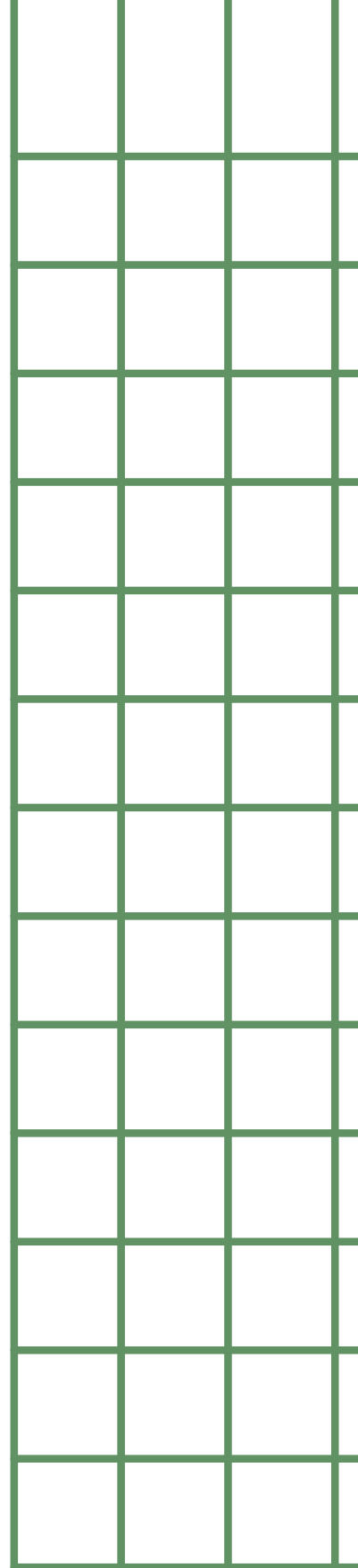# ICTS PhD Program in Mathematics 2024 (Handbook)

Prepared on 01 August 2024. All rules are subject to change and one may refer to https://www.tifr.res.in/academics/academic_procedures.php for additional clarifications.

Table of contents

1

Faculty Mentor, Course Structure, Evaluation and Advisor Preference

a. Faculty Mentor

Each student will be assigned an ICTS faculty mentor. The mentor will discuss and be of assistance on various academic as well as non-academic issues with the students to help them adjust to graduate student life. The mentor will be the main contact for the students before they register for a PhD after which the main contacts will be the PhD supervisor and the thesis committee. Students are required to get in touch with mentors at least once a month and get the academic progress card filled by the 10th of every month. In case the faculty mentor is traveling, the student can get the academic progress card (APC) filled by the Convener, Graduate Cell within the specified time. Please note that the mentor is not the PhD thesis supervisor. The student will be encouraged to find a tentative thesis supervisor before the end of the first year with the assistance of the faculty mentor. The tentative thesis supervisor and mentor (if different from tentative thesis supervisor) will assist the student in designing appropriate coursework for the student.

You can speak to the Convener, Graduate Cell, in case you do not wish to speak to your faculty mentor. For any problems regarding the coursework in the first year, you should approach your faculty mentor or Convener, Graduate Cell if your faculty mentor is away.

b. Course Structure

i. *Credit requirements and courses:* The courses taught in ICTS are 4 credits each. For the PhD program, the student needs to complete 48 credits which may include research projects of 8 credits each. The research project will include presentations. Please find the course structure on page 12.

ii. Course registration: The courses for the autumn semester are to be registered by 10th August and for the spring semester by 10th January every year. The summer project registration should be completed by 6th May every year. The academic office and the faculty mentor should be informed about the course registration within the specified timelines.

iii. *Dropping the course:* The core courses can be dropped with credit if the student passess the qualifying exams or via a drop test at the beginning of the semester. A satisfactory performance (> B i.e. > 65 marks out of 100) allows the student to automatically earn credits for the course.

The elective courses that you have registered for, can be dropped by 15 October for the autumn semester and by the 28th February for the spring semester. The academic office and the respective faculty mentor should be informed about this within the specified timeline. The dropped elective course will not be listed in the final transcript if it is informed to the academic office and the faculty mentor within the specified timeline. A student may register again for a course that he/she has dropped in a previous term/ semester.

Deadlines to register for and drop courses are subject to change. Please check with the ICTS Academic Office for the current deadlines.

c. **Evaluation**

i. *Announcement of grades:* The grades will be announced soon after we receive the grades from the course instructors in ICTS and other institutions. The nominal expected dates for the announcement of grades are given in the Academic Calendar.

Marks	Grades	Grade points	Interpretation
95 – 100	O	10	Outstanding
85 – 94	A+	9	Excellent
75 – 84	A	8	Very Good
65 – 74	B+	7	Good
55 – 64	B	6	Above Average
51 – 54	C	5	Average
50	P	4	Pass
< 50	F	0	Fail

ii. *Fail or Incomplete (I) Grade:* If a student registers for a course, does not drop within the deadline for dropping the course, and does not complete it, he/she is deemed to have failed in that course.

In case a student attends a course but is unable to participate in the final examination for some reason beyond his/her control, he/she will be given an "I" (incomplete) grade. The course instructor, in consultation with the Convener, Graduate Cell, can arrange for one or more special tests/ assignments after the end of the semester to evaluate the performance and convert the Incomplete grade into a regular grade. This procedure is to be adopted only in exceptional situations.

A student who fails a compulsory course has to take a re-exam/ take the course next time it is offered; if he/she passes the course, the new marks replace the old (failed) marks on the student's transcript.

A student who fails an Elective course may replace it with a different Elective.

The marks secured by a student in all the courses taken are used to calculate a Cumulative Grade point average (CGPA) as follows:

$$CGPA = \frac{\sum_n C_n G_n}{\sum_n C_n},$$

where G_n is the grades secured in a course with Credits C_n and both sums run over all the courses taken till the date concerned.

iii. *Academic Probation and Terminations:* The CGPA is the major criterion in determining the performance of a student in the course work. If the CGPA of a student falls below 5.5, they will be placed on Academic Probation and will remain so until the CGPA becomes 5.5 or more, or the course work ends whichever is earlier. Likewise, if a student fails (marks < 50) in one course, they will be placed on Academic Probation and will remain so until they clear the course. A student on Academic Probation may be permitted to take one course less during the ongoing semester in consultation with the Graduate Cell Convenor.

The academic programme of a student will be terminated if any of the following occur:
• they fail in more than two courses, irrespective of CPI,
• they fail in two courses and the CGPA is less than 5.5,
• they fail in one course while on Academic Probation,
• they fail to Register for PhD by the end of the Second Year. This deadline will be extended to compensate for any long-term leave taken by the student (with appropriate permissions from the institute).
• termination is recommended by the institute with sufficient reason(s) and approved by the SBM.
• termination on disciplinary grounds is recommended by a duly constituted authority.
• they have not found a thesis supervisor and passed the qualifying examination by the end of the fourth semester

The coursework requirements should be completed before PhD registration.

d. **PhD advisor preference**

After candidates have completed 3 semesters of coursework, they will have to clear the comprehensive exam. Before that, they have to select a potential advisor (thesis supervisor) who will constitute a comprehensive exam committee.

i) **Guidelines regarding ICTS students having primary advisor in another institute**

- For their PhD work, ICTS students can choose to work with a faculty mentor in another institute provided they can get an ICTS faculty to be their formal thesis advisor. The formal thesis advisor would be responsible for all administrative matters including holding annual TC meetings.

ii) **Eligibility of a research supervisor, co-supervisor and thesis committee**

Eligibility of a Research Supervisor
- A Faculty member who has less than three years left till superannuation at the time of Registration of a student is eligible to be a Research Supervisor provided there is a co-supervisor who is not more than 55 years of age. A Faculty member who has less than six months left till superannuation at the time of Registration of a Ph.D. student is not eligible to be the Research Supervisor.

Eligibility of co-supervisors
- Any faculty member from TIFR or non-TIFR academic institutions
- The CoI rules vis a vis the supervisor and co-supervisors may be relaxed as long as there are 2 additional independent members in the TC.

Eligibility of thesis committee members
- Any faculty member from TIFR or non-TIFR academic institutions
- TIFR Scientific Officers (SO) holding a PhD degree, subject to the Convener's approval of the same and provided they do not report directly to the Research Supervisor/ co-supervisor. It is expected that the officer be at least at SO(F). Exceptional cases of SOs at lower levels may be specifically permitted by the SB Convener.
- PhD-qualified scientists at an appropriate level from the R&D sector

2

Comprehensive exam and PhD registration

a. **Comprehensive exam and PhD registration**
Students have to register for PhD by the end of the second year. Before registering, students are also required to give a comprehensive examination. The comprehensive exam should be taken after 3 semesters of coursework and before the end of the 4th semester. Before a comprehensive exam, students should decide on a potential thesis advisor and a Comprehensive exam committee should be made (The comprehensive exam committee should be preferably the same as the Thesis Committee to be formed after registration). Two chances will be given to pass the exam. The comprehensive exam will be a mix of research topics plus coursework. The exact nature is to be decided by the student's comprehensive exam committee. The assessment panel will comprise of (at the minimum) the student's advisor, an external faculty member from a research institution or university, and another ICTS faculty member. The panel will be chaired by the ICTS faculty member (non-advisor).

Guidelines for extension of registration date beyond July 31
Students should send a request to the GC specifying reasons for asking for an extension. The request has to be supported by the prospective thesis advisor. The GC can recommend an extension up to a maximum period of six months. Fellowships will stop if an extension is not granted.elines

b. **Thesis Committee (TC)**
Every registered student must have a Thesis Committee (TC). The TC should be preferably the same as the Comprehensive Exam Committee. The TC will monitor the progress of the doctoral work, help in mentoring the student, and advise the student as well as the supervisor, if necessary.
The TC must be formed by the Convener, Graduate Cell, in consultation with the Thesis Supervisor, at the time of Registration.
Students should make sure that the coursework requirements, oral comprehensive exam, formation of TC and thesis proposal are completed before the PhD registration deadline. Please note that it is the responsibility of the students to find an advisor (thesis supervisor) and be registered for their PhD before the deadline. Deadlines are subject to change, and the student should contact the Academic Office for the current deadlines.

The students will receive a degree through the Mathematics Subject Board of the TIFR Graduate School. A change of the subject board may be considered in exceptional cases and should be discussed with the academic office, PhD supervisor and the Convener, Graduate Cell.

Please note that students are free to choose any of the ICTS faculty members as a PhD supervisor. Student and advisor should - agree to follow the guidelines of the Maths subject board in which the student is registered.

c. PhD registration procedure

When the student is ready to register for PhD before the end of the second year, the following procedure needs to be followed.

i.   Coursework and comprehensive examination requirements should be completed before PhD registration.

ii. After the comprehensive exam, the student will have to upload the following documents on TIFR's online portal, Datanet: https://datanet2.tifr.res.in/icts:
   • Self-signed research proposal (1-2 pages)
   • Transcript (-download Transcript for registration for Datanet2.0)
   • Thesis committee formation form (available on Datanet2.0)
   • Comprehensive exam report (available on Intranet)

iii. The student will be intimated about their registration number via email and the fellowship will be -enhanced (as per norms) from the date of theirPhD registration.

iv. Please note that the PhD registration is valid for 6 years and the fellowship will be provided only for 5 years from the date of registration. If the student does not submit their thesis within this duration, then they will have to re-register for their PhD.

d. Annual assessment and extension

Assessment before PhD registration: Annual assessment formalities for students before PhD registration will be initiated in the month of June. The annual assessment will be based on the APC report generated every month by the mentor. It is the student's responsibility to ensure that they meet their mentors before the 5th of every month and that the mentors submit their monthly APC report. After completing this formality, the student will be given an extension note by July.

Assessment after PhD registration: Annual assessment formalities for students after PhD registration should be initiated by the Thesis Committee (TC) in May. TCs are expected to monitor the progress of the student till the completion of the student's PhD throughout the year. The TC must meet the student at least once in every academic session, and preferably once every semester. There is no restriction on the maximum number of times the TC can meet the student. TC members are encouraged to interact with the student multiple times in a year, and individual TC members can also talk with the student one-on-one. Students are encouraged to talk to their TC members or the Convener,

Graduate Cell or the subject board representative for any academic problem.

Any serious problems envisaged in the progress of the student should be brought to the attention of the Convener, Graduate Cell and the Subject Board. The TC report should be submitted to the academic office with a cc to the Graduate Cell for the annual extension.

PhD registered students will have an annual assessment in the month of May/ June. Students will have to give a presentation about their project for about 30-40 mins followed by a question and answer session in front of their TC. For every student, a report from the TC is necessary for the annual extension. This report should be submitted to the academic office by the end of June. After completing this formality, the student will be given an extension note by July. If the TC report is not available, the student may be denied an extension of the fellowship for the next academic session.

3

Synopsis submission and its requirements

The PhD thesis should be an advanced exposition of a particular topic. It must demonstrate that the candidate has done a significant amount of original work in the chosen subject.

a. Change in the title before/ during synopsis submission

If the title of the thesis has changed from that in the registration form, this should be indicated by the student and approved by the Guide.

b. Synopsis: Submission, Seminar and Acceptance

Before Synopsis Seminar

i. When the work required for the PhD thesis is completed, the candidate should prepare a synopsis of the would-be thesis and submit it to their thesis supervisor. The contents of the synopsis have to be presented by the candidate in an open seminar (the "synopsis seminar"). A committee of three members will be formed to review the synopsis and synopsis seminar in order to determine whether sufficient work has been done for the candidate to prepare and submit a thesis.

ii. Once the thesis supervisor has approved the synopsis, the candidate will share the synopsis with the Convener, Graduate Cell/ Dean (if the Convener, Graduate Cell is the guide) and the Academic Office.

iii. The thesis supervisor will suggest names of members of the committee that will review the synopsis and the synopsis seminar to the Graduate Cell. In consultation with the supervisor, if so desired that the synopsis committee is different from the TC, the Convener, Graduate Cell/ Dean will constitute the Synopsis Review Committee and share the details with the Academic Office.

iv. In consultation with the thesis supervisor, the student will propose possible dates and times for the synopsis seminar, and share these with the Academic Office. The date of the synopsis seminar must be at least 10 days from the time the synopsis is submitted to the Graduate Cell and the Academic Office.

v. The Academic Office, on behalf of the Graduate Cell, will send out an invitation to the committee members for their confirmation and to fix the date of the synopsis seminar.

vi.   In the meantime, the student needs to share the title and abstract with the Academic Office for the seminar announcement. The announcement will be made once the seminar date is fixed.

vii. The Academic Office will share the required evaluation form with the committee members, prepare an evaluation report and upload it on Datanet2.0.

After the synopsis seminar

i.   The student should get in touch with his/ her thesis supervisor to make the necessary changes suggested by the evaluation committee.

ii.   The student submit the final synopsis after the changes (suggested by the committee members) along with the signed synopsis submission form through the online portal (Datanet2.0).

iii.   In case the synopsis review committee feels that more research work is required in order to proceed further, it may be necessary to repeat the entire synopsis submission and seminar process after the required work is done.

4

PhD thesis submission and its requirements

a. **Thesis submission**

The thesis can be submitted any time starting from the next day following the submission of Synopsis, to a maximum of six months from the date of submission of Synopsis. After consulting with the thesis supervisor, the candidate should submit the thesis (the format of cover page and declaration page is available on Datanet2.0) by uploading it to the online portal (Datanet2.0)

The thesis submission will then go through a series of approvals on Datanet2.0, starting with the thesis supervisor. This sequence of approvals may take several days, and the candidate is advised to keep this in mind. The submission is only complete when it has received all approvals.

When approving the thesis submission on Datanet, the thesis supervisor will be asked to suggest the names of 3 referees from India and 3 from abroad, who can be potential examiners for the thesis. TIFR will choose two examiners.

b. **Fellowship after thesis submission**

The prescribed duration of research stay of students enrolled for PhD at TIFR is 5 years. Students who submit their thesis before the completion of this duration will continue to get their existing fellowship and accommodation till the end of the above duration or Viva-voce, whichever is earlier.

Please note if the student submits the thesis during their extension period, the date of thesis submission is the last date of their fellowship and accommodation.

5

PhD Thesis Defence (viva-voce examination)

a. Approval

If the reports of the external examiners, as well as the report of the Thesis Supervisor, are all positive, and only minor changes or revisions are required, the Convenor, on behalf of the subject board, gives the permission for the Viva-voce Examination (or thesis defense). Till this permission is received, the thesis supervisor should not undertake any communication with external examiners.

Once the permission is received, the following committee is set up to conduct the Viva-voce examination.
• the thesis supervisor (advisor)
• One of the external examiners
• representative of the Subject Board of Mathematics.
The Viva-voce examination will be set up at a date and time convenient to all concerned, with the help of the academic office.
The student needs to share the title and abstract at least one week before the thesis defense for its announcement and necessary arrangements.

b. Viva-voce examination pattern

The viva-voce examination consists of three parts.
i.   An open seminar by the candidate, lasting not less than 45 minutes, which highlights the research work presented in the thesis. This may be attended by any interested person(s), in addition to the examiners.
ii. An open question-and-answer session, during which anyone in the audience of the open seminar (including the examiners) may ask questions or make comments on the material presented in the seminar.
iii. An in-camera examination, where only the candidate and the examiners will be present.

c. Final thesis submission after viva-voce examination

After the viva, the student should incorporate any changes/corrections/additions suggested by the examiners, and prepare a final version of the thesis. The title page should have the revised month and year according to the format available on Datanet 2.0.

The thesis supervisor should check that these changes have been satisfactorily incorporated while approving the thesis on Datanet 2.0.

Please refer to the webpage for the procedure of thesis submission to the University cell: https://www.tifr.res.in/academics/academic_procedures.php

Once the TIFR University Cell receives the final thesis, a provisional degree certificate will be issued. The student can download their provisional degree certificate on Datanet2.0

d. Short-term employment for fresh graduates

Fresh graduates with a provisional/regular PhD degree will be eligible to be appointed as Research Associates (fellowship as per norms) for a maximum of 6 months. For this, the advisor has to make a recommendation to the Postdoc Committee.

6

Extensions beyond 5 years

a. Extension

A student who needs an extension beyond 5 years has to discuss it with their thesis advisor at least 6 months before the tenure end date. The thesis advisor will have to send a request to the Graduate Cell with a justification for the extension. Such requests will be discussed by the Graduate Cell and the extension will be granted as per norms.

Please note that if the student submits the thesis during their extension period, the date of thesis submission is the last date of their fellowship and accommodation.

b. Emoluments

The fellowship in such cases will be reduced to one-half of the original fellowship with no contingency grant.

c. Thesis Fees

The student will have to bear the thesis fees of Rs.4,000/- if he/she fails to submit the thesis beyond the extension approved by the Graduate Cell.

7

Contingency grant, carry forward, grader's grant

a. Contingency grant

Students are eligible for a Contingency Grant for the first five years. The payment will be released against the bills presented. The permissible expenditure under this allowance is listed below. Any expenditure beyond this list needs the approval of the Convener, Graduate Cell.

i.   Purchase of books, journals, cost of photocopying of documents
ii. Payment of fellowships/memberships of professional organisations/ societies
iii. Travel to conferences, workshops and other educational trips
iv. Purchase of Laptops, notebooks, Tablet PC s, e-book readers etc.
v.   Purchase of computer peripherals, accessories and software
vi. Internet bill for students accommodated outside the campus.

If the cost of a particular item is more than the available amount of the Contingency Grant, the extra amount may be claimed from the next year's Contingency Grant once the student becomes eligible for it.

The Contingency Grant has to be utilised during the Academic Year (Aug -July). It may be carried over to the next year only for a known large expenditure in the following year, like a conference that the student plans to attend. The carry-over will be only for this specific purpose, and the permission has to be obtained from the Convener, Graduate Cell (after the recommendation of the faculty mentor/ thesis guide) before the end of the earlier academic year.

The Contingency Grant for an academic year may be claimed at any time during the year. However, if the student leaves the program before the completion of the year, the amount of Grant the student is eligible for will be calculated (proportional to the fraction of the year spent in the Institute) and the extra amount will be recovered.

Please click here for the contingency form. Fill up the form, attach the bills and take your faculty mentor's signature on the form before submitting it to Accounts for processing.

Reimbursement rules are available in item number 16 on the webpage link: https://www.tifr.res.in/academics/academic_procedures.php

b. Contingency carry forward

Students can carry forward their contingency grant for travel costs for attending an identified conference, academic visit, etc. The request should be made by the students mentioning the name of the conference or the planned academic visit to carry forward and it should be recommended by the respective faculty mentor (till PhD registration) and thesis advisor (post PhD registration). The recommendation should be forwarded to the academic office. The academic office will compile the list for the final approval of the Convener, Graduate Cell.

For any queries related to the usage of contingency grants, the accounts team (accounts@icts.res.in) can be contacted.

c. Grader's grant

Any classroom course that is certified as a core/elective course by the Convener, Subject Board, can have a grader who will receive a book grant of Rs.5000/- at the end of the semester. If the course has more than 10 registered students, the faculty can

8

Administration contact information at ICTS

a. **Academic matters and office space:** For all academic matters, including courses, results, comprehensive exams, PhD registration, annual assessment and extensions, synopsis and thesis submission, office space allocation etc., please contact the academic office: academicoffice@icts.res.in; extension: 6042

b. **Accommodation:** For any query or help related to accommodation or hostel, write to the academic office. For personal bookings, go through the housing guidelines on the intranet and contact ICTS guest house: guesthouse@icts.res.in extension: 6400

c. **Laptop/ Desktop/ IT/ Wi-fi:** For any query related to laptop/ desktop/ IT/ wi-fi, please contact ithelp@icts.res.in; extension: 6300

d. **Establishment:** For any query related to your joining, leave, ID card, housing allotment, official letters, medical insurance etc. please contact establishment@icts.res.in; extension: 6016

e. **Accounts:** For any query related to the reimbursements, contingency grants, fellowship etc. please contact accounts@icts.res.in

f. **Useful links**
i. Welcome booklet
ii. Graduate courses
iii. Guidelines for academic ethics
iv. ICTS seminars and colloquia
v. Holiday list

9

Academic and Non-Academic Committees at ICTS

a.   Academic committees

i.   Computing Resource Committee
ii.   Faculty Search and Screening Committee
iii. Graduate Cell
iv. Lectures and Visitors Committee
v.   Library Committee
vi. Outreach Committee
vii. Postdoctoral Committee
viii. Program Cell
ix. Visiting Students Committee

b.   Non-academic committees

i.   Anti-ragging Committee
ii.   Cafeteria and Canteen Committee
iii. Cell for Prevention and Resolution of Sexual Harassment of Women at Workplace (CPRSHWW)
iv. Child Care Cell
v.   Cultural Committee
vi. Engineering Service Committee
vii. General Administration and HR Planning & Management Committee
viii. Grievance Cell
ix. Housing Cell
x.   ICTS Standing Physical Verification Committee
xi. Local Purchase Committee
xii. Medical Committee
xiii. Resource Development and Societal Engagement Wing (RDSEW)
xiv. RTI Cell
xv. Sport and Recreation Committee
xvi. Standing Committee on Admin
xvii. Tender opening committee
xviii. Transport and Security Committee

For the roles, members of these committees and contact details, please check the ICTS intranet here: https://intranet.icts.res.in/content/committees

10

Course Requirements and Syllabi

First two years of the Ph.d. programme: (Tentative Plan)

First-year Course work

Each student will need to take 4 core courses in the first year. The courses will need to be from the list below and offered by ICTS faculty or equivalent courses offered at TIFR-Colaba, TIFR-CAM, ISI or IISc may be taken offline/online based on faculty consent. The elective courses can be taken as topics courses offered by a faculty or reading courses or research projects as mandated by the handbook. Each core/ elective/ reading course will be 4 credits and research project will be 8 credits.

The coursework for the first year will be decided in consultation with the mentor, until the student finds a tentative thesis supervisor. From the beginning of the third semester, or whenever a tentative thesis supervisor is assigned (whichever is earlier), the coursework will be decided in consultation with both the mentor and tentative thesis supervisor.

The student needs to take 48 credits worth of coursework before registration (by the end of the fourth semester). The 48 credits should include a minimum of 4 core courses. Below is the minimum course requirement for the first four semesters.

Semester I : 2 of the core courses and 1 Elective course

Semester II : 2 of the core courses and 1 Elective course.

Semester III: Coursework (prescribed by mentor/tentative thesis supervisor)

Semester IV : Coursework (prescribed by mentor/tentative thesis supervisor)

As per UGC requirement, before registration, the student should complete the courses Research Methodology I (2 credits), Research Methodology II (2 credits), and Research and Publication Ethics (2 credits). These will be offered every year at ICTS; see syllabus below.

Qualifying examinations

In order to be eligible to continue in the program, each student must pass the qualifying examinations before the end of the fourth semester. The qualifying exams will consist of:
Algebra (Syllabus will cover C1 and C4)
Analysis (Syllabus will cover C2 and C6)
Probability (Syllabus will cover C3 and C8)

To pass the qualifying exam, the student has to pass two out of the above three exams. This exam will typically assess how well the student has undergone the study program in their respective core courses. The qualifying exams will be held twice a year (in the first week of August and first week of January). There are two possible results of the qualifying exams- pass or unsatisfactory. A pass performance is a grade > B i.e. >= 65 marks out of 100.

Students of the Ph.D. programme can take the qualifying exam any time from joining. If a student of the Ph.D. programme does not pass the qualifying exam by the beginning of fourth semester, they will be asked to discontinue the programme. The students may attempt the qualifying exam any number of times subject to this deadline.

Core Courses
C1- Algebra 1

- Group theory: Group theory, permutation groups, Cayley's theorem, Sylow theorems.
- Ring theory: Ring theory, modules, integral domains and fraction fields, polynomial rings, matrix rings.
- Spectral theorem: Bilinear forms; Inner product spaces; unitary, self-adjoint, normal, and isometric transformations; Spectral theorem.
- Fields : Field theory: Algebraic and Transcendental extensions; Finite fields, Wedderburn's theorem on finite division rings.

C2- Analysis 1

- Measure Theory : Sigma-algebras, measures, outer measures, completion, construction and properties of the Lebesgue measure, non-measurable sets, Measurable functions, point wise convergence, almost uniform convergence, convergence in measure.
- Integration : Lebesgue integration, limit theorems, comparison with the Riemann integral, relationship with differentiation, functions of bounded variation and absolute continuity.
- Signed Measures : Radon - Nikodym theorem, Lebesgue decomposition theorem, change of variable formula, Product Spaces, Fubini's theorem and applications.
- Lp-Spaces: Hölder and Minkowski inequalities, completeness, convolutions, approximation by smooth functions, duality.
- Riesz representation theorem: Riesz representation theorem for positive linear functionals, Proof of the theorem, construction of the Lebesgue measure via this approach.
- Complex measures and Differentiation of measures.

C3- Probability Theory 1

- Revision of Measure theory: probability spaces, distributions, random variables, standard random variable examples (discrete and continuous distributions), expected value, inequalities (Holder, Cauchy-Schwarz, Jensen, Markov, Chebyshev) , convergence notions(convergence in probability and almost sure, Lp), application of DCT, MCT, Fatou with examples, Revision of Fubinis theorem.
- Independence, sum of random variables, constructing independent random vari ables, weak law of large numbers, Borel-Cantelli lemmas, First and Second Moment methods, Chernoff bounds and some applications.
- Strong law of large number, Kolmogorov 0-1 law. Convergence of random series. Kolmogorovs three series theorem.
- Weak convergence, tightness, characteristic functions with examples, Central limit theorem (iid sequence and triangular array).

C4- Algebra 2

- Topics in Galois Theory, Ring Extensions and their basic properties.
- Semisimple Rings and modules.
- Representation theory of finite groups.

C5- Functional Analysis

- Banach spaces : Banach spaces, dual space, Hahn-Banach theorem.
- Baire category theorem : Baire category theorem and applications (open mapping, closed graph and uniform boundedness theorems).
- Weak* topologies: Weak and Weak* topologies, Banach Alaoglu theorem, separable, reflexive and locally convex topological vector spaces.
- Hilbert spaces: Hilbert spaces, projection theorem, Riesz representation theorem, adjoint operators.
- Compact operators: Spectral theory for compact operators.

C6- Complex Analysis

- Holomorphic functions : Holomorphic functions, power series, exponential and logarithmic functions, Moebius transformations, Cauchy-Riemann equations, conformality, elementary conformal mappings.
- Contour integrals: Contour integration, Cauchy's theorem.
- Cauchy integral formula : Cauchy integral formula, Calculus of residues.
- Open mapping theorem: Zeroes and poles, open mapping theorem, maximum modulus principle, removable singularities, poles, essential singularities. Laurent expansions.
- Harmonic functions: Harmonic functions-Poisson Integral, Jensen's Inequality, idea of analytic continuation.
- Fourier Analysis: Fourier transform and Inverse Fourier transform, Plancherel theorem in R^n.

C7- Topology

- Rapid review of relevant point-set topology.
- Covering spaces and fundamental groups, van Kampen's theorem and classification of surfaces.
- CW complexes, basics of homology and cohomology (singular and cellular); the Mayer-Vietoris sequence, excision, Kunneth and universal coefficient theorems, isomorphism with de Rham cohomology.
- Cup and cap products, Poincare duality,, Alexander duality, Lefschetz fixed point theorem.
- Optional topics that can be included at the discretion of the instructor: Higher homotopy groups, Whitehead's theorem, fibrations and cofibrations, the long exact sequence of a fibration, the Freudenthal suspension theorem, the Blakers-Massey theorem.

C8- Probability Theory 2

- Discrete time Markov chains: for countable state space, classification of states.
- Discrete parameter martingales: conditional expectation, optional sampling theorems, Doob's inequalities, martingale convergence theorems.
- Brownian motion: construction, continuity properties. Markov and strong Markov property and applications, Donsker's invariance principle, sample path properties.

C9- Partial Differential Equations

- Basics of ODE: (local as well as global) existence and uniqueness results, Picard iteration, Gronwall's inequality, solving some first order and second order equations.
- Introduction to PDE: order of a PDE, classification of PDEs into linear, semilinear, quasi-linear, and fully nonlinear equations, Examples of equations from Physics, Geometry, etc. The notion of well-posed PDEs
- First order PDEs: Method of characteristics, existence and uniqueness results of the Cauchy problem for quasilinear and fully nonlinear equations. [9 lectures] Second order linear PDEs in two independent variables: classification into hyperbolic, parabolic and elliptic equations, canonical forms.
- Laplace equation: Definition of Harmonic functions. Mean-value property, Strong Maximum principle for harmonic functions, Liouville's theorem, smoothness of harmonic functions, Poisson's formula. Harmonic functions in rectangles, cubes, circles, wedges, annuli.
- Heat equation: Fundamental solution of heat equation, Duhamel's principle, weak and strong maximum principles, smoothness of solutions of heat equation, ill-posedness of backward heat equation.
- Wave equation: well-posedness of initial and boundary value problem in 1D and d'Alembert formula. method of descent in 2D and 3D. Duhamel's principle, domain of dependence, range of influence, finite speed of propagation.
- Boundary problems: Separation of variables, Dirichlet, Neumann and Robin conditions. The method of seperation of variables for Laplace, Heat and Wave equations.

C-10 Linear Algebra

- Quick review of solutions of a system of linear equations, vector spaces, subspaces, linear independence and span, Zorn's lemma and existence of basis, quotient spaces and direct sum of vector spaces, exact sequences and splittings, linear maps and matrices, matrix of a linear map in a basis, invertibility, rank and determinant, linear functionals, dual space, annihilator, transpose of a linear map.
- Eigenvalues, algebraic and geometric multiplicities, characteristic and minimal polynomials, upper triangularization, diagonalizability and semisimplicity, decomposition into nilpotent and semisimple matrices, Cayley-Hamilton Theorem.
- Tensor product of vector spaces, extension of scalars, complexification, tensor product of linear maps, symmetric and exterior algebra, determinant as a multilinear map and Laplace expansion.
- Inner-product spaces, orthogonality, Gram-Schmidt orthogonalization, Bessel's inequality, projection and orthogonal projection, symmetric and Hermitian operators, orthogonal and unitary diagonalizability, normal operators, spectral theorem, bilinear and quadratic forms, positive definite operator, square-root of a positive operator, polar decomposition, isometry, rigid motions, the rotation group.
- Structure theory of finitely generated modules over PID and application to canonical forms.

C11: Differential Geometry

- Smooth manifolds: Manifolds in R^n , submanifolds, manifolds with boundary. Smooth maps between manifolds. Regular values. Examples of manifolds: A) Curves and surfaces in R^2 and R^3 . B) Level surfaces in R^{n+1}, C) Inverse image of regular values. Tangent spaces, derivatives of smooth maps, smooth vector fields, Existence of integral curves of a vector field near a point.
- Geometry of curves and surfaces: Parametrized curves in R^3 , length, integral formula for smooth curves, regular curves, parametrization by arc length. Osculating plane of a space curve, Frenet frame, Frenet formula, curvature, invariance under isometry and reparametrization. Discussion of the cases for plane curves, rotation number of a closed curve, osculating circle, 'Umlaufsatz'. •
- Surfaces in R^3 : Existence of a normal vector of a connected surface. Gauss map. The notion of a geodesic on a surface. The existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof. Covariant derivative of a smooth vector field. Parallel vector field along a curve. Existence and uniqueness theorem of a parallel vector field along a curve with a given initial vector. The Weingarten map of a surface at a point, 16 its self-adjointness property. Normal curvature of a surface at a point in a given direction. Principal curvatures, first and second fundamental forms, Gauss curvature and mean curvature. Gauss-Bonnet theorem.

- Differential forms and orientation: Differential Forms, Orientation of manifolds, Integration of forms, Stokes' Theorem. (proof to be given if time permits). Proof of Gauss-Bonnet theorem (if time permits).

- Differentiable functions, inverse and implicit function theorems. Theory of manifolds: differentiable manifolds, charts, tangent bundles, transversality, Sard's theorem, vector and tensor fields and differential forms: Frobenius' theorem, integration on manifolds, Stokes' theorem in n dimensions, de Rham cohomology. Introduction to Lie groups and Lie group actions.

C12: Computational PDE (MTH-206.4)
- Review of basic numerical analysis
- Finite differences for linear equations
- Linear hyperbolic equations, finite differences, theoretical concepts of stability and consistency, order of accuracy, upwind, Lax-Fredrichs and Lax-Wendroff schemes.
- Linear parabolic equations-explicit and implicit schemes, Crank-Nicholson method, introduction to multi-dimensional problems.
- Linear elliptic equations-finite difference schemes, Finite Difference schemes for nonlinear equations.
- One dimensional scalar conservation laws, review of basic theory, solutions of the Riemann problem and entropy conditions. First order schemes like Lax Fredrichs, Godunov, Enquist Osher and Roe's scheme. Convergence results, entropy consistency and numerical viscosity. Introduction to higher order schemes-Lax Wendroff scheme, Upwind schemes of Van Leer, ENO schemes, Central schemes, Relaxation methods. Introduction to finite volume methods. Convection-Reaction-Diffusion equations,Extension to the above methods. Splitting schemes for multi-dimensional problems.
- Finite element methods for linear equations.
- Review of elliptic equations, weak formulation and Lax-Milgram lemma, Galerkin approximation, basis functions, energy methods and error estimates, Cea's estimate and Babuska Brezzi theorem.
- Finite elements for parabolic equations - Galerkin approximation and error estimates. A posteriori error estimates for Elliptic and Parabolic equations.
- Spectral Methods, Fast Fourier transformation, introduction to Fourier, spectral and pseudo spectral methods.
- Implementation of algorithms on computers is an integral part of this course.

C13: Differential Geometry -II
- Real and complex manifolds with examples
- Differential forms, vector bundles, fiber bundles, connections, curvature, etc
- Group action on manifolds, especially adjoint and coadjoint action of Lie groups on the lie algebra, coadjoint orbits.
- Introduction to symplectic and Kähler geometry, with examples.

1. Research Methodology and Ethics Courses

The three courses below (RM1, RM2 and RPE) are compulsory for all students in the mathematics PhD program. Students must complete these courses before registering for their PhD (at the latest by the fourth semester).

RM1: Research Methodology I

Course description: Each session will consist of a lecture given by one faculty member about the work done in their group(s). Students are supposed to interact and actively engage in discussions during the lectures with the speakers, and their colleagues. Each student will be required to submit a short summary essay of what was discussed in these classes. As part of their essays, students are encouraged to identify an interesting open problem that they found interesting in the research area summarized during that class.

RM2: Research Methodology II

The Craft of Research (Research Methodology) and Critical Thinking: What constitutes science, the idea of so called scientific method and the limitations of thinking about science that way. What constitutes a good research problem and the art of identifying one. Critical thinking and critical reading of scientific literature. Use and abuse of statistics within scientific literature.
Scientific writing: General attitudes to writing including how to deal with the "writers' block". Content and structure of scientific documents of various kinds (proposals, reports, papers, reviews etc.). The nature of abstracts, the canonical IMRaD structure and sensible deviations from it. Style in scientific writing. Other forms of writing including research summaries, reply to referees, etc.
Scientific Talk and Presentation: Introduction to rhetoric or the art of persuasive argumentation. What a scientific talk should and should not include given a specific audience. Structure of the conceptual structure and the structure of visual aids. Delivery of talks and presentations.

RPE: Research and Publication Ethics

1. Frameworks useful in analysis of ethical problems.
2. Ethical misconduct involving fabrication, falsification of data as well as writing useless papers
3. Our ethical commitments to the rest of the scientific community. This involves an array of topics including authorship, citation, plagiarism, mentorship, conflicts of interest, confidentiality in peer review, etc.
4. Scientists role in a society with some special attention to the Indian context (diversity issues within scientific community, interactions with society, approach towards superstition and pseudo-science and the impact of political, religious and socio-economic ideologies on scientific research, etc)

Reading Courses

Reading courses can be taken by students with any faculty member at ICTS. These courses are graded through either regular assignments, or through two exams (mid-term and final), or a combination of these.

Projects

Projects can be done with any faculty member at ICTS. Students are encouraged to do projects before choosing the thesis advisor. The student is graded based on a project report to be examined by the project guide, and a seminar presentation to be examined by a two-member committee. The final grade will be based on these two evaluations (equal weightage).

The committee constituted to evaluate the seminars will not only grade the performance of the candidate but also determine whether a sufficient amount of work has been done in the Project under review. The committee may, for example, decide that the work done is insufficient to earn full credit (i.e., 8 credits), in which case the student will be given an Incomplete (I) grade. The scope of the project will have to be extended according to the recommendations by the committee with the approval of the graduate cell. The seminar will be held again at the end of the following semester.

Contact Us

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