A proposed steering criterion using Generalised Uncertainty Relation

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Otherwise it is Entangled

e.g.

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$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle \quad \Rightarrow\Rightarrow\Rightarrow \quad \text{Pure bipartite Entangled state.}$$

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A mixed state, ρ_{AB} is separable iff, $\quad \rho_{AB} = \Sigma p_i |a_i\rangle\langle a_i| \otimes |b_i\rangle\langle b_i|$
Otherwise it is entangled.

e.g.

$$\rho_{AB} = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|) \quad \Rightarrow\Rightarrow\Rightarrow \quad \text{Bipartite mixed separable state}$$
$$\rho_{AB} = (1-p)\frac{I}{4} + p|\phi+\rangle\langle\phi+| \quad \Rightarrow\Rightarrow\Rightarrow \quad \text{Bipartite mixed entangled state with } \frac{1}{3} < p < 1$$

Different aspects of correlations present in Entanglement

1. **Nonlocality**- Correlation that can not be explained under the conjunction of realism and Locality.
Any pure bipartite entangled state is Non local. (violates Bell-CHSH inequality)

2. **Steering** - Ability to infer an observable of one system from the result of measurement performed on a second system spatially separated from the first.

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Here we want to emphasize on Reid's proposal to analyse steering in an operationally meaningful way or empirical testable way. we want to find in form of inequalities so that experiment can be done, which, when violated, imply steering.

What is Steering

Reid's sufficient condition of reality: "If, without in any way disturbing a system, we can predict with some specified uncertainty the value of a physical quantity, then there exists a probabilistic element of physical reality which determines this physical quantity with at most that specific uncertainty." (*PRA 80,032112*)

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Outcomes: $x_A(or \quad p_A)$ $\quad$ $x_B(or \quad p_B)$

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Based on the measurement of a quantity, say, $\hat{X}_A$ on her subsystem, Alice tries to infer the outcomes of Bob's measurement of the same quantity($\hat{X}_B$).

Alice can make an estimate of the result for Bob's outcome corresponding to the measurement of, X_B, $\Rightarrow X_B^{est}(X_A)$

Average inference variance of $\hat{X_B}$ and $\hat{P_B}$ defined as,

$$\Delta^2_{inf}\hat{X_B} = \langle(\hat{X_B} - \hat{X_B}^{est}(\hat{X_A}))^2\rangle \qquad \Delta^2_{inf}\hat{P_B} = \langle(\hat{P_B} - \hat{P_B}^{est}(\hat{P_A}))^2\rangle$$

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Applying Reid's reality criterion, Alice's choice of the observable that is measured can not affect the elements of reality of Bob, then there must be simultaneous probabilistic elements of reality which determine X_B and P_B with at most those uncertainties.

Now, by Heisenberg's Uncertainty relation. Quantum mechanics imposes a limit to the precession with which one can assign the values to observables corresponding to non-commuting operators $\hat{X}$ and $\hat{P}$.

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Now with sufficient condition for reality defined by Reid, the limit with which one could determine the average inference variance,

$$\Delta_{inf}X_B\Delta_{inf}P_B \geqslant \frac{1}{4}$$

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Violation of this inequality can be interpreted as signature of the correlations embodied in an Entanglement that is called Steering.

Limitations of reid criterion

Reid constructed his inequality using Heisenberg's Uncertainty principle.
Which fails to detect the steerability of the states having higher than second order correlation.
E.g. Non- Gaussian entangled states of the Orbital angular momentum, i.e. HG and LG beams., Photon subtracted squeezed vacuum state.

Here we propose a new steering criterion based on Generalised Uncertainty Relation in continuous variable scenario which is tighter than Reid's steering criterion.

Furthermore, our proposed Steering criterion can detect the steerability of the non Gaussian states having higher than the second order correlation.

Uncertainty Relation

Heisenberg's Uncertainty relation (1927): *"The more precisely the position is determined, the less precisely the momentum is known and conversely."* (*Quantum Theory and Measurement - J.A.Wheeler and W.H. Zurek, Princeton University Press*)

We can not determine position and momentum of a particle simultaneously.

Let ϵ_x and ϵ_p be the precision (mean error) with which the value x and p be known, By the famous γ ray experiment Heisenberg derived,

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$$\sigma(x)\sigma(p) \geq \frac{h}{4\pi} \qquad \Longrightarrow \text{limitations of state preparations.}$$

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E.U. Condon - Kennard's Uncertainty relation is only based on for *Conjugate variables* (FT duals of one another), What happen if they are not conjugate. (1929)

H.P.Robertson: "The uncertainty principle for two such (Hermitian) variables A, B, having $[A, B] = i\frac{h}{2\pi}$, is expressed by

$$\triangle A \cdot \triangle B \geq \frac{h}{4\pi}$$

(half the absolute value of the mean of their commutator)" - *Phys. Rev. 34,163* (1929)

Generalised Uncertainty Relation

Schrödinger generalised the uncertainty relation (1930) for any two arbitrary Observables (Hermitian Operator).

The average error or the mean uncertainty of the value of a operator A, is defined as,
$\Delta A = \sqrt{\bar{A^2} - (\bar{A})^2}$

To find out the lower bound of the product of the uncertainties of two random variables A and B, we need to denote the following statements,

(1) A and B must be hermitian so that the expectation value is always real.

(2)The product of two Hermitian Operators is in general not Hermitian.

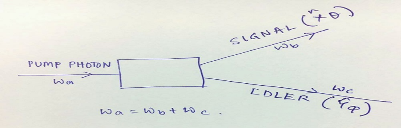
$$AB = (AB + BA)/2 + (AB - BA)/2$$

(3) Swartz inequality should satisfy. i.e.

$$(\sum a_i a_j^*)(\sum b_i b_j^*) \geq \mid (\sum a_i b_i) \mid^2$$

From this it can be proved,

$$(\Delta A)^2(\Delta B)^2 \geq \mid 1/2i\langle[A, B]\rangle \mid^2 + \mid 1/2\langle\{A, B\}\rangle - \langle A\rangle\langle B\rangle \mid^2$$



For a plane wave field, in quantum theory the *amplitudes are characterised by non hermitian anhilation operator a*.
Let consider two single mode field E_a and E_b with frequency ω_a and ω_b,

$$E_\lambda = C[\hat{\lambda} e^{-i\omega_\lambda t} + \hat{\lambda}^\dagger e^{i\omega_\lambda t}],$$

where $\lambda \in a, b$ are the Bosonic operators for two different modes, C is a constant incorporating spatial factors.
The general quadrature phase amplitudes for two modes are defined as,

$$\hat{X}_\theta = \frac{\hat{a}e^{-i\theta} + \hat{a}e^{i\theta}}{\sqrt{2}} \qquad \hat{Y}_\phi = \frac{\hat{b}e^{-i\phi} + \hat{b}e^{i\phi}}{\sqrt{2}}$$

CSI: $|\langle \hat{X}_\theta \hat{Y}_\phi \rangle|^2 \leq \langle (\hat{X}_\theta)^2 \rangle \langle (\hat{Y}_\phi)^2 \rangle$ (Violation of CSI $\Rightarrow$ Non classicality)
We define,
The correlation coefficient $\Longrightarrow C_{\theta\phi} = \frac{(X_\theta Y_\phi)}{[(X_\theta)^2 (Y_\phi)^2]^{1/2}}$

Steering:

Result of measurement performed on Idler amplitude, $Y_\phi \Rightarrow$ Infer the result for Signal amplitude, X_θ

Let us propose the Inferred estimate of the signal amplitude $\Rightarrow \hat{X_\theta}^{est} = g\hat{Y_\phi}$
g $\longrightarrow$ Possible scaling parameter, which one can adjust to allow for greatest accuracy in the determination of $\hat{X_\theta}$

Average inference error $\Rightarrow$ Deviation of the estimated value from the true signal amplitude

$$\triangle^2_{inf} X_\theta = \langle [\hat{X_\theta} - \hat{X_\theta}^{est}]^2 \rangle = \langle [\hat{X_\theta} - g\hat{Y_\phi}]^2 \rangle$$

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Best Estimation: To minimise $\triangle^2_{inf} X_\theta \Rightarrow$
(i) $C_{\theta\phi}$ should be maximum, and (ii) $\frac{\partial(\triangle^2_{inf} X_\theta)}{\partial g} = 0$ (Condition for extremisation)
We deduce that, $g = \frac{\langle \hat{X}_\theta \hat{Y}_\phi \rangle}{\langle Y^2_\phi \rangle}$

$$[\triangle^2_{inf} X_\theta]_{min} = \langle \hat{X_\theta}^2 \rangle - \frac{\langle \hat{X_\theta} \hat{Y_\phi} \rangle^2}{\langle \hat{Y_\phi}^2 \rangle}$$

Measurement of $\hat{Y_{\phi 1}} \Longrightarrow$ Simultaneously specify $\hat{X_{\theta 1}}$ with error $\triangle^2_{inf} X_{\theta 1}$
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Product of the inferred uncertainty $\geq$ measured uncertainty $\geq$ lower bound given by GUR

Steering Criterion using GUR

Here we take a **Gaussian distribution** to calculate the lower bound as it gives the **minimum uncertainty**.

We consider the **coherent state**, [Field produced by a single mode laser has well defined amplitude and phase. The appropriate state of the radiation field is coherent state satisfies, $a|\alpha\rangle = \alpha|\alpha\rangle$; (*R. J. Glauber, Phys Rev. 131, 2766 (1963)*)]

$$| \alpha >= e^{-\frac{|\alpha|^2}{2}} \sum \frac{\alpha^n}{\sqrt{n!}} \mid n >$$

For two general quadrature amplitudes, $\hat{X_{\theta 1}}$ and $\hat{X_{\theta 2}}$,

$$\langle[\hat{X_{\theta 1}}, \hat{X_{\theta 2}}]\rangle = (-i)sin(\theta 1 - \theta 2)$$

$$\langle\{\hat{X_{\theta 1}}, \hat{X_{\theta 1}}\}\rangle = \alpha^2 e^{i(\theta 1+\theta 2)} + (\alpha^*)^2 e^{-i(\theta 1+\theta 2)} = r^2 cos(\theta 1 + \theta 2 + 2\xi),\ \alpha = re^{i\xi}$$

$$\langle X_\theta \rangle = \frac{1}{\sqrt{2}}[\alpha e^{i\theta} + \alpha e^{-i\theta}],\ \theta \in \{\theta 1, \theta 2\}$$

The lower bound, $| 1/2i\langle[A, B]\rangle |^2 + | 1/2\langle\{A, B\}\rangle - \langle A\rangle\langle B\rangle |^2 \geq \frac{5}{4}$

The new steering inequality is given by,

$$S_{EPR} \equiv (\Delta_{inf} X_{\theta 1})^2 (\Delta_{inf} X_{\theta 2})^2 < 5/4$$

Results: (1)Two mode squeezed vacuum state

A common resource of the quantum entanglement in CV is the two-mode squeezed vacuum state generated by means of spontaneous parametric down-conversion in the non-degenerate optical parametric amplifier (NOPA).
Two mode squeezed state can be generated by applying two mode squeezing operator on the two mode vacuum state, $|0,0\rangle$

$$|\xi\rangle = S(\xi)|0,0\rangle = exp(\xi \dagger a \dagger b - \xi^* ab)|0,0\rangle; \quad \xi = re^{i\phi}$$
$$= \sqrt{1-\lambda^2}\sum \lambda^n \mid n, n >; \quad \lambda = \tanh r \in [0,1]$$

We calculate,

$$\langle \hat{X}_\theta \hat{Y}_\phi \rangle = 2cosh(r)sinh(r)sin(\theta+\phi)$$
$$\langle \hat{X_\theta}^2 \rangle = \hat{\langle} Y_\phi{}^2 \rangle = cosh^2(r) + sinh^2(r)$$
$$C_{\theta\phi} = sin(\theta+\phi);$$

The inferred uncertainty is given by,

$$(\Delta_{inf} X_\theta)^2 = \frac{1}{2}cosh(2r) - \frac{1}{2}tanh(2r)sinh(2r)cos^2(\theta+\phi),$$

For two different values of θ i.e. $\theta_1 = 0$ and $\theta_2 = \pi/2$,

$$(\Delta_{inf} X_{\theta 1})^2 = \frac{1}{2\cosh[2r]} \text{ (Minimized when } \phi = 0)$$
$$(\Delta_{inf} X_{\theta 1})^2 = \frac{1}{2\cosh[2r]} \text{ (Minimized when } \phi = \tfrac{\pi}{2})$$
$$(\Delta_{inf} X_{\theta 1})(\Delta_{inf} X_{\theta 2}) = \frac{1}{4\cosh[2r]} \longrightarrow 0 \text{ for } r \longrightarrow \infty$$

Two mode squeezed vacuum state shows EPR steering (from Reid's condition) for all values of r except $r = 0$.

(2) Photon-subtracted squeezed vacuum state:

Non-Gaussian states can be derived from Gaussian states by the subtraction of photons. (Process of subtraction can not make a classical field non classical.)

Considering single-photon reduction from each mode, the state becomes

$$|\alpha> = \sqrt{1-\lambda^2}\sum \lambda^n \sqrt{n}|n-1, n> + (-1)^k|n, n-1>$$

$$(\Delta inf X_\theta)^2 = \cosh(2r) - \sinh(r)\cosh(r)\cos(2\theta) - \frac{[\cosh(2r)\cos(\theta-\phi) - 2\sinh(2r)\cos(\theta+\phi)]^2}{4[\cosh(2r) - \sinh(r)\cosh(r)\cos(2\phi)]}$$

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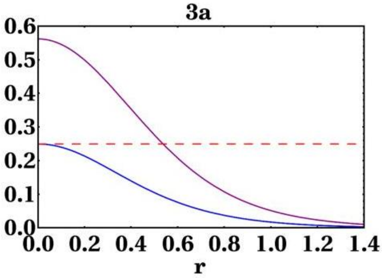
The product of uncertainties turns out to be, (Calculating the minimum value for two different values of θ, i.e. $\theta 1 = 0$ and $\theta 2 = \frac{\pi}{2}$. The minimum occurs when $\phi 1 = \frac{\pi}{2}$ and $\phi 2 = 0$ respectively)

$$(\Delta inf X_{\theta 1})^2 (\Delta inf X_{\theta 2})^2 = \frac{9}{2[3\cosh 4r + 5]}$$



(3)Non-Gaussian entangled states of the OAM - HG and LG beams:

OAM is the important property of the light (2D fields, also can be realised as 2D HO). In paraxial optics (Propagation vectors are close to the axis, i.e., Transverse component of momentum $<$) longitudinal or axial component) two types of beams can be considered, (A)HG modes and (B) LG modes.

$$\textbf{(A)}\ u_{nm}(x,y) = \sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{2^{n+m}w^2 n!m!}}H_n(\frac{\sqrt{2}x}{w})H_m(\frac{\sqrt{2}y}{w})e^{-\frac{(x^2+y^2)}{w^2}}; 0 \le n,m \le \infty$$

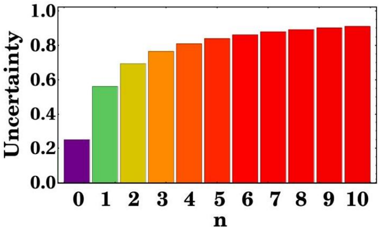
$$\textbf{(A)}\ \phi_{n,m}(\rho,\theta) = e^{i(n-m)\theta}e^{-\frac{\rho^2}{\omega^2}}(-1)^{min(n,m)}(\frac{\rho\sqrt{2}}{\omega})^{|n-m|}\sqrt{\frac{2}{\pi n!m!\omega^2}}L^{|n-m|}_{min(n,m)}(\frac{2\rho^2}{\omega^2})[min(n,m)]!$$

$N = n+m \Rightarrow$ Order of the mode; $l = n-m \Rightarrow$ Azimuthal index; $min(n,m) \Rightarrow$ Radial index
LG modes carry an OAM $\hbar l$ per photon.
For $n = m = 0 \implies u_{00}$ and ϕ_{00} both are Gaussian beams.
For $n+m = 1 \implies$ The two modes differ for both beams.
Let the pump field is taken to be u_{00}, Signal and idler fields can be given in terms of the ϕ_{01} and ϕ_{10}
LG modes are linear combination of HG modes for the same value of N. So Entangled states can be constructed from HG mode . Considering the special case,

$$\phi_{10} = \frac{2}{\sqrt{\pi}w^2}(x+iy)e^{-\frac{(x^2+y^2)}{w^2}} \qquad \phi_{01} = \frac{2}{\sqrt{\pi}w^2}(x-iy)e^{-\frac{(x^2+y^2)}{w^2}}$$

w $\Rightarrow$ Beam waist.
$(\Delta_{inf}X_\theta)^2 = \langle X_\theta^2\rangle[1-(C^{max}_{\theta,\phi})^2]$; $|C^{max}_{\theta,\phi}| = \frac{1}{2}$, occurs for $\phi - \theta = \frac{k\pi}{2}$, $k \in$ Odd Integer

We plot the product of uncertainties $(\Delta_{inf} X_{\theta 1})(\Delta_{inf} X_{\theta 2})$ vs the angular momentum n



Conclusions

1. In the present work we have studied EPR steering by several examples of non-gaussian entangled pure states which are failed to reveal steering through Reid criterion for wide ranges of parameter. Steering with such states is demonstrated using our proposed steering criterion.

2. The states we considered are also steerable according to the entropic steering inequality (*Phy. Rev. A, 89, 012104 - 2014*). It will be interesting to investigate the robustness between these two inequalities.

3. Investigation of Steerability of Werner class states in continuous variable scenario will be one of the future direction of our work.

4. This idea can be applied for discreet spin systems to achieve the steerability conditions for different class of states.

5. Fluctuations can be defined in different way to obtain new kind of inequalities which will be our future study.

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Thank You

Appendix

Paraxial Laguerre-Gauss (LG) beams carry a well defined orbital angular momentum associated with their spiral wave fronts. The energy eigenfunctions of the 2D HO may be expressed in terms of HG functions. Entangled states may be constructed from superpositions of HG wave functions,

$$\phi_{n,m}(\rho,\theta) = \Sigma_{k=0}^{m+n} u_{n+m-k,k}(x,y)\frac{f_k^{n,m}}{k!}(-1)^k\sqrt{\frac{k!(n+m-k)!}{n!m!2^{n+m}}};\ f_k^{n,m} = \frac{d^k}{dt^k}[(1-t)^n(1+t)^m]|_{t=0}$$

General LG functions is given by,

$$\phi_{n,m}(\rho,\theta) = e^{i(n-m)\theta}e^{-\frac{\rho^2}{\omega^2}}(-1)^{min(n,m)}(\frac{\rho\sqrt{2}}{\omega})^{|n-m|}\sqrt{\frac{2}{\pi n!m!\omega^2}}L_{min(n,m)}^{|n-m|}(\frac{2\rho^2}{\omega^2})[min(n,m)]!$$

With $\int|\phi_{n,m}(\rho,\theta)|^2dxdy = 1$ Where, $L_p^l(x)$ is general Laguerre polynomial.
The index determines the dependence of the modes on the azimuthal phase, ϕ, and each mode carrying an Orbital angular momentum (OAM) of $m\hbar$ per photon. LG modes form a complete Hilbert basis and can thus be used to represent the spatial quantum photon states within the paraxial regime. In this regime, the LG modes are eigenmodes of the quantum mechanical orbital angular momentum operator, $L_z|m,p\rangle = m\hbar|m,p\rangle$. Photons represented by a single LG mode, $|\psi\rangle = |m,p\rangle$, are in a quantum state with a well-defined value of the orbital angular momentum $(m\hbar)$.

We consider the dimensionless Quadratures for two mode be $\{X, P_X\}$ and $\{Y, P_Y\}$, given by,

$$x(y) \rightarrow \frac{\omega}{\sqrt{2}} X(Y), \qquad p_x(p_y) \rightarrow \frac{\sqrt{2}\hbar}{\omega} P_X(P_Y)$$

The Wigner function corresponding to the LG wave function in terms of the scaled variables,

$$W_{nm}(X, P_X; Y, P_Y) = \frac{(-1)^{n+m}}{\pi^2} L_n[4(Q_0 + Q_2)] L_m[4(Q_0 - Q_2)] e^{4Q_0}$$
$$Q_0 = \frac{1}{4}[X^2 + Y^2 + P_X^2 + P_Y^2], \text{ and } Q_2 = \frac{XP_Y - YP_X}{2}$$

To check Steerability criterion we calculate the inferred variance uncertainty and performed the minimisation by maximizing the correlation coefficient $C_{\theta,\phi}$.