University of Amsterdam

Many-body strategies for multi-qubit gates

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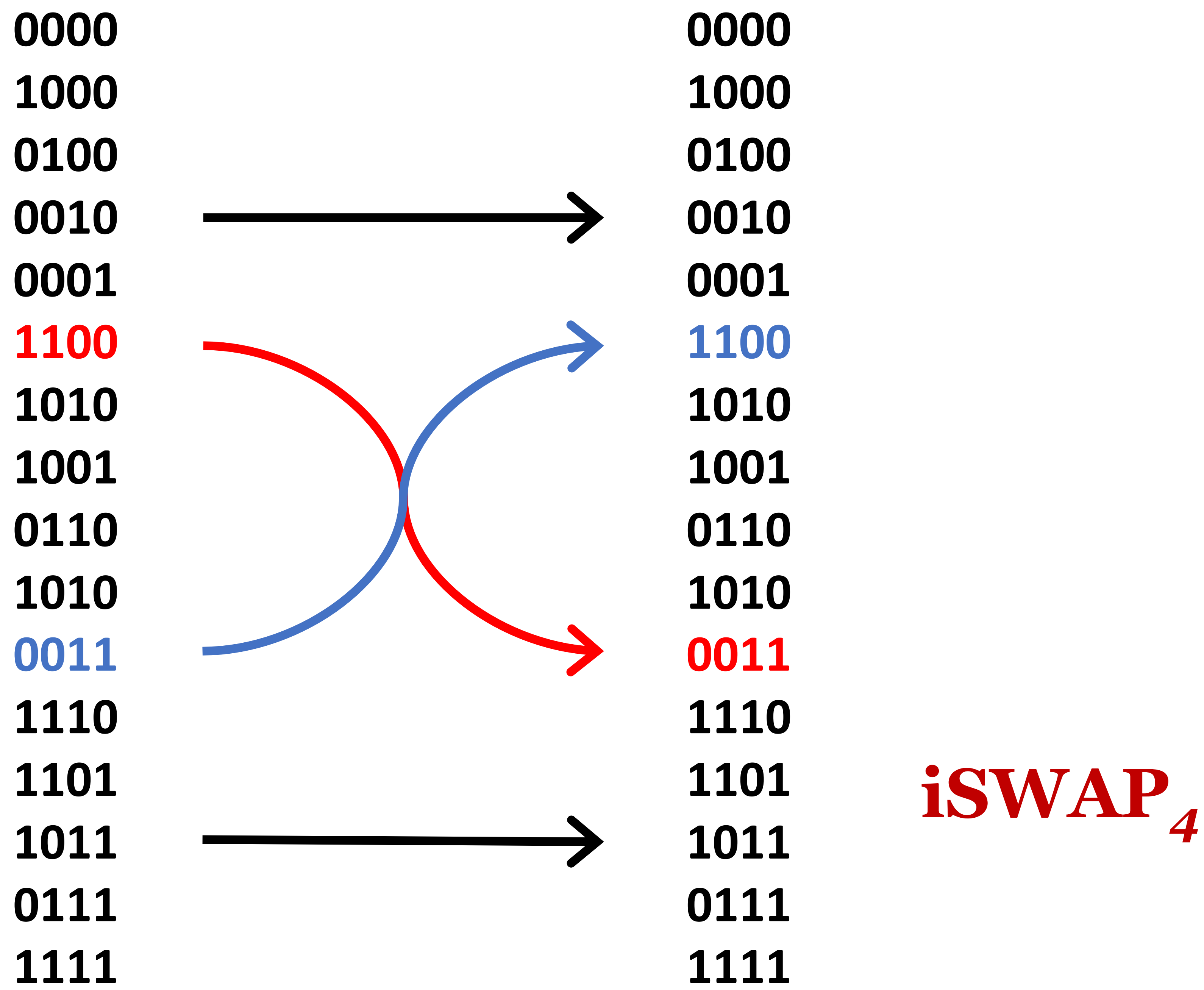
ICTS Bangalore, 31 July 2018

QuSoft

Delta Institute for Theoretical Physics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$iSWAP_4$

Summary

we propose many-body strategies to realize **multi-qubit** quantum gates on *N*-qubit registers

we use 2-qubit interactions only

our protocol combines **eigengates** with selective **resonant driving**

we demonstrate results for **Krawtchouk** qubit chain and **Polychronakos** chain with inverse square interaction

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0010	0010
0001	0001
1100	1100
1010	1010
1001	1001
0110	0110
1010	1010
0011	0011
1110	1110
1101	1101
1011	1011
0111	0111
1111	1111

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Many-body strategies for multiqubit gates: Quantum control through Krawtchouk-chain dynamics

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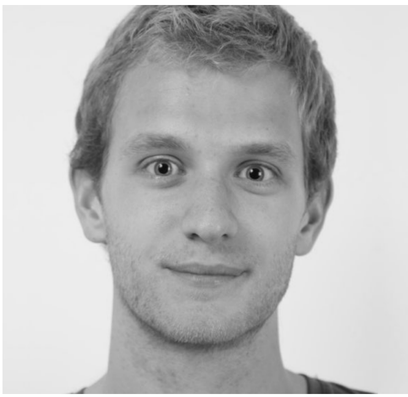
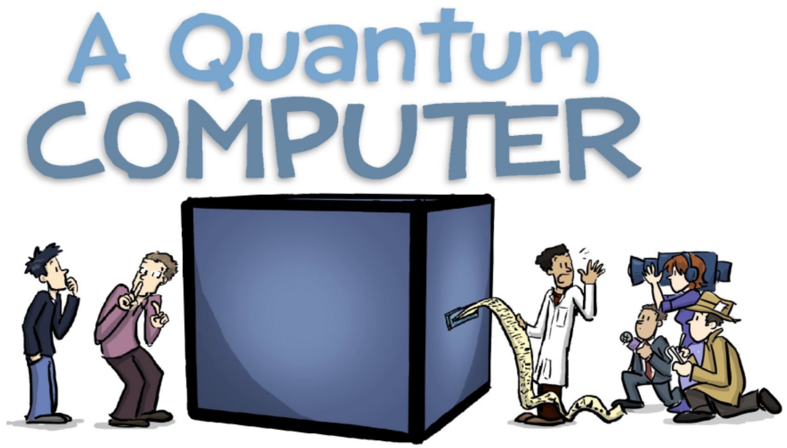
We propose a strategy for engineering multiqubit quantum gates. As a first step, it employs an *eigengate* to map states in the computational basis to eigenstates of a suitable many-body Hamiltonian. The second step employs resonant driving to enforce a transition between a single pair of eigenstates, leaving all others unchanged. The procedure is completed by mapping back to the computational basis. We demonstrate the strategy for the case of a linear array with an even number N of qubits, with specific $XX + YY$ couplings between nearest neighbors. For this so-called Krawtchouk chain, a two-body driving term leads to the iSWAP_N gate, which we numerically test for $N = 4$ and 6.



QuSoft

outline



- **background and motivation**
- many-body strategies for multi-qubit gates
- quantum control on the Krawtchouk and Polychronakos chains
- multi-qubit gates

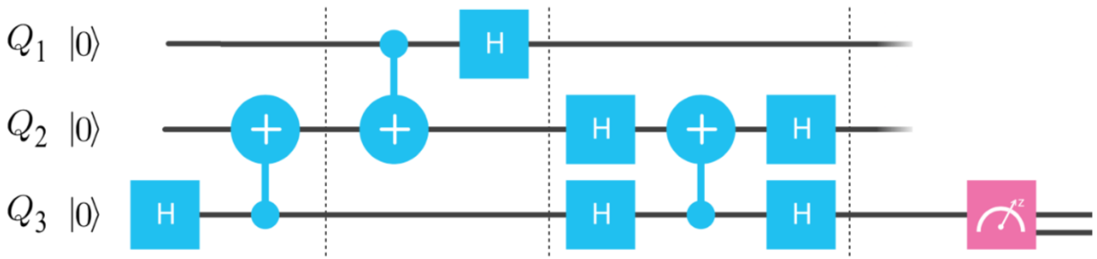
quantum algorithms

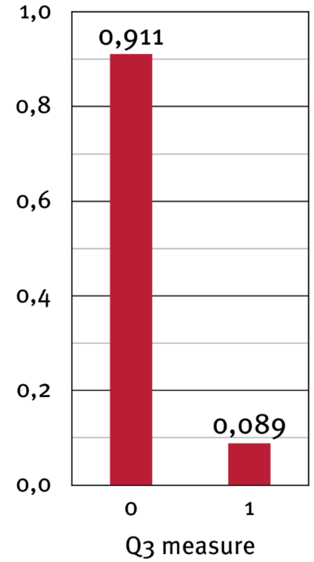
For specific problems quantum algorithms can be made to outperform classical computers by cunningly combining quantum parallelism with interference.

quantum circuit

3-step implementation of quantum algorithm on N-qubit quantum register

- **initialization**
- **unitary evolution** via quantum gates
- read-out through **measurement**

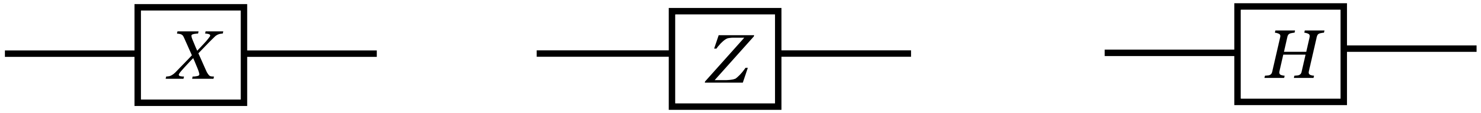




IBM Quantum Experience

quantum gates

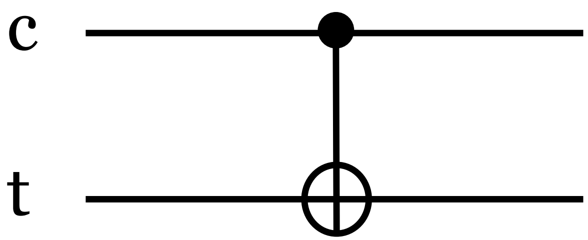
- **1-qubit gates:** *X*, *Z*, *H*, ...



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- **2-qubit gates:** CNOT, *XX(θ)*, SWAP, ...

CNOT:



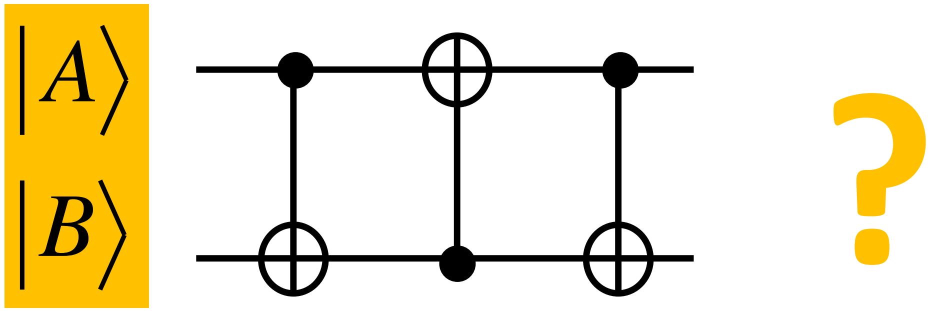
flips target qubit t iff control qubit c is in state |1>

universal gate sets

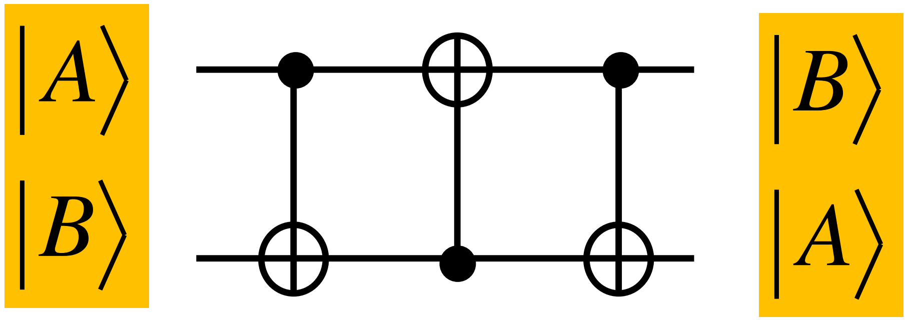
strong universality

all N-qubit unitaries can be built from CNOTs
plus sufficiently many 1-qubit gates

universal gate sets



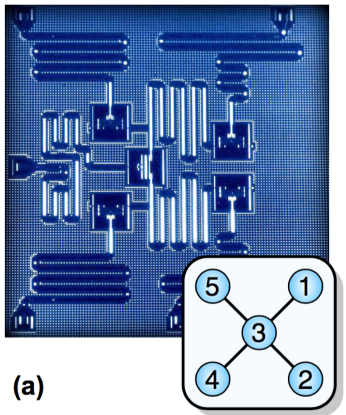
universal gate sets



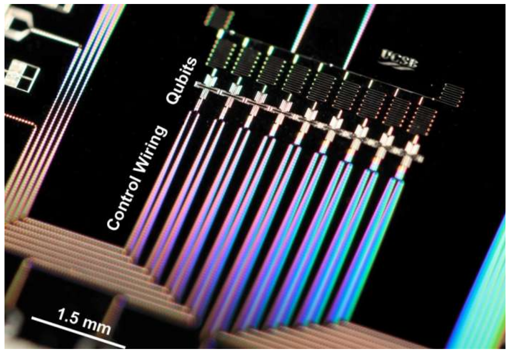
SWAP gate

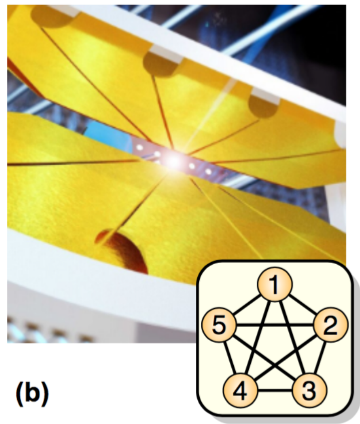
state of the art

quantum hardware has progressed to the point that programmable qubit platforms with up to some 20 qubits are available
→ real-world testing of few-qubit quantum algorithms!



(a)





(b)

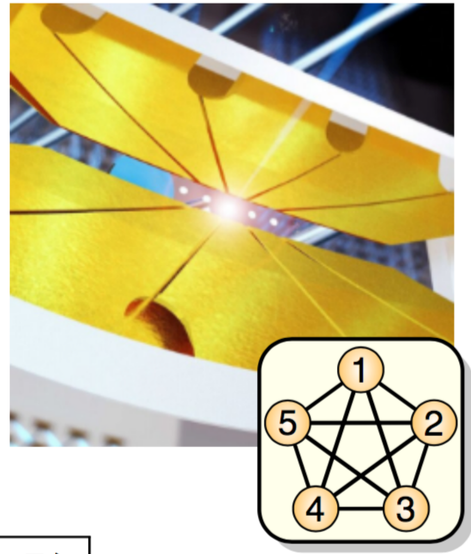
native gates and quantum compiling

- **native gate libraries**

  the 1-qubit and 2-qubit interactions that are natural for a given qubit platform lead to a `native gate library'.

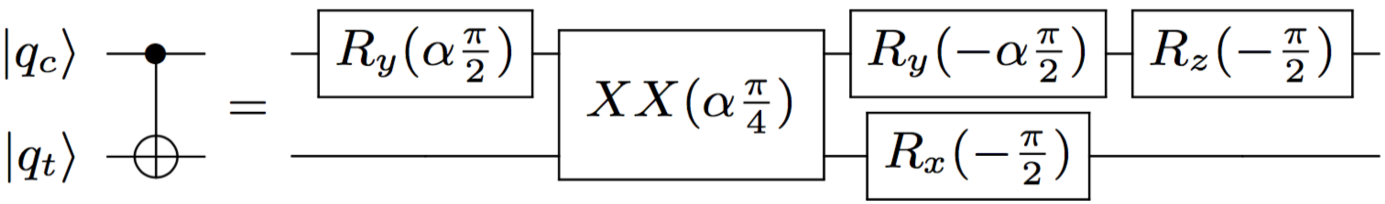
- **quantum compiling**

  expressing universal gates in native gates



example: native gate library for trapped ions

- all 1-qubit rotations $R_\alpha(\theta)$
- 2-qubit gates $X_i X_j (\theta)$



Complete 3-Qubit Grover Search on a Programmable Quantum Computer

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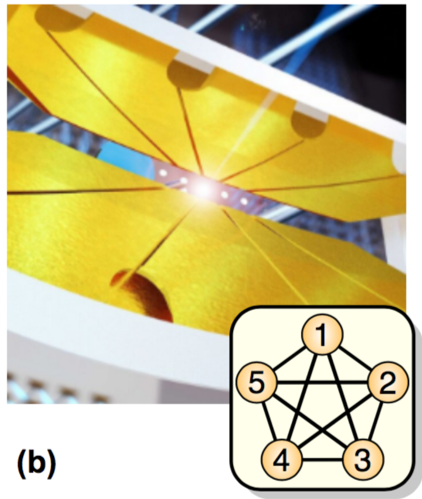
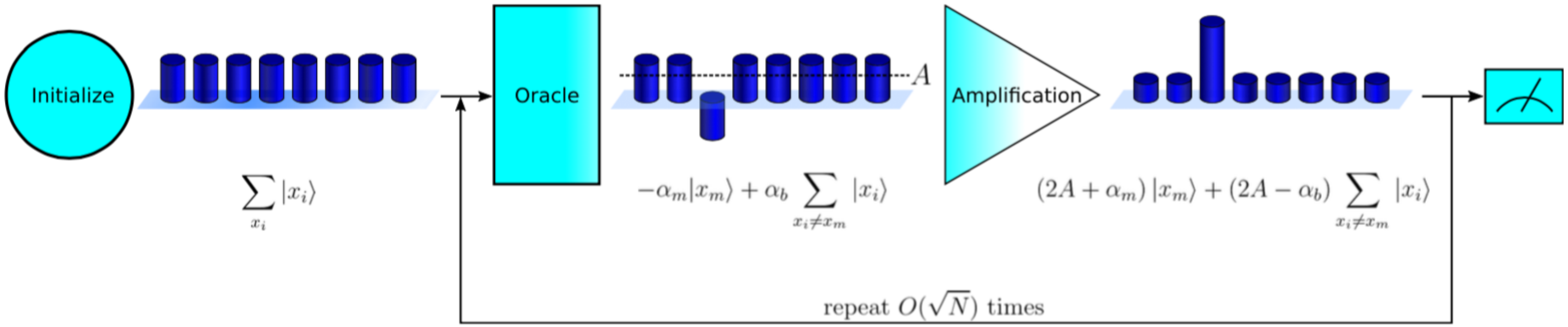
[2] *National Science Foundation, Arlington, VA 22230, USA*

[3] *IonQ Inc., College Park, MD 20742, USA*

(Dated: March 31, 2017)



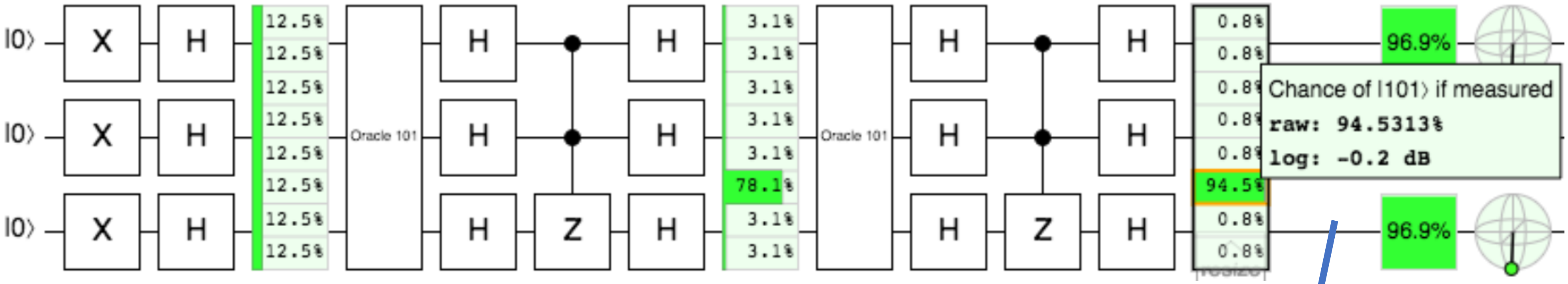
Grover search: finding tagged element in size-*N* database in $O(\sqrt{N})$ steps



3-qubit Grover search

finds 1 tagged element out of 8 in two steps

quantum circuit on Quirk simulator:



Oracle tagging the element |101>

Initializing the qubits to |0>

read-out gives tagged element |101> with 94.5% chance

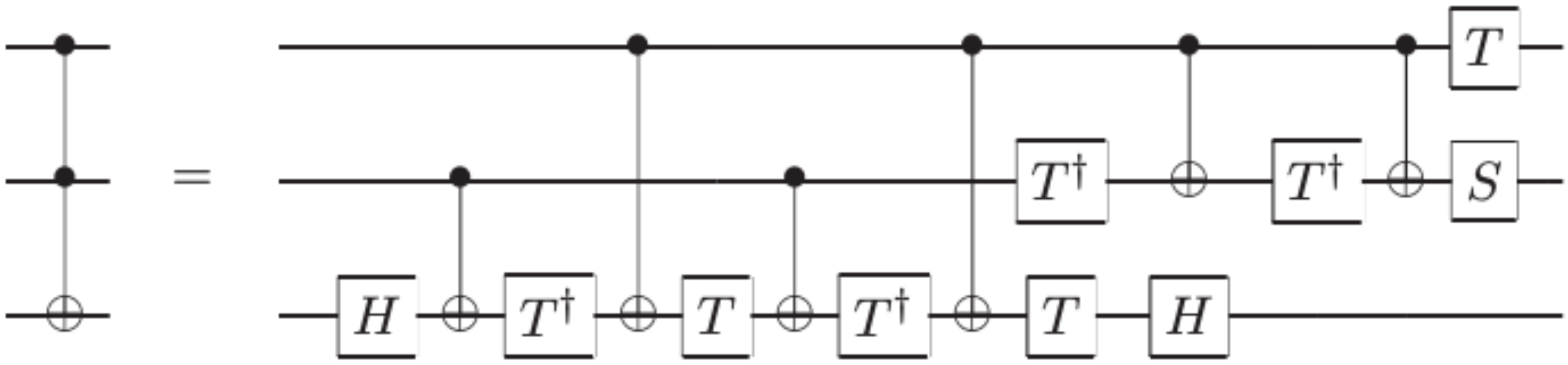
multi-qubit gates

- quantum algorithms such as Grover search use gates like

CCNOT (Toffoli), CCZ, ... , C^{N-1}NOT, C^{N-1}Z, etc

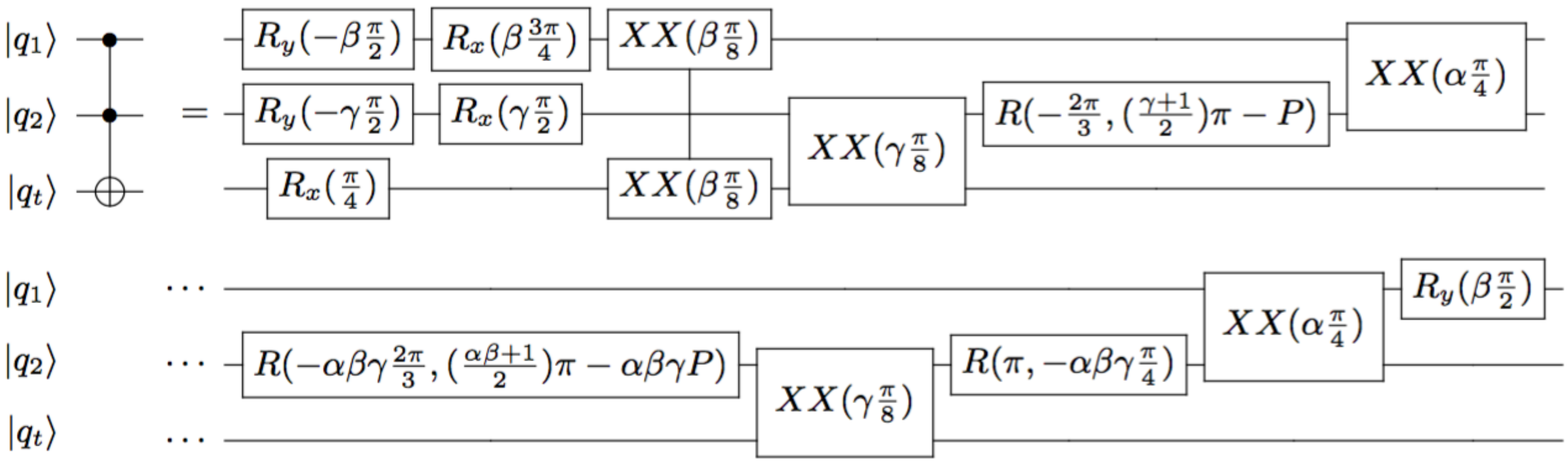
- building these from 1-qubit and 2-qubit gates requires lengthy circuits

multi-qubit gates



Toffoli-3 using standard Clifford + T gate library

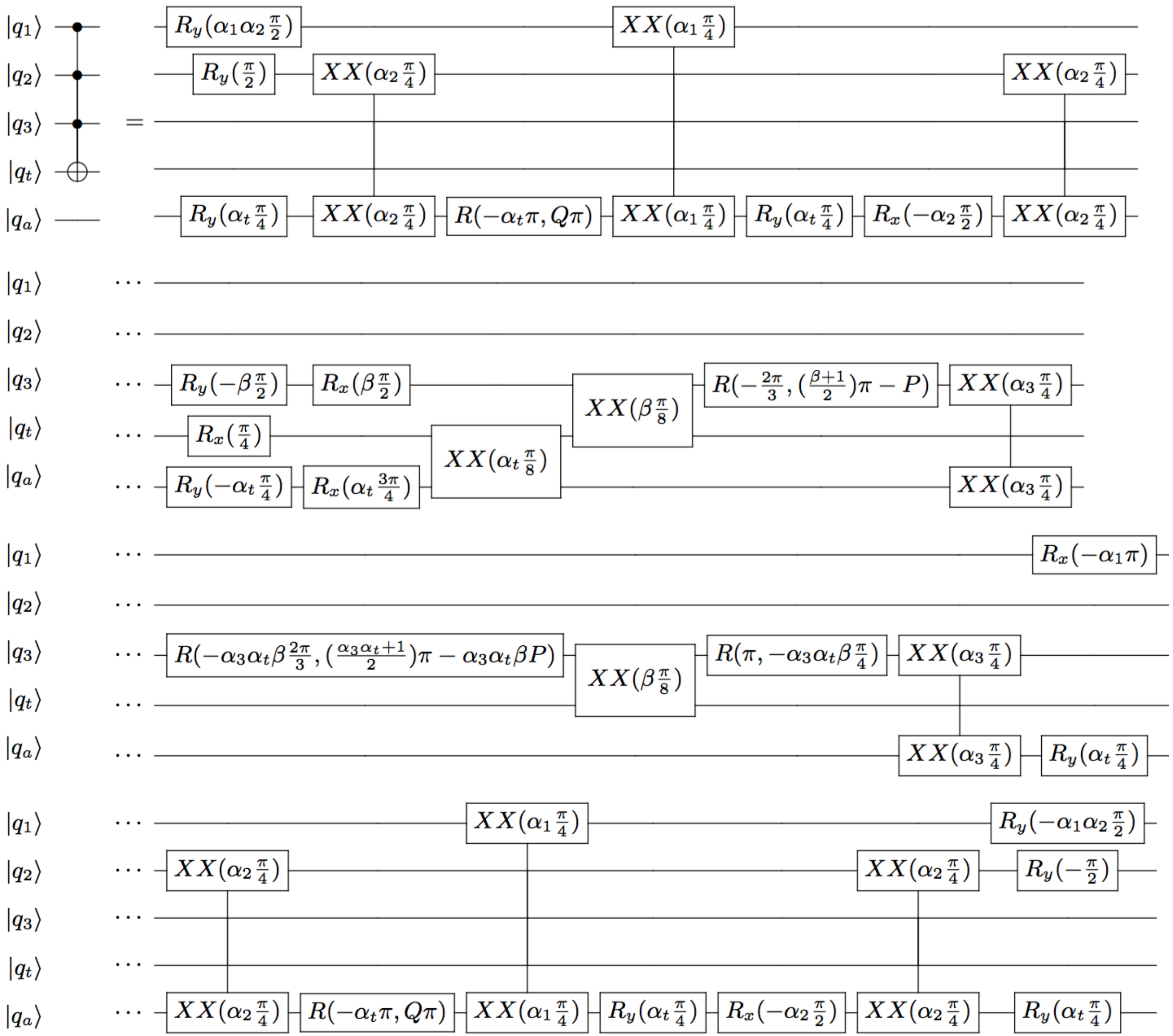
multi-qubit gates



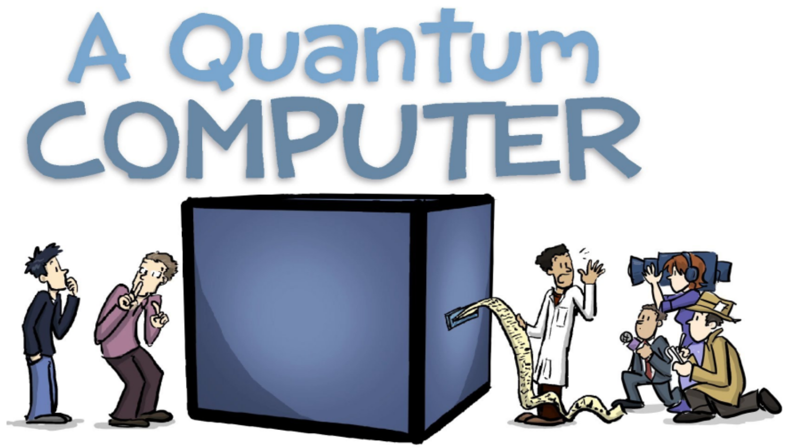
Toffoli-3 using XX/R gate library

multi-qubit gates

Toffoli-4 using XX/R gate library



outline



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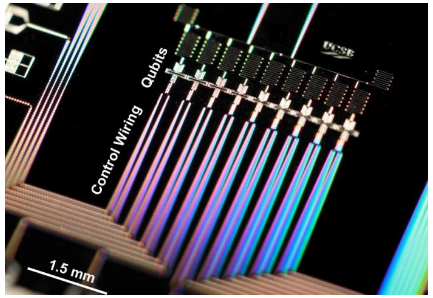
- multi-qubit gates

many-body strategies



idea

- couple N qubits, via 2-body interactions
- use many-body time evolution to realize quantum gates

proposed protocol

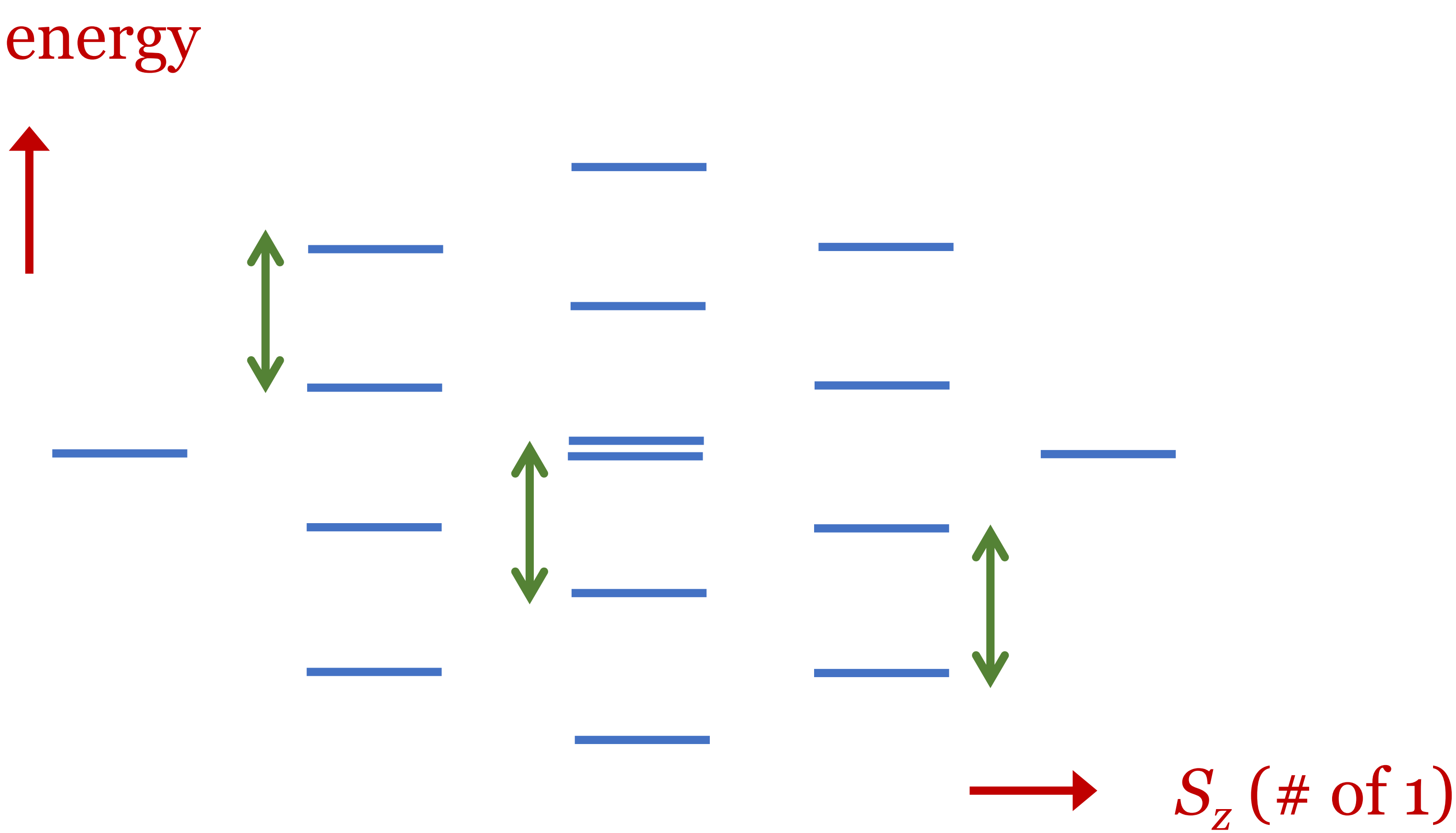
Step 1. Apply quantum circuit for *eigengate* to produce eigenstates from states in computational basis.

Step 2. Use resonant driving to selectively couple and interchange 2 out of 2^N eigenstates.

Step 3. Apply eigengate to return to computational basis.

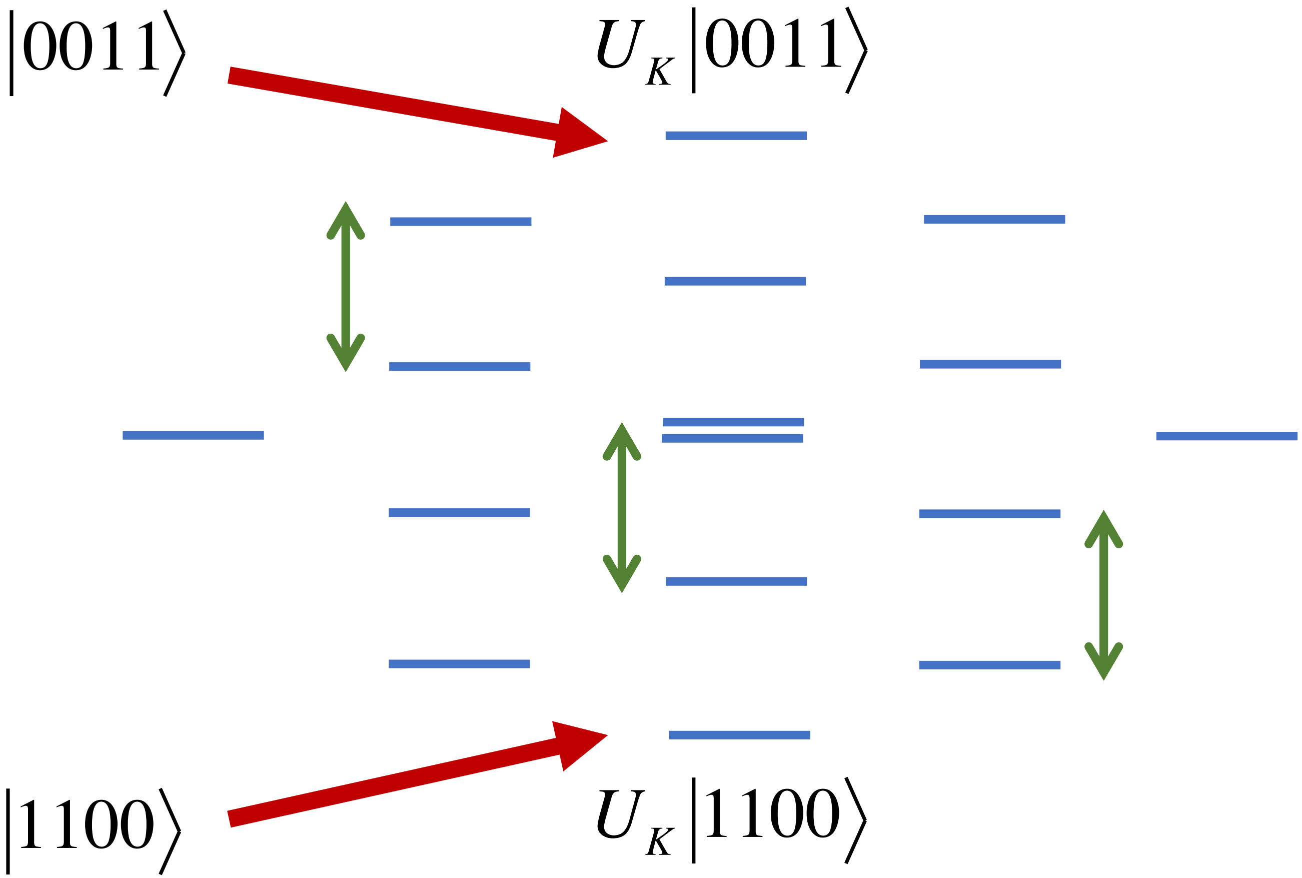
step 0:

many-body energy spectrum (*N*=*4*)



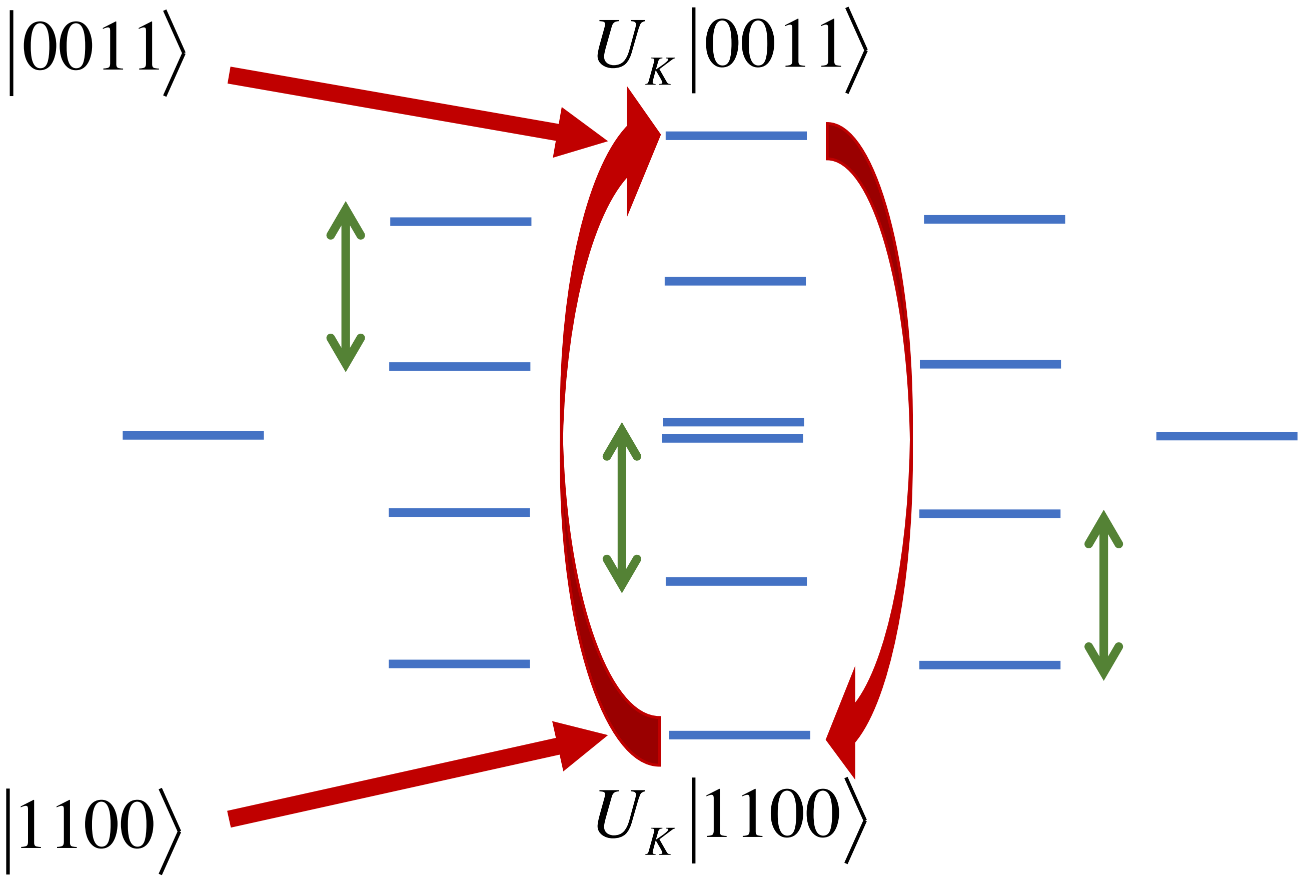
step 1:

eigengate U_K maps computational basis to eigenstates



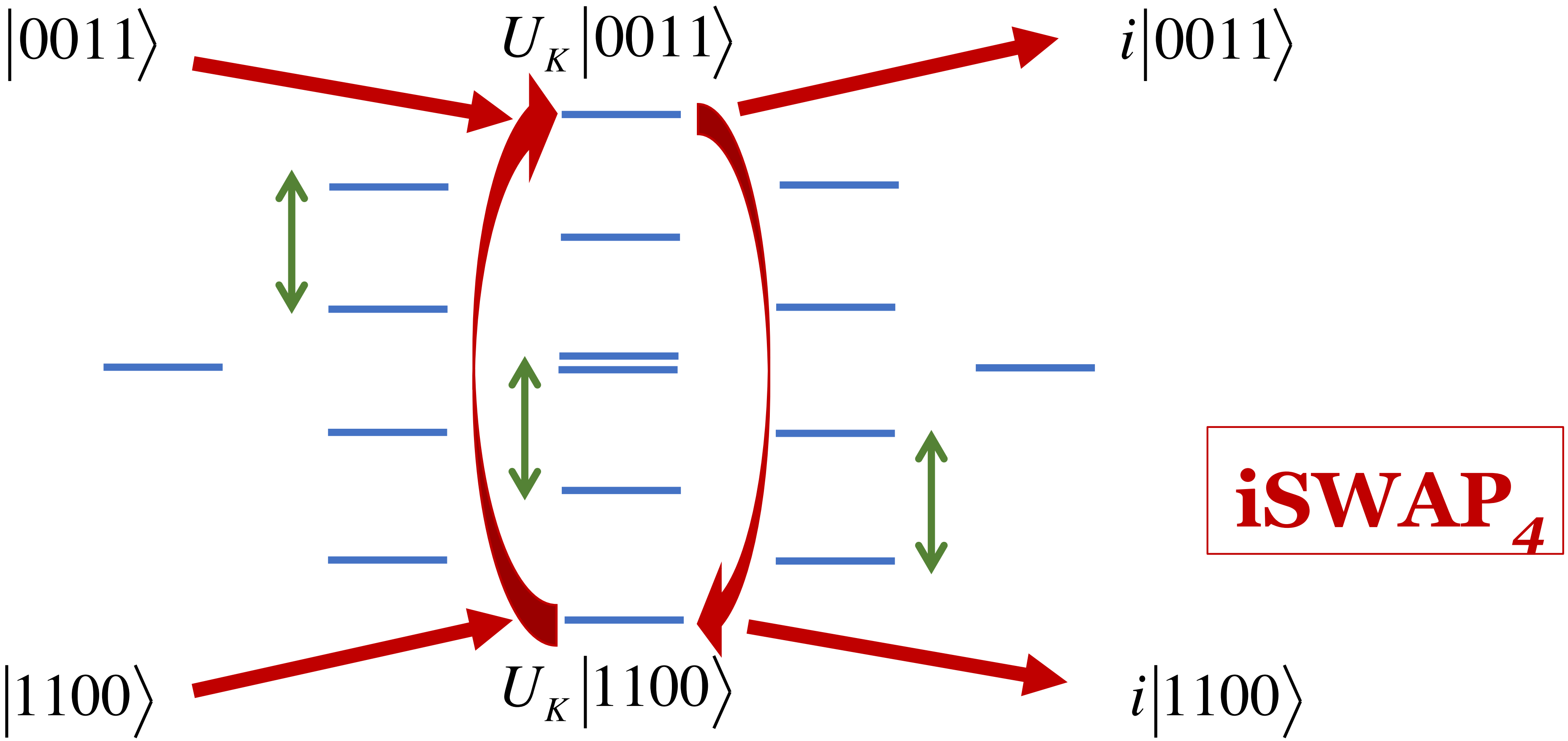
step 2:

resonant driving interchanges a single pair of eigenstates



step 3:

inverse eigengate *U_K* maps back to computational basis



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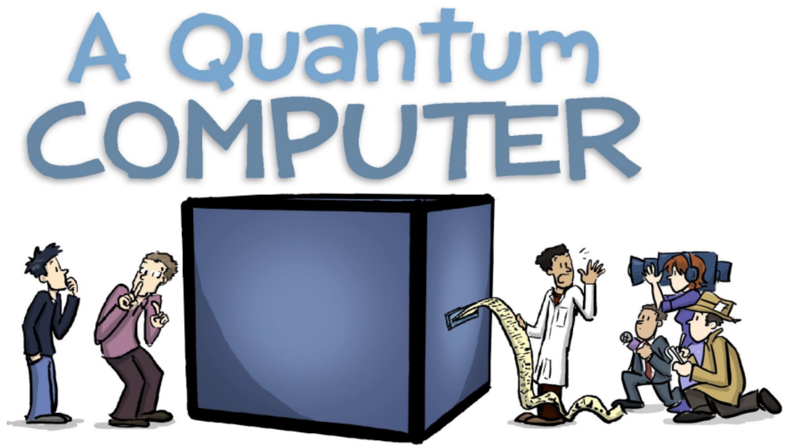

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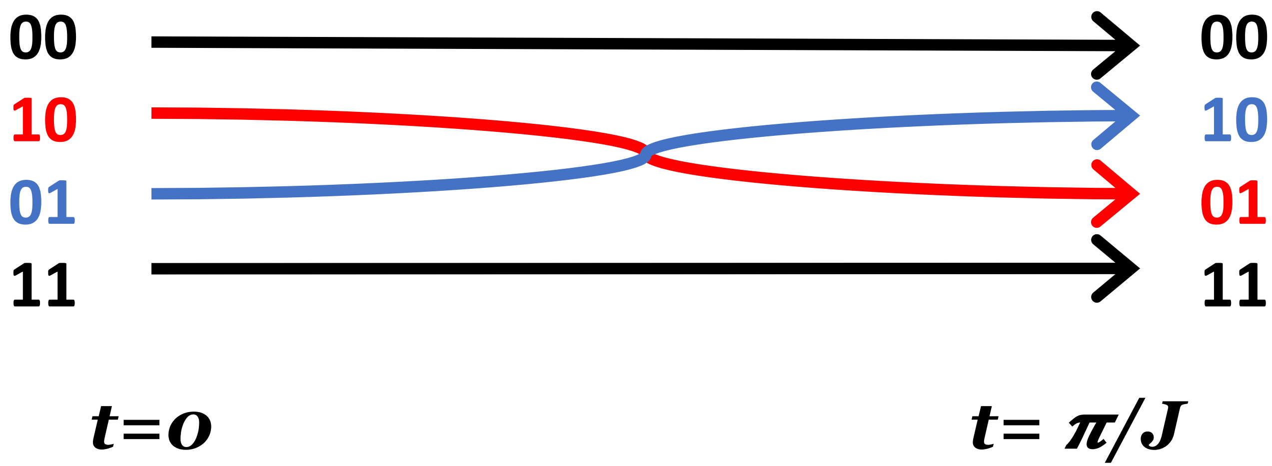
- multi-qubit gates

Krawtchouk chain: 2 qubits

$$H^{(2)} = -\frac{J}{2}(X_1 X_2 + Y_1 Y_2)$$

$t=\pi/J$ pulse of $H^{(2)}$ gives gate iSWAP$_2$

$|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow i\,|10\rangle$, $|10\rangle \rightarrow i\,|01\rangle$, $|11\rangle \rightarrow |11\rangle$



Krawtchouk chain, *N=n+1* qubits

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

- 1-body energies are all commensurate

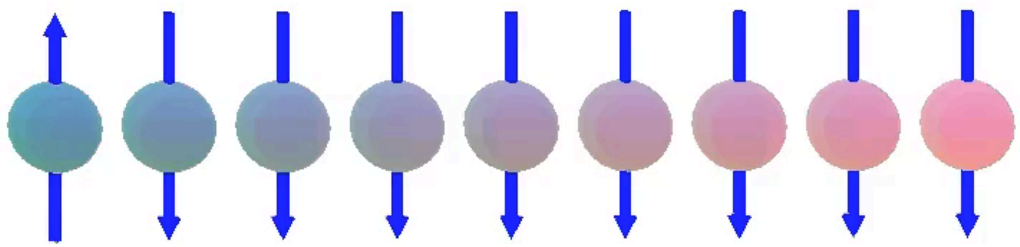
$$\lambda_k = J(k - \frac{N-1}{2}), \qquad k = 0, 1, .., n$$

- many-body energies are (free) sums of 1-body energies thanks to mapping to free fermions

Krawtchouk chain dynamics, I

Time evolution over time $t^{*}=\pi/J$ gives Perfect State Transfer (PST) for state with single `particle' or `spin-flip'

Christandl-Datta-Ekert-Landahl 2004



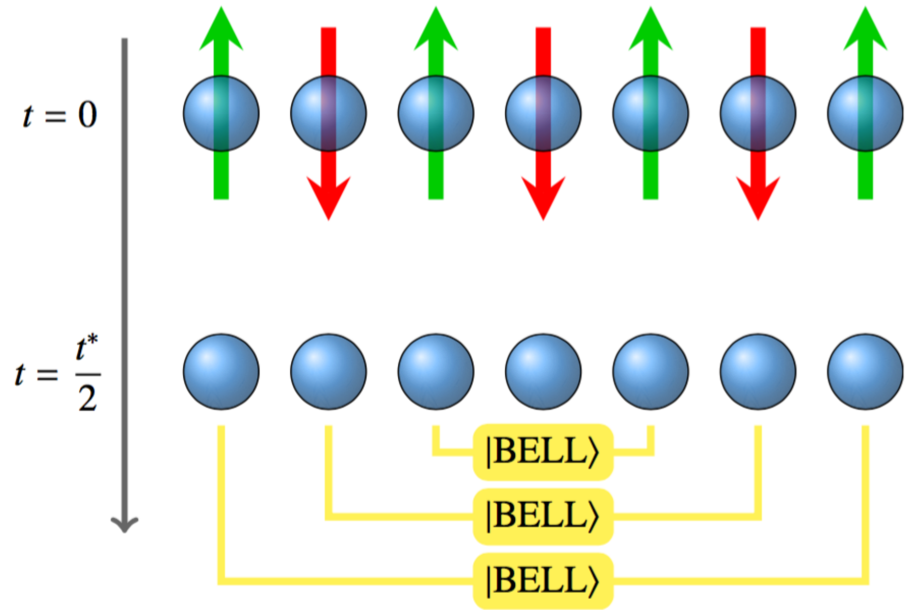
animation:
van der Jeugt

Krawtchouk chain dynamics, II

pulse of time $t^*/2 = \pi/(2J)$

on Néel state |1010..>

generates **Rainbow state**:



nested Bell pairs with

maximal block entanglement

entropy $S_{LR} = N/2 \ln(2)$

Alkurtass-Banchi-Bose 2014

Krawtchouk chain, details (*N*=*n*+1)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

- 1-body spectrum

$$\lambda_k = J(k - \frac{N-1}{2}), \qquad k = 0, 1, .., n$$

- eigenstates

$$\left|\{k\}\right\rangle_{H^K} = \sum_{x=0}^{n}\phi_{k,x}^{(n)}\left|\{x\}\right\rangle \qquad \phi_{k,x}^{(n)} = K_{k,x}^{(n)}\sqrt{\frac{\binom{n}{x}}{\binom{n}{k}2^n}}$$

with $K^{(n)}$ the **Krawtchouk polynomials**

$$K_{k,x}^{(n)} = \sum_{j=0}^{k}(-1)^j\binom{x}{j}\binom{n-x}{k-j}$$

Krawtchouk chain, details (*N*=*n*+1)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

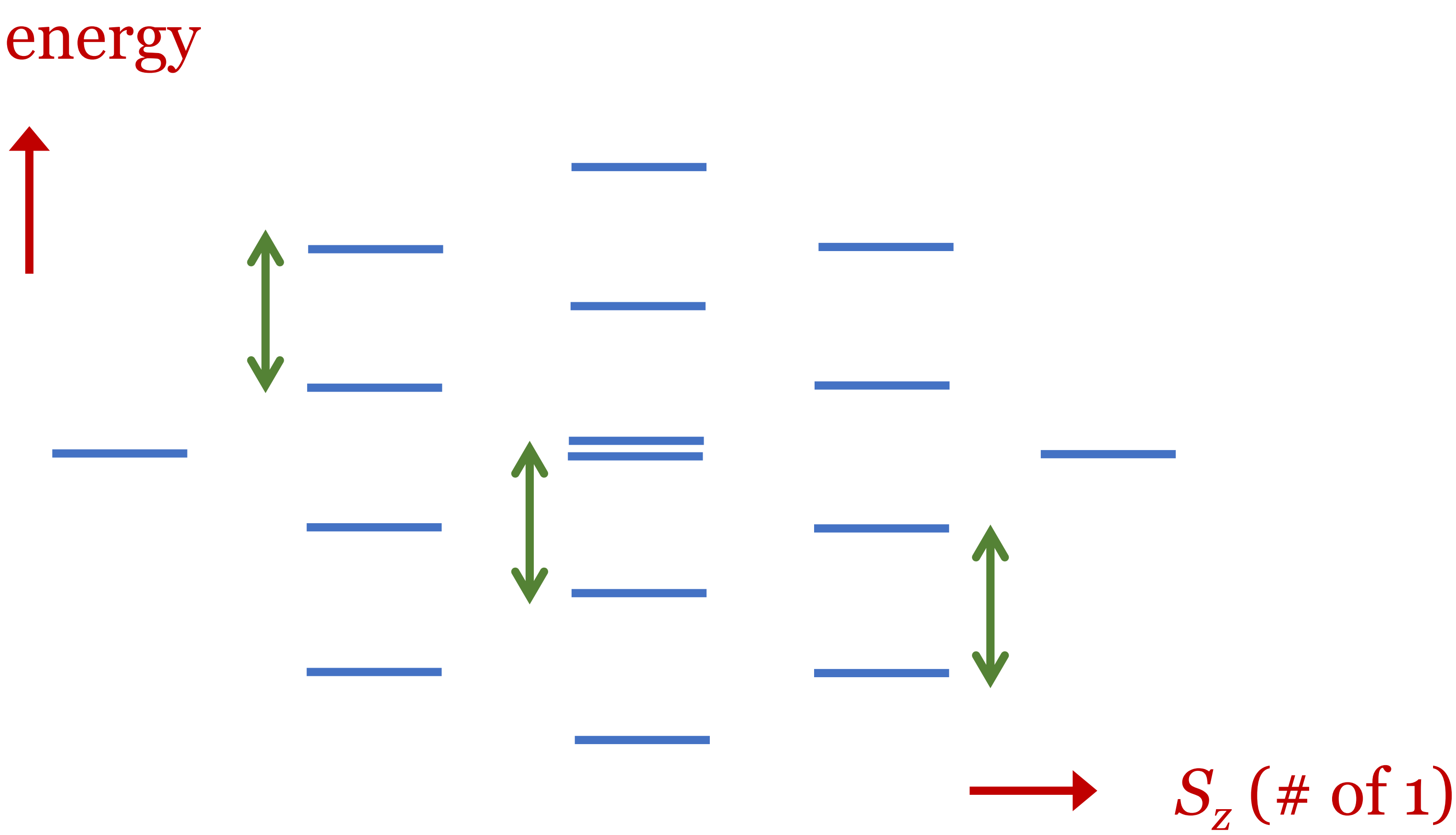
- mapping to free fermions through Jordan-Wigner transformation

$$\frac{1}{2}\left(X_j + iY_j\right) = \prod_{i=0}^{j-1}(1-2n_i)f_j \qquad \frac{1}{2}\left(X_j - iY_j\right) = \prod_{i=0}^{j-1}(1-2n_i)f_j^+$$

- many-body eigenstates built from fermionic eigenmodes

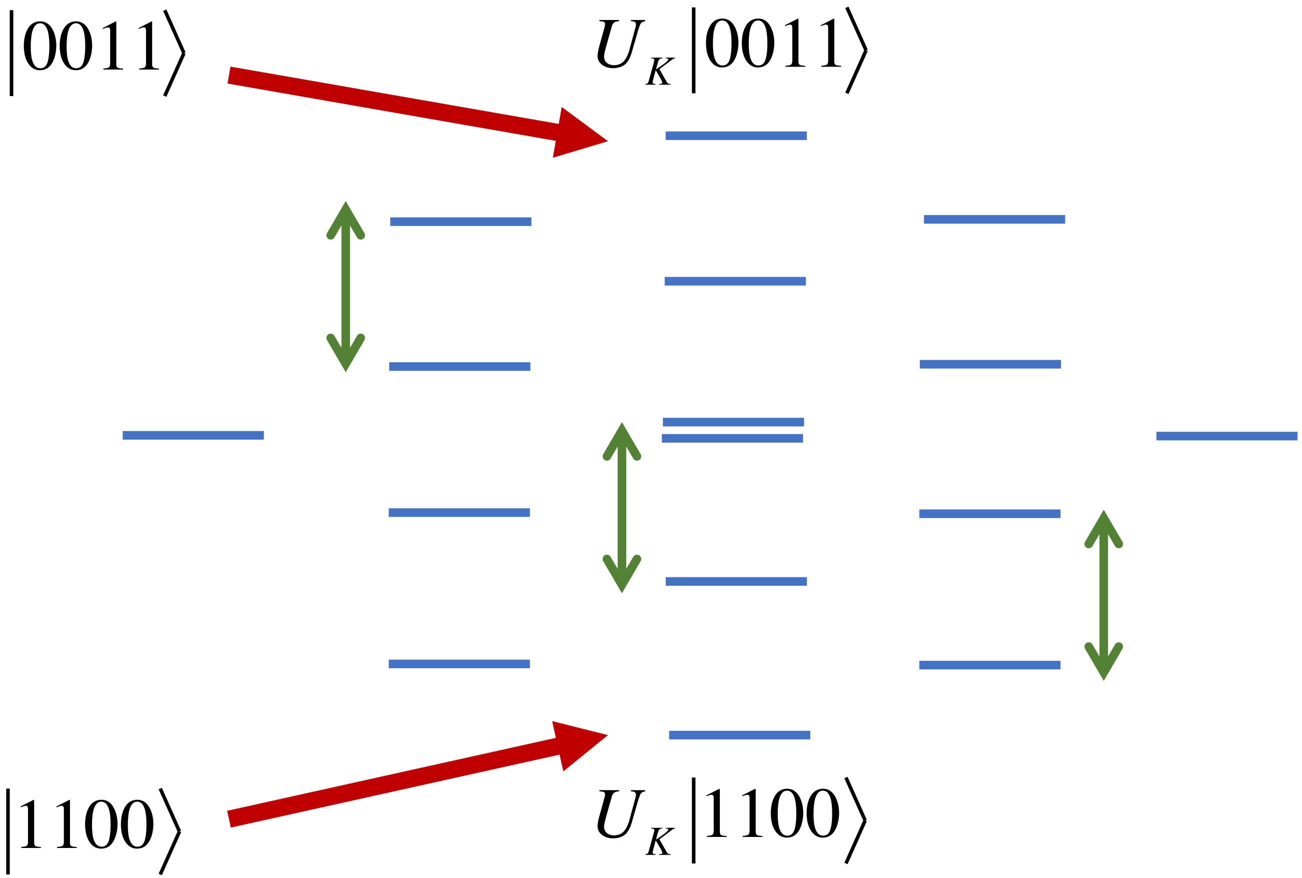
$$c_k^+ = \sum_{j=0}^{n}\phi_{k,j}^{(n)} f_j^+$$

Krawtchouk chain energy spectrum (*N=4*)



step 1:

eigengate U_K maps computational basis to eigenstates



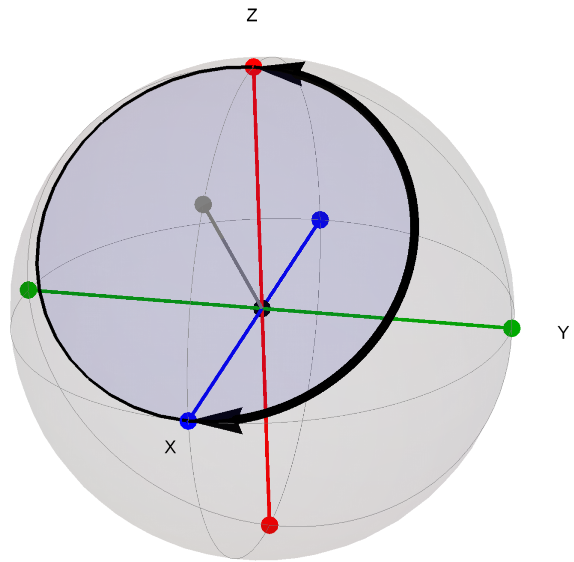
Krawtchouk eigengate

- exact *eigengate* for Krawtchouk chain eigenstates

$$U_K = \exp\left(-i\frac{\pi}{J}\frac{(H^K + H^Z)}{\sqrt{2}}\right)$$

with

$$H^Z = \frac{J}{2}\sum_{x=0}^{n}(x - \frac{n}{2})(I - Z)_x$$

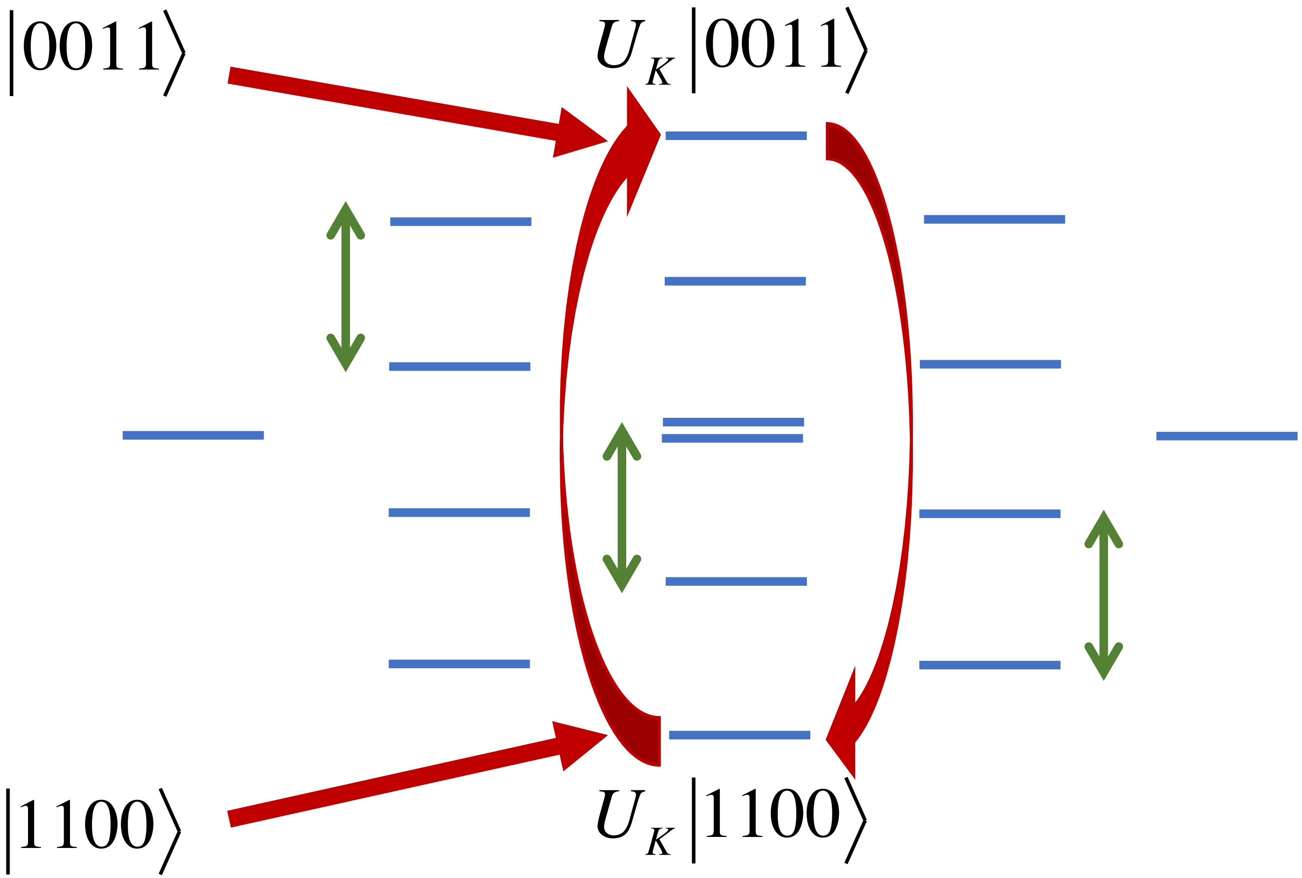


- proof: use angular momentum commutation relations of Krawtchouk operators $L_X = H^K$ and $L_Z = H^Z$ to show that

$$U_K H^Z = H^K U_K$$

step 2:

resonant driving interchanges a single pair of eigenstates



multi-qubit gate: $\mathbf{iSWAP}_N$

- need driving term $H_D(t)$ that resonantly couples the

  highest energy state $U_K|00...01...11>$

  to the

  lowest energy state $U_K|11...10...00>$

- need to annihilate the fermionic modes with $\lambda_k>0$ and create the fermionic modes with $\lambda_k<0$ (or $\lambda_k \leq 0$)

- can be done by suitable 1-qubit or 2-qubit operator (!)

multi-qubit gate: iSWAP$_N$

- N odd, $N=n+1$, need to couple

  $U_K\,|0^{n/2+1}\,1^{n/2}>$ with $U_K\,|1^{n/2+1}\,0^{n/2}>$

- need to

  annihilate the $n/2$ fermionic modes with $\lambda_k>0$

  and

  create the $n/2+1$ modes with $\lambda_k\leq 0$

- can be done by the 1-qubit operator

$$\sigma^-_{n/2} = [1-2f_1^+ f_1]\ldots[1-2f^+_{n/2-1}f_{n/2-1}]f^+_{n/2}$$

multi-qubit gate: iSWAP$_N$

- N odd, N=$n+1$, matrix element for single qubit resonant driving

$$\left\langle 1^{n/2+1}0^{n/2}\left|U_K\,\sigma^-_{n/2}\,U_K\right|0^{n/2+1}1^{n/2}\right\rangle$$
$$= 2^{n/2}\left|\phi^{(n)}_{\{0,\ldots,n/2\},\{0,\ldots,n/2\}}\right|\left|\phi^{(n)}_{\{0,\ldots,n/2-1\},\{n/2+1,\ldots,n\}}\right|$$
$$= \ldots$$
$$= (-2)^{-n^2/4}$$

- exponential decay implies that driving time for resonant transition grows quickly with N

multi-qubit gate: iSWAP_N

- N even, need to couple

  $U_K\,|0^{N/2}\,1^{N/2}>$ to $U_K\,|1^{N/2}\,0^{N/2}>$

- need to

  annihilate the $N/2$ fermionic modes with $\lambda_k>0$

  and

  create the $N/2$ fermionic modes with $\lambda_k<0$

- can be done by the 2-qubit operator

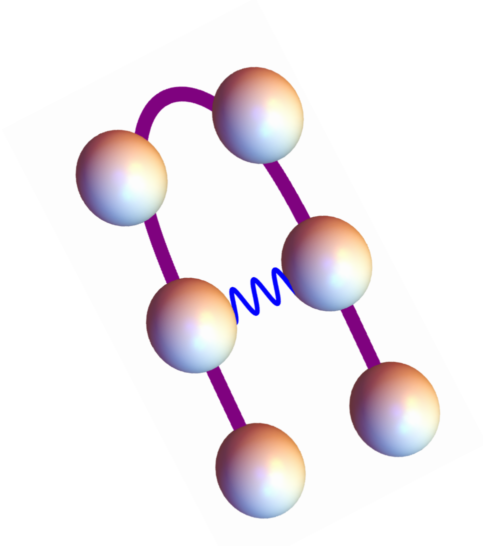
$$\sigma_j^-\sigma_{j+N/2}^+ = f_j^+[1-2f_{j+1}^+f_{j+1}]\ldots[1-2f_{j+N/2-1}^+f_{j+N/2-1}]f_{j+N/2}$$

multi-qubit gate: iSWAP$_N$

- for N=6: matrix element

$$\langle 111000 | U_K(\sigma_1^+\sigma_4^- - \sigma_4^+\sigma_1^-)U_K | 000111\rangle = \frac{5}{32}$$

- resonant driving term



$$H_D^{(1,-)}(t) = i\,J_D \cos[9Jt]\,[\sigma_1^+\sigma_4^- - \sigma_4^+\sigma_1^-]$$

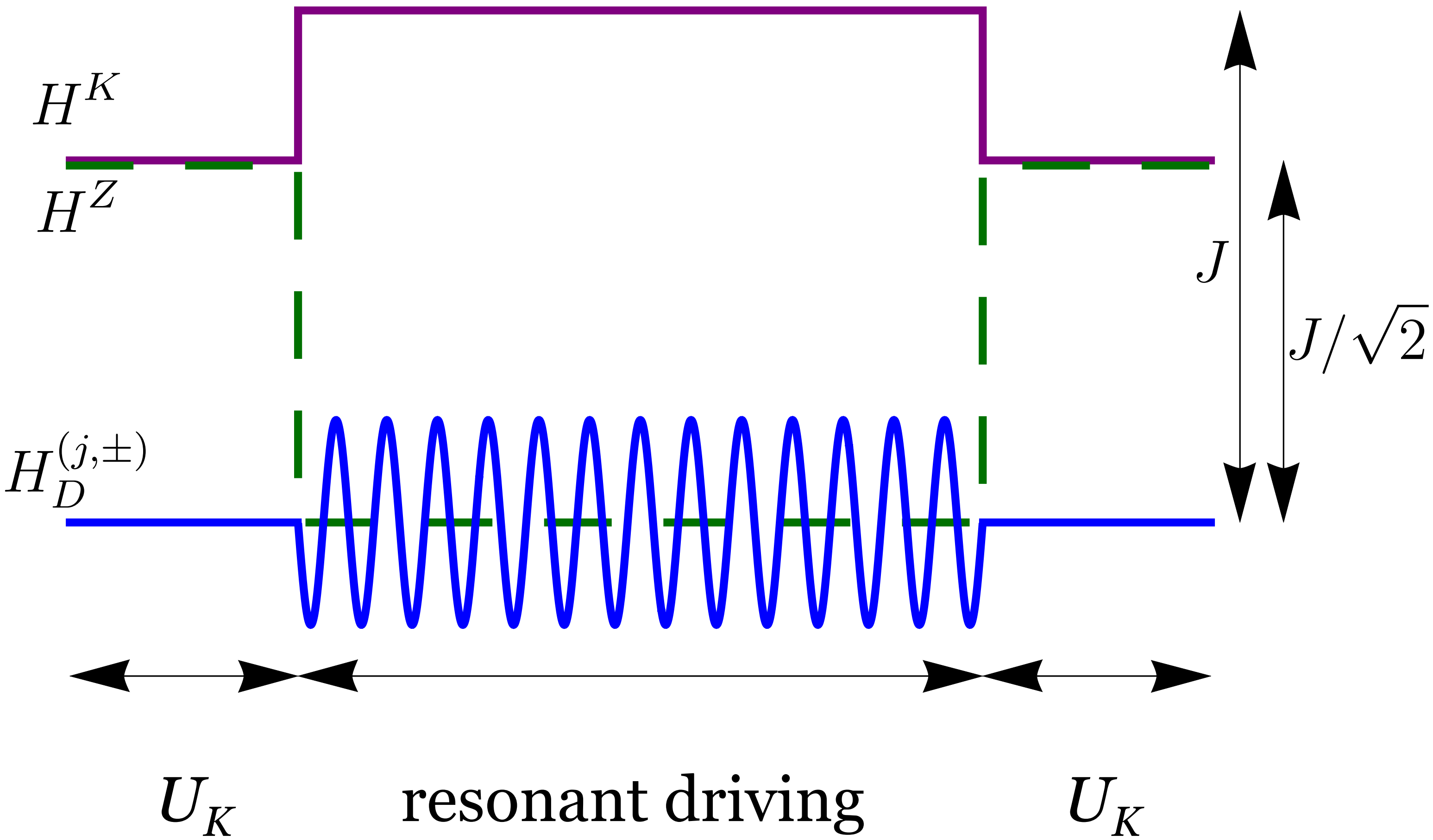
- conditions on driving time τ_D

$$\tau_D(5J_D/64) = \pi/2 \qquad \tau_D = M(2\pi/J)$$

so that (in leading order) |000111> and |111000> are interchanged and all dynamical phases return to 1

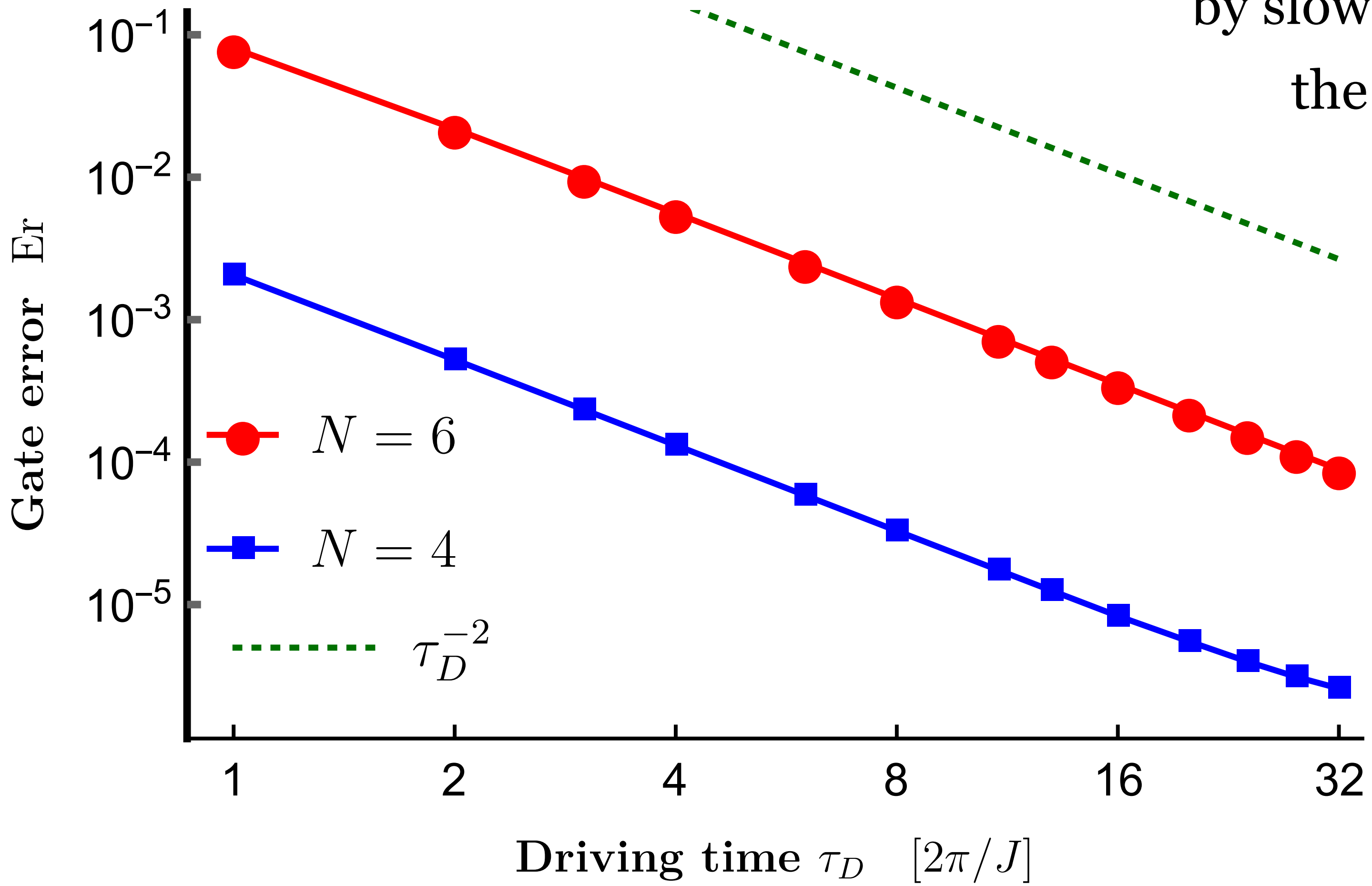
many-body protocol for iSWAP$_6$

$$|000111\rangle \rightarrow i|111000\rangle, \qquad |111000\rangle \rightarrow i|000111\rangle$$



fidelities for iSWAP_4 and iSWAP_6

gate fidelity enhanced
by slowing down
the resonant
driving



Polychronakos chain

$$H_{\mathrm{P}} = \sum_{j<k} h_{jk} P_{jk}$$

$$h_{jk} = \frac{1}{(x_j - x_k)^2},$$

$$P_{jk} = \frac{1}{2}\left(\mathbb{1}_j \mathbb{1}_k + \sum_{\alpha=x,y,z} \sigma_j^\alpha \sigma_k^\alpha\right)$$

- linear chain with inverse square interaction
- x_j tuned to be roots of Hermite polynomial $H_N(x)$

Polychronakos 1993

Polychronakos chain - symmetries

$$L_0^\alpha = \frac{1}{2}\sum_{j=1}^{N} x_j \vec{\sigma}_j^\alpha \qquad L_1^\alpha = \frac{1}{4}\sum_{j\neq k} w_{jk}\epsilon^{\alpha\beta\gamma}\vec{\sigma}_j^\beta \vec{\sigma}_k^\gamma \qquad w_{jk} = \frac{1}{x_j - x_k}$$

- both are vectors with respect to SU(2) symmetry
- commutation relations with hamiltonian

$$[H_{\mathrm{P}}, L_0^\alpha] = iL_1^\alpha \qquad [H_{\mathrm{P}}, L_1^\alpha] = -iL_0^\alpha$$

Frahm 1993

Polychronakos chain - symmetries

L_0^z is diagonal in computational basis

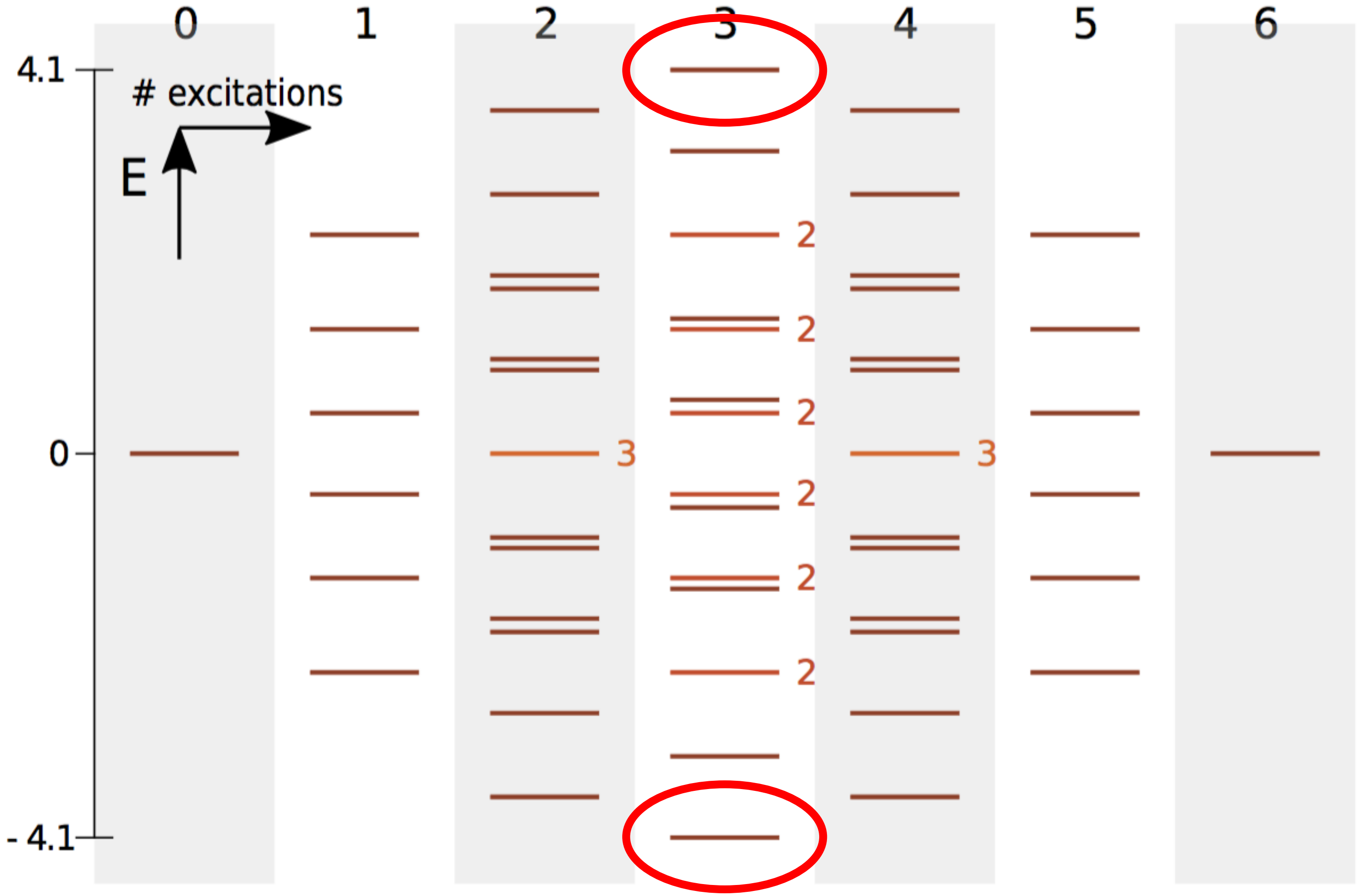
$$L_0^z |\{k_1, k_2, \ldots, k_p\}\rangle = \sum_{j=1}^{p} x_{k_j} |\{k_1, k_2, \ldots, k_p\}\rangle.$$

Using commutation relations show that

$$e^{-i\frac{\pi}{2}H_P} L_0^z \, e^{i\frac{\pi}{2}H_P} = L_1^z$$

Conclude that $U_P = e^{-i\frac{\pi}{2}H_P}$ *is eigengate* for L_1^z

Polychronakos chain - spectrum of L_1^z



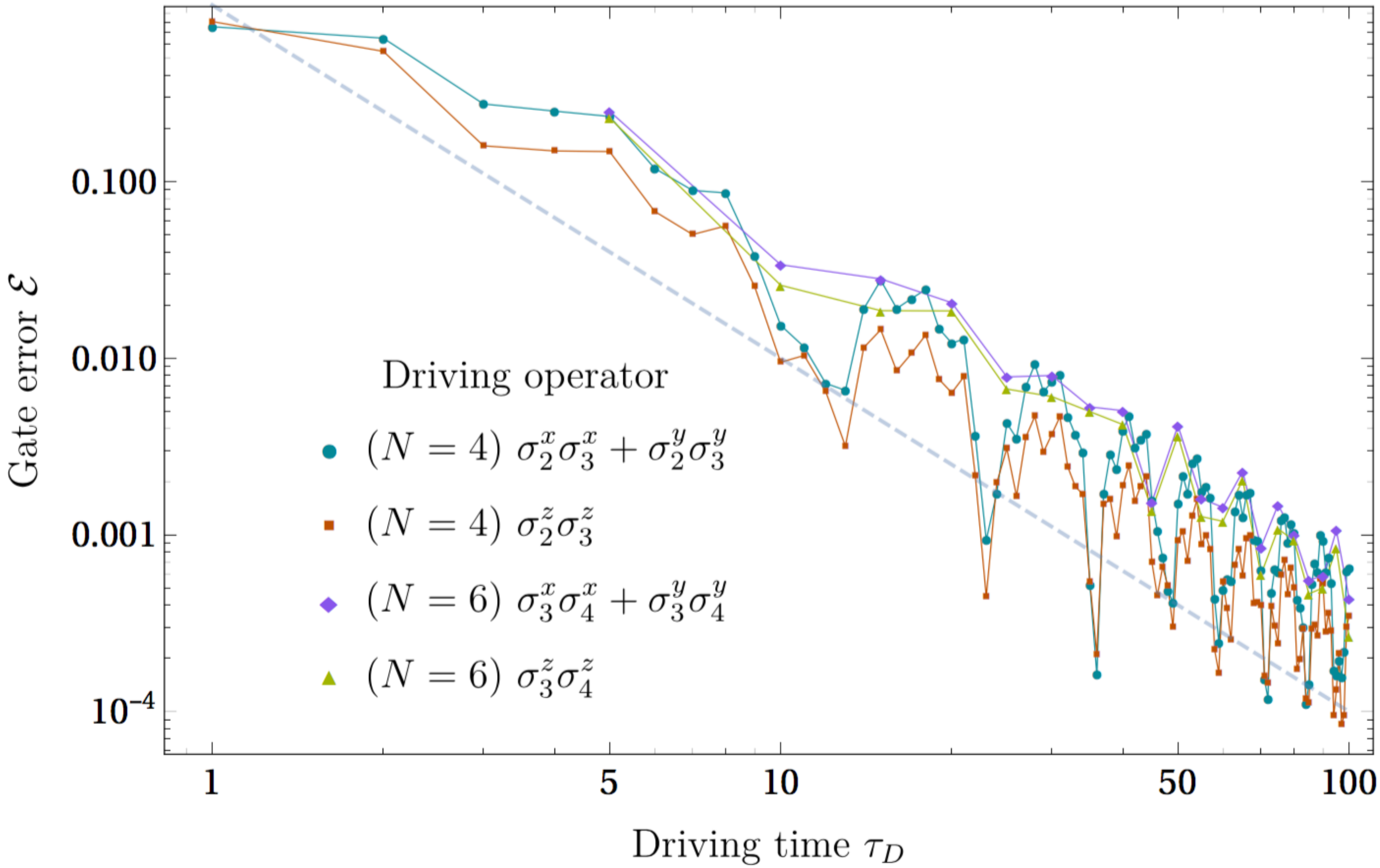
Polychronakos chain – resonant driving

- top and bottom states of N-body spectrum of L_1^z can be connected by 1- and 2-body spin-operators

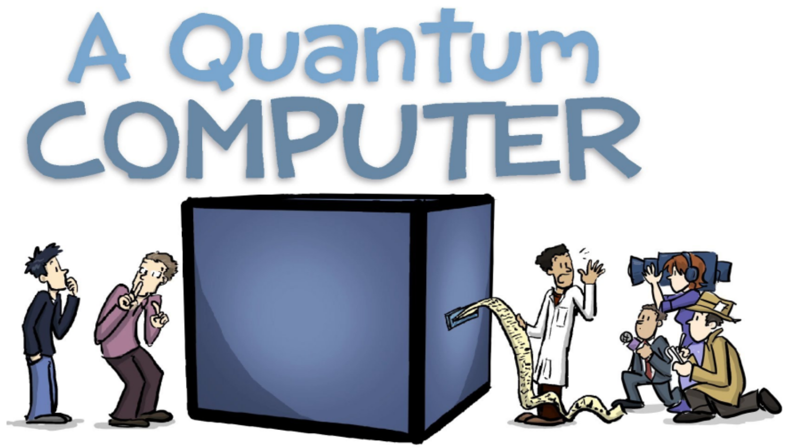
$N=6$	
H_{drive}	${}_{L_1^z}\langle t_1 \vert H_{\text{drive}} \vert t_2 \rangle_{L_1^z}$
σ_3^z	$0.116012i$
$\sigma_3^z \sigma_4^z$	-0.327919
$\sigma_3^x \sigma_4^x$	0.200378

- resonant driving step needs `half way inversion' to cancel unwanted dynamical phases

Polychronakos chain – fidelity of iSWAP$_N$



outline



- background and motivation
- many-body strategies for multi-qubit gates
- quantum control on the Krawtchouk and Polychronakos chains
- **multi-qubit gates**

multi-qubit gates

combining *eigengates* with resonant driving produces N-qubit gate $\mathbf{iSWAP}_N$

$$|000...111...> \rightarrow i\ |111...000...>,$$

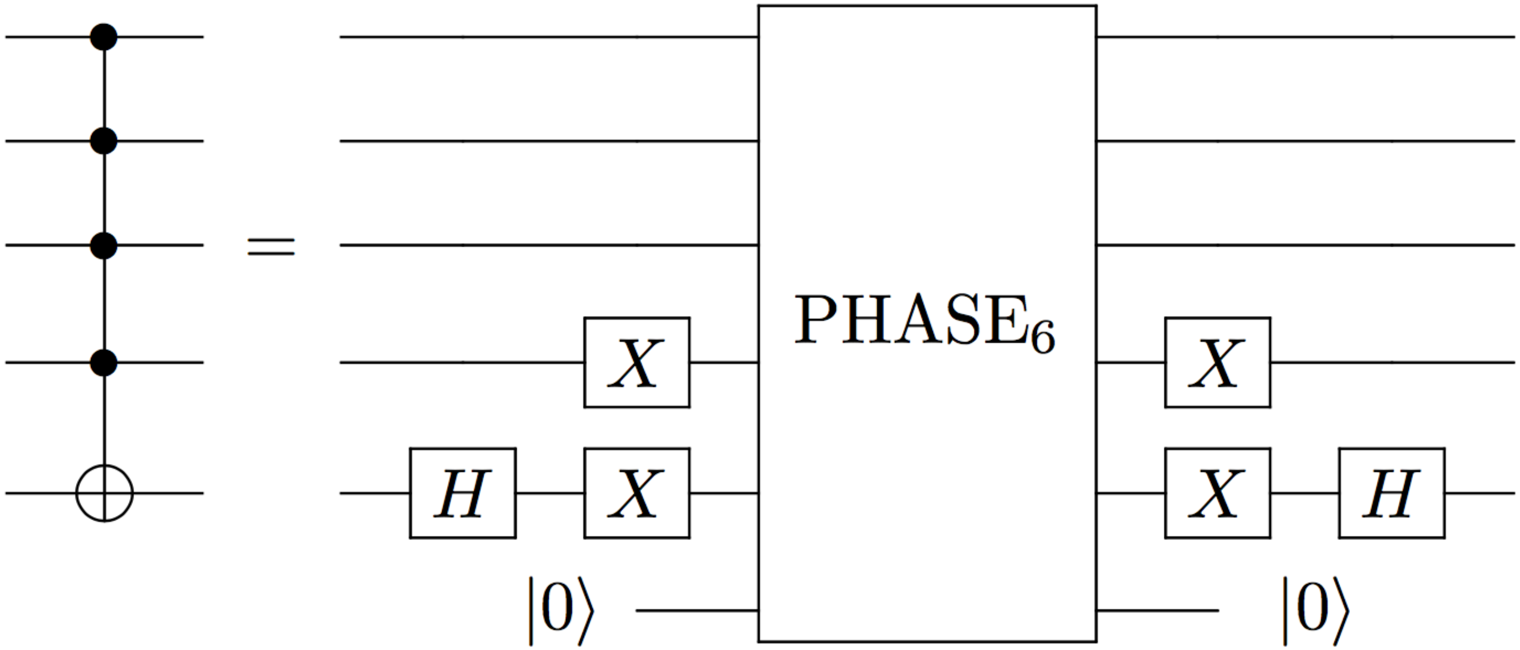
$$|111...000...> \rightarrow i\ |000...111...>$$

double–time $\mathbf{iSWAP}_N$ gives $\mathbf{PHASE}_N$

$$|000...111...> \rightarrow -\ |000...111...>,$$

$$|111...000...> \rightarrow -\ |111...000...>$$

multi-qubit gates



Toffoli-5 using double strength $iSWAP_6$ gate called $PHASE_6$

Outlook

further results (with Koen Groenland)

- sensitivity to noise
- various optimizations
- ...

Outlook

questions, questions …

- for large *N*, our gate times grow rapidly due to suppression of matrix elements – can this be avoided?
- fundamental `speed limits' for quantum gates – given the maximum strength of 2-qubit interactions, how much time is needed to achieve a gate?