

Supermassive Black Holes (SMBHs)

SMBHs are found at the centre of almost all the galaxies, ranging from masses of about $(10^6 - 10^9)M_\odot$. But very little is known about their origin and subsequent evolution to such high masses within the age of the universe.

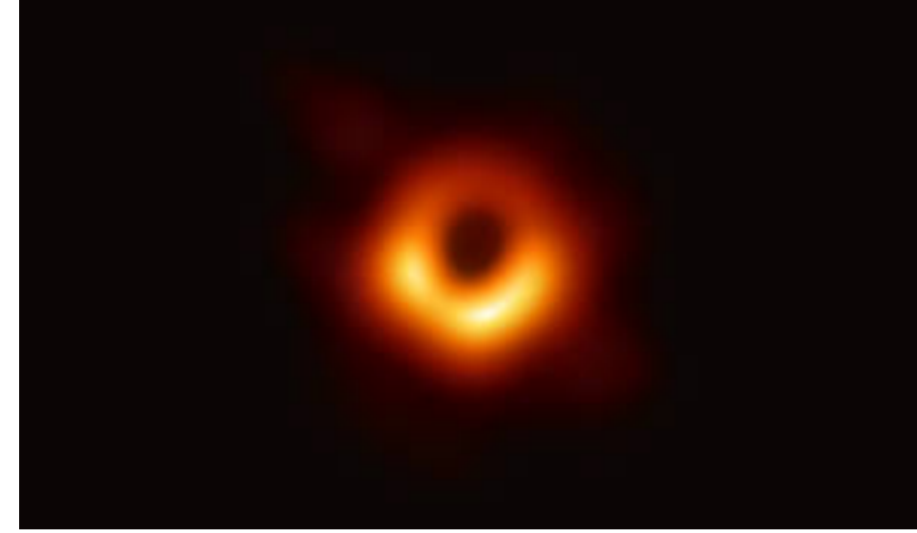


Figure 1. Sagittarius A* - black hole at the centre our galaxy (EHT)

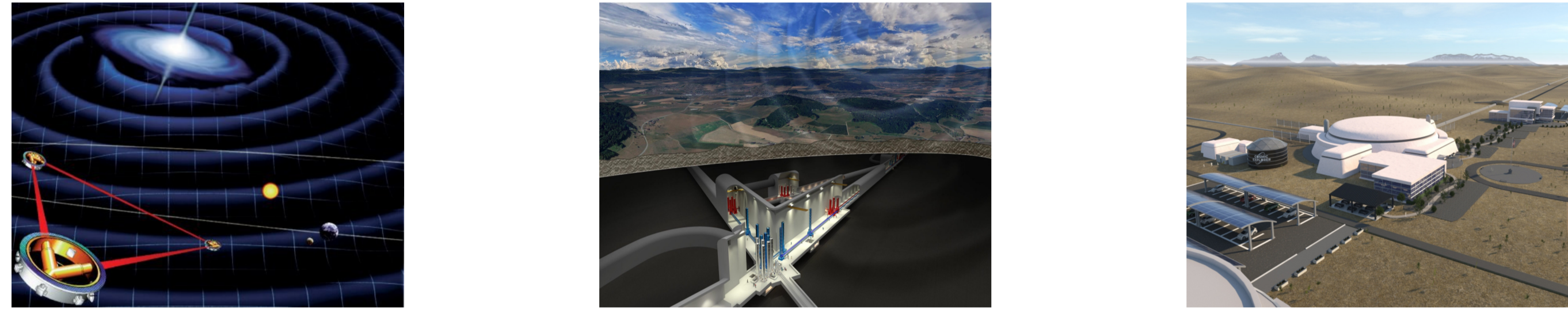


Figure 2. Future detectors - (left to right) - LISA, ET, CE (ESA, NIKHEF, MDPI)

Formation of SMBHs

- **Light seeds** - BH remnants of pop III stars $(10^2 - 10^3)M_\odot \rightarrow (10^6 - 10^9)M_\odot$.
- **Heavy seeds** - BHs formed from DCBH scenario $(10^4 - 10^5)M_\odot \rightarrow (10^6 - 10^9)M_\odot$.
- Successive accretion from the surrounding environment and merger with other black holes lead to their mass growth across the cosmic timescale.

Cosmic growth history of the SMBHs

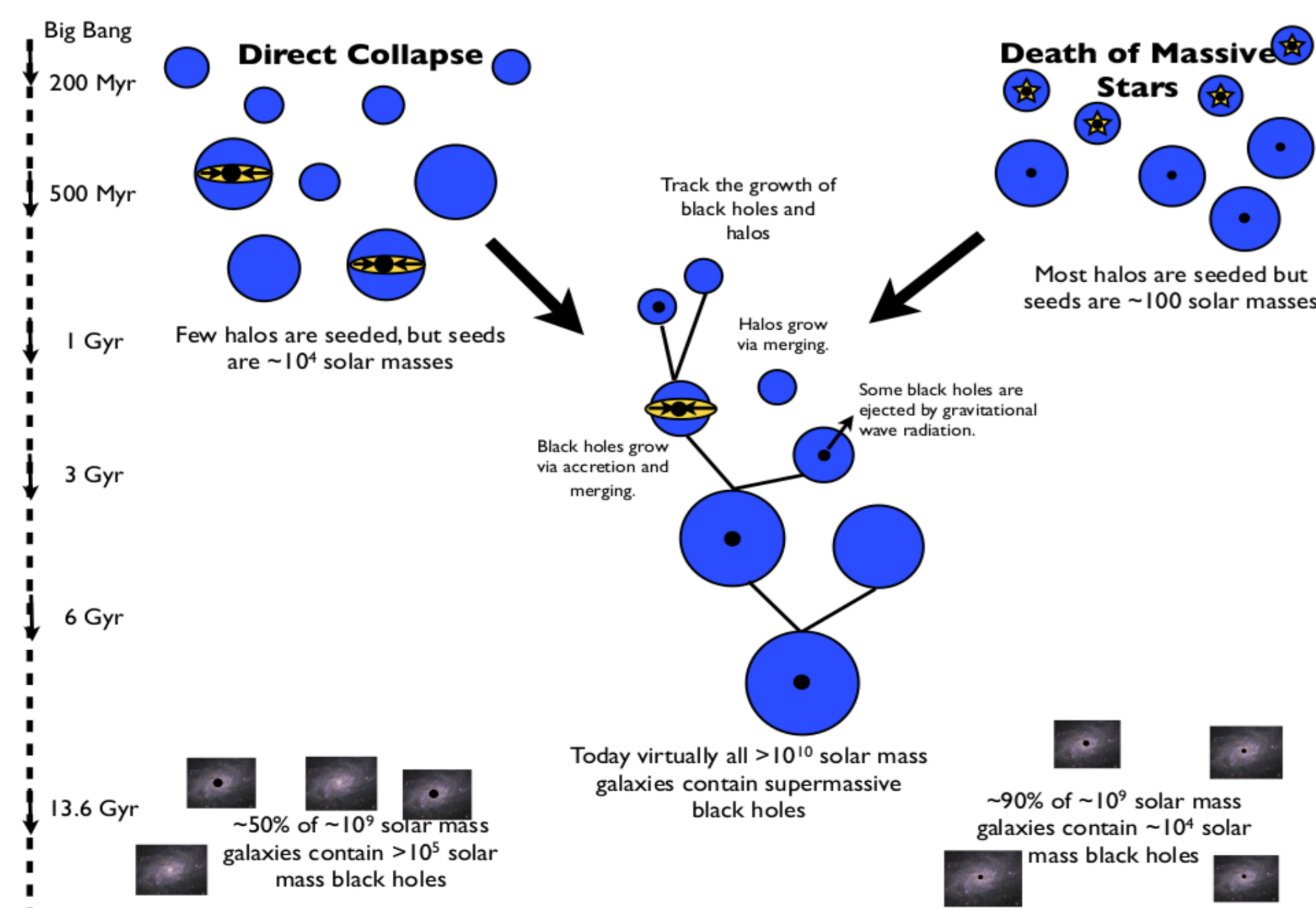


Figure 3. Merger Tree for SMBH formation(J.E.Green, 2012)

Future third generation detectors

- **Laser Interferometer Space Antenna (LISA)** - to be operating in $(10^{-4} - 10^{-1})$ Hz \rightarrow looking out to $z > 20$.
- **Lunar Gravitational-wave Antenna (LGWA)** - to be operating in $(10^{-3} - 4)$ Hz \rightarrow looking out to $z \sim 10$.
- **Einstein Telescope (ET)** - to be operating in $(3 - 10^3)$ Hz \rightarrow looking out to $z \sim 15$.
- **Cosmic Explorer (CE)** - to be operating in $(10 - 10^3)$ Hz \rightarrow looking out to $z > 20$.

Why Multibanding?

- Probing mass function at high z and probable link between the growing seeds and SMBHs.
- Probe the physical processes of black hole mass growth across all z .

SMBH merger event in the detectors

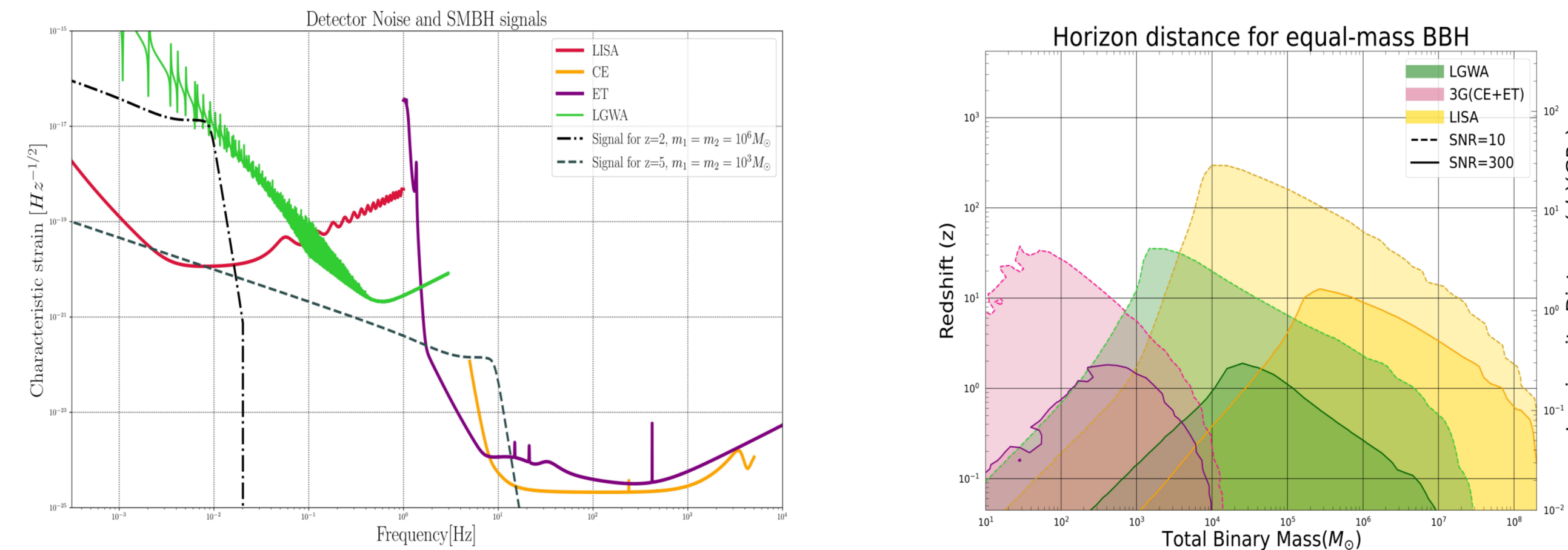


Figure 4. Power spectral densities and horizon of detectors

Fisher Matrix Analysis (FMA) of detectable events

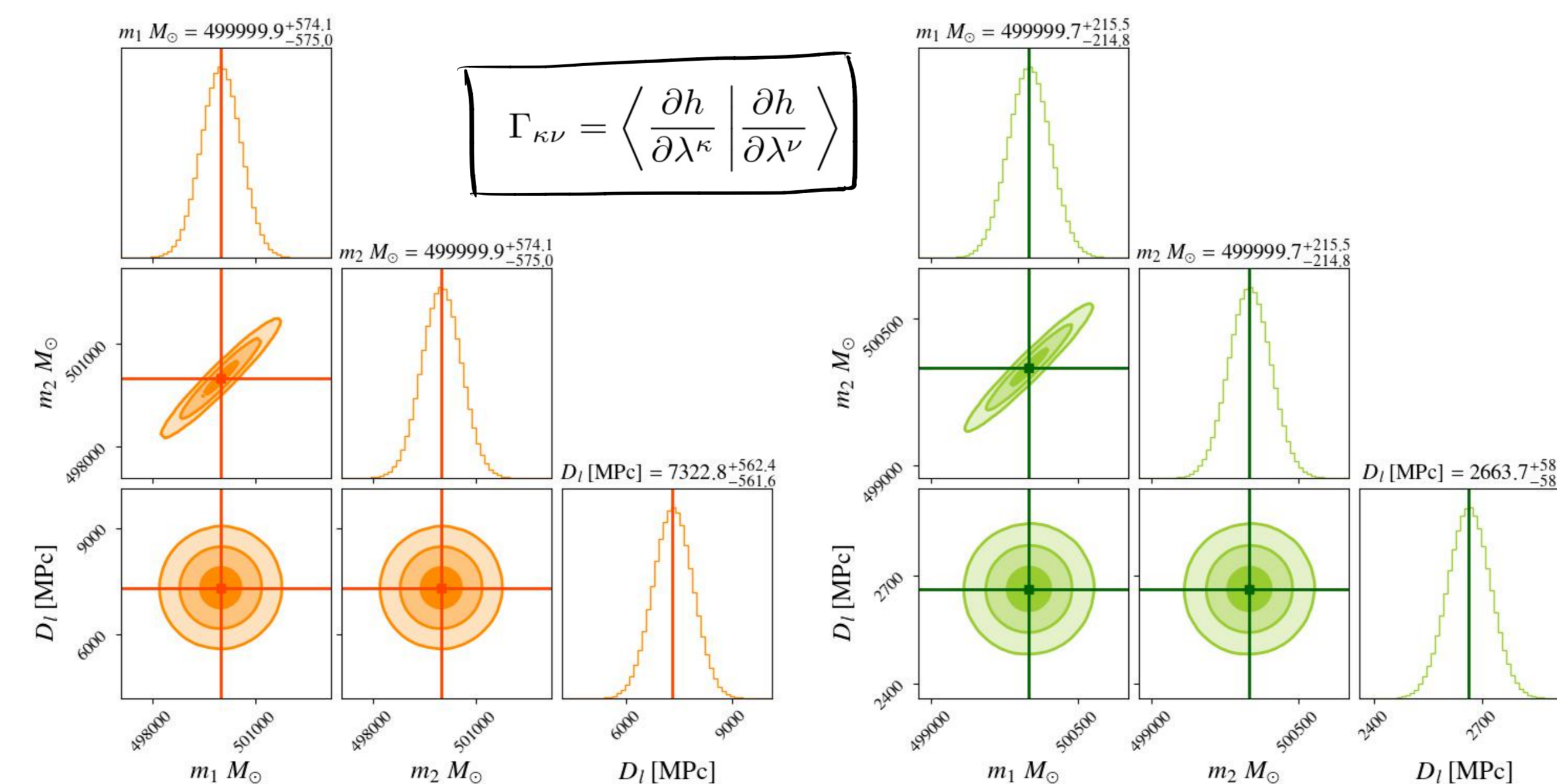
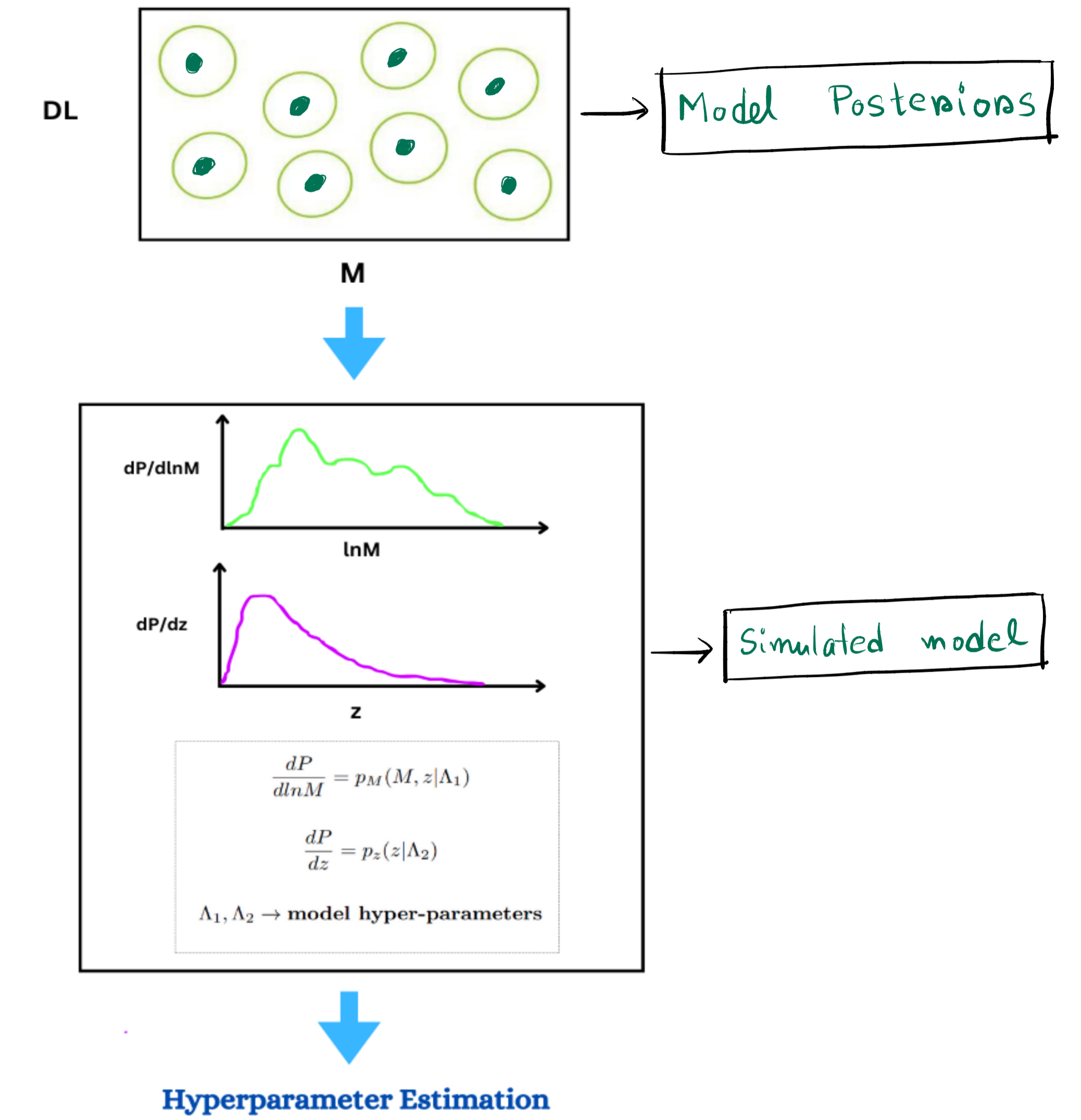


Figure 5. Parameter errors for a mid-SNR (left) and a high SNR (right) event using FMA

Model dataset and hyperparameters



Hierarchical Bayesian Inference

- Λ can be spectral index of black hole masses, exponent in z -distribution or the mixing fraction of models etc.
- For this hyper-parameters, marginalized likelihood is,

$$\mathcal{L}(d|\Lambda) = \int d\theta \mathcal{L}(d|\theta) \pi(\theta|\Lambda) \quad (1)$$

and the hyper-posterior is,

$$p(\Lambda|d) = \frac{\mathcal{L}(d|\Lambda) \pi(\Lambda)}{\int d\Lambda \mathcal{L}(d|\Lambda) \pi(\Lambda)} \quad (2)$$

where, $\mathcal{Z}_\Lambda = \int d\Lambda \mathcal{L}(d|\Lambda) \pi(\Lambda) =$ hyper-evidence.

Future Prospects

- Finding the posteriors for the hyperparameter estimation is currently under progress.
- LISA, after it is in operation around 2035, the real event posteriors would allow us to further pin-down on a particular mass-redshift evolution model for the SMBHs.