Instability of columnar vortices

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Columnar vortices

A tornado approaching Elie, Manitoba. (Wiki)

Trailing edge vortices in an aircraft. (Wiki)

Stability of axissymetric vortices with swirl

- Leibovich & Stewartson (1983)
- The instability of incompressible, inviscid, concentrated vortex flows.
- The base flow :

- Distribution of vorticity decays with radial distance from the axis of symmetry.

- Numerical experiments have suggested that the most dangerous modes are those with positive azimuthal wavenumber.
- These modes for large azimuthal wavenumber are associated with a critical radius and are concentrated near that. Hence these are also called ring modes.
- An asymptotic theory is employed to study growth rates of such modes.

Formulation

- Incompressible, inviscid fluid filling all space with velocity (0,V(r),W(r)), in cylindrical coordinates.
- Perturbations are assumed to be of the form:
- 'n' is taken as an integer
- Euler equations become:
- where:
- Boundary conditions are :

Asymptotic analysis for large 'n'

- To study the dynamics for large 'n' the problem is reformulated as an eigen value problem:

- where:

- Now in the neighborhood of 'r' where the Re(K) reaches its -ve minimum the function K is expanded to get:

- Now for lowest order in 1/n

- For the requirement of minimum for K we get the value of critical r.

- Which for the trailing edge vortex can be calculated from the equation:

- Now the functions can be expanded near this critical r and the growth rate till the leading order can be obtained to get the instability criteria:

Including Viscosity

- Stewartson (1982) includes viscosity in this calculation and gives the growth rate as:

- Landman & Saffman (1980) use the Kelvin waves formulation and get the viscous term as:

- where k is the wavevector.