lambda_x;\Lambda_y;\Lambda_z)$

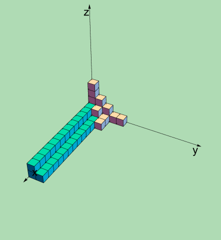
$$\boxed{\text{character of } \mathcal{W}_{1+\infty} = \text{generating function of plane partition}}$$

Plane partition as representations of W



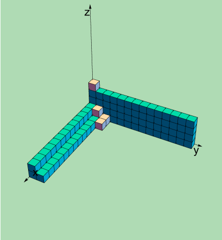
Trivial b.c.

vacuum



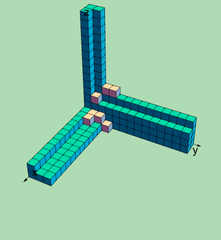
$(\Lambda_x; 0) = (\Lambda; 0)$

perturbative
in Vasiliev



$(\Lambda_x; \Lambda_y) = (\Lambda_+; \Lambda_-)$

non-perturbative
in Vasiliev

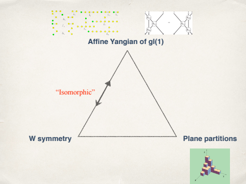


$(\Lambda_x; \Lambda_y; \Lambda_z)$

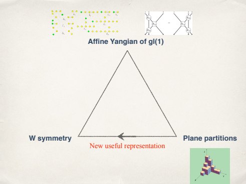
new representation

character of $\mathcal{W}_{1+\infty}$ = generating function of plane partition

Application



- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



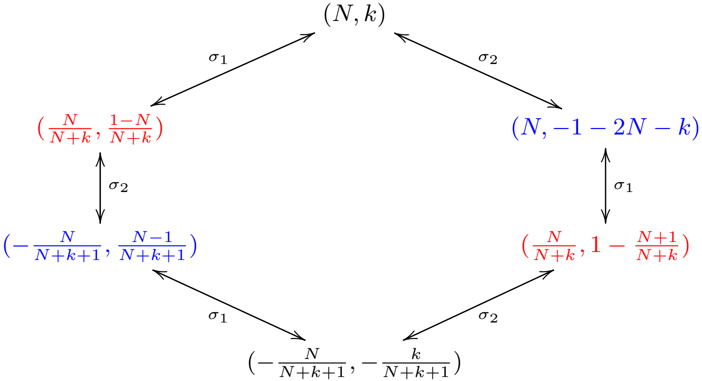
- Character computation more transparent

$\mathcal{S}_3$ action on $\mathcal{W}_{N,k}$ coset

$\mathcal{W}_{N,k}$ coset

$$\frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

had hidden $\mathcal{S}_3$

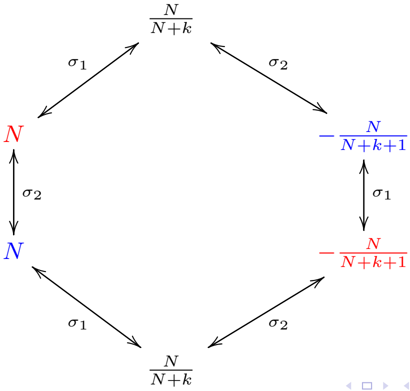


$\mathcal{S}_3$ action on 't Hooft coupling

$\mathcal{W}_{N,k}$ coset

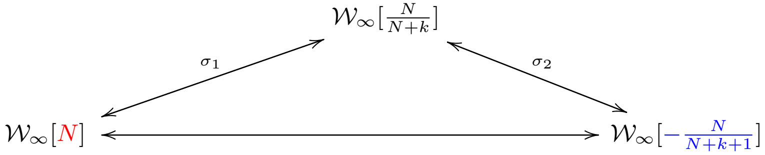
$$\frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

't Hooft coupling $\lambda = \frac{N}{N+k}$ transform under $\mathcal{S}_3$



Triality symmetry for higher spin holography

For fixed c, three $\mathcal{W}_\infty[\lambda]$ are isomorphic *Gaberdiel Gopakumar '12*



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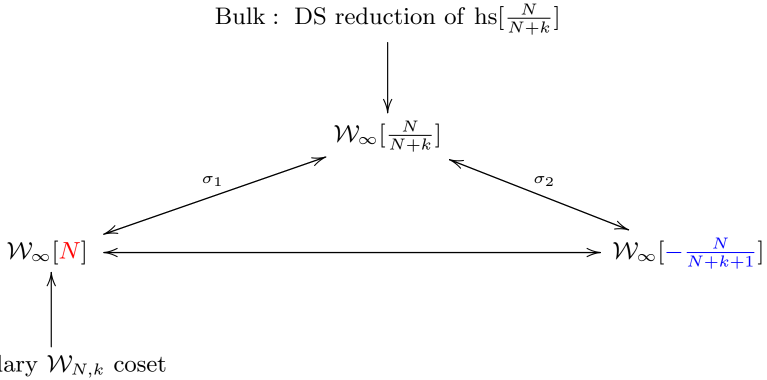

Crucial in Higher spin AdS_3/CFT_2 (Vasiliev theory in AdS_3 = $\mathcal{W}_{N,k}$ coset)

- $\mathcal{S}_3$ symmetry in $\mathcal{W}_\infty$CFT is highly non-trivial
  - hard to check/prove *Gaberdiel Gopakumar '12, Linshaw '17*
  - UV — IR
- Manifest in $\mathcal{Y}[\widehat{\mathfrak{gl}_1}]$

$\mathcal{Y}[\widehat{\mathfrak{gl}_1}]$ depends on (h_1, h_2, h_3) symmetrically

$$h_1 = -\sqrt{\frac{N+k+1}{N+k}} \qquad h_2 = \sqrt{\frac{N+k}{N+k+1}} \qquad h_3 = \frac{1}{\sqrt{(N+k)(N+k+1)}}$$

Procházka '15, *Gaberdiel Gopakumar Li Peng '17*

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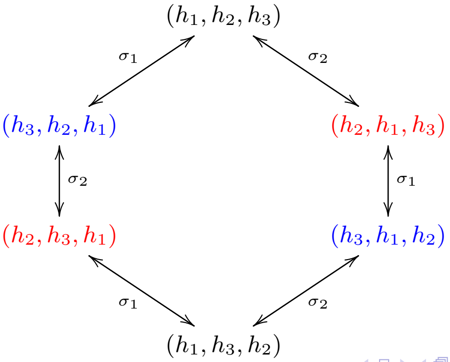
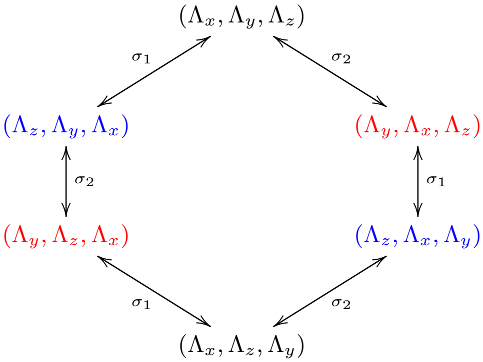
Procházka '15, *Gaberdiel Gopakumar Li Peng '17*

Under $\mathcal{S}_3$ transformation on (N, k)

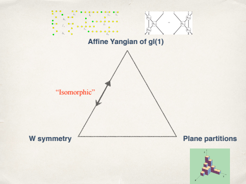


$\mathcal{S}_3$ symmetry of plane partition

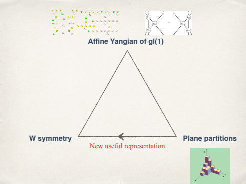
The representations of $\mathcal{W}_\infty$ comes in $\mathcal{S}_3$ family



Application



- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



- Character computation more transparent

1. Manifest $\mathcal{S}_3$ symmetry
2. Good representation theory
   - manifest $\mathcal{S}_3$ symmetry
   - describe new representations invisible in coset
   - easier to compute $\mathcal{W}_\infty$ characters via counting boxes

Datta Gaberdiel Li Peng '16

1. Manifest $\mathcal{S}_3$ symmetry

2. Good representation theory

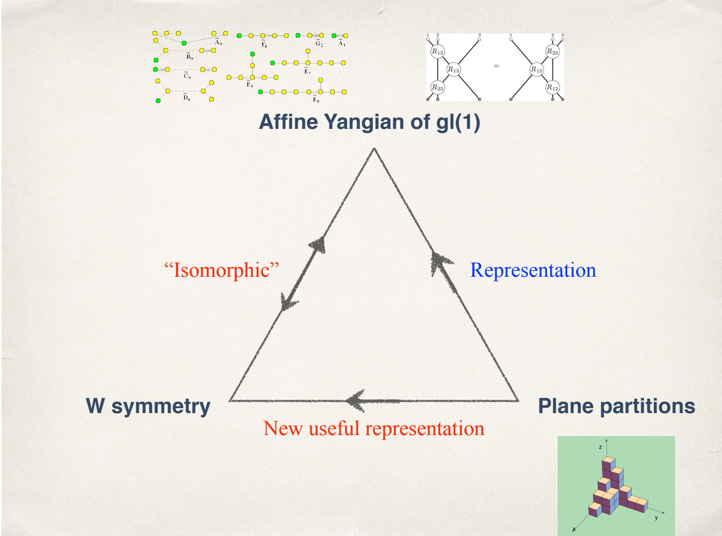
   - manifest $\mathcal{S}_3$ symmetry
   - describe new representations invisible in coset
   - easier to compute $\mathcal{W}_\infty$ characters via counting boxes

     Datta Gaberdiel Li Peng '16

3. Connect to integrable structure?

Outline

Bosonic W and affine Yangian



Two questions

1. Supersymmetrize $\triangle$?
2. $\triangle$ as lego pieces for new VOA/affine Yangian?

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A surprising (partial) answer

Glue two $\triangle$ to get $\mathcal{N}=2$ version of $\triangle$

Two questions

1. Supersymmetrize $\triangle$?

2. $\triangle$ as lego pieces for new VOA/affine Yangian?

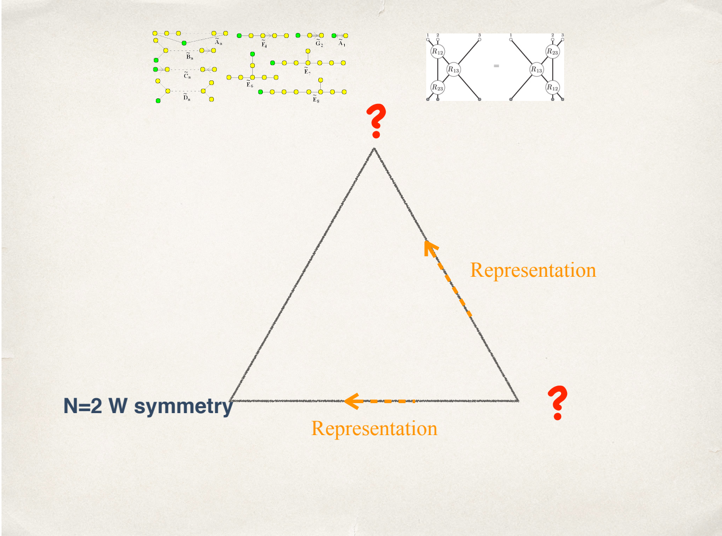
Gaiotto Rapcak '17, Rapcak Prochazka '17

A surprising (partial) answer

Glue two $\triangle$ to get $\mathcal{N}=2$ version of $\triangle$

Gaberdiel Li Peng Zhang'17

$\mathcal{N}=2$ version?



Constructing $\mathcal{N}=2$ version

1. Rewrite representations of $\mathcal{N}=2\ \mathcal{W}_\infty$ in terms of (some version) of plane partitions

Twin plane partition

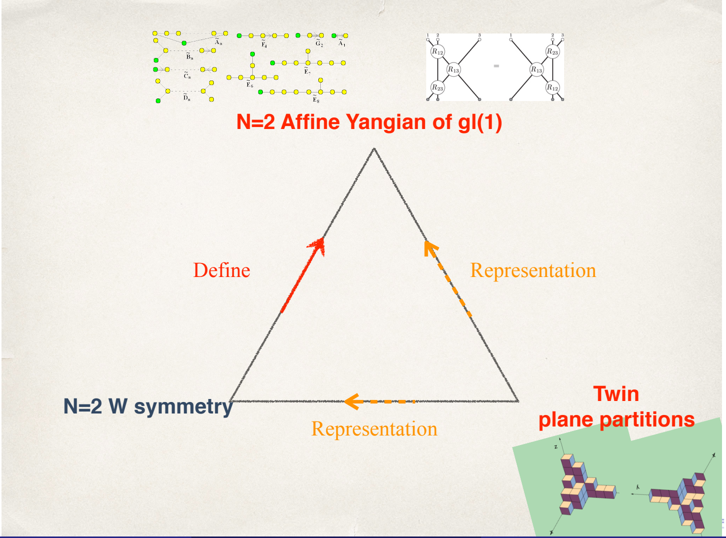
Constructing $\mathcal{N}=2$ version

1. Rewrite representations of $\mathcal{N}=2$ $\mathcal{W}_\infty$ in terms of (some version) of plane partitions

Twin plane partition

2. Define $\mathcal{N}=2$ affine Yangian such that
   - twin plane partitions are faithful representations
   - reproduce $\mathcal{N}=2$ $\mathcal{W}_\infty$ charges

$\mathcal{N}=2$ version



$\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ algebra

- One $\mathcal{N}=2$ multiplet per spin

Creutzig, Hikida, Ronne '11

Candu Gaberdiel '12

$$\begin{pmatrix} & T & \\ G^- & & G^+ \\ & J & \end{pmatrix} \quad \begin{pmatrix} & W^{(2)1} & \\ W^{(2)-} & & W^{(2)+} \\ & W^{(2)0} & \end{pmatrix} \quad \begin{pmatrix} & W^{(3)1} & \\ W^{(3)-} & & W^{(3)+} \\ & W^{(3)0} & \end{pmatrix} \dots$$

$\mathcal{N}=2\ \mathcal{W}_\infty[\lambda]$ algebra

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$$\begin{pmatrix} & T & \\ G^- & & G^+ \\ & J & \end{pmatrix} \quad \begin{pmatrix} & W^{(2)1} & \\ W^{(2)-} & & W^{(2)+} \\ & W^{(2)0} & \end{pmatrix} \quad \begin{pmatrix} & W^{(3)1} & \\ W^{(3)-} & & W^{(3)+} \\ & W^{(3)0} & \end{pmatrix} \ldots$$

- Rearrange by spin

4								$W^{(3)1}$	
$\frac{7}{2}$							$W^{(3)-}$		$W^{($
3					$W^{(2)1}$			$W^{(3)0}$	
$\frac{5}{2}$				$W^{(2)-}$		$W^{(2)+}$			
2		T			$W^{(2)0}$				
$\frac{3}{2}$	G^-		G^+						
1		J							

$\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ algebra

- One $\mathcal{N}=2$ multiplet per spin *Creutzig, Hikida, Ronne '11* *Candu Gaberdiel '12*

$$\begin{pmatrix} G^- & \begin{matrix} T \\ J \end{matrix} & G^+ \end{pmatrix} \begin{pmatrix} W^{(2)-} & \begin{matrix} W^{(2)1} \\ W^{(2)0} \end{matrix} & W^{(2)+} \end{pmatrix} \begin{pmatrix} W^{(3)-} & \begin{matrix} W^{(3)1} \\ W^{(3)0} \end{matrix} & W^{(3)+} \end{pmatrix} \ldots$$

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4								$W^{(3)1}$	
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$\frac{5}{2}$					$W^{(2)-}$		$W^{(2)+}$		
2			T			$W^{(2)0}$			
$\frac{3}{2}$		G^-		G^+					
1	U		J						

- $\mathfrak{u}(1) \oplus \mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has 2 fields per spin

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — bosonic part

- Conjecture: $\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has two bosonic $\mathcal{W}_\infty$ subalgebra

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$\mathcal{N}=2 \quad \mathcal{W}_\infty[\lambda]$

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — bosonic part

- Conjecture: $\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has two bosonic $\mathcal{W}_\infty$ subalgebra

$$\text{Vasiliev shs}[\lambda] \quad \supset \quad \text{hs}[\lambda] \oplus \text{hs}[1-\lambda]$$

Prokushkin Vasiliev '98

wedge subalgbra $\uparrow$

$$\mathcal{N}=2 \quad \mathcal{W}_\infty[\lambda]$$

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — bosonic part

- Conjecture: $\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has two bosonic $\mathcal{W}_\infty$ subalgebra

Vasiliev shs$[\lambda]$ $\supset$ hs$[\lambda] \oplus$ hs$[1-\lambda]$ *Prokushkin Vasiliev '98*

↑ wedge subalgbra

$\mathcal{N}=2\quad \mathcal{W}_\infty[\lambda]$

↓ Truncation

$\mathcal{N}=2\quad \mathcal{W}_3$ $\supset$ Virasoro $\oplus$ Virasoro *Romans '92*

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — bosonic part

- Conjecture: $\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has two bosonic $\mathcal{W}_\infty$ subalgebra

$$
\begin{array}{ccccc}
\text{Vasiliev shs}[\lambda] & \supset & \text{hs}[\lambda]\oplus\text{hs}[1-\lambda] & & \textit{Prokushkin Vasiliev '98} \\
\text{wedge subalgbra}\uparrow & & \downarrow & & \\
\mathcal{N}=2 \quad \mathcal{W}_\infty[\lambda] & \supset & \mathcal{W}_\infty[\lambda]\oplus\mathcal{W}_\infty[1-\lambda] & & \\
\downarrow\text{Truncation} & & \uparrow & & \\
\mathcal{N}=2 \quad \mathcal{W}_3 & \supset & \text{Virasoro}\oplus\text{Virasoro} & & \textit{Romans '92}
\end{array}
$$

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — bosonic part

- Conjecture: $\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]$ has two bosonic $\mathcal{W}_\infty$ subalgebra

Vasiliev $\mathrm{shs}[\lambda]$ $\supset$ $\mathrm{hs}[\lambda] \oplus \mathrm{hs}[1-\lambda]$ *Prokushkin Vasiliev '98*

wedge subalgbra ↑ ↓

$\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ $\supset$ $\mathcal{W}_\infty[\lambda] \oplus \mathcal{W}_\infty[1-\lambda]$

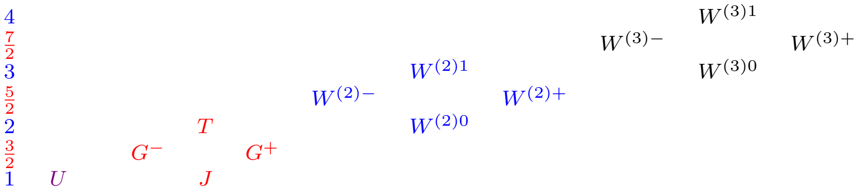
↓ Truncation ↑

$\mathcal{N}=2$ $\mathcal{W}_3$ $\supset$ Virasoro $\oplus$ Virasoro *Romans '92*

- Checked up to $s=3$ (non-trivial!) *Gaberdiel Li Peng Zhang '17*

to appear

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$ — fermionic part



- Bosonic sub-algebra

$$\mathfrak{u}(1) \oplus \mathcal{W}_\infty^{\mathcal{N}=2}[\lambda] \supset \mathcal{W}_{1+\infty}[\lambda] \oplus \mathcal{W}_{1+\infty}[1-\lambda]$$

- How do fermions fit in?

Decomposing $\mathcal{W}_{\infty}^{\mathcal{N}=2}[\lambda]$ vacuum character

- Vacuum character of $\mathfrak{u}(1) \oplus \mathcal{W}_{\infty}^{\mathcal{N}=2}[\lambda]$

$$\chi_0^{\text{Full}}(q,y) = \prod_{n=1}^{\infty} \frac{(1+yq^{n+\frac{1}{2}})^n (1+\frac{1}{y}q^{n+\frac{1}{2}})^n}{(1-q^n)^{2n}}$$

$$= \chi_{\text{pp}}(q) \left(\sum_{\text{R}} y^{|\text{R}|} \chi_{\text{R}}^{(\text{wedge})\,[\lambda]}(q) \cdot \chi_{\bar{\text{R}}^T}^{(\text{wedge})\,[1-\lambda]}(q) \right)$$

$$\cdot \left(\sum_{\text{S}} \frac{1}{y^{|\text{S}|}} \chi_{\bar{\text{S}}}^{(\text{wedge})\,[\lambda]}(q) \cdot \chi_{\text{S}^T}^{(\text{wedge})\,[1-\lambda]}(q) \right) \chi_{\text{pp}}(q)$$

- Fermions transform as $(\lambda, \bar{\lambda}^T)$ and $(\bar{\lambda}^T, \lambda)$ of $\mathcal{W}_{1+\infty}[\lambda] \oplus \mathcal{W}_{1+\infty}[1-\lambda]$

Decomposing $\mathcal{W}_{\infty}^{\mathcal{N}=2}[\lambda]$ vacuum character

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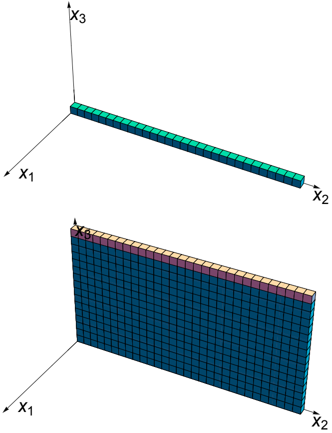
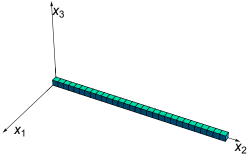
$$\cdot\left(\sum_{\text{S}} \frac{1}{y^{|\text{S}|}}\chi_{\bar{\text{S}}}^{(\text{wedge})\,[\lambda]}(q)\cdot\chi_{\text{S}^T}^{(\text{wedge})\,[1-\lambda]}(q)\right)\chi_{\text{pp}}(q)$$

- Fermions transform as $(\lambda, \bar{\lambda}^T)$ and $(\bar{\lambda}^T, \lambda)$ of $\mathcal{W}_{1+\infty}[\lambda] \oplus \mathcal{W}_{1+\infty}[1-\lambda]$
  - only need to label left representation
  - How to describe $\bar{\lambda}$ as plane partition?

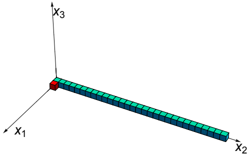
Aside: $\square$ v.s. $\bar{\square}$

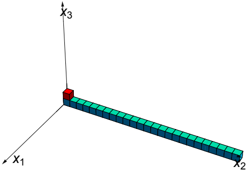


Aside: $\square$ v.s. $\bar{\square}$ ... and their descendents

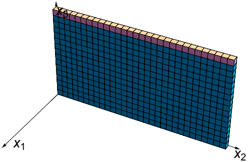


ground state of $\square$

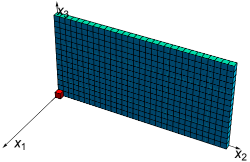


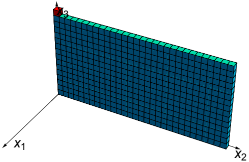


level-1 descendents



ground state of $\bar{\square}$





level-1 descendents

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$

4									$W^{(3)1}$	
$\frac{7}{2}$								$W^{(3)-}$		$W^{(3)+}$
3						$W^{(2)1}$			$W^{(3)0}$	
$\frac{5}{2}$					$W^{(2)-}$		$W^{(2)+}$			
2			T			$W^{(2)0}$				
$\frac{3}{2}$		G^-		G^+						
1	U		J							

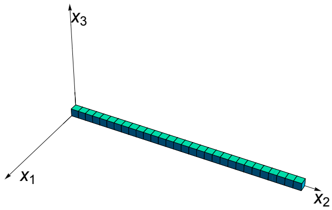
- Bosonic sub-algebra

$$\boxed{\mathfrak{u}(1)\oplus\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]\supset\mathcal{W}_{1+\infty}[\lambda]\oplus\mathcal{W}_{1+\infty}[1-\lambda]}$$

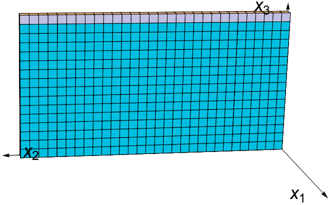
- Fermions:

$$\boxed{(\lambda,\bar{\lambda}^T)\qquad\qquad(\bar{\lambda},\lambda^T)}$$

Representation □

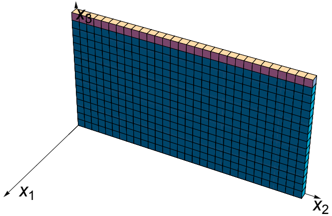


seen from left $\mathcal{W}_{1+\infty}$

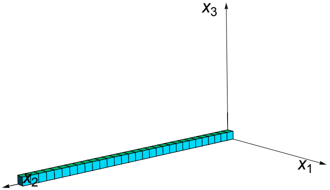


seen from right $\mathcal{W}_{1+\infty}$

Representation $\bar{\square}$

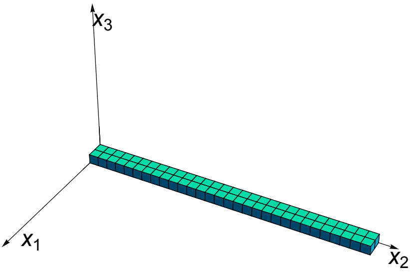


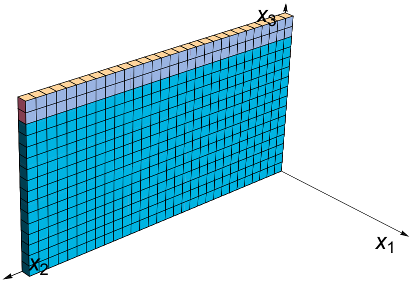
seen from left $\mathcal{W}_{1+\infty}$



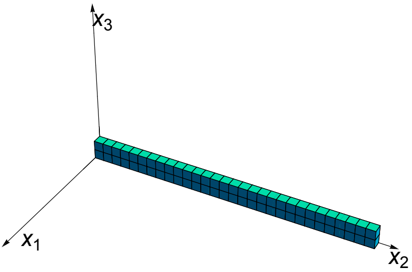
seen from right $\mathcal{W}_{1+\infty}$

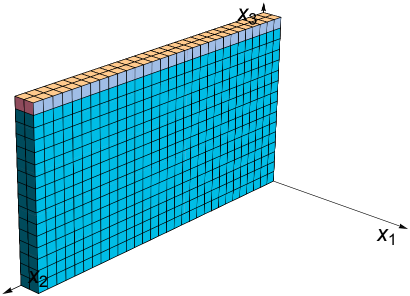
Representation □□





Representation $\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$





Building blocks

Gaberdiel Li Peng Zhang '17

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$

- Bosonic sub-algebra

$$\boxed{\mathfrak{u}(1)\oplus\mathcal{W}_\infty^{\mathcal{N}=2}[\lambda]\supset\mathcal{W}_{1+\infty}[\lambda]\oplus\mathcal{W}_{1+\infty}[1-\lambda]}$$

- Fermions:

$$\boxed{(\lambda,\bar{\lambda}^T)\qquad\qquad(\bar{\lambda}^T,\lambda)}$$

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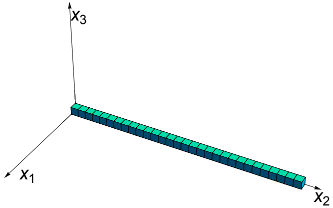
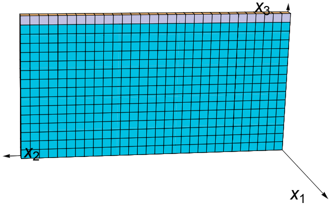
Building blocks of $\mathcal{N}=2$ Yangian

- Vacuum: a pair of plane partition (left and right)
- $\mathrm{x}\equiv(\square,\bar{\square})$: a pair of plane partition with asymptotics $(\square,\bar{\square})$
- $\bar{\mathrm{x}}\equiv(\bar{\square},\square)$: a pair of plane partition with asymptotics $(\bar{\square},\square)$
- single boxes (created by e and $\hat{e}$) for descendents

Fermionic building block-1: $\mathrm{x} \equiv \square$

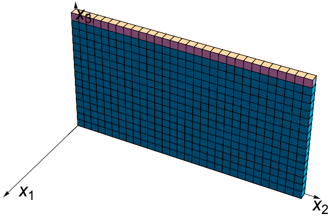


x seen from left $\mathcal{W}_{1+\infty}$

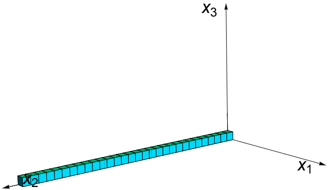


x seen from right $\mathcal{W}_{1+\infty}$

Fermionic building block-2: $\bar{\text{x}} \equiv \bar{\square}$

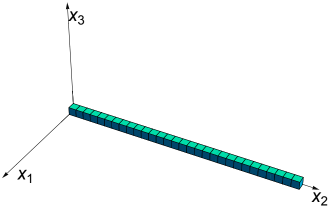


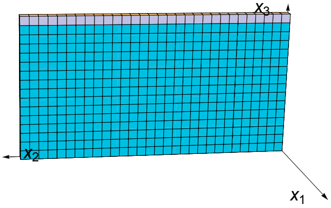
$\bar{\text{x}}$ seen from left $\mathcal{W}_{1+\infty}$



$\bar{\text{x}}$ seen from right $\mathcal{W}_{1+\infty}$

Fermionic building block-1: $\mathrm{x} \equiv \square$

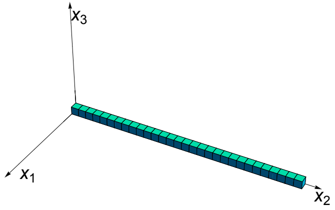


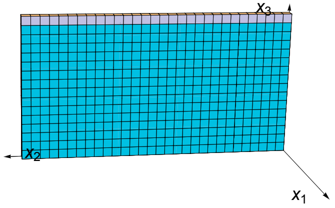


$$\psi(z) = \left(1+\frac{\psi_0\sigma_3}{z}\right)\prod_{n=0}^{\infty}\varphi_3(z-nh_2) = \left(1+\frac{\psi_0\sigma_3}{z}\right)\frac{z(z+h_2)}{(z-h_1)(z-h_3)}$$

$$\boxed{\varphi_2(z) = \frac{z(z+h_2)}{(z-h_1)(z-h_3)}}$$

Fermionic building block-1: $\mathrm{x} \equiv \square$





$$h = \frac{1}{2}(1+\lambda) \qquad \hat{h} = \frac{1}{2}\big(1+(1-\lambda)\big)$$

$$\boxed{h + \hat{h} = \frac{3}{2}}$$

Building blocks of bosonic affine Yangian of $\mathfrak{gl}_1$

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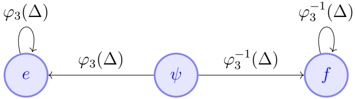
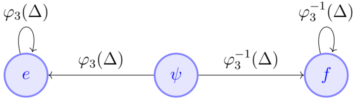
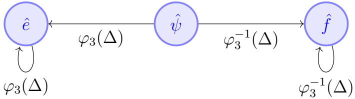
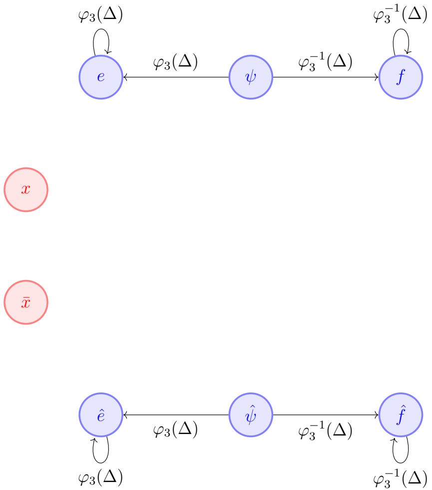


A pair of bosonic affine Yangian of $\mathfrak{gl}_1$

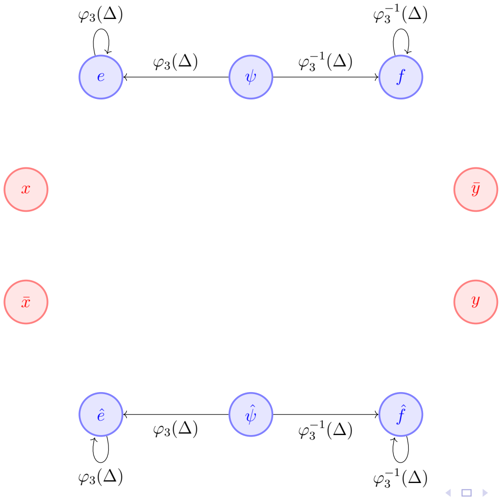




Fermionic creators



Building blocks of $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$



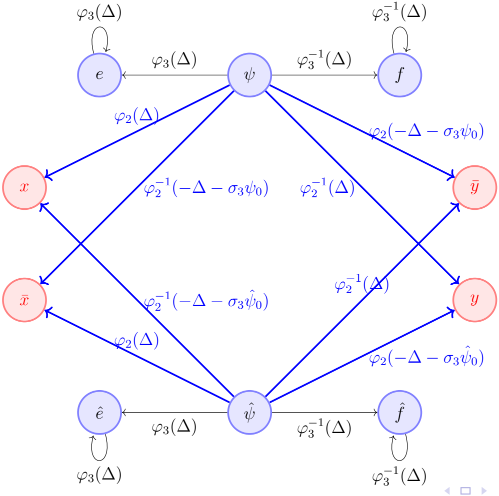
Constructing $\mathcal{N}=2$ version

1. Rewrite representations of $\mathcal{N}=2$ $\mathcal{W}_\infty$ in terms of (some version) of plane partitions

   Twin plane partition

2. Define $\mathcal{N}=2$ affine Yangian such that
   - twin plane partitions are faithful representations
   - reproduce $\mathcal{N}=2$ $\mathcal{W}_\infty$ charges

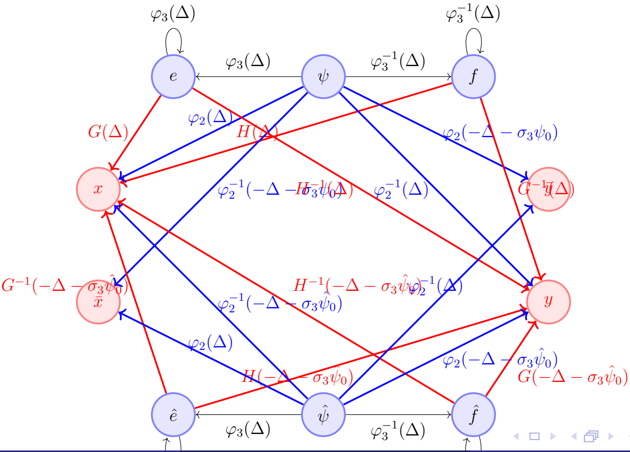
Building up $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

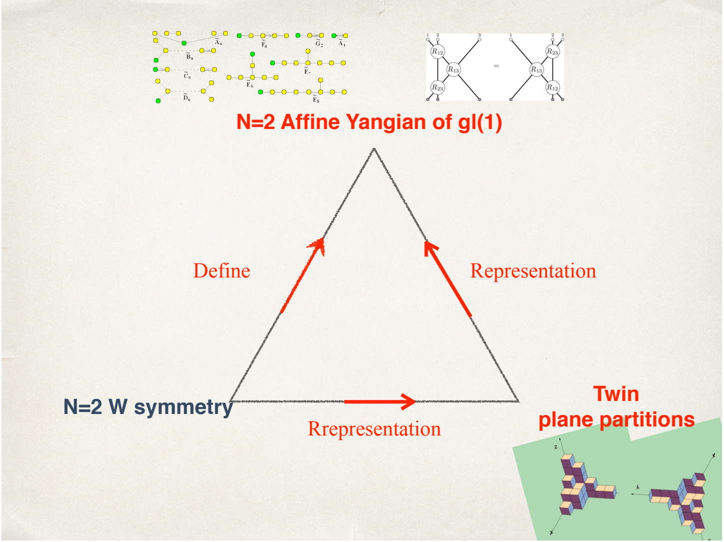


Building up $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

Gaberdiel Li Peng Zhang'17

Gaberdiel Li Peng '18



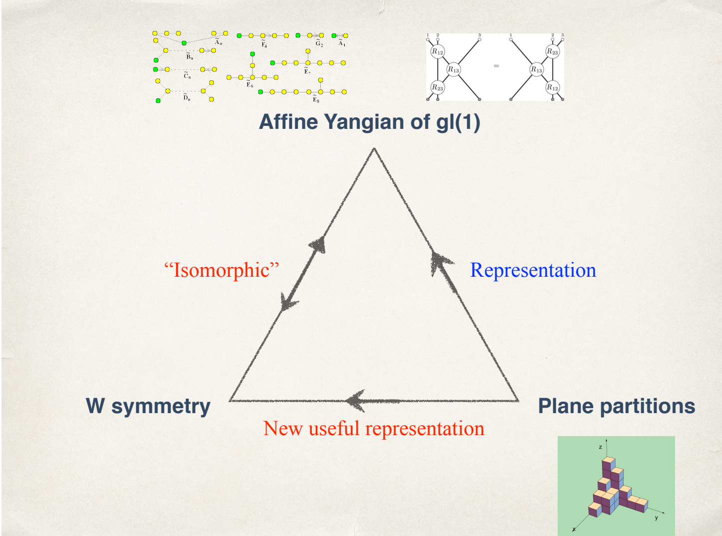

N=2 Affine Yangian of gl(1)
Define
Representation
N=2 W symmetry
Rrepresentation
Twin
plane partitions

Lessons

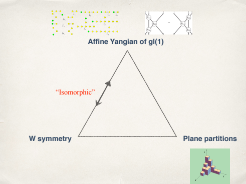
- plane partition is also very useful in the gluing process
  - visualize Fock space
  - Define algebra by faithful representation

Outline

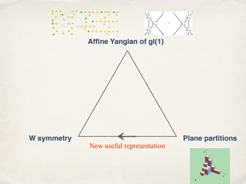
Part 1


Affine Yangian of gl(1)
"Isomorphic"
Representation
W symmetry
Plane partitions
New useful representation

Application

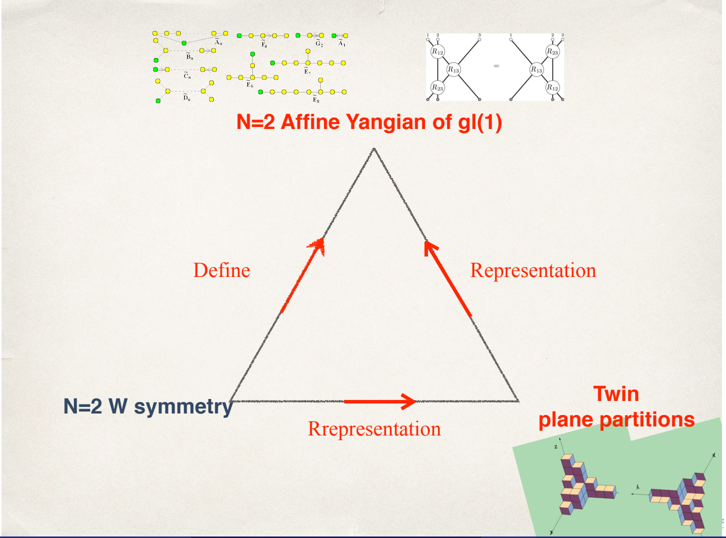


- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



- Character computation more transparent

Part 2


N=2 Affine Yangian of gl(1)
Define
Representation
N=2 W symmetry
Rrepresentation
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plane partitions

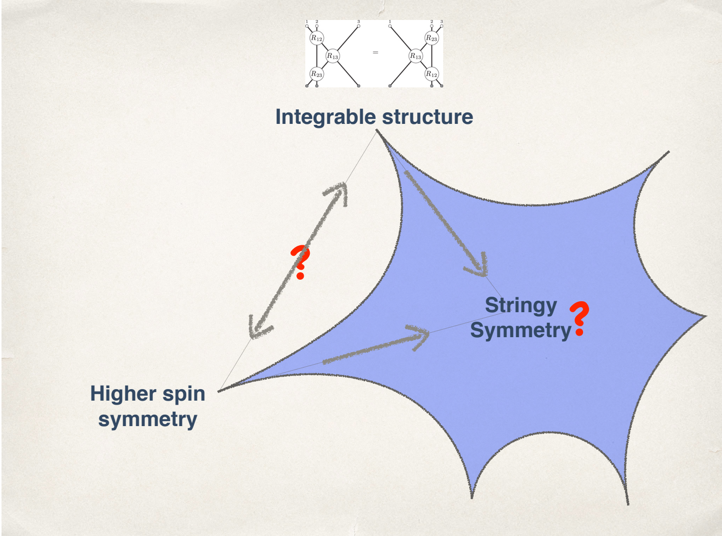
- affine Yangian is useful for $\mathcal{W}_\infty$ computation
- Can define new VOA/affine Yangian via gluing plane partitions

Open problems

1. large $\mathcal{N}=4$ $\mathcal{W}_\infty[\lambda]$

2. alternate ways of gluing

Different manifestation of stringy symmetry



More open problems

1. What is the relation between higher spin symmetry and integrable structure ?

2. What is stringy symmetry?

3. Application of stringy symmetry?

Thank you very much !