Droplets in Turbulent Flows

Lessons for the Microphysics of Clouds

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21^{st} **July, 2016**

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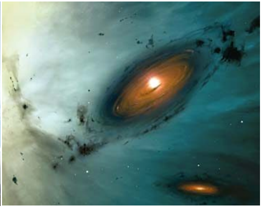
Summer Research Program on Dynamics of Complex Systems

International Centre for Theoretical Sciences, Bangalore

Where do we find them?

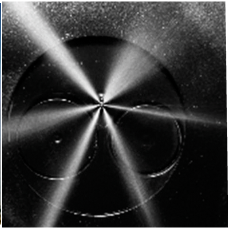
Cloud formation

Pyroclastic flows



Planetary formation

Pollutant dispersion

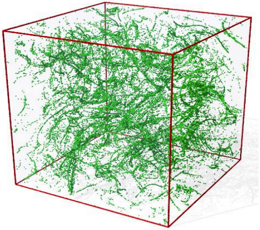


Industry

Planktons and marine biology

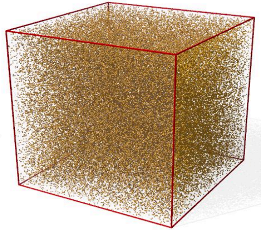
Types of Particles

$$\boxed{\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}}$$



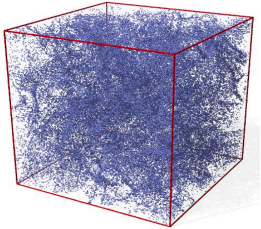
Light particles

$\rho_p \ll \rho_f$
$\beta = 3$



Tracers

$\rho_p = \rho_f$
$\beta = 1$



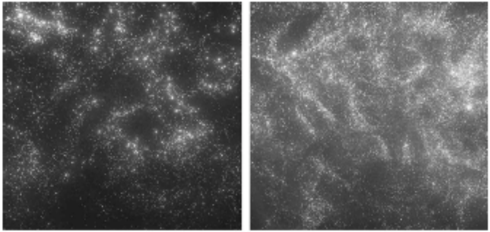
Heavy particles

$\rho_p \gg \rho_f$
$\beta \ll 1$

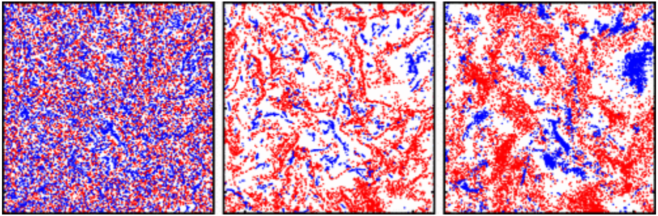
$$\boxed{\tau_p = \frac{2a^2\rho_p}{9\nu\rho_f}}$$

Effect of Inertia: Preferential Concentration

Experiments:



Simulations:



A. M. Wood, *et al.*, Int. J. Multiphase Flow, **31** (2005).
E. Calzavarini, *et al.*, Phys. Rev. Lett., **101** (2008).

Single particle dynamics

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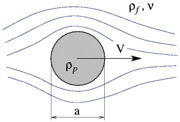
Single, passive, spherical, inertial, particle of radius a, mass m_p.

$$
\begin{aligned}
\rho_p \frac{d\mathbf{v}}{dt} = \rho_f \frac{D\mathbf{u}}{Dt} + (\rho_p - \rho_f)\mathbf{g} \\
- \frac{9\nu\rho_f}{2a^2}\left(\mathbf{v} - \mathbf{u} - \frac{a^2}{6}\nabla^2\mathbf{u}\right) \\
- \frac{\rho_f}{2}\left(\frac{d\mathbf{v}}{dt} - \frac{D}{Dt}\left[\mathbf{u} + \frac{a^2}{10}\nabla^2\mathbf{u}\right]\right)
\end{aligned}
$$



$$\boxed{\frac{a(u-V)}{\nu} \ll 1 \qquad a \ll \eta}$$

M. R. Maxey & J. J. Riley, Phys. Fluids **26**, 883 (1983).

Stokes Drag Model

Setting:

- small sized particles;
- dilute suspensions;
- passive particles.

Simplifications:

- The Faxen correction $a^2\nabla^2\mathbf{u} \approx \mathcal{O}(a^2 u/L) \ll 1$.
- $\frac{D\mathbf{u}}{Dt} \approx \frac{d\mathbf{v}}{dt}$
- Buoyancy effects negligible.

Working Equations (for heavy particles):

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{v}; \\ \frac{d\mathbf{v}}{dt} &= -\frac{\mathbf{v}-\mathbf{u}}{\tau_p}. \end{aligned}$$

Introduction

- In warm clouds, turbulence in the airflow enhances the collision rate of the water droplets.
- It thus influences the evolution of droplet sizes and the timescale for rain formation.
- Two mechanisms are at play:
  - preferential concentration;
  - very large approach velocities explained in terms of the *sling effect* and the subsequent formation of *caustics*.
- Open question regarding the coalescence rate of droplets.
  - Collisions that are too violent can cause particle fragmentation.
- Developing an understanding:
  - Experiments
  - Theory
  - Direct Numerical Simulations

G. Falkovich, *et al*, Nature **419**, (2002).
R. Shaw, Ann. Rev. Fluid Mech. **35** (2003).
E.-W. Saw, *et al.*, Phys. Rev. Lett. **100** (2008).
M. Wilkinson, *et al*, Phys. Rev. Lett. **97** (2006).
E. Balkovsky, *et al.*, Phys. Rev. Lett. **86** (2001).
J. Bec, *et al*, Phys. Rev. Lett. **98** (2007).
G. P. Bewley, *et al.*, New J. Phys. **15** (2013).
G. Falkovich & A. Pumir, J. Atmos. Sci. **64** (2007).

Questions

- How fast do droplets collide?
- How frequently do droplets collide?
- How fast do droplets settle under gravity?
- How fast do droplets grow through coalescence?

Saw, Bewley, Bodenschatz, Ray, and Bec, Physics of Fluids Letters, **26**, 111702 (2014).
Bec, Homann, and Ray, Physical Review Letters **112**, 184501 (2014).
Bec, Ray, Saw, and Homann, Physical Review E (Rapid) **93**, 031102(R) (2016).
James and Ray, ArXiv: 1603.05880 (under review) (2016).
James and Ray, (under review) (2016).

Our Approach

- **The Fluid**
  - The fluid velocity $\mathbf{u}$ is a solution of the incompressible Navier–Stokes equation and obtained via pseudo-spectral, direct numerical simulations.
  - Statistically steady, homogeneous, isotropic turbulence is maintained by a large-scale forcing.
- **The Droplets**
  - Inertial particles which obey the Stokes drag model.
    - Particles are finite-sized, much smaller than the Kolmogorov scale, much heavier than the surrounding fluid, and with a small Reynolds number associated to their slip velocity.
    - Friction (Stokes) and other forces result in their velocities different from the underlying fluid velocity.

The Model: Equations

- **The Fluid**
  - The incompressible, forced Navier–Stokes equation:

$$\begin{aligned} \partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} &= -\nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}; \\ \nabla \cdot \boldsymbol{u} &= 0. \end{aligned}$$

    - ν is the fluid kinematic viscosity and $\boldsymbol{f}$ a large scale forcing.

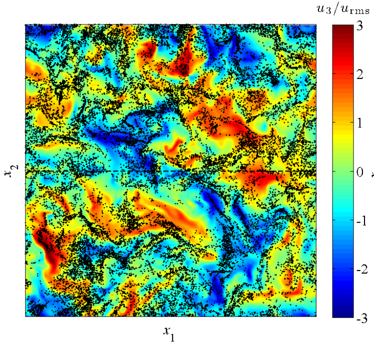
- **The Particles**
  - Stokes drag and gravity:

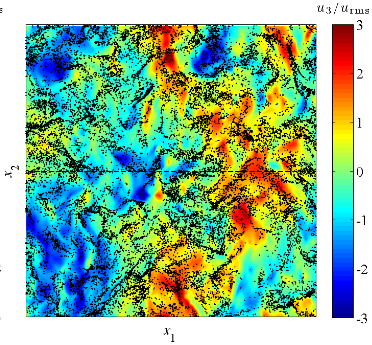
$$\begin{aligned} \frac{d\mathbf{x}_{\rm p}}{dt} &= \mathbf{v}_{\rm p}; \\ \frac{d\mathbf{v}_{\rm p}}{dt} &= -\frac{1}{\tau_{\rm p}}\left[\mathbf{v}_{\rm p} - \mathbf{u}(\mathbf{x}_{\rm p}, t)\right] + \mathbf{g}. \end{aligned}$$

    - $\mathbf{u}(\mathbf{x}_{\rm p}, t)$ is evaluated by linear interpolation.

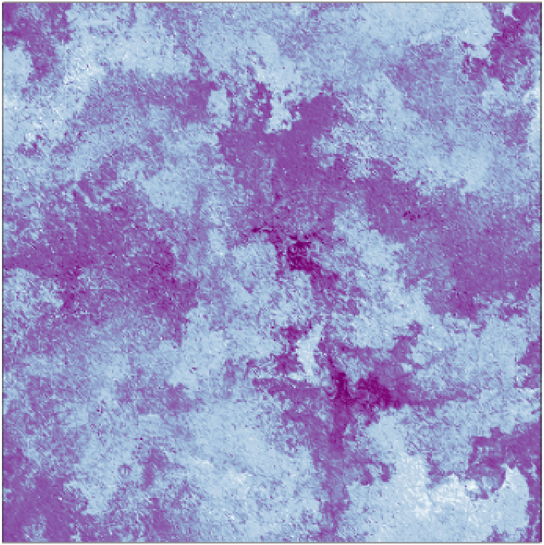
Simulation: Example

Re_λ	$u_{\rm rms}$	Δt	η	τ_η	L	T_L	N^3	N_p
460	0.189	0.0012	1.45×10^{-3}	0.083	1.85	9.9	2048^3	10×10^8
290	0.185	0.003	2.81×10^{-3}	0.131	1.85	9.9	1024^3	1.28×10^8
127	0.144	0.02	1.12×10^{-2}	0.45	2.11	14.6	256^3	0.08×10^8





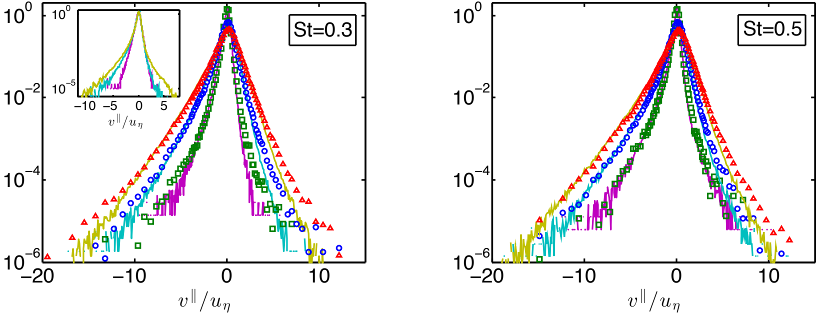
Turbulent Mixing



Distance traveled by fluid elements in a 3D turbulent flow during a large-eddy turnover time. Long (white) and short (purple) distances, represented here as a function of the final position in a 2D slice.

Bec, Ray, Saw, and Homann, Phys. Rev. E (Rapid) **93**, 031102(R) (2016).

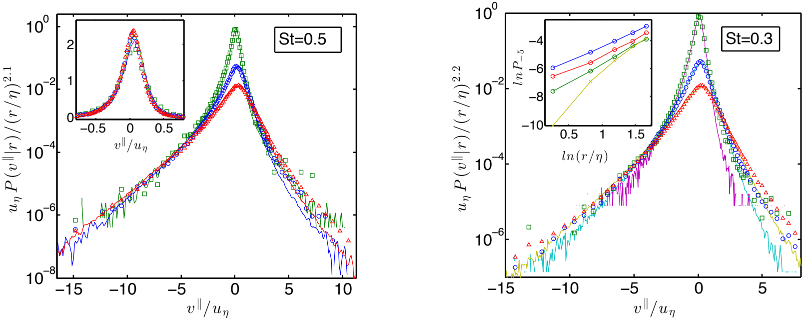
Validating the Stokes Drag Model



Probability distribution functions of the longitudinal velocity differences conditioned on different separations r for particles with (left) $St = 0.3$ and (right) $St = 0.5$. The symbols are the experimental data and solid lines are the DNS data. In all panels, for the experiment (DNS) data, squares (purple) correspond to $r = 1 - 1.6\eta$, circles (cyan) to $r = 3 - 3.6\eta$, and triangles (gold) to $r = 5 - 5.6\eta$. The inset shows the variation with respect to St, with the separation fixed to $r = 1 - 1.6\eta$. From the bottom to the top curve, $St = 0.05$, 0.3, 0.5.

Saw, Bewley, Bodenschatz, Ray, and Bec, Phys. Fluids Lett., **26**, 111702, (2014).

Relative Velocity: Rescaled PDF

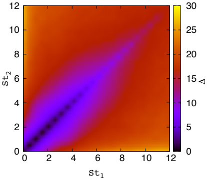


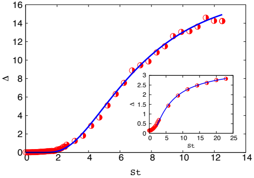
Rescaled probability distributions of the longitudinal velocity difference conditioned on different separations r for both the experimental (symbols) and DNS (solid lines) data for (left) $St = 0.5$ with $\beta = 2.1$ and (right) $St = 0.3$, with $\beta = 2.2$. Green corresponds to $r = 1 - 1.6\eta$, blue to $r = 3 - 3.6\eta$, and red to $r = 5 - 5.6\eta$. Inset (left): r-scaling of the distribution bulk; collapse is attained by $r \times p(v^{\|}|r)$ and $(1/r) \times v^{\|}/u_\eta$. Inset (right): plots of $\ln[\Pr(v^{\|}/u_\eta < 5\,|r)]$ (denoted as lnP_{-5}) versus $\ln(r/\eta)$ for different St from the experiment. Unambiguous values of β could not be obtained at such low St.

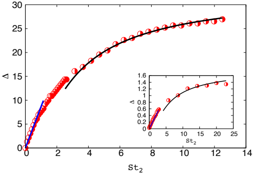
Saw, Bewley, Bodenschatz, Ray, and Bec, Physics of Fluids Letters, **26**, 111702, (2014)

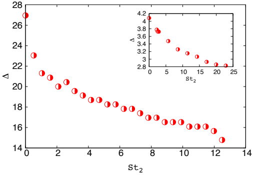
Impact Velocity: Inhomegeneous Suspension

Case	St_1	St_2	Prediction
Case 1	–	St_1	$\boldsymbol{\Delta} = \boldsymbol{\Delta}_0 \exp\left(-\tau/St\right)$
Case 2	$St_1 \ll 1$	$St_2 \lesssim 1$	$\boldsymbol{\Delta} \sim St_2$
Case 3	$St_1 \ll 1$	$St_2 \gg 1$	$\boldsymbol{\Delta} = \boldsymbol{\Delta}_0 \exp\left(-\tau/St_2\right)$
Case 4	$St_1 \gtrsim 1$	$St_2 \neq St_1$	None









James and Ray, ArXiv: 1603.05880 (under review) (2016).

Coalescence: Why are we bothered?

$$\dot{n}_i = \frac{1}{2}\sum_{j=1}^{i-1} \lambda^{\infty}_{i-j,j}\, n_{i-j}\, n_j - \sum_{j=1}^{\infty} \lambda^{\infty}_{i,j}\, n_i\, n_j.$$

- For explaining the formation of large particles in a dilute suspension, these timescales are in general not sufficiently separated.
- The sudden appearance of sizable aggregates requires a brisk sequence of coalescences that are very likely to be correlated to each other.
- When, in addition, the coalescing species are transported by a turbulent flow, such correlations speed up the growth of large particles.

Turbulent Collision Rates: Our Prediction

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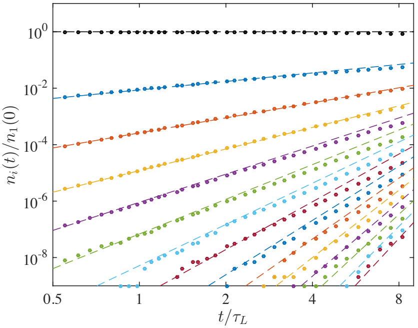
$$\boxed{\lambda_{i,j}^{\mathrm{turb}}(\tau) \propto (1/\tau_L)\,(\tau/\tau_L)^{-\frac{3}{2}\delta_3}}$$

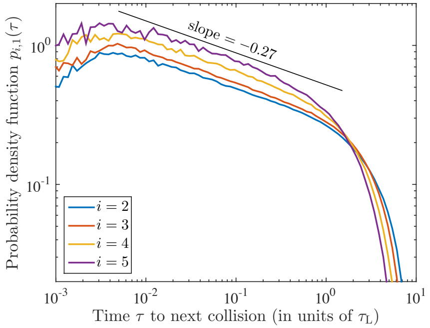
$$\boxed{n_i(t) \simeq n_1^i\,(t/\tilde{t}_i)^{(1-\frac{3}{2}\delta_3)(i-2)+1}}$$

- $\delta_3 = 0.18$ is universal for all turbulent flows.
- For $\delta_3 > 0$ the algebraic exponent is smaller than that obtained in from Smoluchowski's kinetics.
- The intermittency of turbulence mixing thus enhances the short-time growth by coalescence.
- In addition, the larger is the aggregate size considered, the stronger is this enhancement.

Bec, Ray, Saw, and Homann, Phys. Rev. E (Rapid) **93**, 031102(R) (2016).

Validating Theory: Time Evolution





$$\boxed{n_i(t) \simeq n_1^i \, (t/\tilde{t}_i)^{(1-\frac{3}{2}\delta_3)(i-2)+1}}$$

$$\boxed{\lambda_{i,j}^{\mathrm{turb}}(\tau) \propto (1/\tau_L)\,(\tau/\tau_L)^{-\frac{3}{2}\delta_3}}$$

Bec, Ray, Saw, and Homann, Phys. Rev. E (Rapid) **93**, 031102(R) (2016).

Collision Rates: Transients & Steady States

- It is important to note that much of the work in this field have dealt with mono-disperse (same-sized droplets) suspensions in flows where both the particle dynamics and the turbulent flow itself is in a statistically stationary regime.
- In nature, however, particle suspensions are typically inhomogeneous and, because of processes such as nucleation and droplet-droplet interactions, would often be characterised by a non-stationary (transient) measures, at least on short time scales.
- In is important to explore, numerically, the intriguing possibility of a further enhancement in collision rates in transient regimes as well as the possibility of accelerated droplet growths when the suspension itself is poly-disperse.

James and Ray (under review) (2016).

Collision Rates: Effect of Transients

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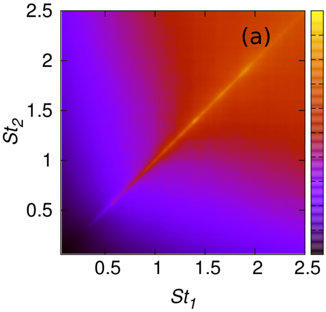

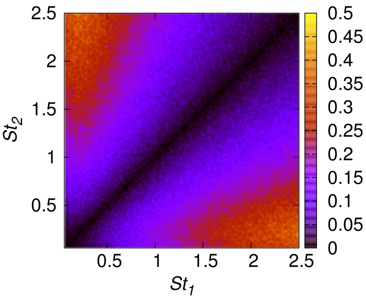

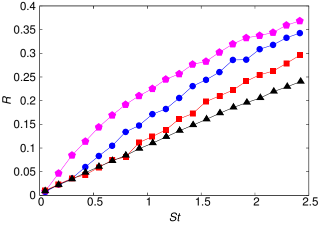

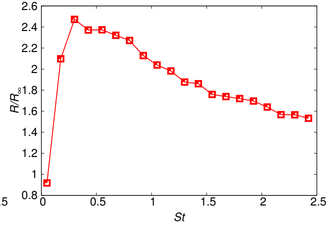

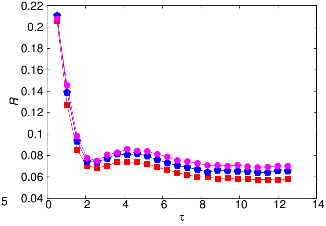

James and Ray (under review) (2016).

Particle Distribution: Effect of Gravity

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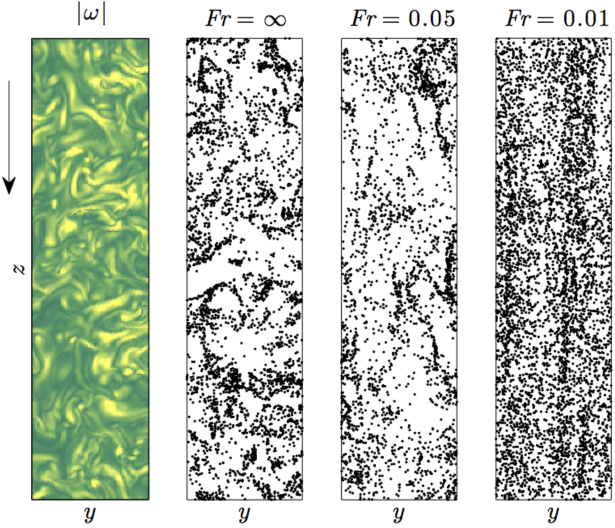

Snapshot of the vorticity modulus (Left; yellow = low values, green = high values) and of the particle positions for $R_\lambda = 130$, $\mathrm{St} = 1$ and three different values of the Froude number in a slice of thickness 10η, width 130η, and height 520η. The vertical arrow indicates gravity.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Settling Velocity: Qualitative Understanding

- Define: The average settling velocity $V_g = -\langle \boldsymbol{V}_{\rm p} \cdot \hat{\boldsymbol{e}}_z \rangle$.
- Statistical stationarity $\implies$ $V_g = \tau_{\rm p} g - \langle u_z(\boldsymbol{X}_{\rm p}, t) \rangle$.
- Define: The relative increase in settling velocity:

$$\Delta_V = (V_g - \tau_{\rm p} g)/(\tau_{\rm p} g) = -\langle u_z(\boldsymbol{X}_{\rm p}, t) \rangle / (\tau_{\rm p} g)$$

- *If* settling particles in a turbulent flow sample regions where the vertical fluid velocity is aligned with gravity, we expect an enhancement of the average settling velocity.

M. Maxey, J. Fluid Mech. **174**, (1987).
L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

Settling Velocity: Qualitative Understanding

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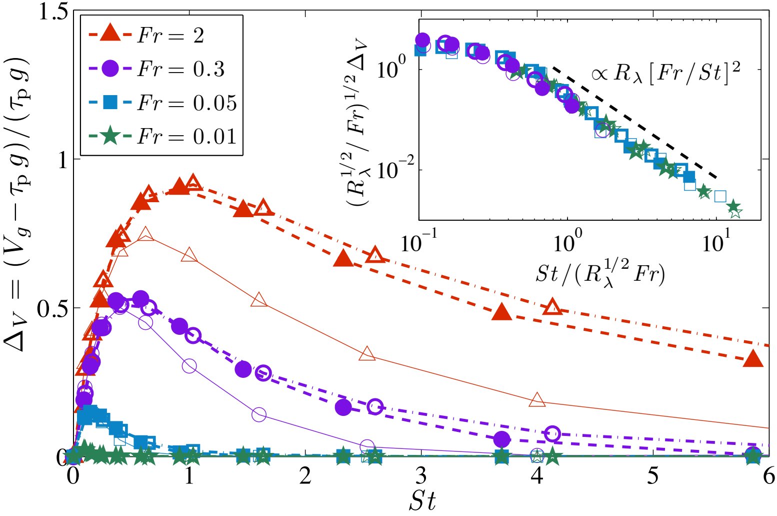
  - What is its dependence on the particle Stokes number and for different values of Fr and R_λ?
- *If* settling particles in a turbulent flow sample regions where the vertical fluid velocity is aligned with gravity, we expect an enhancement of the average settling velocity.
  - Is there a way to see this preferential sampling from the equations of motion?

M. Maxey, J. Fluid Mech. **174**, (1987).
L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

Settling Velocity



Relative increase of the settling velocity Δ_V as a function of the Stokes number St for various Froude numbers, as labeled, and $R_\lambda = 130$ (thin symbols, plain lines), $R_\lambda = 290$ (filled symbols, dashed lines) and $R_\lambda = 460$ (open symbols, broken lines). Inset: $[R_\lambda^{1/2}/Fr]^{1/2}\Delta_V$ as a function of $St/[R_\lambda^{1/2} Fr]$ for the same data.

Bec, Homann, and Ray, Phys. Rev. Lett. **112**, 184501 (2014).

Settling Velocity: Preferential Sampling

Small Stokes Asymptotics

- **Why is there an enhancement?**
  - To leading order, the particles advected by an effective compressible velocity field:

$$\boldsymbol{v} = \boldsymbol{u} - \tau_{\mathrm{p}} \left[\partial_t \boldsymbol{u} + \left(\boldsymbol{u} + \tau_{\mathrm{p}} \, \boldsymbol{g} \right) \cdot \nabla \boldsymbol{u} \right].$$

  - Focus on the (x, y) plane.
  - By using isotropy and incompressibility, we obtain:

$$\langle u_z \nabla_\perp \cdot \boldsymbol{v}_\perp \rangle = \tau_{\mathrm{p}}^2 g \left\langle (\partial_z u_z)^2 \right\rangle > 0.$$

Settling Velocity: Preferential Sampling

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Small Stokes Asymptotics

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  - Focus on the (x, y) plane.
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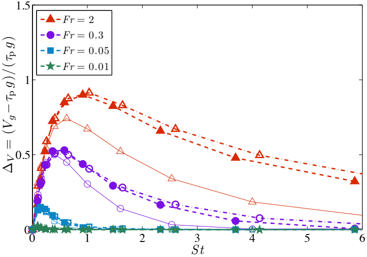
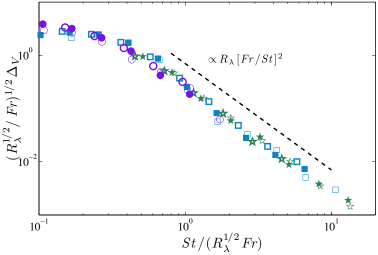
$$\langle u_z \nabla_\perp \cdot \boldsymbol{v}_\perp \rangle = \tau_{\mathrm{p}}^2 g \left\langle (\partial_z u_z)^2 \right\rangle > 0.$$

  - *Particles preferentially cluster (negative divergence), on average, in the (x, y) plane, at points where the fluid velocity is vertically downwards ($u_z < 0$).*

L.-P. Wang & M. Maxey, J. Fluid Mech. **256**, 27 (1993).
K. Gustavsson, *et al.*, Phys. Rev. Lett. **112**, 214501 (2014).

Settling Velocity: Quantitative Understanding





Small Stokes Asymptotics

$$\boxed{\Delta_V \propto \tau_\eta \tau_{\mathrm{p}} \left\langle (\partial_z u_z)^2 \right\rangle \propto St}$$

Assumptions & Algorithm:

- Relate V_g to $\langle u_z \nabla_\perp \cdot \boldsymbol{v}_\perp \rangle$.
- Hence $\langle u_z(\boldsymbol{X}_{\mathrm{p}}, t)\rangle \propto \tau_\eta \langle u_z \nabla_\perp \cdot \boldsymbol{v}_\perp \rangle$.

G. Falkovich, *et al*, Nature **419**, (2002).

Large Stokes Asymptotics

$$\boxed{\Delta_V \propto R_\lambda^{3/4} Fr^{5/2} St^{-2}}$$

Assumptions & Algorithm:

- Ballistic motion vertically: $L/V_g \ll \tau_L$.
- Effective horizontal dynamics.

Valid:

- $St \gg R_\lambda^{1/2} Fr$ and $Fr \ll R_\lambda^{1/2}$.

I. Fouxon & P. Horvai, Phys. Rev. Lett. **100**, (2008).

Conclusions and Open Questions

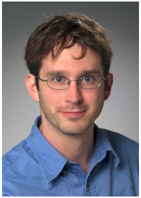
Conclusions

- Validating the limits of the linear Stokes drag model.
- Theoretical and numerical understanding of the settling of heavy particles under gravity and their approach rates.
- Intermittency of turbulent mixing is responsible for an enhanced growth of dilute coalescing aggregates.

Open Questions

- The role of intermittency of turbulent velocity statistics and non-trivial Reynolds number dependencies of particle relative velocity and coalescence statistics.
- Modelling collision kernels.
- Large particles and realistic geometry.

Acknowledgements



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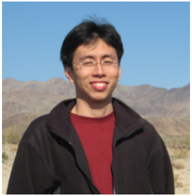

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