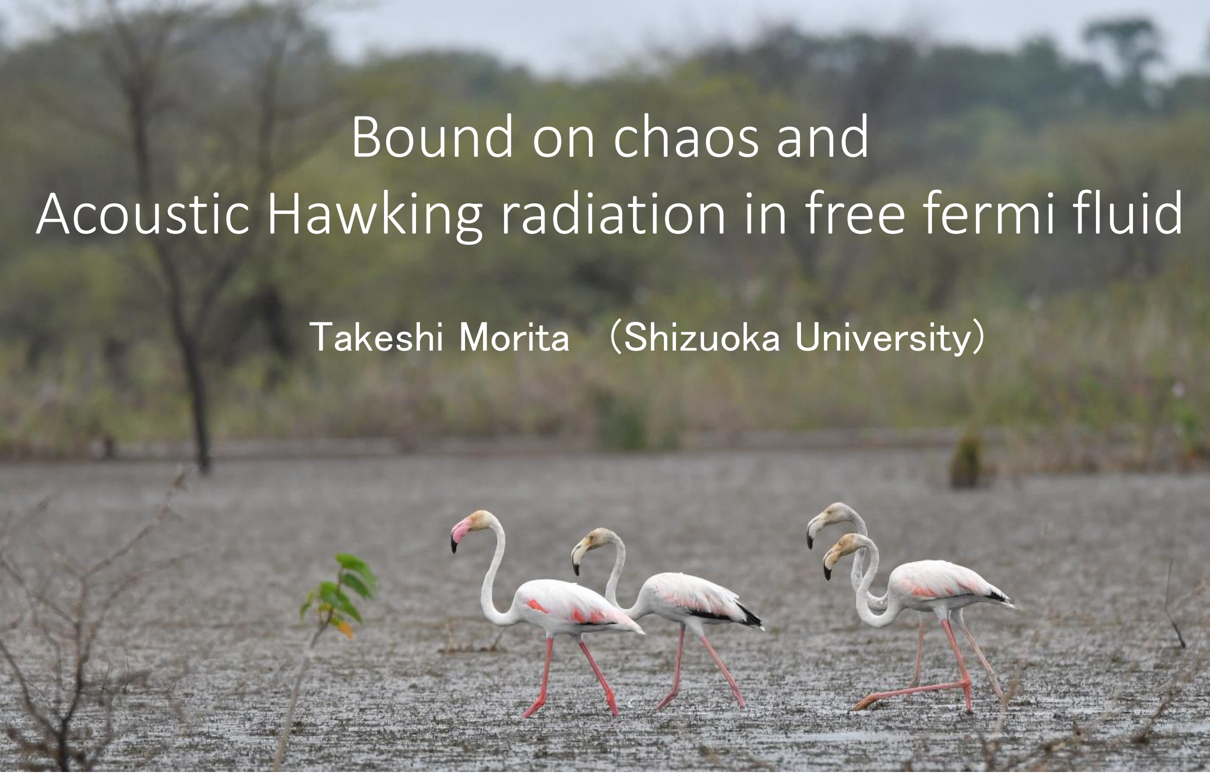
Bound on chaos and Acoustic Hawking radiation in free fermi fluid

Takeshi Morita (Shizuoka University)



Reference :arXiv:1801.00967

+ Work in progress

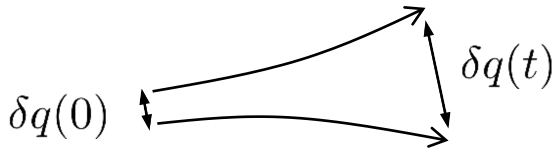
Introduction:

bound on chaos — Maldacena-Shenker-Stanford (2015)

$$\lambda_L \le \frac{2\pi T}{\hbar}$$

λ_L : Lyapunov exponent

T : Temperature



$$\delta q(t) \sim \delta q(0) \exp(\lambda_L t)$$

Examples of the saturation of the bound $\lambda_L = \frac{2\pi T}{\hbar}$

- AdS/CFT (dual black holes)
- SYK model

→ Most chaotic systems? Fast scramblers?
Signature of Quantum Gravity?

Introduction:

bound on chaos

Maldacena-Shenker-Stanford (2015)

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λ_L : Lyapunov exponent

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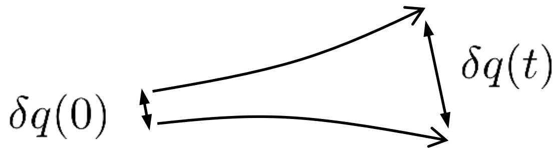

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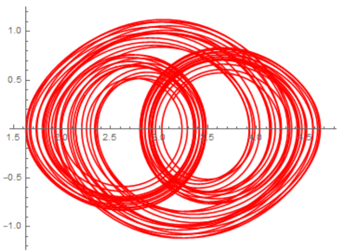
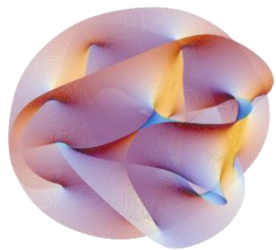


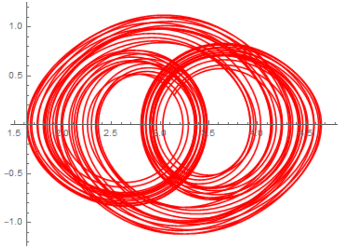
Chaos



Quantum Gravity

(Image courtesy of Wikipedia.)

Introduction:



$$\lambda_L \leq \frac{2\pi T}{\hbar}$$

$$\Leftrightarrow \quad T \geq \frac{\hbar}{2\pi}\lambda_L$$

“Temperature is bounded from below in chaotic systems.”

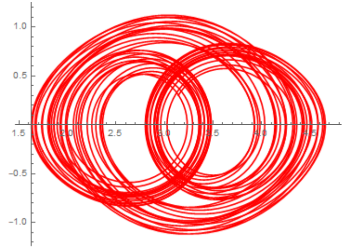
Kurchan 2016

TM 2018

Today’s talk:

This relation may be interesting in semi-classical regime.

Temperature bound in chaotic system



A classical chaotic system with λ_L at T=0.
(Hamiltonian system)

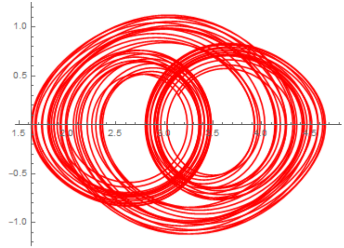
Turn on quantum correction.
(semi-classical)

$$\lambda_L \leq \frac{2\pi T}{\hbar}, \qquad T \geq \frac{\hbar}{2\pi}\lambda_L$$

Two possibilities:

- $\lambda_L \to 0$
- $T \neq 0$

Temperature bound in chaotic system

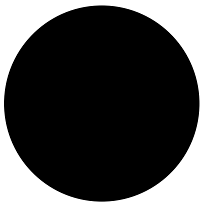


A classical chaotic system with λ_L at T=0.
(Hamiltonian system)

↓ Turn on quantum correction.
(semi-classical)

$$T \geq T_L := \frac{\hbar}{2\pi}\lambda_L$$

cf. Black hole



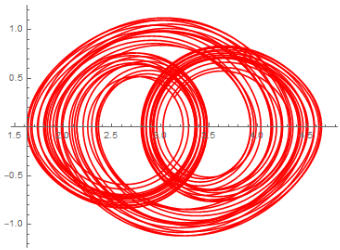
Quantum mechanically thermal

$$T_H = \frac{\hbar}{8\pi G M}$$

Classical chaos becomes thermal quantum mechanically similar to BH?

<u>Today's topic</u>

- Emergence of thermal behavior in some chaotic systems may indeed occur semi-classically.



$$T \geq T_L := \frac{\hbar}{2\pi}\lambda_L$$

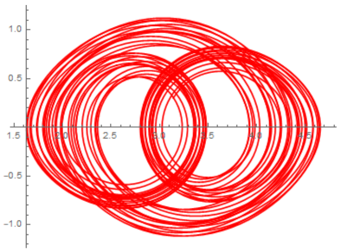
- The bound is saturated even in integrable systems.

$$\lambda_L = \frac{2\pi T}{\hbar}$$

- Acoustic Hawking radiation in supersonic fermi fluid is related to this emergent thermodynamics.

Section 1

Emergence of thermal behavior in semi-classical chaotic systems

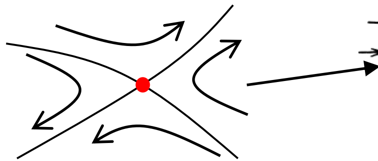


$$T \geq T_L := \frac{\hbar}{2\pi}\lambda_L$$

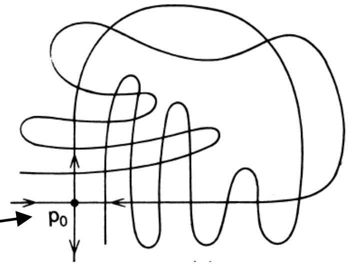
Classical chaos and hyperbolic fixed point

Typical two ingredients of chaotic motion

- Hyperbolic fixed point
- Disturbance

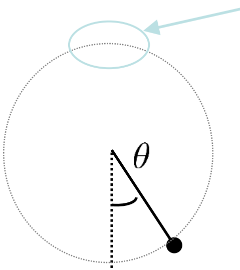


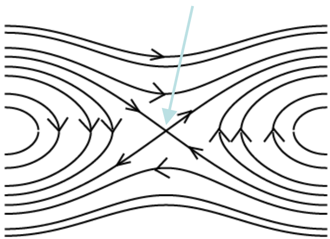
particle trajectory



Wiggins's textbook

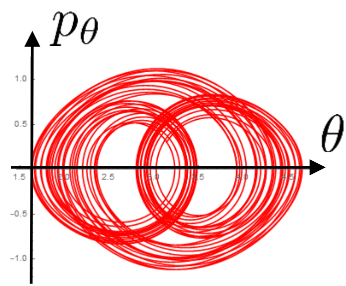
ex) damped driven pendulum motion





particle trajectories in (p,q) phase space
(without damping and driven force)

driven force
damping



chaotic motion

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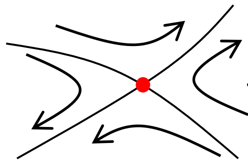

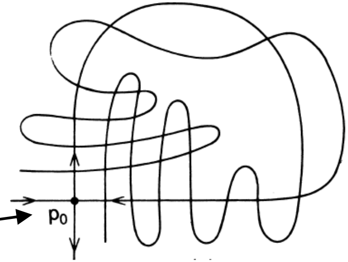

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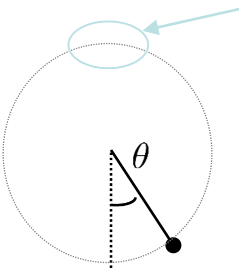
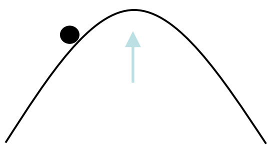
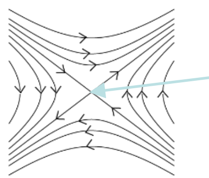


Particle motion near hyperbolic fixed point

≃ 1 dim inverse harmonic potential

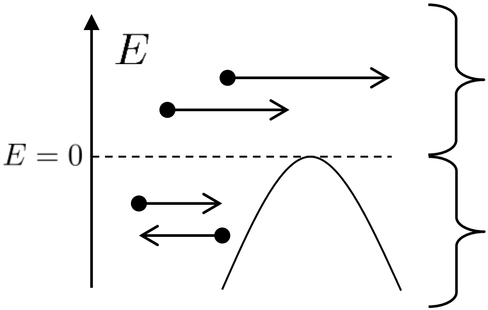




Thermal behavior will appear around there.

Thermal emission from inverse harmonic potential

- Particle motion in an inverse harmonic potential (classical mechanics)



$E > 0$ Go through the potential

$E < 0$ Reflected by the potential

$$V(x) = -\frac{\alpha}{2}x^2.$$

$$m\ddot{x}(t) = -V'(x) = \alpha x,$$

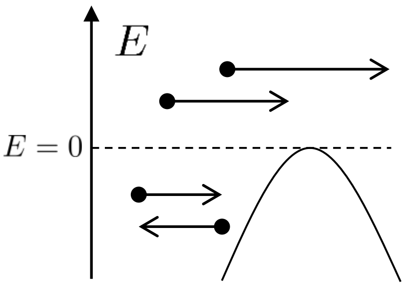
$$\longrightarrow \qquad x(t) = c_1 e^{\sqrt{\alpha/m}t} + c_2 e^{-\sqrt{\alpha/m}t},$$

Lyapunov exponent $\lambda_L := \sqrt{\dfrac{\alpha}{m}}$

Thermal emission from inverse harmonic potential

TM 2018

◆ Particle motion in an inverse harmonic potential (classical mechanics)

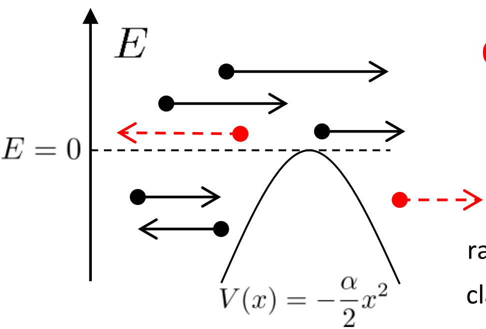


$E>0$ Go through the potential

$E<0$ Reflected by the potential

$$x(t)=c_1 e^{\lambda_L t}+c_2 e^{-\lambda_L t}, \quad \lambda_L := \sqrt{\frac{\alpha}{m}}$$

Quantum correction



Quantum reflection

$1:\exp(-\beta_L E) \quad (E>0)$

Quantum tunneling

$1:\exp(\beta_L E) \quad (E<0)$

ratio ↑

classical : quantum

Similar to Boltzmann factor

$$\begin{cases} \beta_L := 1/T_L \\ T_L = \frac{\hbar}{2\pi}\lambda_L \end{cases}$$

$\lambda_L \leq \frac{2\pi T}{\hbar}$ is saturated.

Thermal emission from inverse harmonic potential

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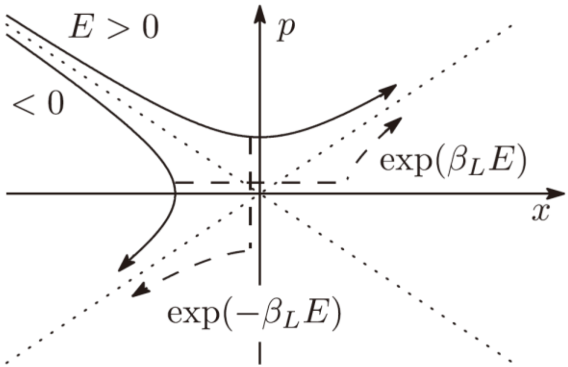
◆ Particle motion in an inverse harmonic potential (classical mechanics)

phase space



E

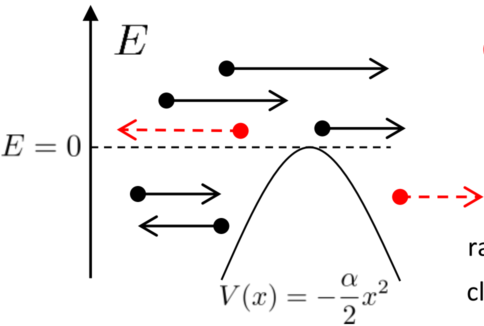
$E = 0$

ential

tential

t, $\lambda_L := \sqrt{\dfrac{\alpha}{m}}$

Quantum …



Quantum reflection

$1 : \exp\left(-\beta_L E\right) \quad (E > 0)$

Quantum tunneling

$1 : \exp\left(\beta_L E\right) \quad (E < 0)$

ratio ↑

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$$\begin{cases} \beta_L := 1/T_L \\ T_L = \dfrac{\hbar}{2\pi}\lambda_L \end{cases}$$

$\lambda_L \leq \dfrac{2\pi T}{\hbar}$ is saturated.

Thermal emission from inverse harmonic potential

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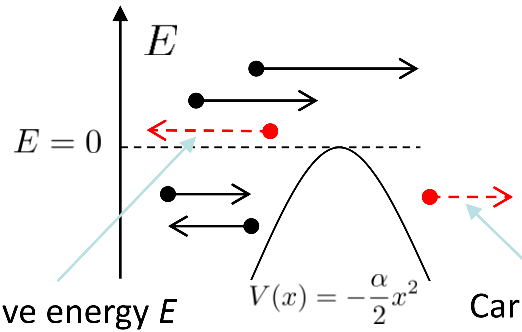
◆ Energy transfer: How much energy is transferred by the quantum effects comparing with the classical motion?



Carry positive energy *E*

Energy in the left region increases.

probability: $\exp\left(-\beta_L E\right)$

Carry negative energy *E*

Negative energy in the left decreases.

→ Increase positive energy $(-E)$
(``hole'' creation)

probability: $\exp\left(\beta_L E\right) = \exp\left(-\beta_L(-E)\right)$

Energy $|E|$ is transferred by the probability $\exp\left(-\beta_L |E|\right)$.

→ It can be regarded as a thermal excitation at $T_L = \frac{\hbar}{2\pi}\lambda_L$.

Thermal emission from inverse harmonic potential

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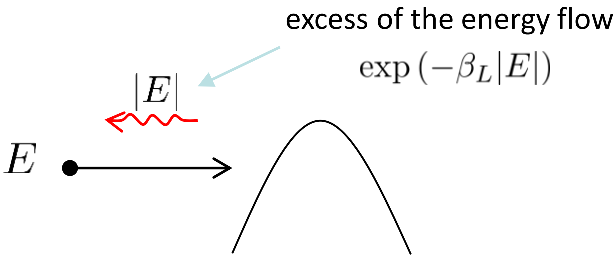
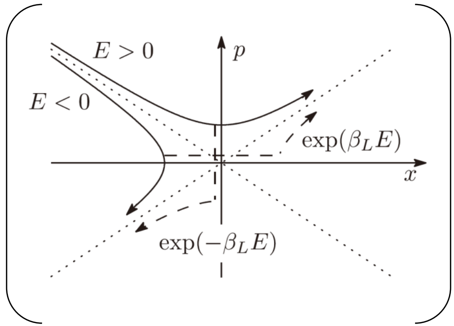
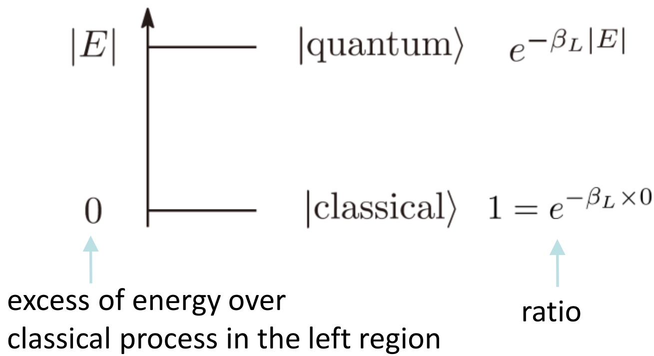
◆ Connection to two level system





Analogous to the thermal excitation of a two level system.

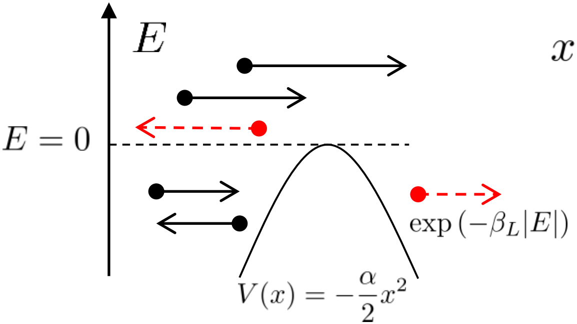


Particle motion near hyperbolic fixed point

$\simeq$ Two level system

at $T_L=\dfrac{\hbar}{2\pi}\lambda_L$

Remarks

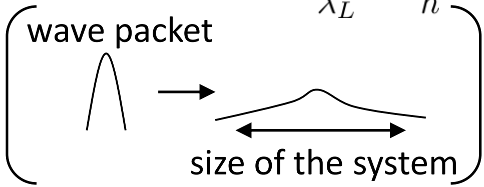


$$x(t) = c_1 e^{\sqrt{\alpha/m}t} + c_2 e^{-\sqrt{\alpha/m}t},$$

Lyapunov exponent $\lambda_L := \sqrt{\frac{\alpha}{m}}$

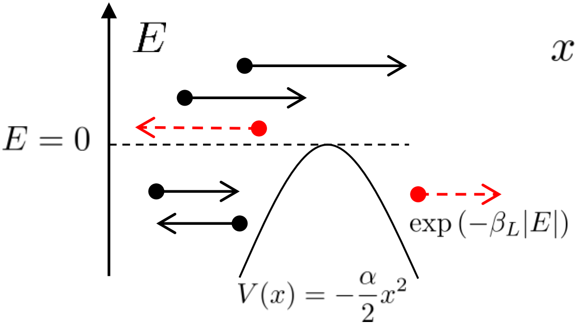
$\lambda_L \leq \frac{2\pi T}{\hbar}$ is saturated.

- The particle scattering occurs within the Ehrenfest's time $t \ll \frac{1}{\lambda_L}\log\frac{1}{\hbar}$
  → The Lyapunov exponent defined
  by the classical motion may be valid.
  (OTOC may not be necessary.)



- Inverse harmonic potential saturates the bound. But it's integrable!
  ex) c=1 matrix model (integrable) saturates the bound.

Remarks

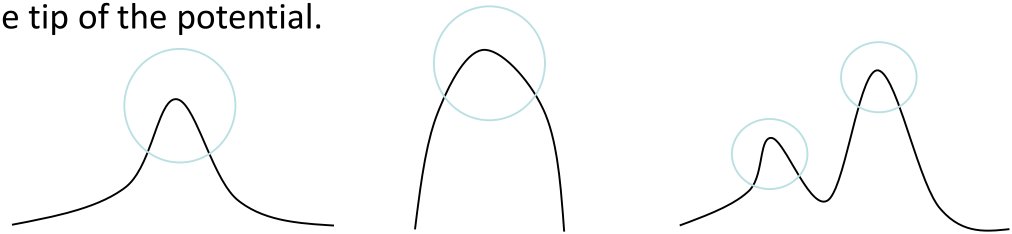


$$x(t) = c_1 e^{\sqrt{\alpha/m}t} + c_2 e^{-\sqrt{\alpha/m}t},$$

Lyapunov exponent $\lambda_L := \sqrt{\frac{\alpha}{m}}$

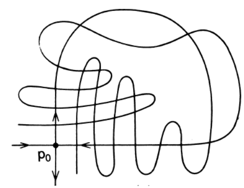
$\lambda_L \leq \frac{2\pi T}{\hbar}$ is saturated.

- The result would be universal, since the excitation mainly occurs near the tip of the potential.



- In the chaotic system, the Lyapunov exponent of the hyperbolic fixed point and the Lyapunov exponent of the system is different.

  → Connection to the bound on chaos is unclear.

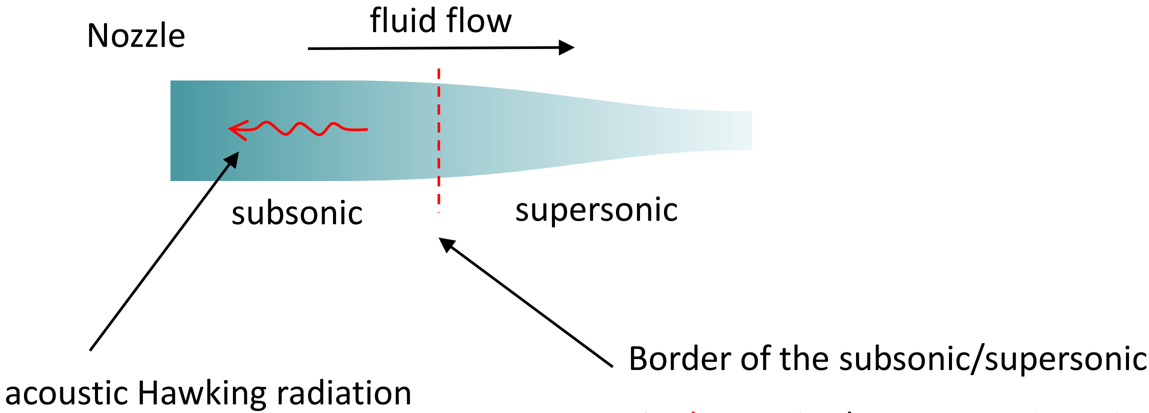


Section 2

Connection to Acoustic Hawking radiation

Connection to acoustic Hawking radiation

◆ Acoustic Hawking radiation: Unruh 1981



→ Phonon in the supersonic region cannot propagate into the subsonic region.

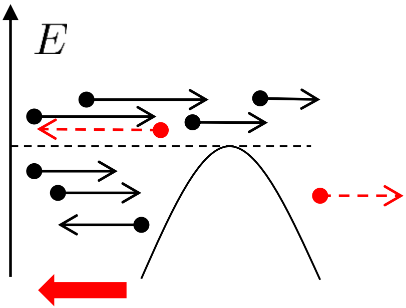
→ Play the role of an Event Horizon for phonon

Connection to acoustic Hawking radiation

Giovanazzi 2004, TM 2018

Inject many free fermions from the left

cf. c=1 matrix model



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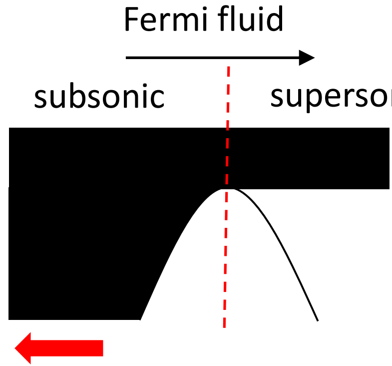


Quantum energy transfer

Thermal emission obeys
Fermi-Dirac distribution.

Acoustic Hawking radiation
(Horizon is at the tip of the potential)

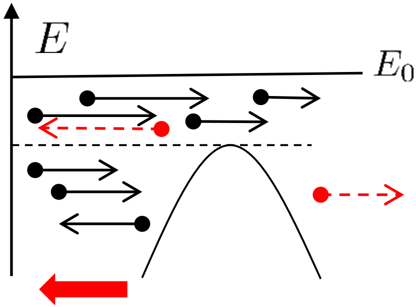
Microscopic description of HR

Thermal emission from the hyperbolic fixed point

$\simeq$ Hawking radiation

Acoustic Hawking radiation from quantum mechanics

Giovanazzi 2004, TM 2018

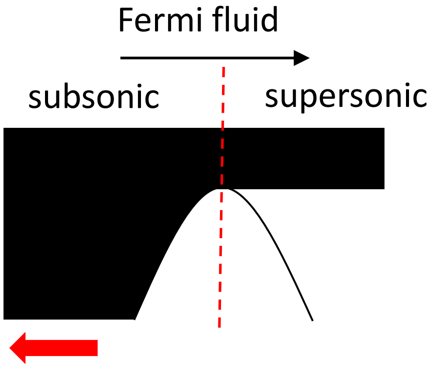


$$\frac{m}{2\pi\hbar}\left(\int_{-\infty}^{0}\frac{dE}{|p(E,x)|}\frac{-E}{e^{-\beta_L E}+1}+\int_{0}^{E_0}\frac{dE}{|p(E,x)|}\frac{E}{e^{\beta_L E}+1}\right)=\frac{1}{(-x)}\frac{T_L}{24}\left(1+O\left(\frac{T_L}{\alpha x^2}\right)\right)$$

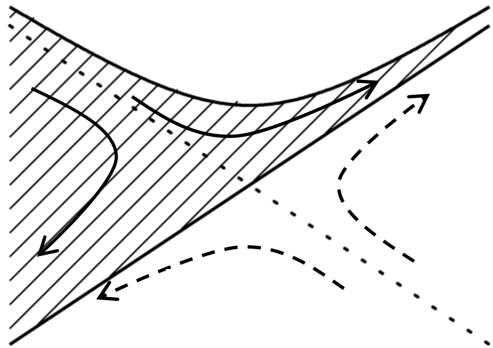
density of states — tunneling — reflection

Acoustic Hawking radiation from fluid mechanics

Giovanazzi 2004, TM 2018



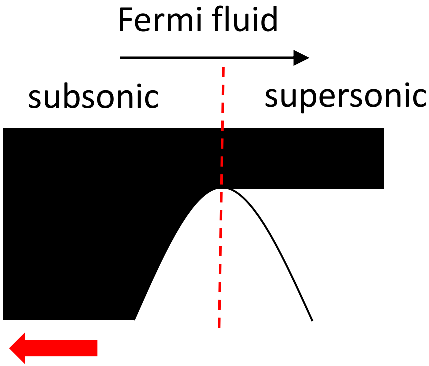
Free fermions in the phase space(classical)



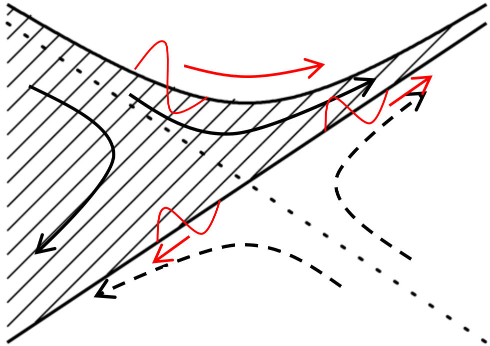
phonon = (small) fluctuation of the surface of the droplet

Acoustic Hawking radiation from fluid mechanics

Giovanazzi 2004, TM 2018



Free fermions in the phase space(classical)



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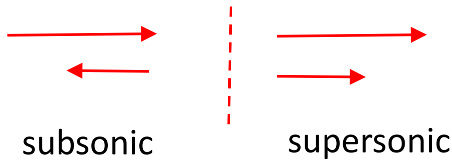
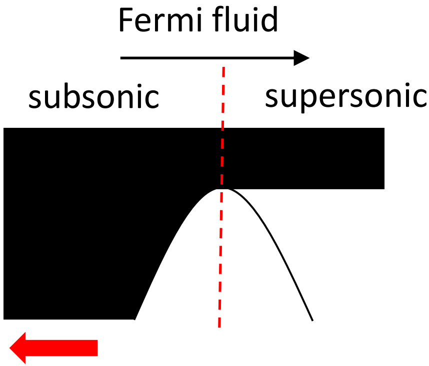
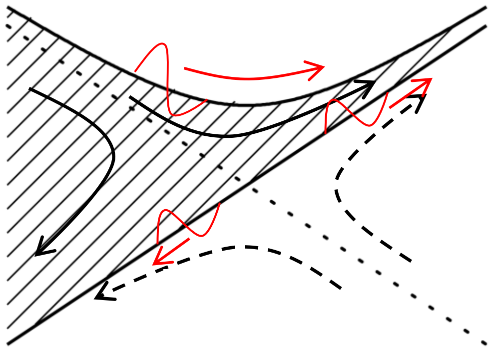


Acoustic Hawking radiation from fluid mechanics

Giovanazzi 2004, TM 2018



Free fermions in the phase space(classical)



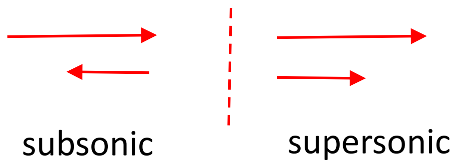
Bogoliubov transformation

$$T_L = \frac{\hbar}{2\pi}\lambda_L$$

$$\frac{1}{(-x)}\frac{T_L}{24}\left(1 + O\left(\frac{T_L}{\alpha x^2}\right)\right)$$

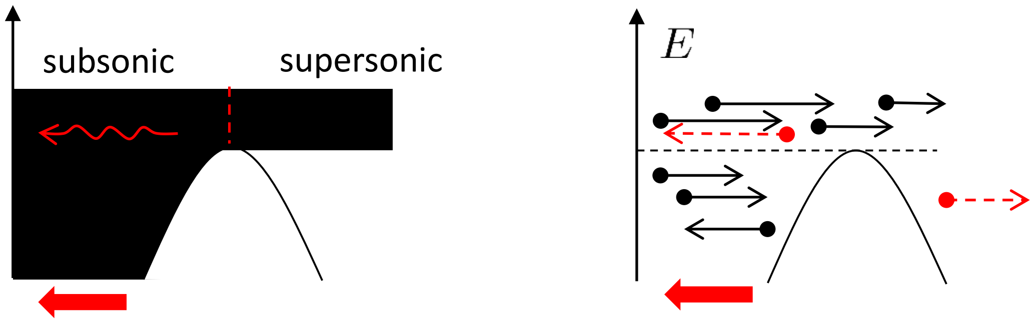
consistent with QM

phonon = (small) fluctuation of the surface of the droplet



Acoustic Hawking radiation from fluid mechanics

◆ Two descriptions of the acoustic Hawking radiation



This agreement is natural but these two descriptions are quite different.

Dynamical variables	Phonon (macroscopic)	Elementary fermions (microscopic)
Statistics	Boson	Fermion
Descriptions	QFT	Quantum mechanics
choice of vacuum	Unruh vacuum	A natural boundary condition

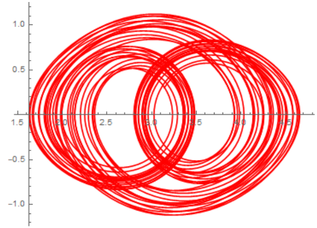
→ Deeper understanding of Hawking radiation.

Summary

Summary:

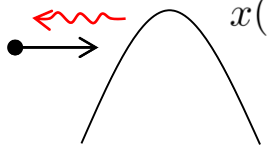
Maldacena-Shenker-Stanford (2015)

Classical chaotic systems may become thermal semi-classically.



$$T \geq \frac{\hbar}{2\pi}\lambda_L$$

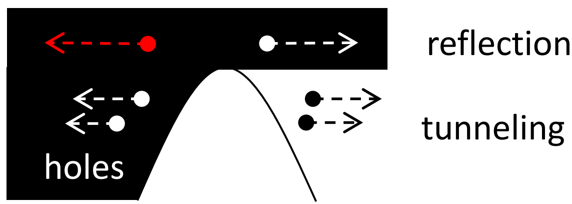
Hyperbolic fixed point (Inverse harmonic potential) may explain this mechanism.



$$x(t) = c_1 e^{\lambda_L t} + c_2 e^{-\lambda_L t},$$

$$\exp\left(-\beta_L |E|\right) \qquad T_L = \frac{\hbar}{2\pi}\lambda_L$$

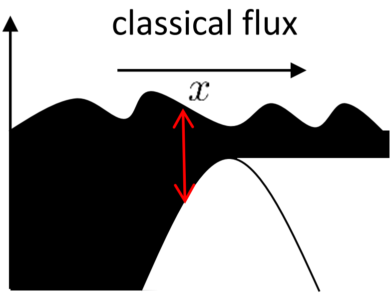
The thermal emission may be related to Hawking radiation.



Thanks!

Appendix

Acoustic Hawking radiation from fluid mechanics



Fluid variables

$$\begin{cases} \rho(x,t) & \text{density of the fermions} \\ v(x,t) & \text{velocity} \\ p(x,t) = \dfrac{\hbar^2\pi^2}{3m^2}\rho(x,t)^3 & \text{pressure} \end{cases}$$

Fluid variables obey hydrodynamic equations: Dhar-Mandal-Wadia 1992

$$\partial_t\rho + \partial_x(\rho v) = 0, \qquad \rho\left(\partial_t v + v\partial_x v\right) = -\partial_x p - \rho\partial_x V(x).$$

Assume an expansion
$$\begin{cases} \rho(x,t) = \rho_0(x,t) + \epsilon\rho_1(x,t) + \epsilon^2\rho_2(x,t) + \cdots\,, \\ v(x,t) = v_0(x,t) + \epsilon v_1(x,t) + \epsilon^2 v_2(x,t) + \cdots\,, \end{cases}$$

ϵ : a small parameter

background flow (e.g. stationary flow) — ρ_0, v_0

fluctuation (phonon) — $\epsilon\rho_1, \epsilon v_1$

Acoustic Hawking radiation from fluid mechanics

◆ Acoustic geometry

$$\partial_t \rho + \partial_x(\rho v) = 0, \qquad \rho\left(\partial_t v + v\partial_x v\right) = -\partial_x p - \rho \partial_x V(x).$$

Expansion
$$\begin{cases} \rho(x,t) = \rho_0(x,t) + \epsilon\rho_1(x,t) + \epsilon^2\rho_2(x,t) + \cdots, \\ v(x,t) = v_0(x,t) + \epsilon v_1(x,t) + \epsilon^2 v_2(x,t) + \cdots, \end{cases}$$

$v_1 = -\partial_x \psi$: define velocity potential ψ

At ϵ^1 order, $0 = \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \psi$

$$g_{\mu\nu} = \frac{1}{\rho_0}\begin{pmatrix} c^2 - v_0^2 & v_0 \\ v_0 & -1 \end{pmatrix} \qquad c(x,t) := \sqrt{\frac{\partial p}{\partial \rho}} = \frac{\hbar\pi}{m}\rho_0(x,t).$$

Acoustic metric — speed of sound

→ ψ obeys a wave equation on a curved geometry.

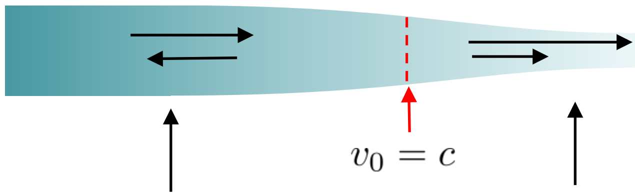
g_{tt} vanishes if $v_0 = c$. → acoustic event horizon

Acoustic Hawking radiation from fluid mechanics

◆ Acoustic event horizon

fluid flow $v_0(x)$

subsonic $(|v_0| < c)$ supersonic $(v_0 > c)$



$v_0 = c$

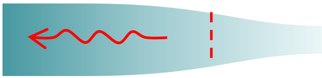
Phonon can propagate to both directions.

Phonon cannot propagate to the left directions.

Causally Disconnected

If we quantize phonon, Hawking radiation is induced toward left

Unruh 1981



$$T_H = \frac{c\hbar}{4\pi} \partial_x f|_{v_0=c} \qquad f := 1 - \frac{v_0^2}{c^2}$$