Non-Hermitian matter-wave mixing in Bose-Einstein condensates: *Dissipation-induced amplification*

PRA **96** (2017) 013605

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MAX-PLANCK-GESELLSCHAFT

Geography

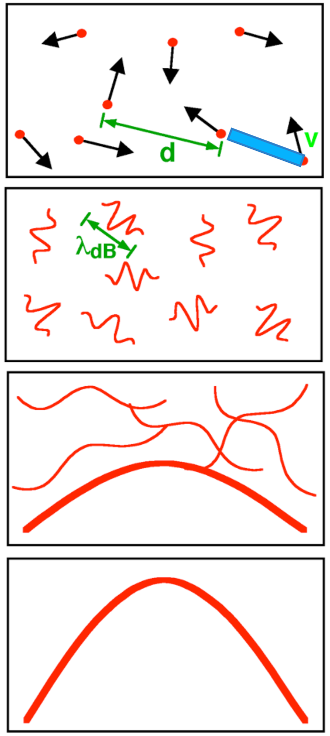
Ultra-cold and Rydberg atom theory in Bhopal: Max-Planck-IISER Partner Group

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH BHOPAL

Map data ©2018 Google, Mapa GISrael, ORION-ME 500 km

Motivation: BEC/ atomic collisions

What is Bose-Einstein condensation (BEC)?

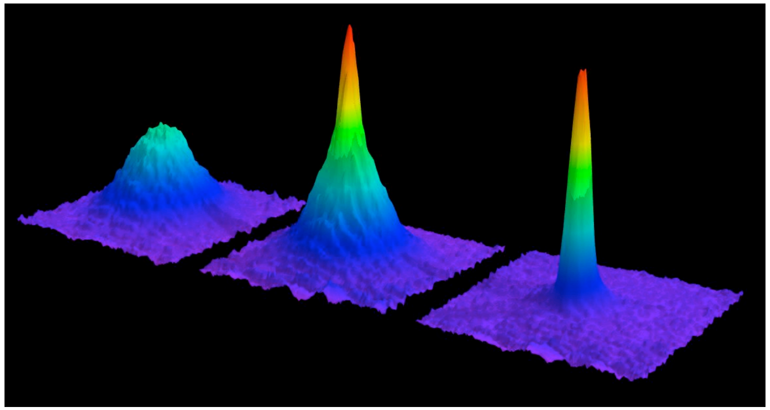


High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

Low Temperature T:
De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$
"Wave packets"

$T=T_{crit}$:
Bose-Einstein Condensation
$\lambda_{dB} \approx d$
"Matter wave overlap"

T=0:
Pure Bose condensate
"Giant matter wave"



Images Ketterle group

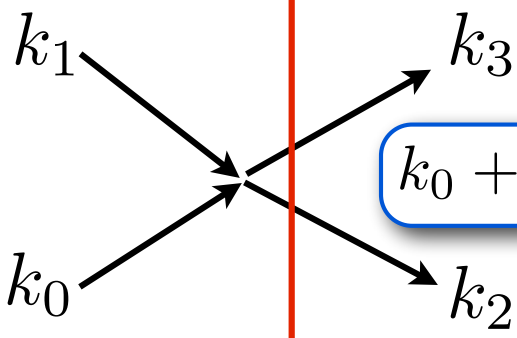
K.B. Davis *et al.*, Phys. Rev. Lett. **75** (1995) 3969.

One wave function for all atoms

GPE: $$i\hbar\frac{\partial\Psi(x)}{\partial t} = \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + U|\Psi(x)|^2\right]\Psi(x)$$

Gross Pitaevskii equation (GPE)

Weak but important s-wave collisions



$$k_0 + k_1 \to k_2 + k_3$$

Non-interacting atom, dispersion relation

$$E(k) = \frac{\hbar^2 k^2}{2m}$$

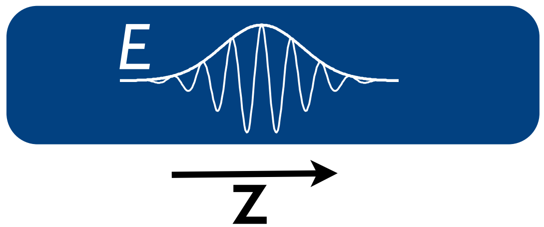
$$E(k_0) + E(k_1) = E(k_2) + E(k_3)$$

Motivation: wave mixing

Analogy to non-linear optics with cubic (Kerr) nonlinearity

$$i\hbar\frac{\partial\Psi(x)}{\partial t} = \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + U|\Psi(x)|^2\right]\Psi(x) \quad \longleftrightarrow \quad i\frac{\partial E}{\partial z} = \left[-a\frac{\partial^2}{\partial \tau^2} + b|E|^2\right]E$$

GPE *NLSE*



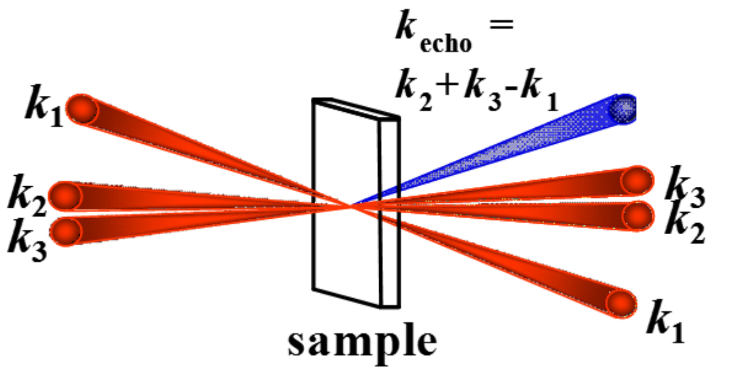
Nonlinear optics, wave mixing

$$E(t) = \frac{1}{2}E_1 e^{-i\omega_1 t} + \frac{1}{2}E_2 e^{-i\omega_2 t} \text{ +c.c.}$$

$$P(t) = \epsilon_0\chi^{(2)}E^2(t) = \frac{\epsilon_0\chi^{(2)}}{4}\left[E_1^2 e^{-i2\omega_1 t} + E_2^2 e^{-i2\omega_2 t} + 2E_1E_2 e^{-i(\omega_1+\omega_2)t} + 2E_1E_2 e^{-i(\omega_1-\omega_2)t} + \cdots\right]$$

Phase matching

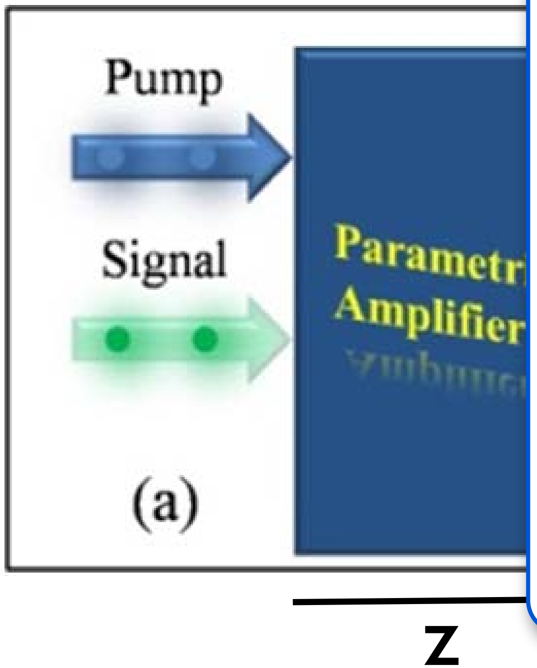
$$\exp\left[i(\omega_n t - k_n x)\right]$$



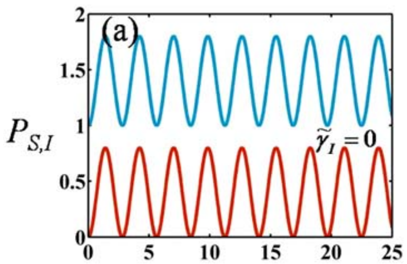
(img: Fayer lab webpage, Standford)

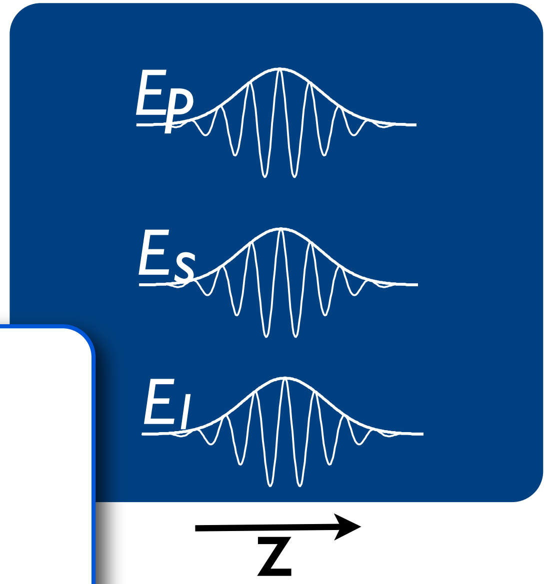
Motivation: non-Hermitian optical amplification

Optical parametric amplification (OPA) / three-wave mixing



No amplification





$\exp(i\Delta\beta z)$

phase-matching

$\exp(i\Delta\beta z)$

$\exp(-i\Delta\beta z)$

$$\Delta\beta = \beta_P - (\beta_S + \beta_I)$$

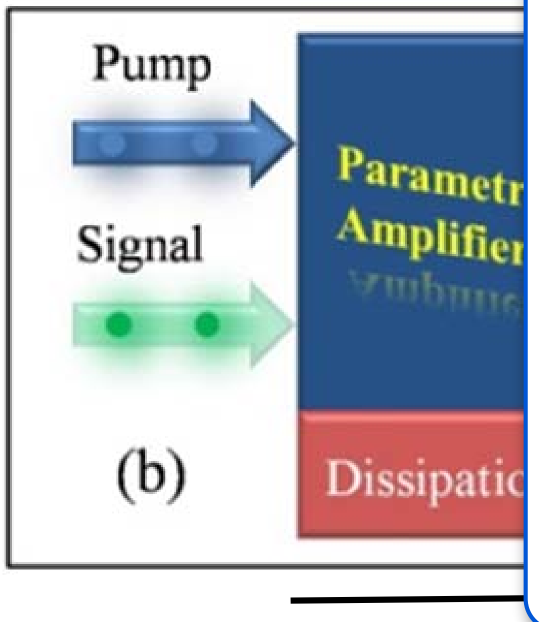
β *dimless wavenumbers*

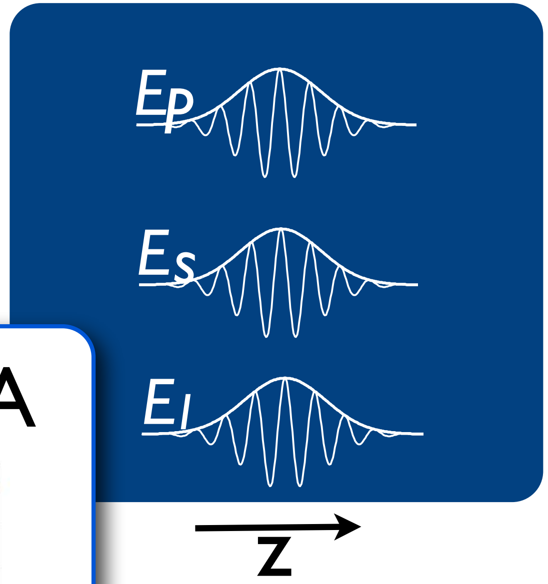
R. El-Ganainy *et al.*, Opt. Lett. **40** (2015)

Motivation: non-Hermitian optical amplification

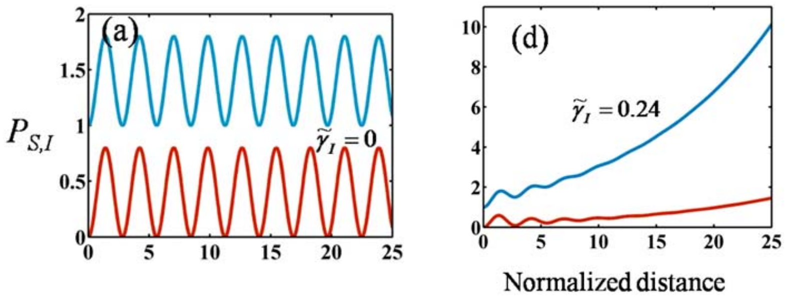
Optical parametric amplification (OPA) / three-wave mixing

$\omega_p = \omega_s + \omega_i$





Non-Hermitian OPA



R. El-Ganainy *et al.*, Opt. Lett. **40** (2015) 5086.
see also:
T. Wasak *et al.*, Opt. Lett. **40** (2015) 5291.
D.A. Antonosyan *et al.*, Opt. Lett. **40** (2015) 4575.

$\exp(i\Delta\beta z)$

phase-matching

$\exp(i\Delta\beta z) - \gamma_I E_I,$

$\exp(-i\Delta\beta z),$

$- (\beta_S + \beta_I)$

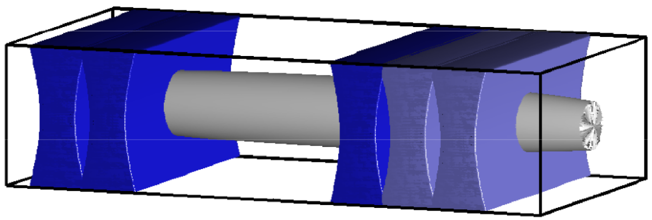
wavenumbers

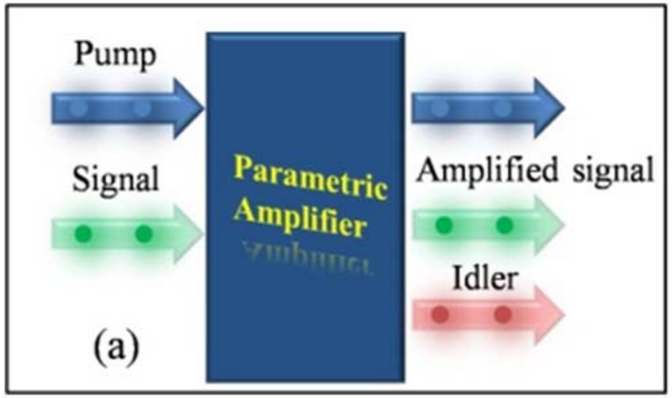
R. El-Ganainy *et*

Ramifications for BEC?

Outline

1. Motivation
2. Introduction, BEC four-wave mixing
3. Matter-wave amplification in homogeneous 1D condensate
4. Two-dimensional wave mixing
5. Engineering atomic losses
6. Summary and Outlook





BEC four-wave mixing

GPE again, position space

$$i\hbar \frac{\partial \Psi(x)}{\partial t} = \left[-\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + U|\Psi(x)|^2 \right] \Psi(x)$$

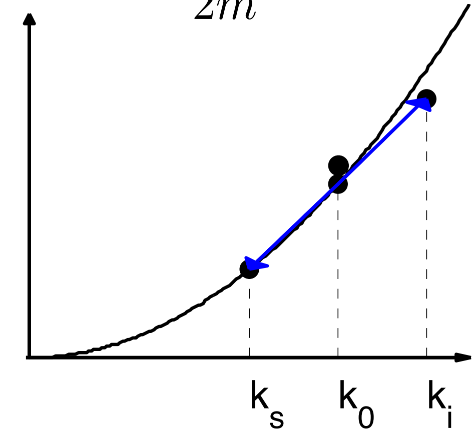
BEC four-wave mixing

GPE in momentum-space and rotating frame

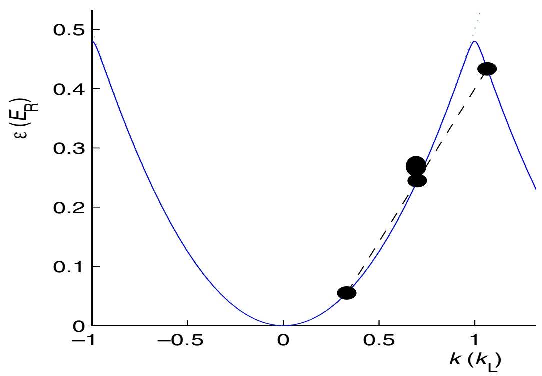
$$i\hbar\frac{\partial\phi_p}{\partial t} = U \sum_{n,m,l} \delta_{k_p+k_n-k_m-k_l}$$

$$\times\ \phi_n^*\phi_m\phi_l e^{i(E_p+E_n-E_m-E_l)t/\hbar}$$

$$E(k) = \frac{\hbar^2 k^2}{2m}$$



- for large Δk = four-wave mixing
- 1D: no energy-momentum conservation / phase matching

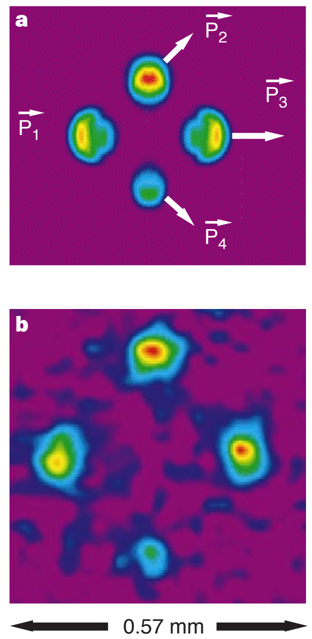


K. Mølmer, NJP **8** (2006) 170.

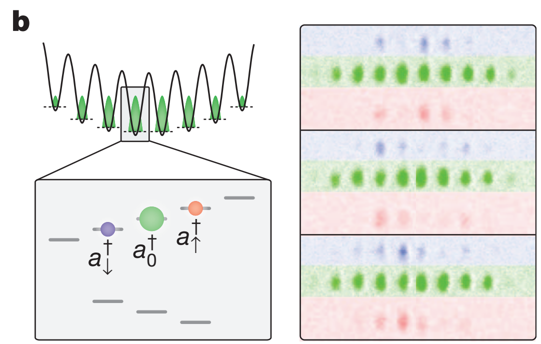
G. K. Campbell *et al.*
PRL **96** (2006) 020406.

change dispersion

L. Deng *et al.*
Nature **398** (1999) 218.



2D



C. Gross *et al.*
Nature **480** (2011) 219.
R. Bückner *et al.*
Nat. Phys. **7** (2011) 608.

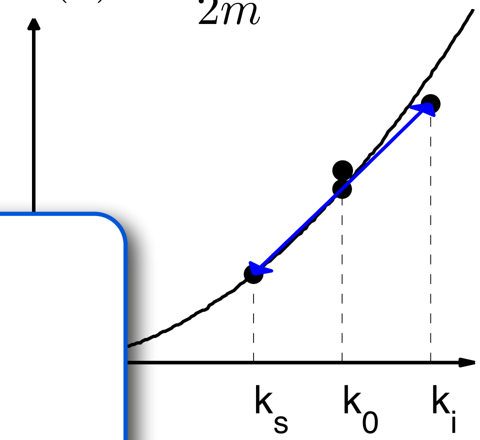
internal /trap energy

BEC four-wave mixing

GPE in momentum-space and rotating frame

$$i\hbar\frac{\partial\phi_p}{\partial t} = U \sum_{n,m,l} \delta_{k_p+k_n-k_m-k_l} \times \phi_n^*\phi_m\phi_l e^{i(E_p+E_n-E_m-E_l)t/\hbar}$$

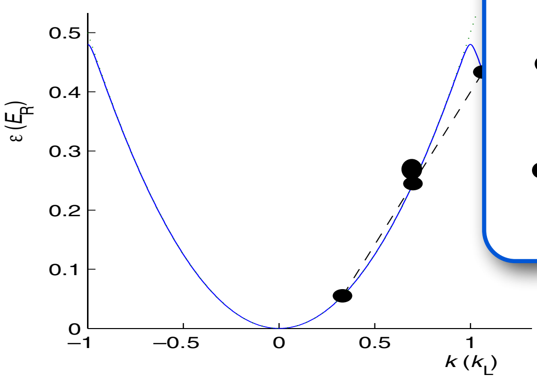
$$E(k) = \frac{\hbar^2 k^2}{2m}$$



- for large Δk = four-wave mixing
- 1D: no energy-m
  phase matching

Four-wave mixing applications

- fundamental studies
- EPR entangled atom pairs
- Atom lasers and interferometry



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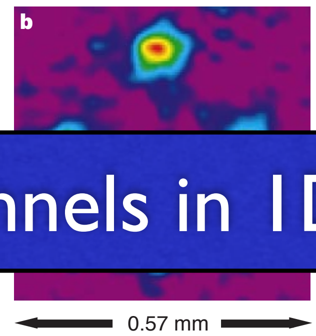
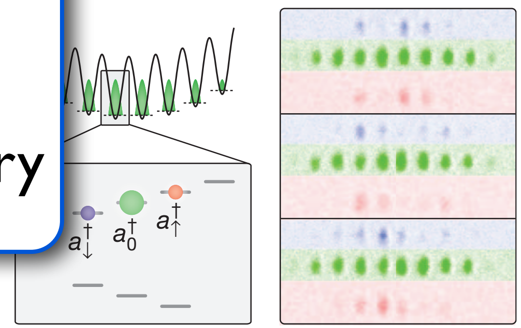


2D



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internal /trap energy

New channels in 1D?

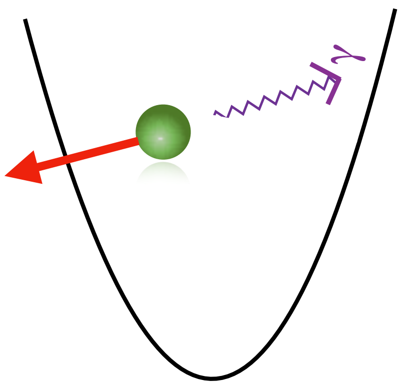
BEC - Non-Hermiticity

GPE: $i\hbar\frac{\partial\Psi(x)}{\partial t} = \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + U|\Psi(x)|^2\right]\Psi(x)$

Many-body H
Hermitian

Unavoidable atom losses:

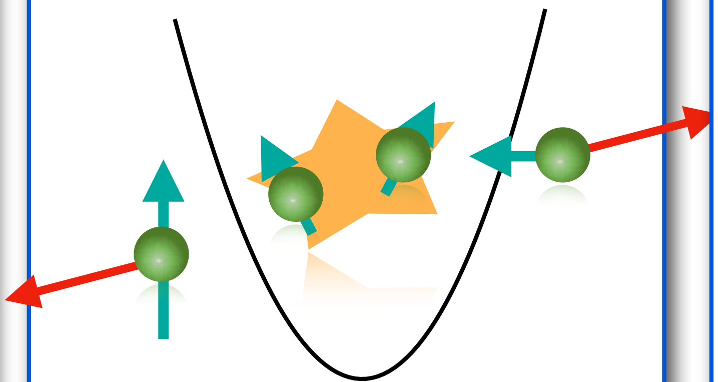
One-body loss



- vacuum imperfections
- BB-photons
- trap photon recoil

$$\dot{\Psi} = -\gamma\Psi$$

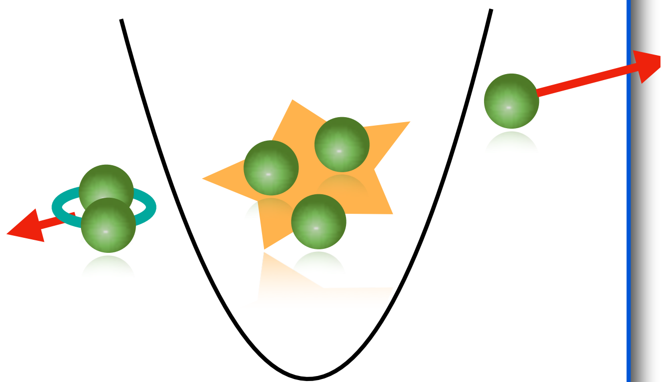
Two-body loss



- spin-flip collisions

$$\dot{\Psi} = -K_2|\Psi|^2\Psi$$

Three-body loss



- molecule formation
- main density & life-time limitation

$$\dot{\Psi} = -K_3\frac{|\Psi|^4}{2}\Psi$$

BEC - Non-Hermiticity

GPE: $$i\hbar\frac{\partial\Psi(x)}{\partial t}=\left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2}+U|\Psi(x)|^2\right]\Psi(x)$$

Many-body H
Hermitian

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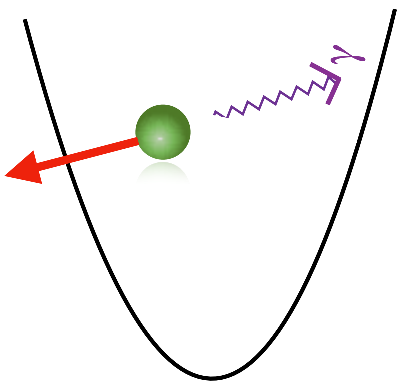


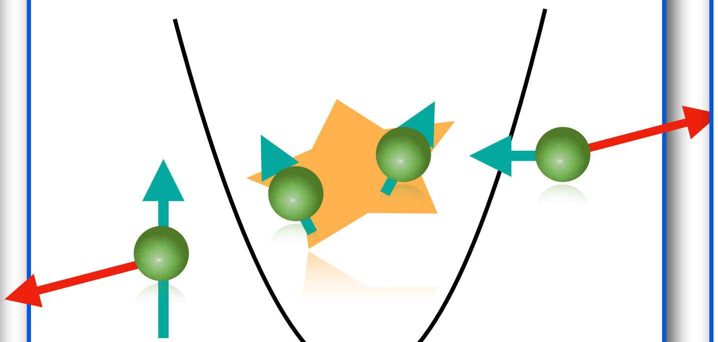
- vacuum

Here & Laser Cooling

- trap photon recoil

$$\dot{\Psi}=-\gamma\Psi$$

Two-body loss



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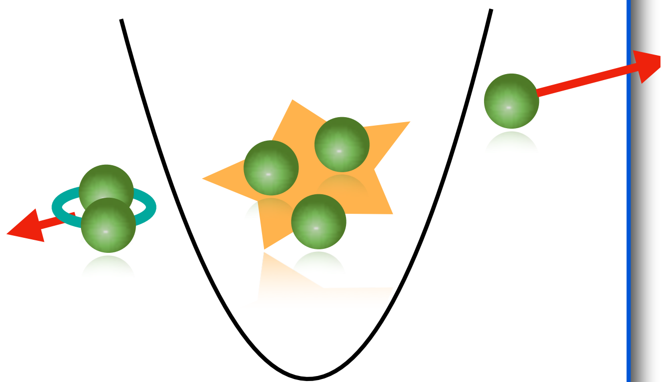
$$\dot{\Psi}=-K_2|\Psi|^2\Psi$$

Three-body loss



- molecule formation
- main density & life-time limitation

$$\dot{\Psi}=-K_3\frac{|\Psi|^4}{2}\Psi$$

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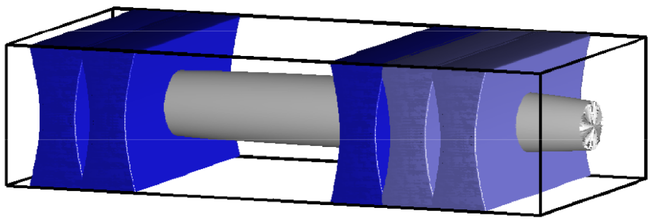
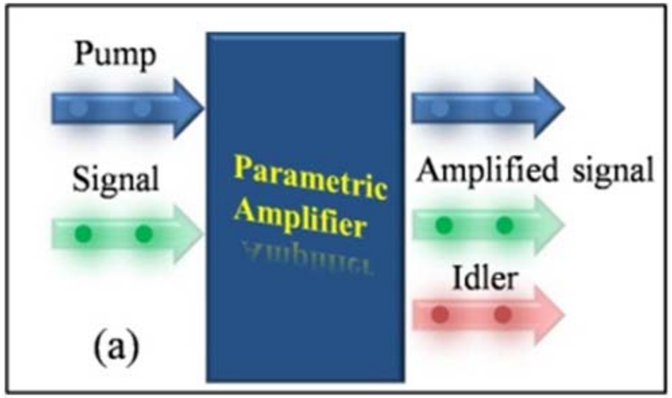
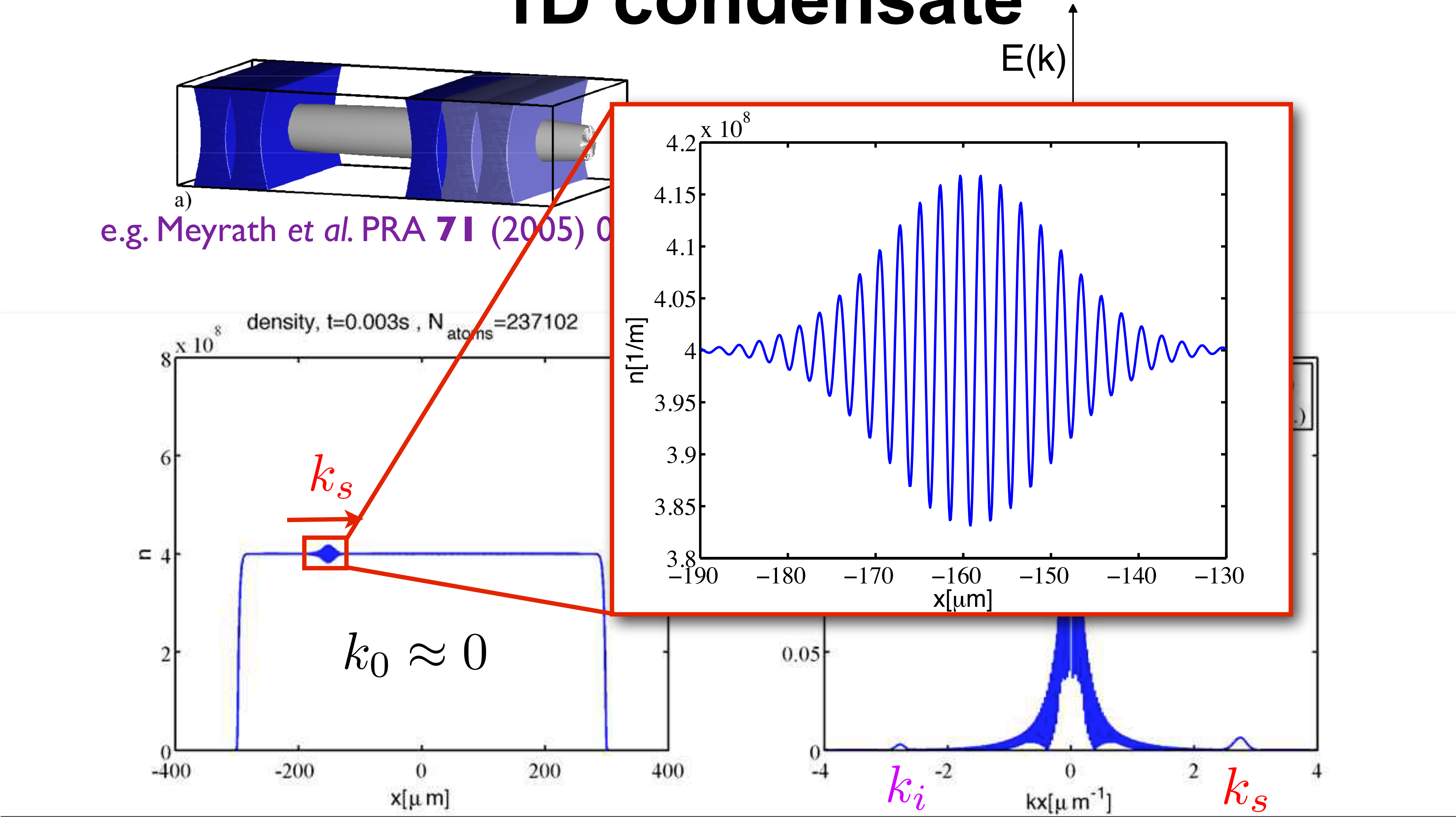




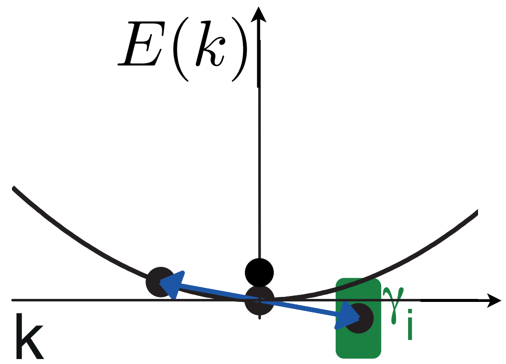
No wave mixing in homogeneous 1D condensate

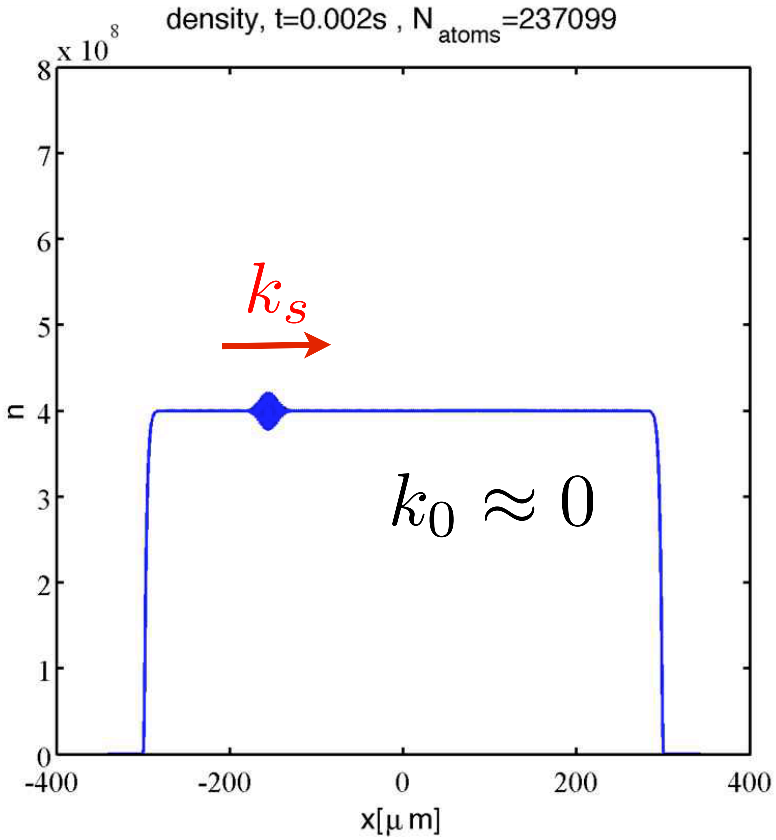


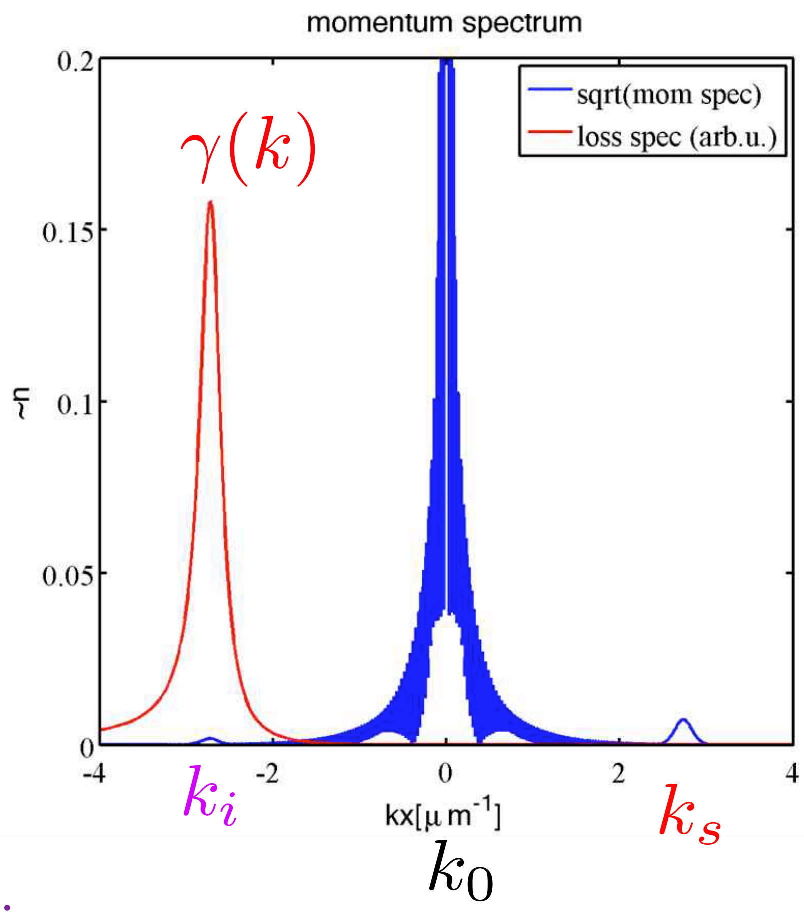
Matter-wave amplification in homogeneous 1D condensate

$$i\hbar\frac{\partial\phi(k)}{\partial t} = \frac{\hbar^2k^2}{2m}\phi(k) + U\int_{-\infty}^{\infty}dk_1\int_{-\infty}^{\infty}dk_2\int_{-\infty}^{\infty}dk_3$$

$$\times\,\delta(k_2+k_3-k_1-k)\phi^*(k_1)\phi(k_2)\phi(k_3) - i\hbar\gamma(k)\phi(k)$$







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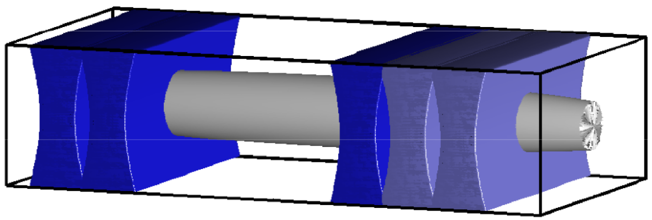

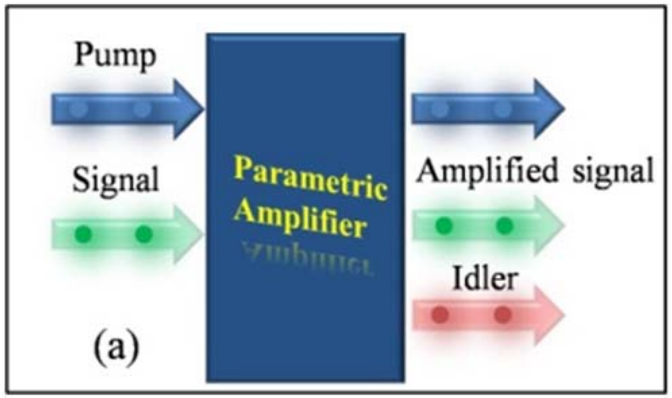

Two dimensional wave mixing

L. Deng *et al.*
Nature **398**
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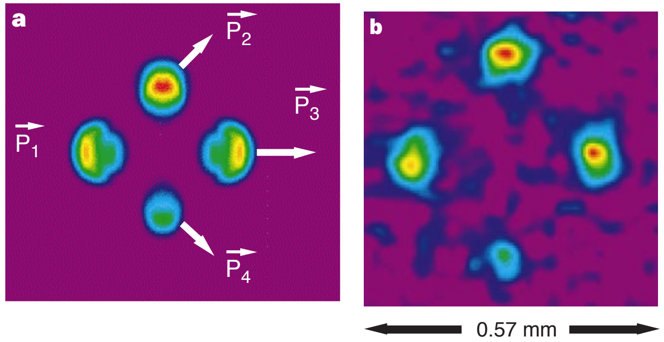
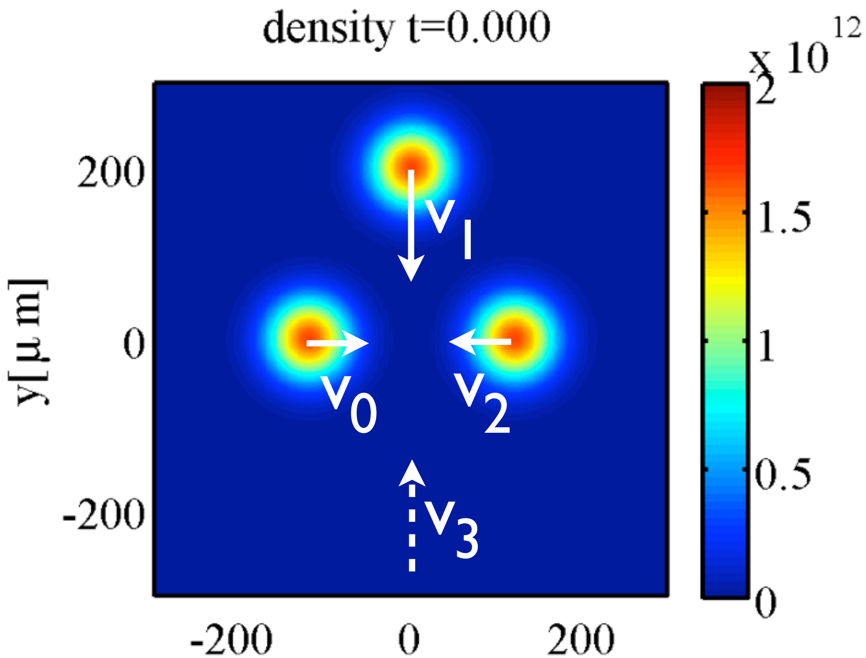
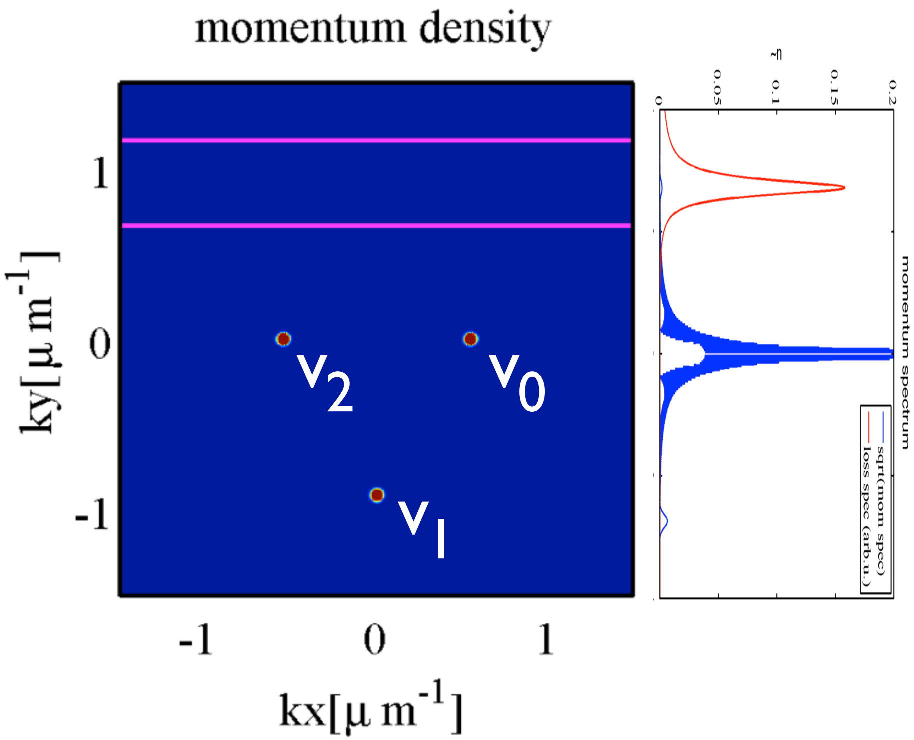
2D



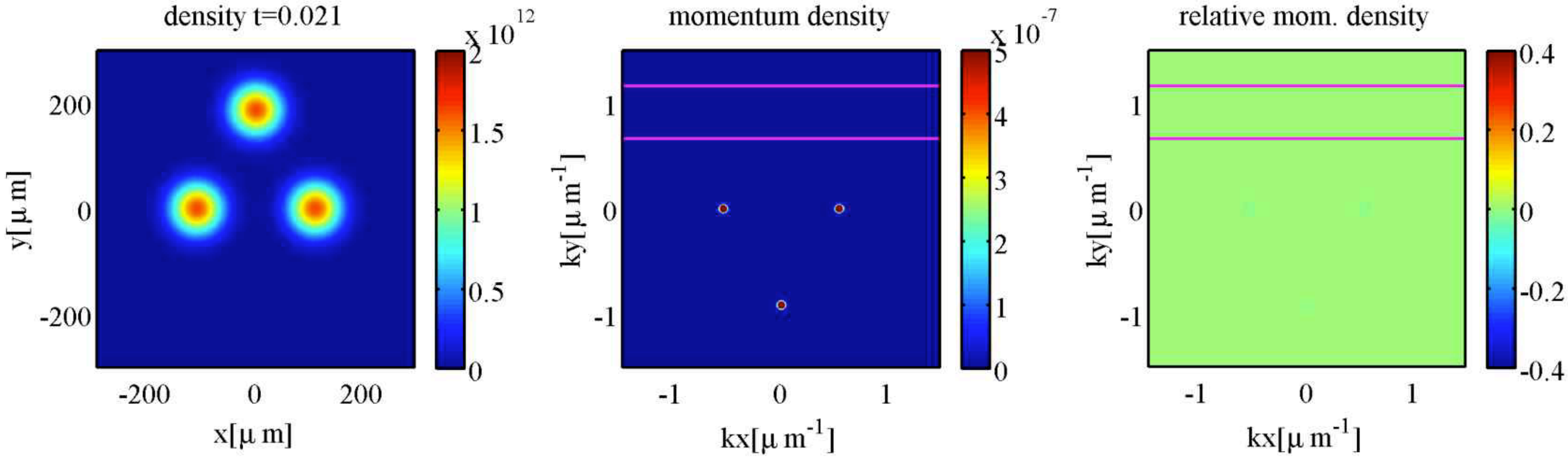




$$\mathbf{p}_0 + \mathbf{p}_2 = \mathbf{p}_1 + \mathbf{p}_3 = 0$$

$$E_0 + E_2 \neq E_1 + E_3$$

Two dimensional wave mixing



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New channels in 1D and 2D, phonon amplifier!

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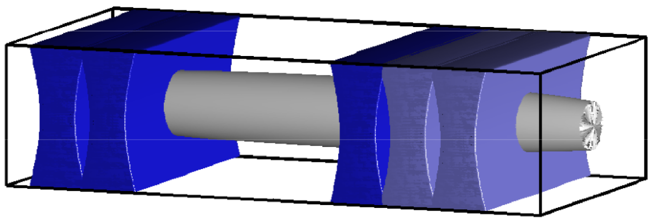
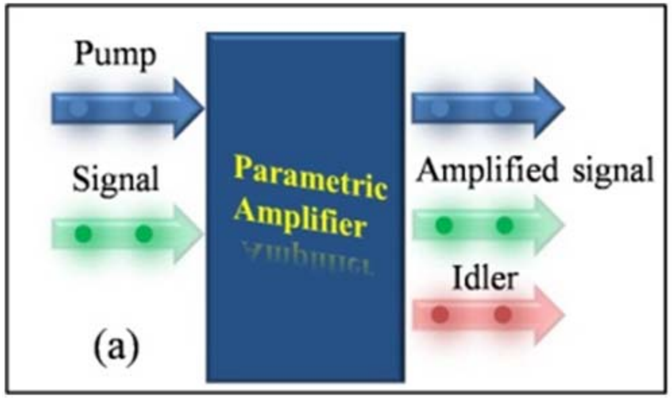
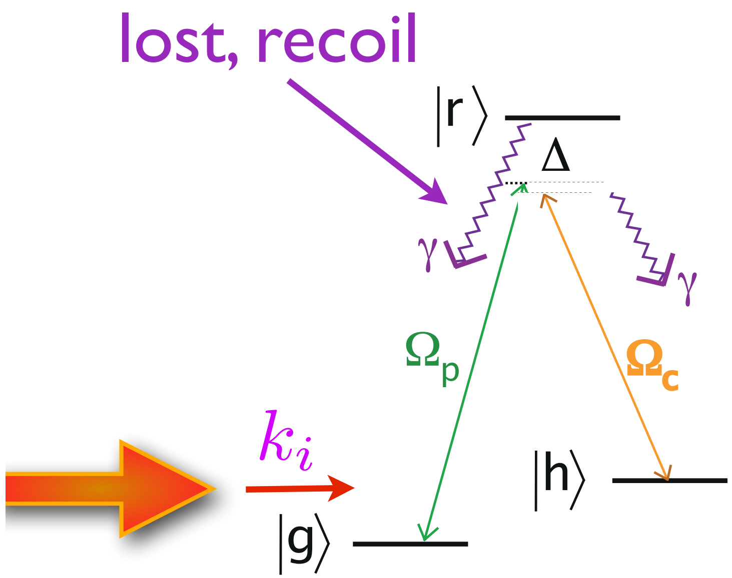


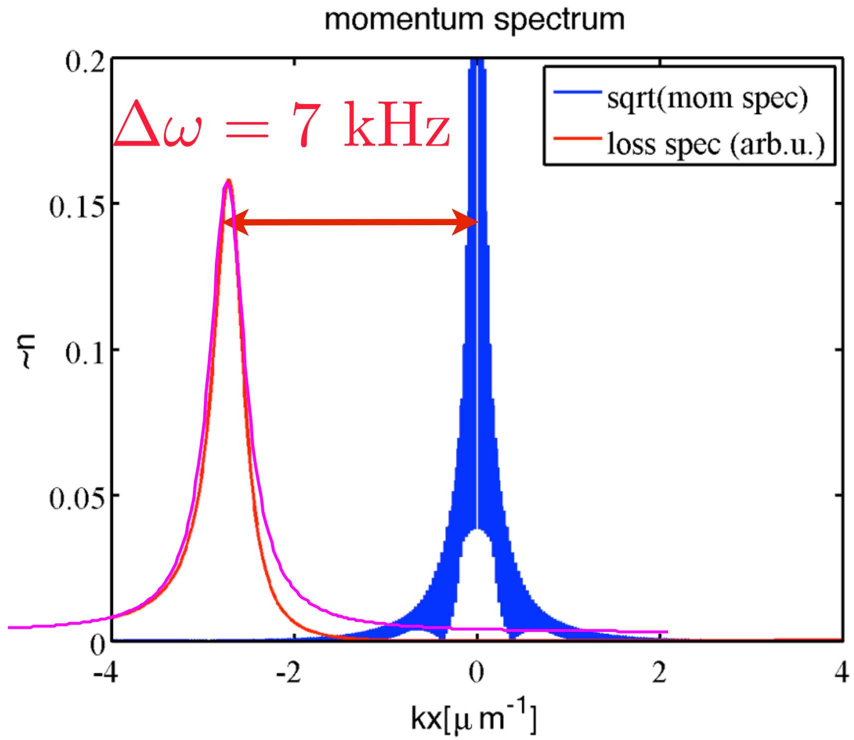


Engineering atomic losses

Challenge: γ(k) should act on *small range of k* only



Doppler shift: $\omega' = \omega - \mathbf{v} \cdot \mathbf{q}_{las}$



(1) BEC velocities small, need relatively long-lived state |r>
(metastable triplett or Rydberg n=70,p γ= 5 kHz)

(2) Tackle "tails" of Lorentzian: quantum interference, EIT

adaptation of [theory] G. Morigi *et al.* PRL **85** (2000) 4458. [expt] C. F. Roos *et al.* ibid. 5547.

Engineering atomic losses

Challenge: $\gamma(k)$ should act on *small range of k* only

(2) Tackle "tails" of Lorentzian: quantum interference, EIT adaptation of [theory] G. Morigi *et al.* PRL **85** (2000) 4458. [expt] C. F. Roos *et al.* ibid. 5547.

Here:

- Metastable excited states
- Decaying atoms lost from system
  - Weak trap
  - Hierarchy of time-scales: $T_{BEC} >> T_{loss} >> T_{\gamma}$

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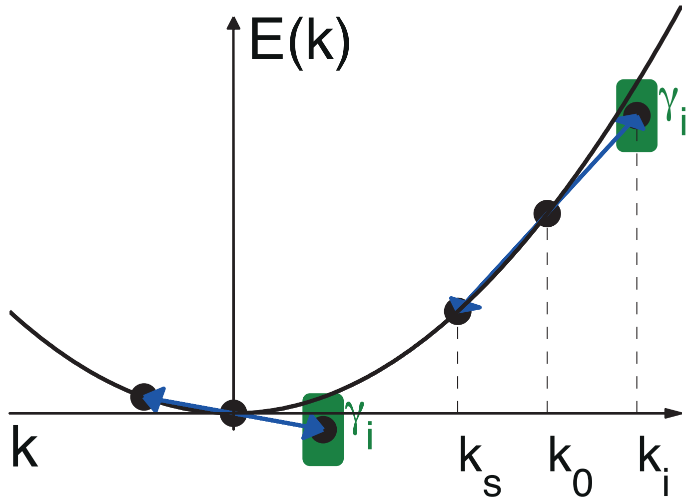
Few mode model

Analytical understanding of amplification mechanism

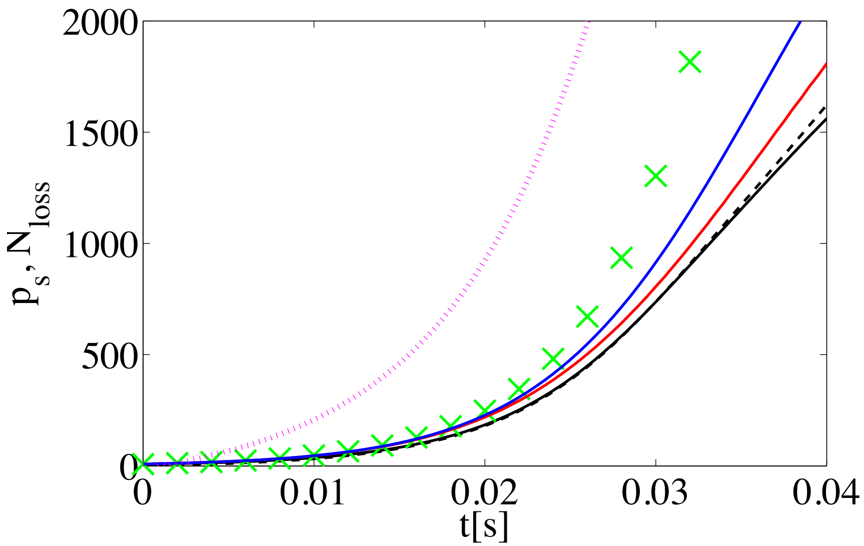


$$i\hbar\frac{\partial \phi_p}{\partial t} = U \sum_{n,m,l} \delta_{k_p+k_n-k_m-k_l}$$
$$\times\, \phi_n^* \phi_m \phi_l e^{i(E_p+E_n-E_m-E_l)t/\hbar} - i\hbar\gamma_p\phi_p$$

- Consider only three modes (0,s,i).
- Growth rates of solutions

$$\lambda'' = \frac{(U\rho/\hbar)^2\gamma}{\gamma^2 + (\Delta E/\hbar)^2}$$

$$\Delta E = 2E_0 - E_s - E_i$$

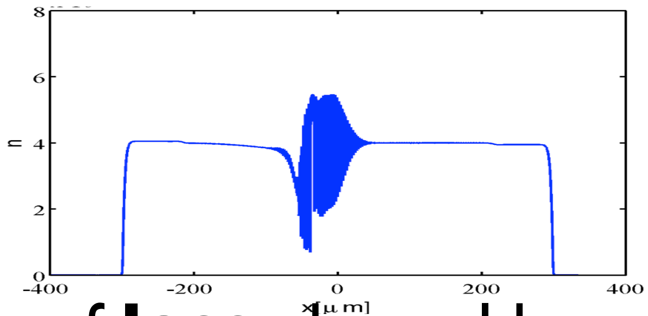


- Max for $\frac{\Delta E}{\hbar} = \gamma$, but "frequency" $\lambda'' \ll (U\rho/\hbar)$

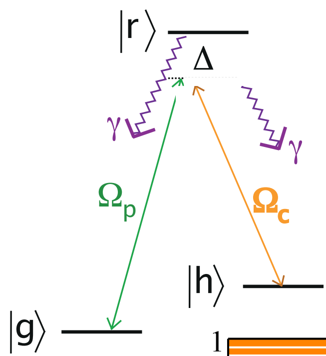
Summary

S. Wüster and R. El-Ganainy, PRA **96** (2017) 013605.

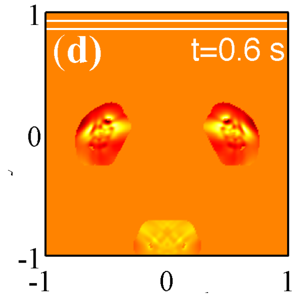
Spectrally selective **atom loss** in BEC **enables** otherwise forbidden **scattering** processes in **1D**



Engineering of **loss**-channel by adaptation of laser cooling techniques with **Rydberg** states



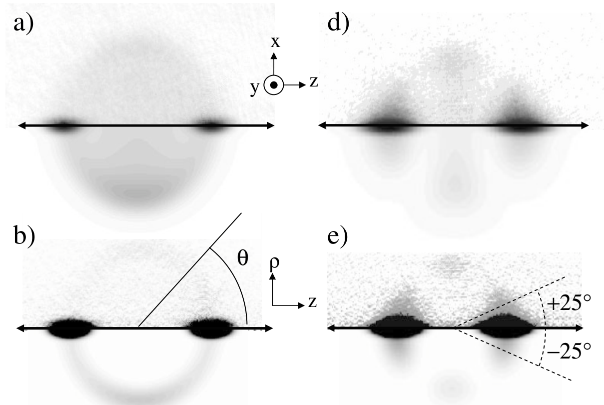
Simple studies of Non-Hermitian wave mixing in 2D



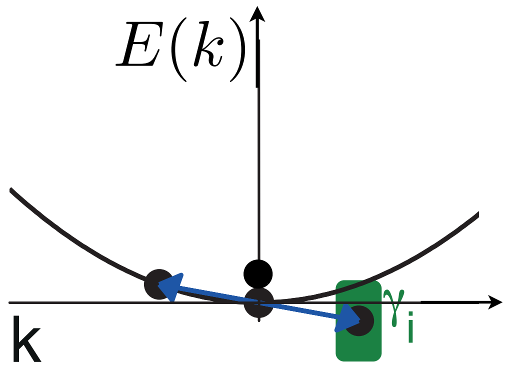
Outlook

Amplify scattering seeded by **vacuum fluctuations** (TWA)?

Ch. Buggle *et al.* PRL **93** (2004) 173202.



Quantum correlations / **entanglement** between pump and signal (TWA)?



BEC naturally exhibit **non-linear** loss (e.g. three-body recombination)

$$-i\hbar K_n |\Psi|^{2(n-1)}\Psi$$

Thanks for your attention