Basics of quantum measurement with quantum light

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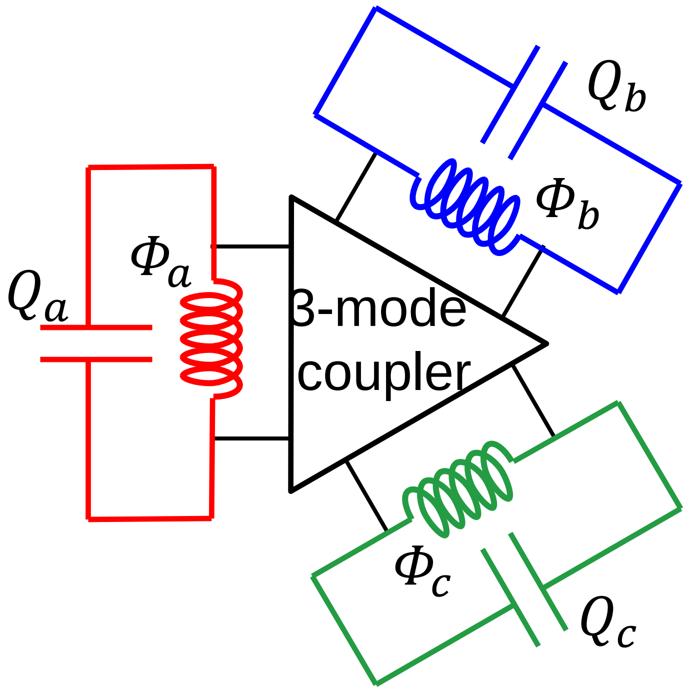
TIFR

A quick review:

Parametric coupling overview



$$H_{couple} \propto \Phi_a \Phi_b \Phi_c$$

Re-write in terms of a, b, c

$$H_{couple} = \hbar g_3 (a + a^\dagger)(b + b^\dagger)(c + c^\dagger)$$

$$= \hbar g_3 (abc^\dagger + a^\dagger b^\dagger c + ab^\dagger c + a^\dagger bc^\dagger + \cdots)$$

- If frequencies $\omega_{a,b,c}$ all very different, all terms die in the rotating wave approx.
- Drive one mode (c) at $\omega_p = \omega_a + \omega_b \neq \omega_c$
- Stiff pump: $c \to \langle c \rangle = |c| e^{i\phi_p}$

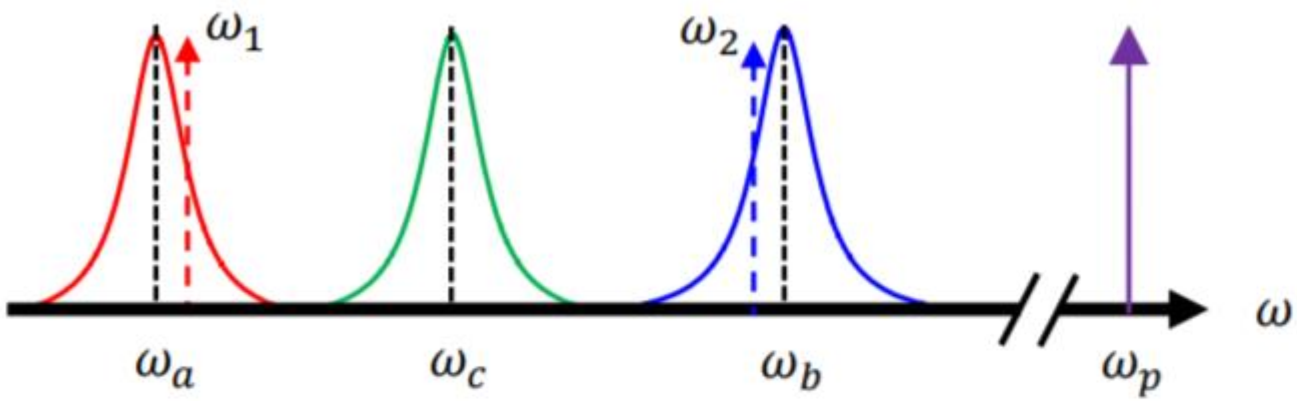
$$H_G = \hbar g \left(a^\dagger b^\dagger e^{i\phi_p} + abe^{-i\phi_p}\right)$$

Physical Implementations: Josephson junctions, opto-mechanics, diodes, optical fibers….

System Dynamics – Phase preserving amplification

$$\frac{\mathcal{H}}{\hbar} = \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_c c^\dagger c + g\left(a^\dagger b^\dagger c + abc^\dagger\right)$$

Pay careful attention to coupling, this form destroys one c 'pump' photon to create one photon each in a and b

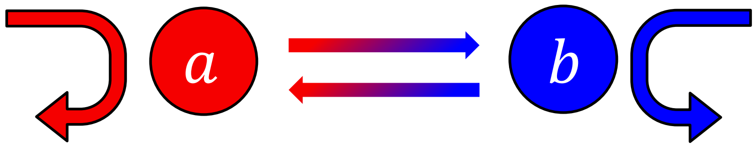


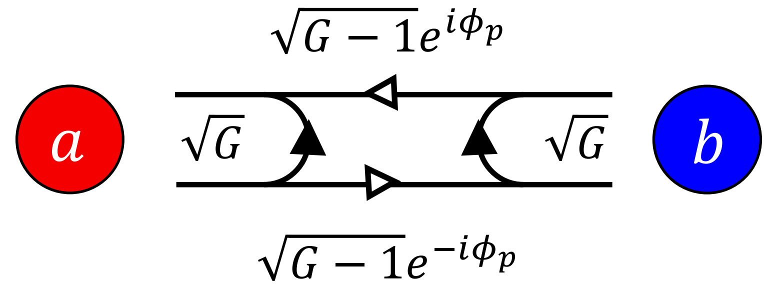
- Modes we use for quantum signals should be driven near their resonance frequency
- We need the third 'pump' mode to be far away from the pump frequency so the pump can be 'stiff' and c becomes a number

Phase preserving gain (Gain)

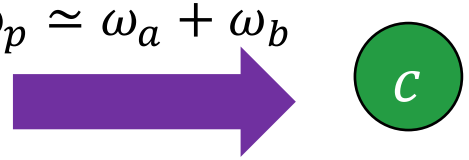
$$\omega_p = \omega_a + \omega_b \neq \omega_c$$

$$H_G = \hbar g\left(a^\dagger b^\dagger e^{i\phi_p} + ab e^{-i\phi_p}\right)$$

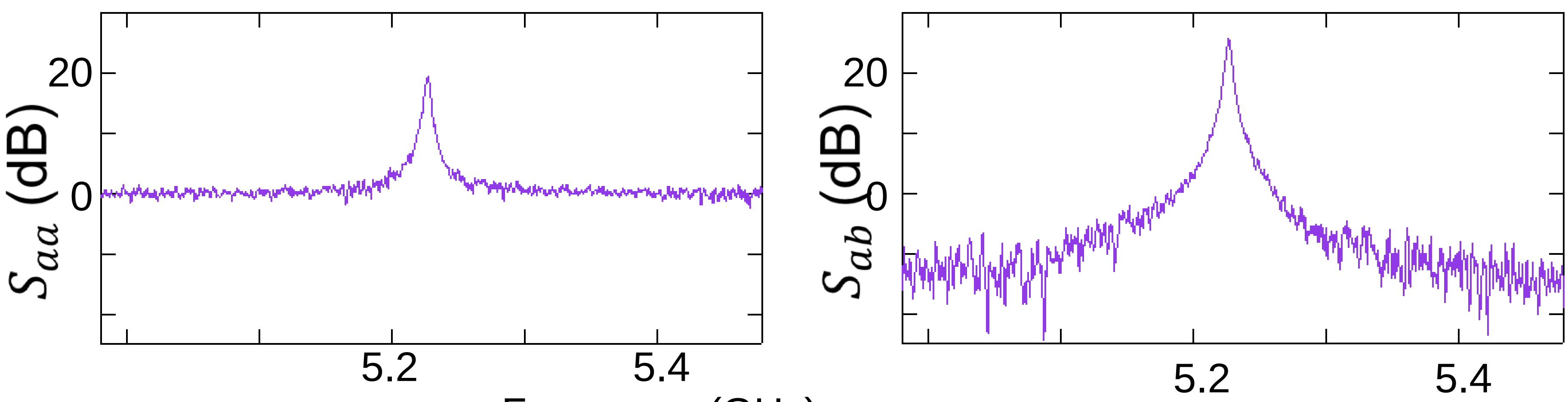




$$\omega_p \simeq \omega_a + \omega_b$$

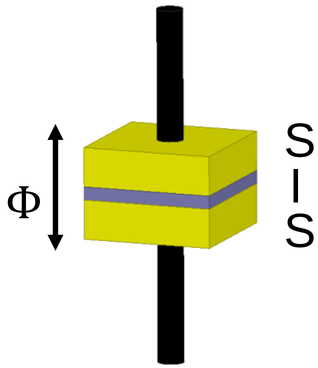


$$G = \frac{\left(1 + {P_P}/{P_C}\right)^2}{\left(1 - {P_P}/{P_C}\right)^2}$$



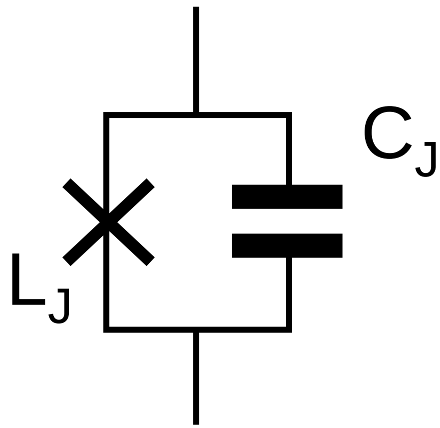
Sliwa *PRX* (2015)

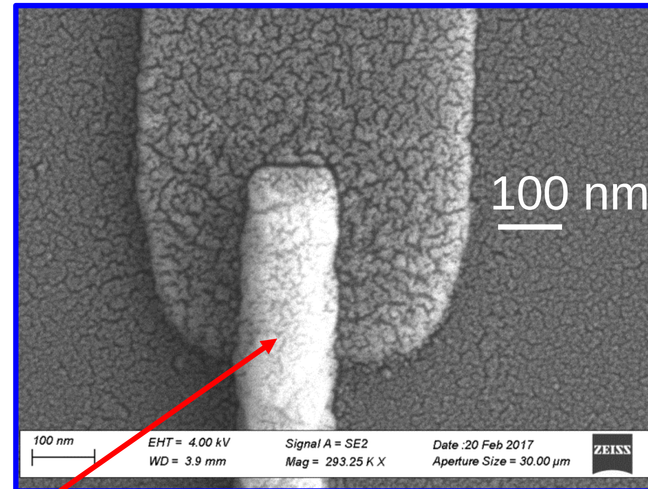
The Josephson tunnel junction



SUPERCONDUCTING TUNNEL JUNCTION

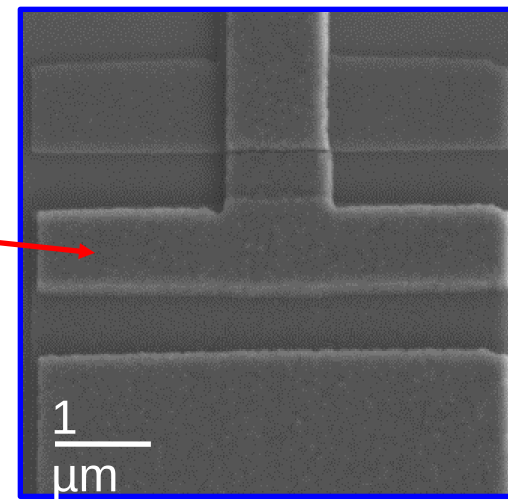
$$I = I_0 \sin\left(\frac{2\pi}{\Phi_0}\Phi\right)$$





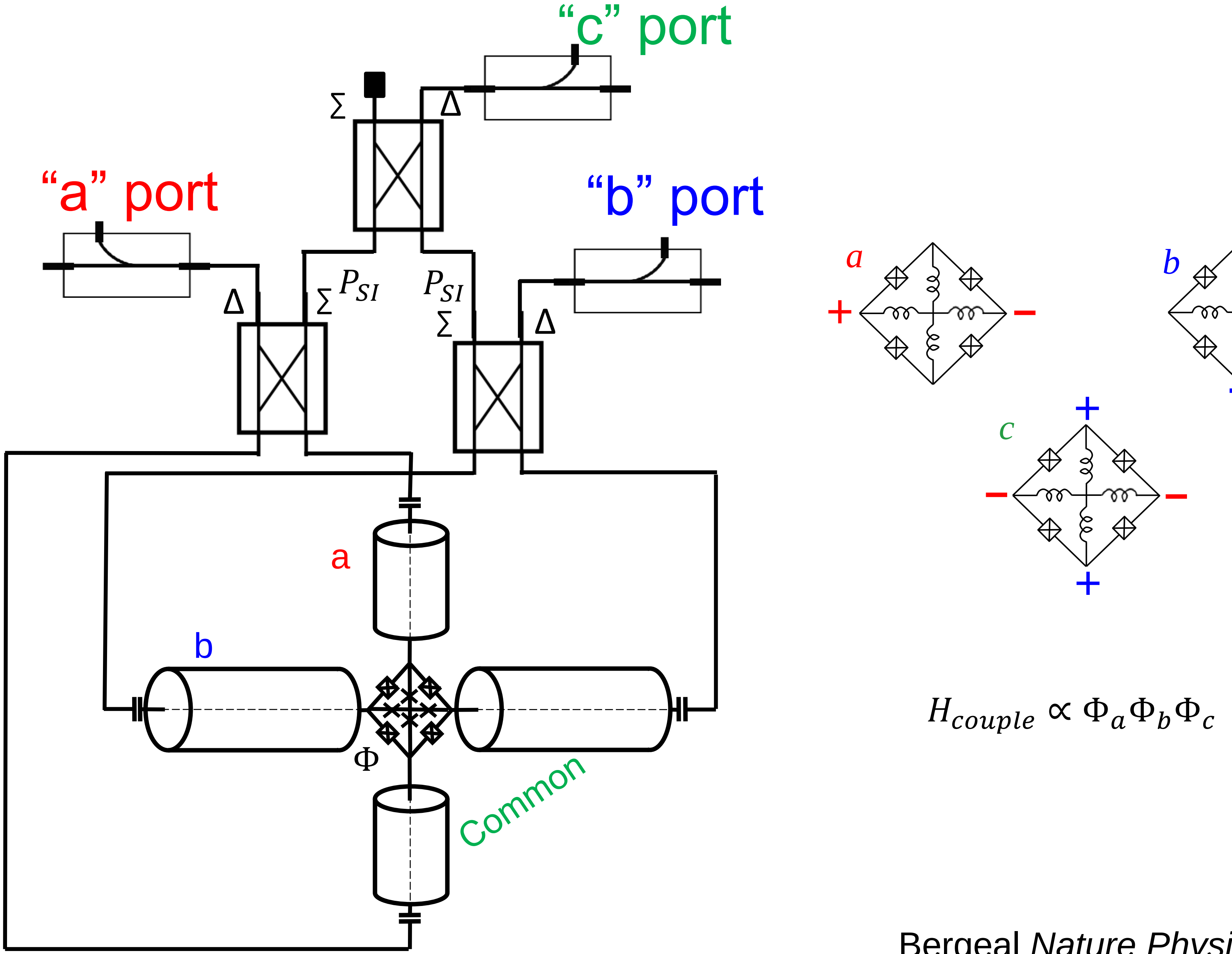
$Al/AlO_x/Al$ tunnel junction

T ~ 20 mK



$$H = \frac{Q^2}{2C} - E_J \cos\left(\frac{2\pi}{\Phi_0}\Phi\right) = \hbar\omega_0 b^\dagger b - \lambda\left(b^\dagger b\right)^2 + \cdots$$

The three mode Josephson Parametric Converter

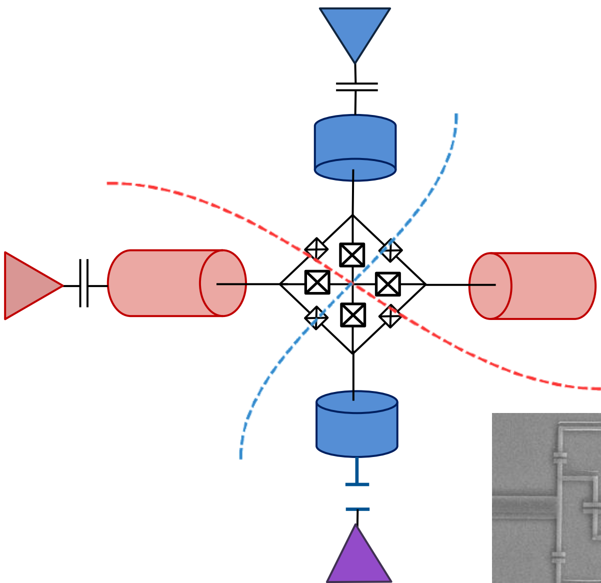


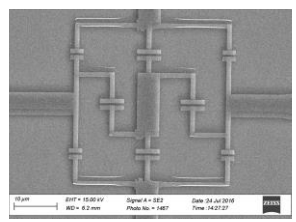
$$H_{couple} \propto \Phi_a \Phi_b \Phi_c$$

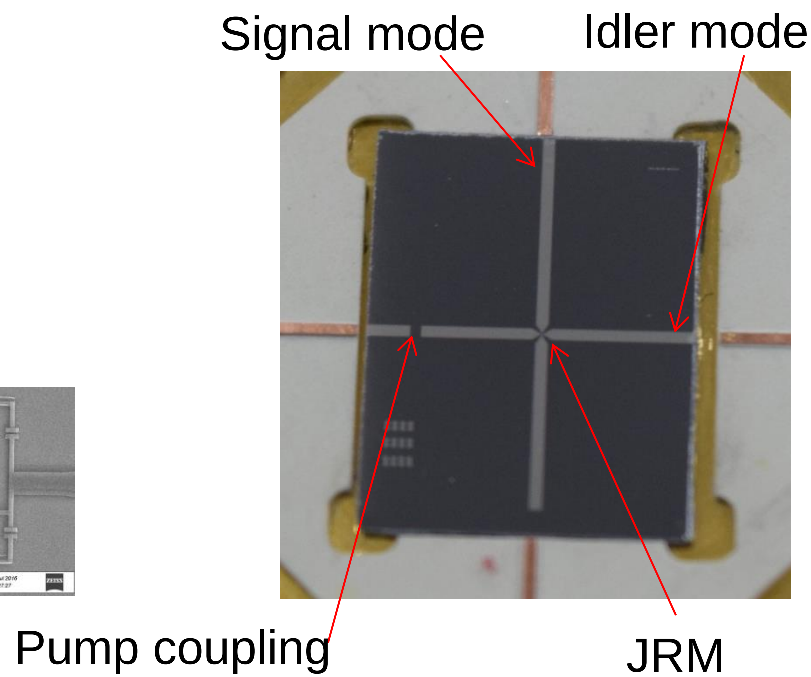
Bergeal *Nature Physics* (2010

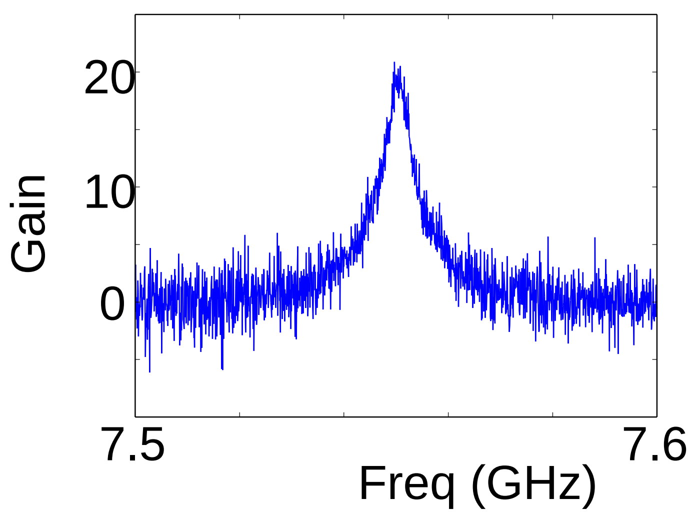
The 8-junction Josephson Parametric Converter

(a)





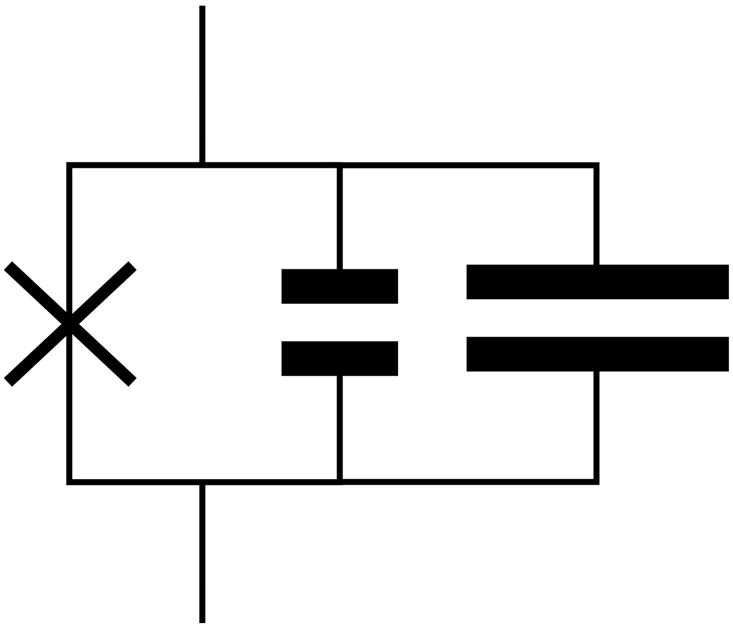




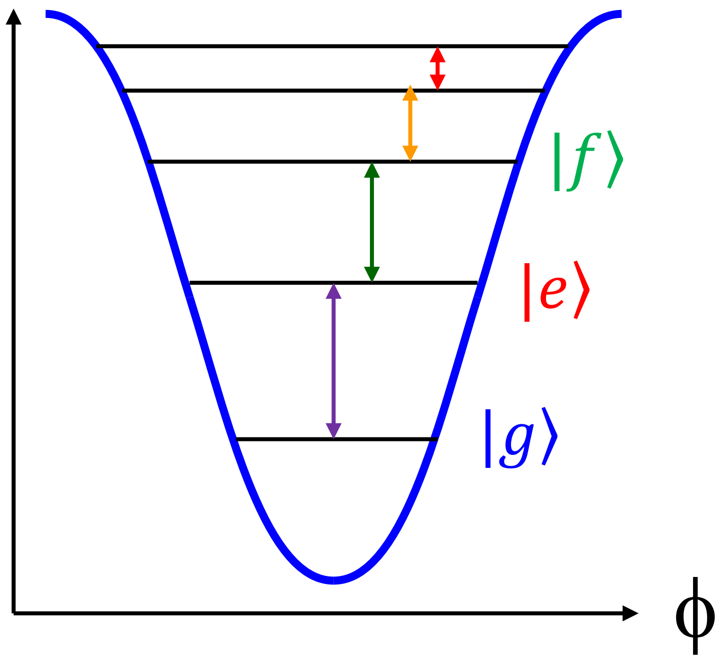
Bergeal *et al* Nature (2010)
See also Roch *et al* PRL (2012)

Superconducting transmon qubit

Josephson junction with shunting capacitor ➔ anharmonic oscillator



Potential energy

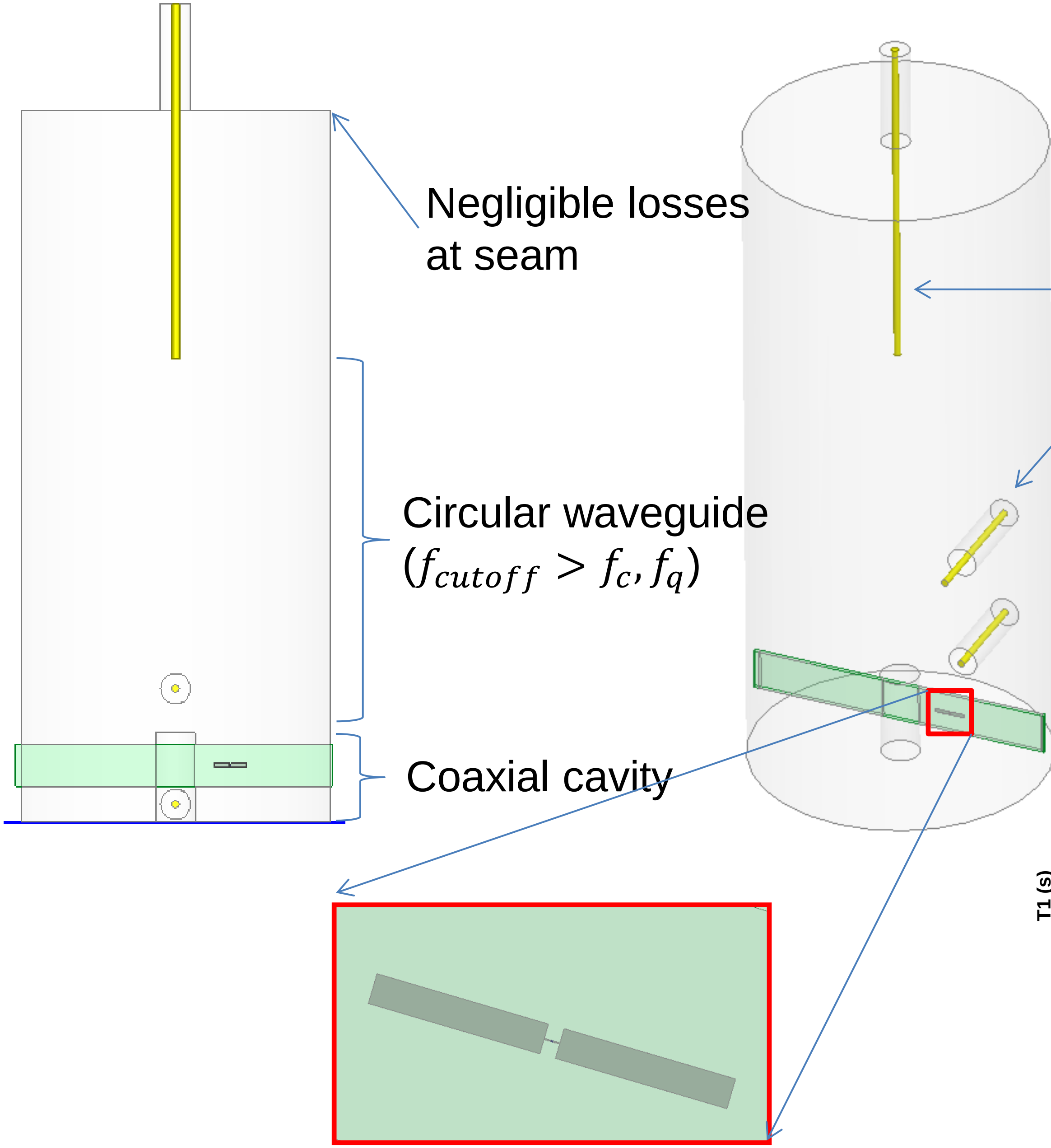


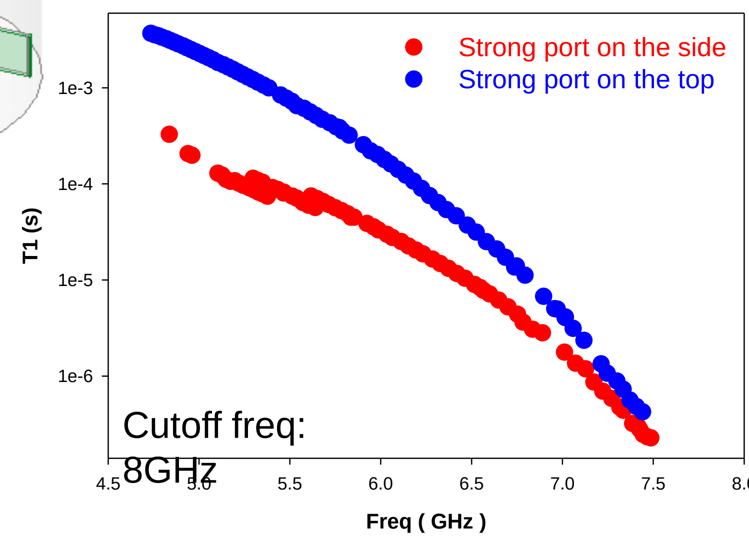
lowest two levels form qubit

f_{ge} ~ 5.025 GHz, f_{ef} ~ 4.805 GHz

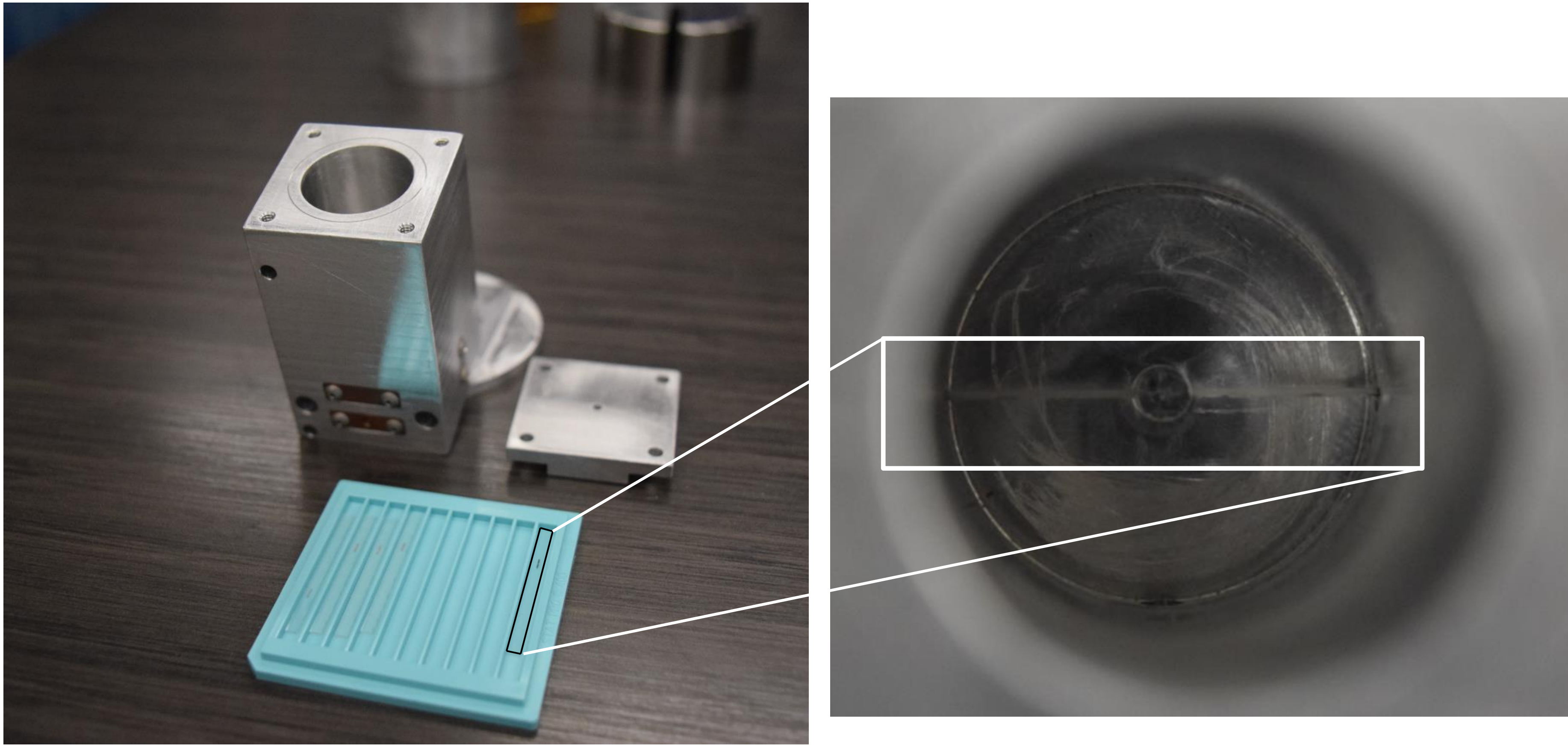
Koch et al., Phys. Rev. A (2007)

Coaxial cavity + transmon





Isolating the transmon from the environment



Cavity
$f_{c,g}$ = 7.4817 GHz
$1/\kappa$ = 30 ns

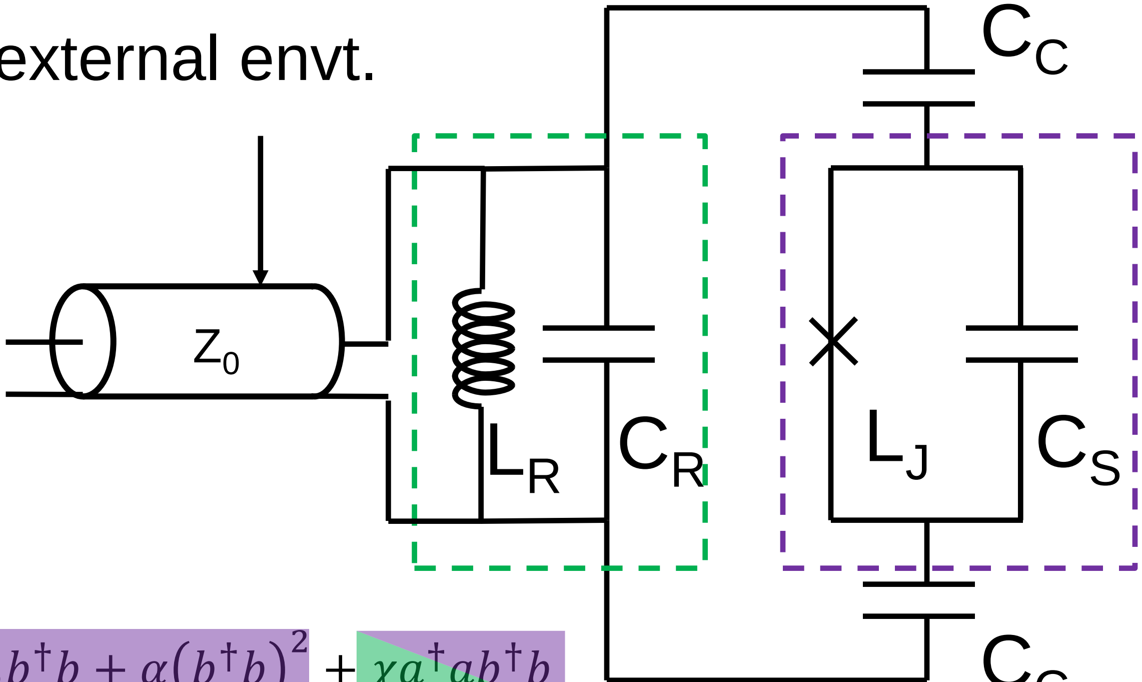
Qubit
f_Q=5.0252 GHz
T_1 = 30 μs
T_{2R} = 8 μs

Qubit environment: circuit QED

Strongly couple to a resonator (harmonic oscillator)

Blais et al., Phys. Rev. A (2004)

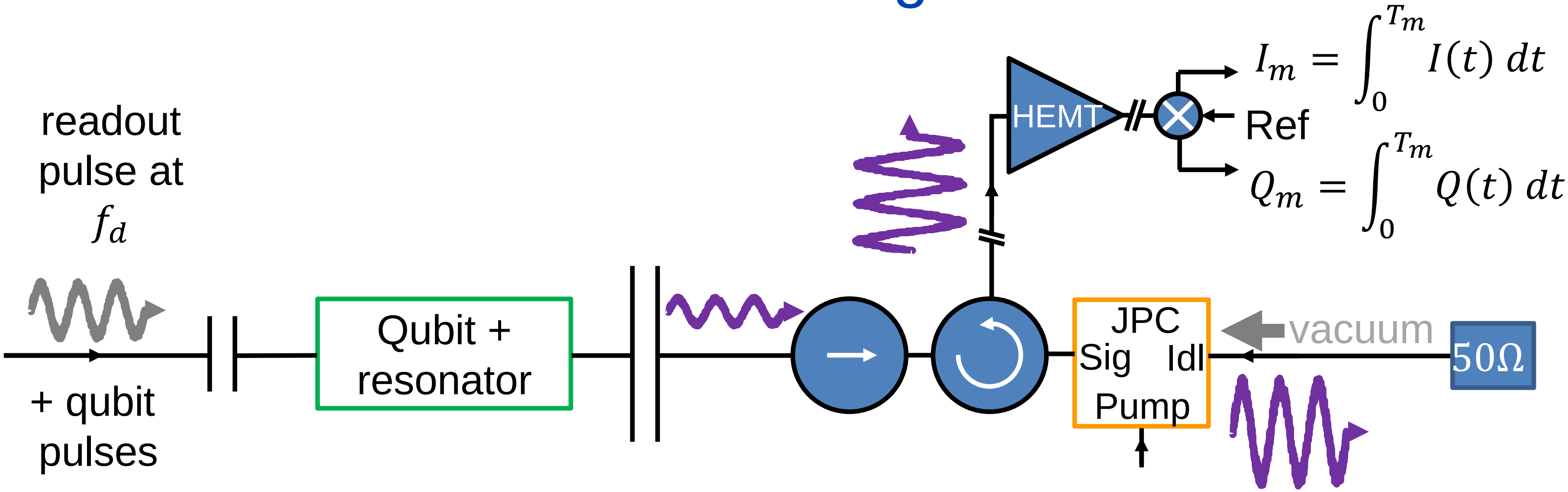
Cavity filters external envt.

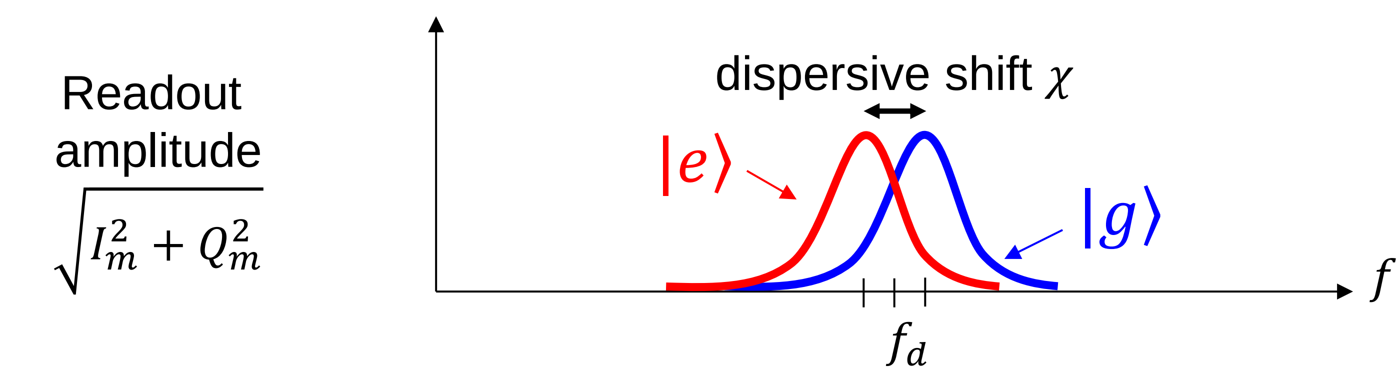


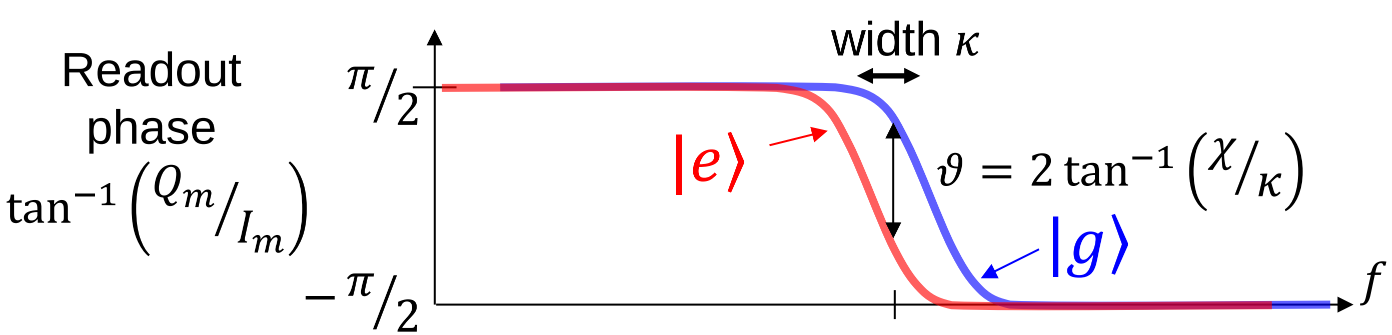
$$\frac{H_{eff}}{\hbar} = \omega_R a^\dagger a + \omega_Q b^\dagger b + \alpha\left(b^\dagger b\right)^2 + \chi a^\dagger a b^\dagger b$$

$\omega_R \pm \chi / 2$ transition frequency of resonator depends on qubit state

Measurement configuration

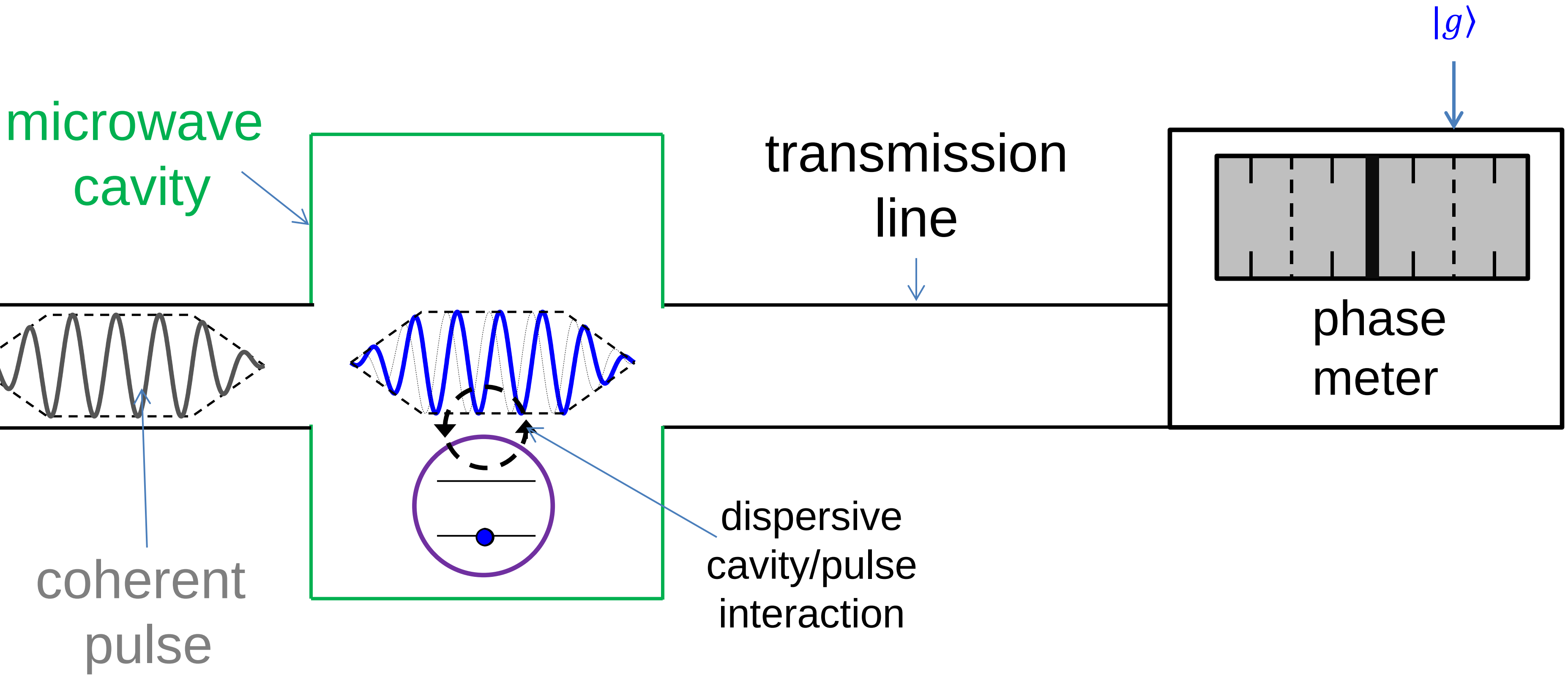




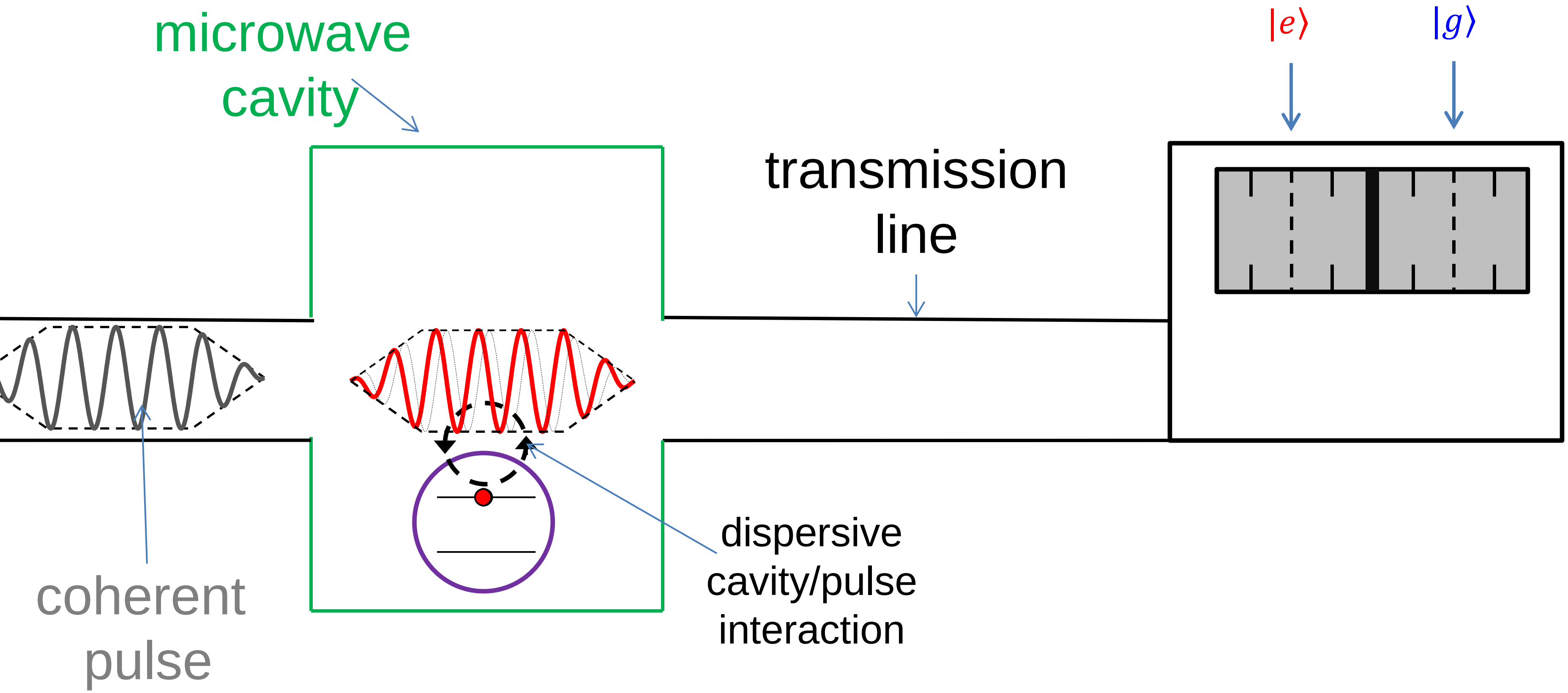


Part 1: Measurement with coherent states

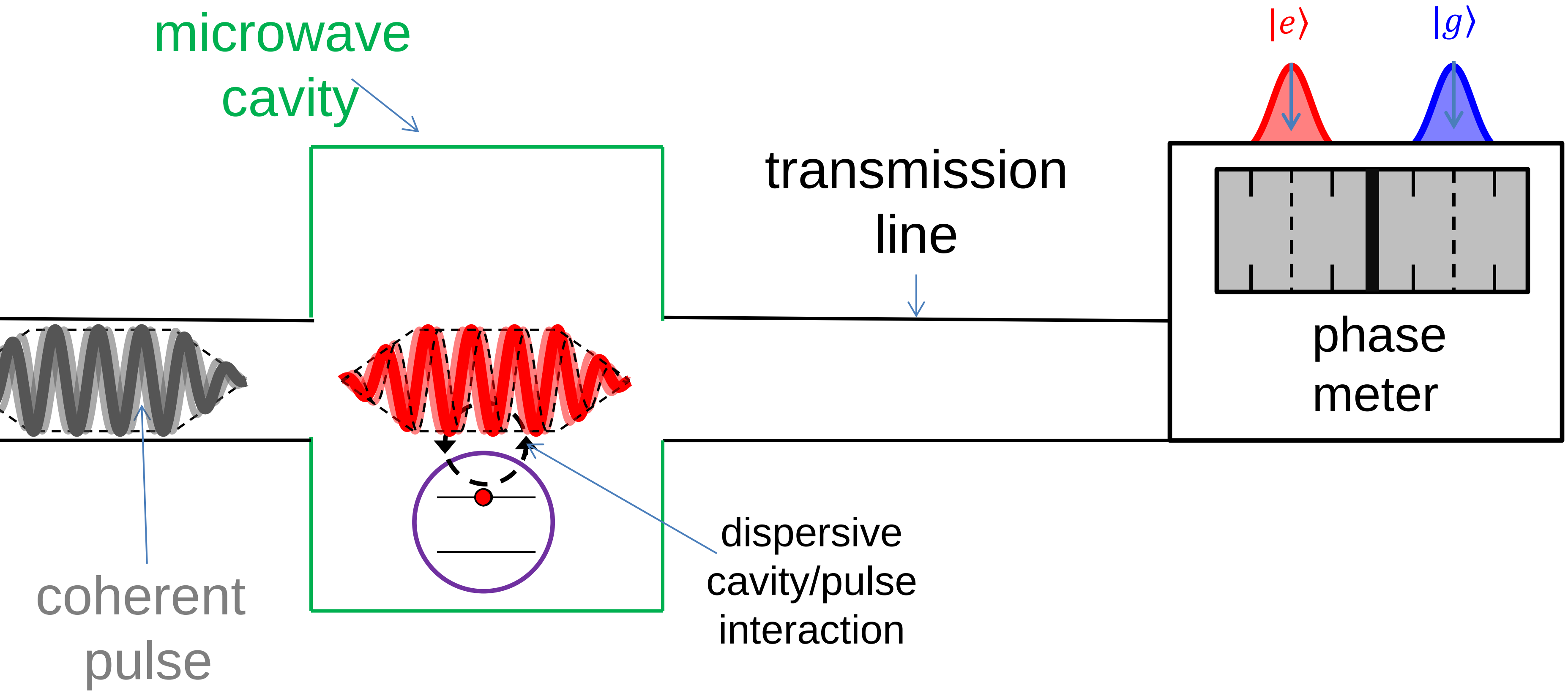
Dispersive measurement: classical version



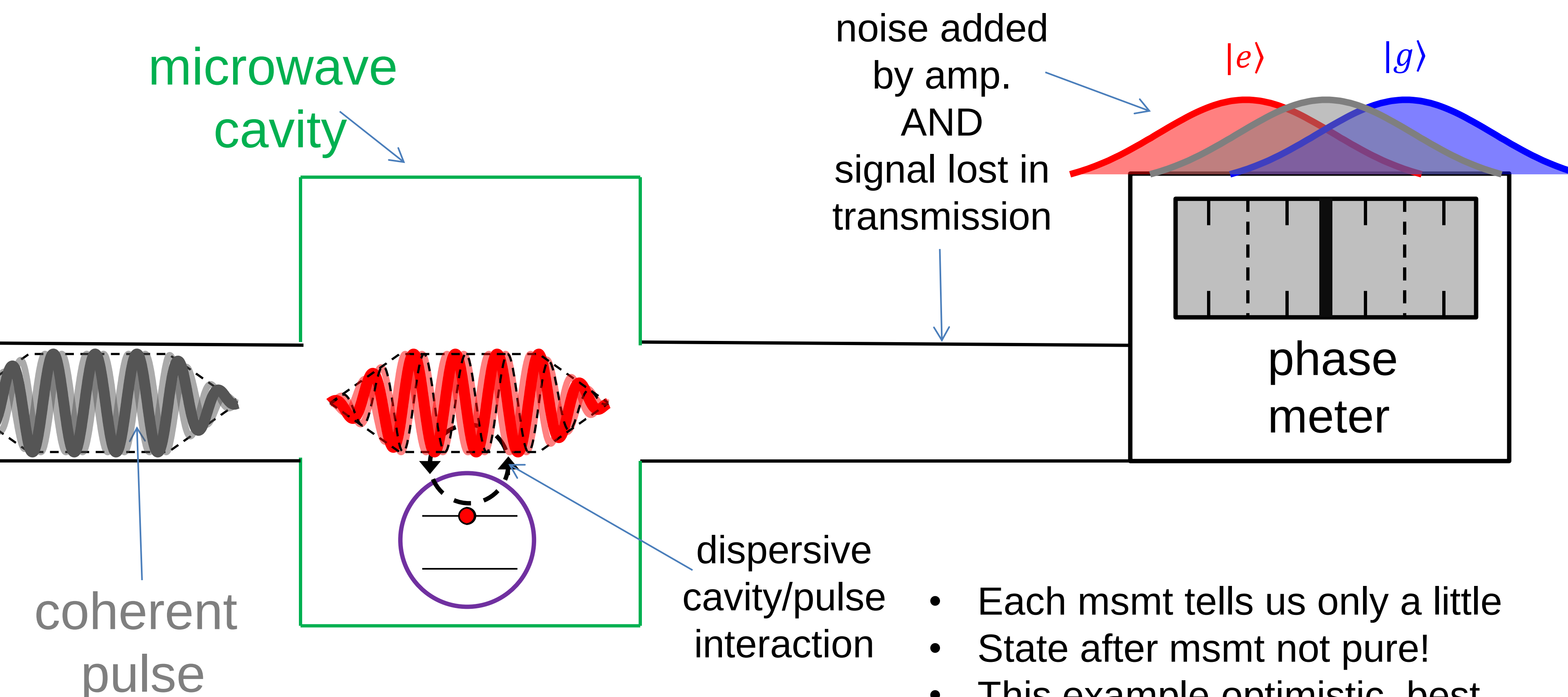
Dispersive measurement: classical version



Now a wrinkle: finite phase uncertainty

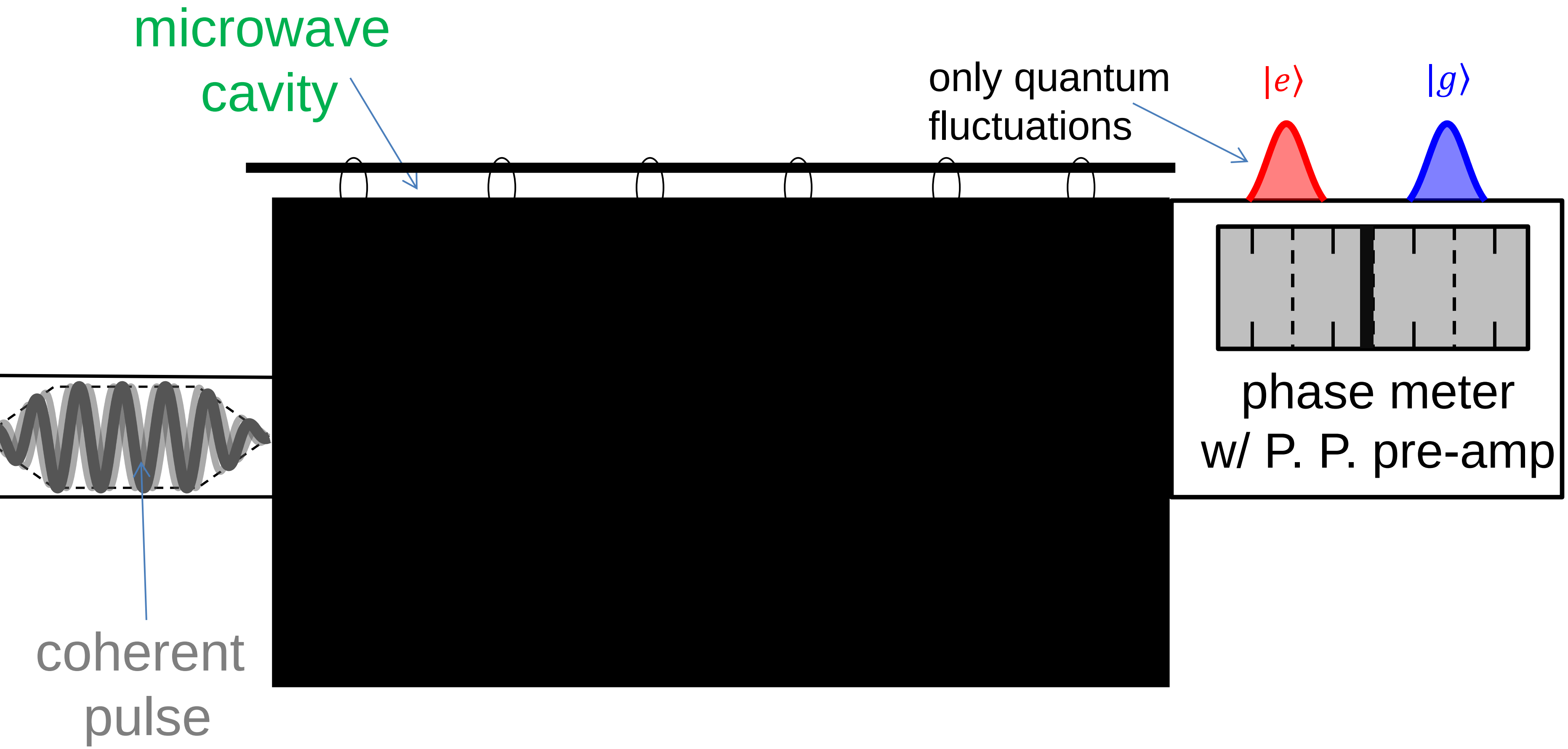


Measurement with bad meter (still classical)



- Each msmt tells us only a little
- State after msmt not pure!
- This example optimistic, best commercial amp adds 20-30x noise
- We fix this with quantum-limited amplification

Quantum-limited amplification: projective msmt

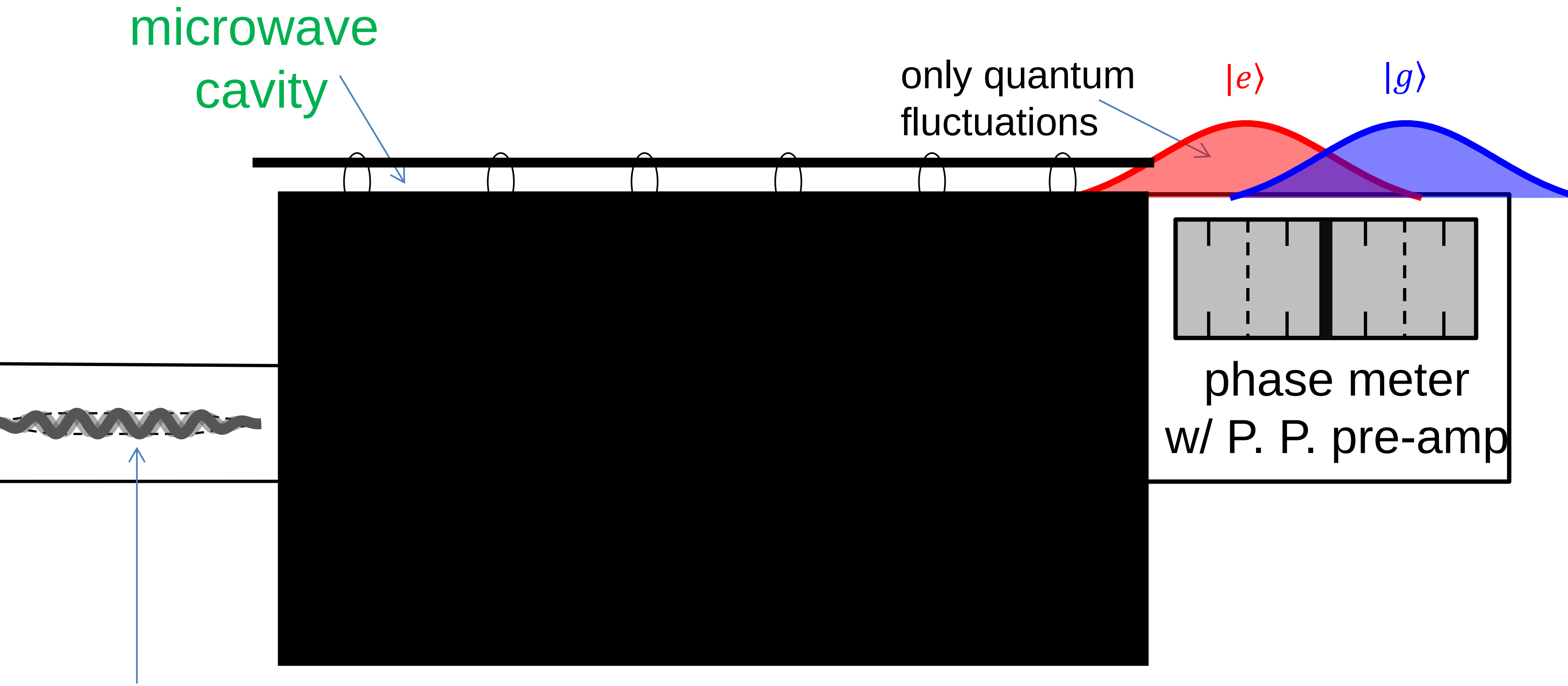


- state of qubit pure after each msmt
- For unknown initial state

$c_g|g\rangle + c_e|e\rangle$, repeat

many times to estimate $|c_g|^2$, $|c_e|^2$

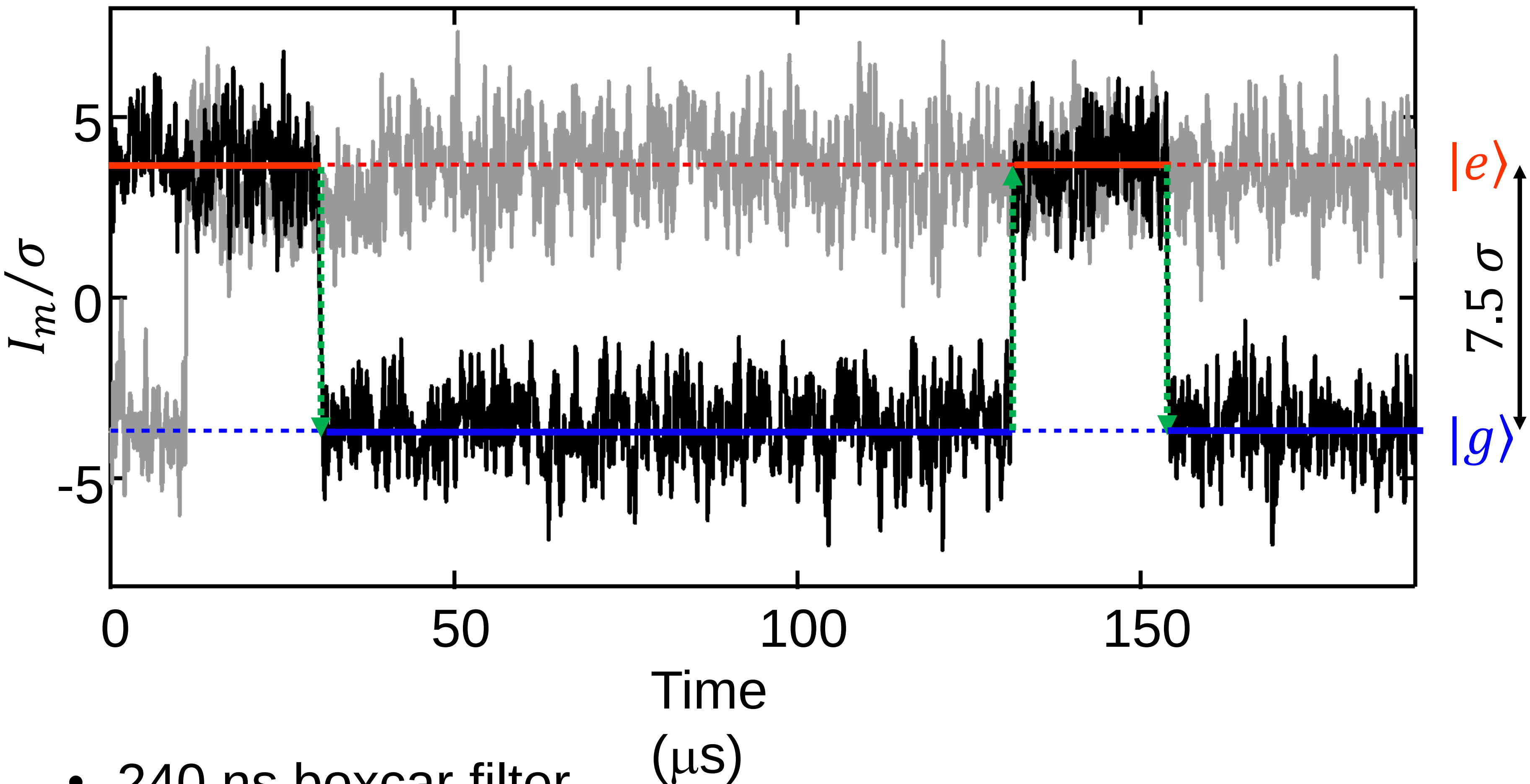
Quantum-limited amplification: 'partial' msmt



- state of qubit pure after each msmt
- counter-intuitive, but is achievable in the laboratory

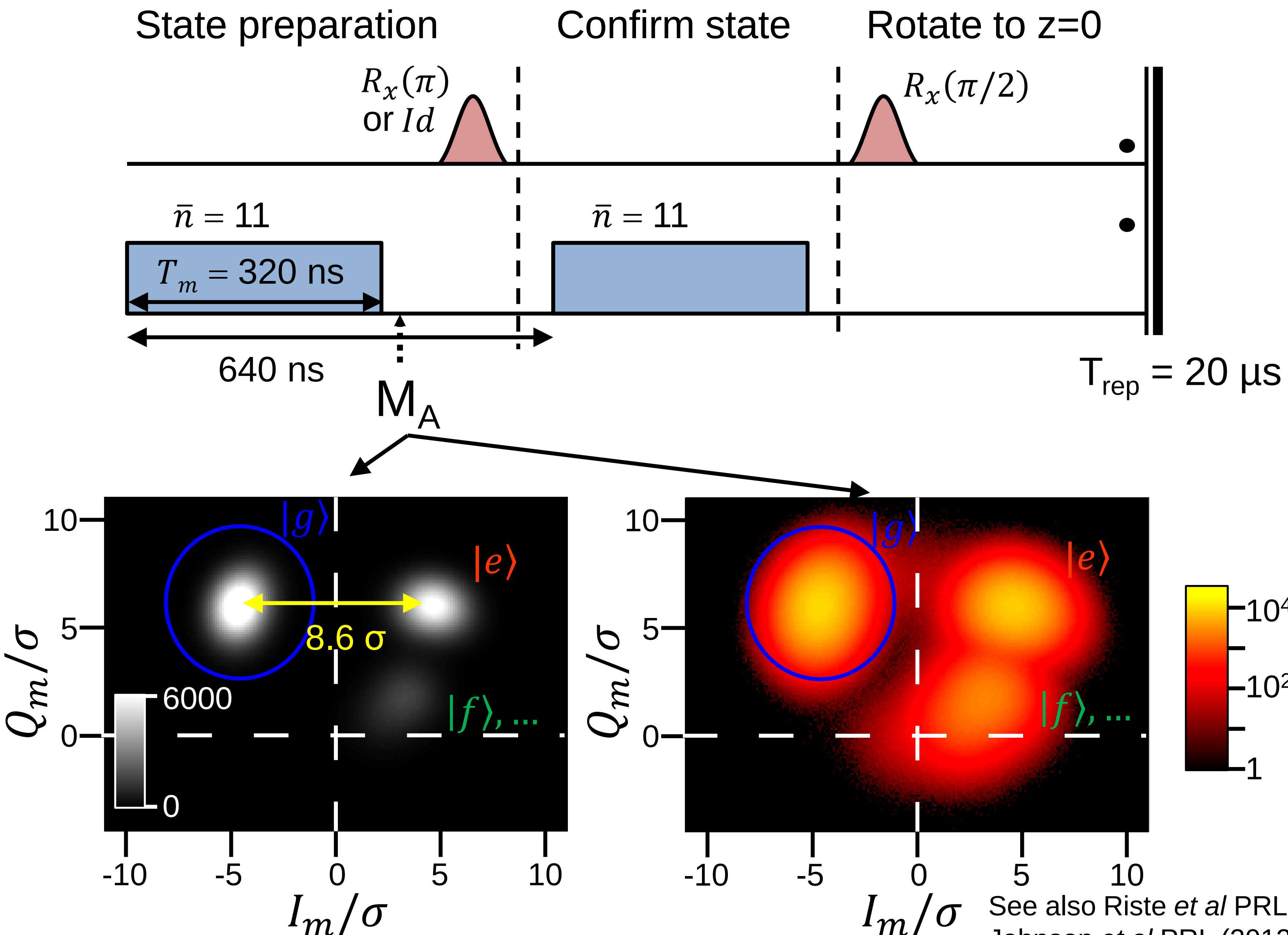
Part 2: Partial measurement with transmon qubit and JPC

Quantum jumps



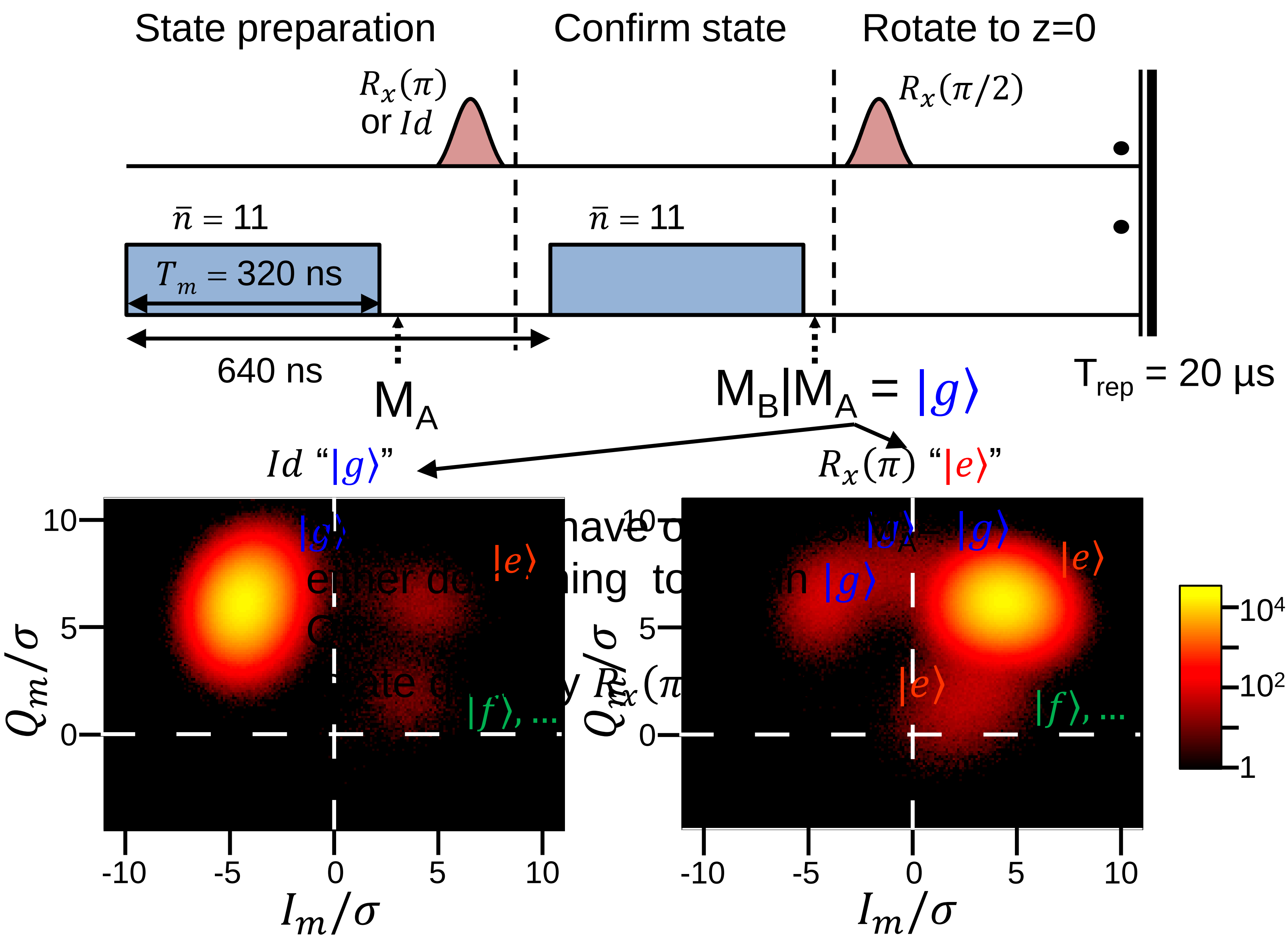
- 240 ns boxcar filter
- $T_1(\bar{n} = 10) \cong 50\mu s$
- Fully linear (can see $|f\rangle, |h\rangle \ldots$ in IQ plane)

Preparation by measurement + post-selection

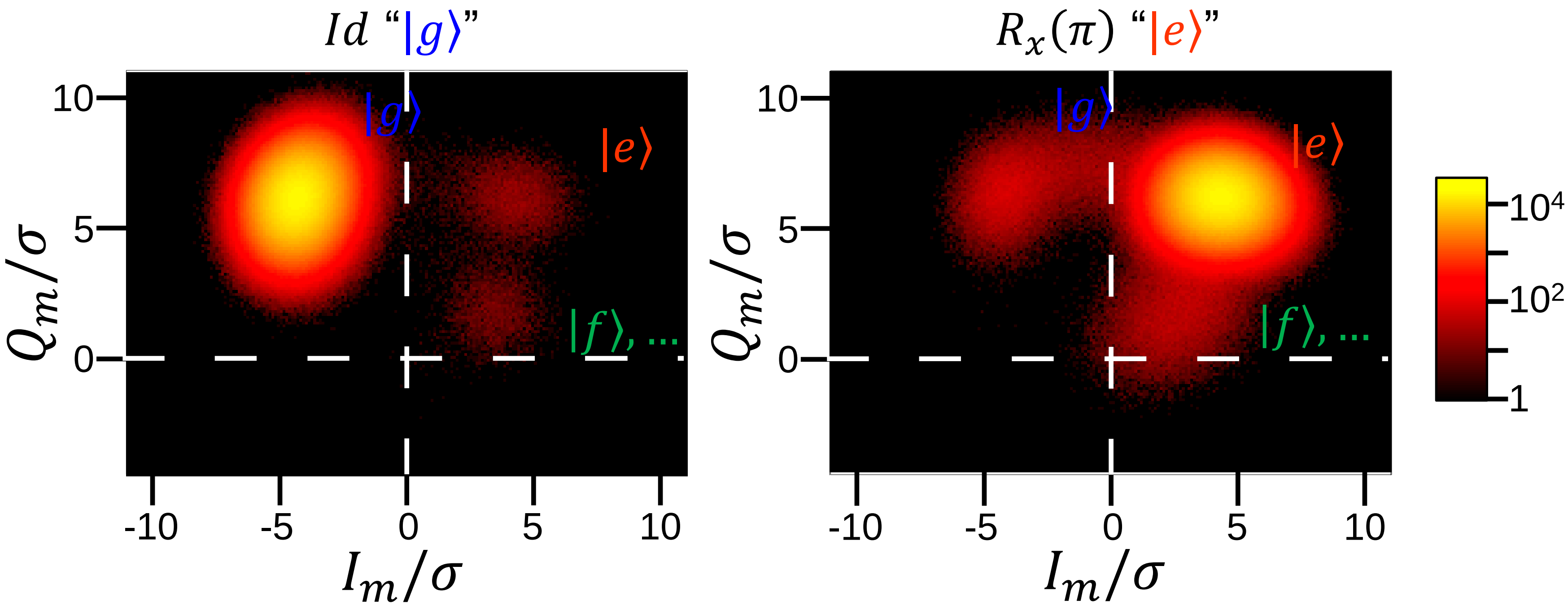


See also Riste *et al* PRL (2012)
Johnson *et al* PRL (2012)

Preparation by measurement + post-selection



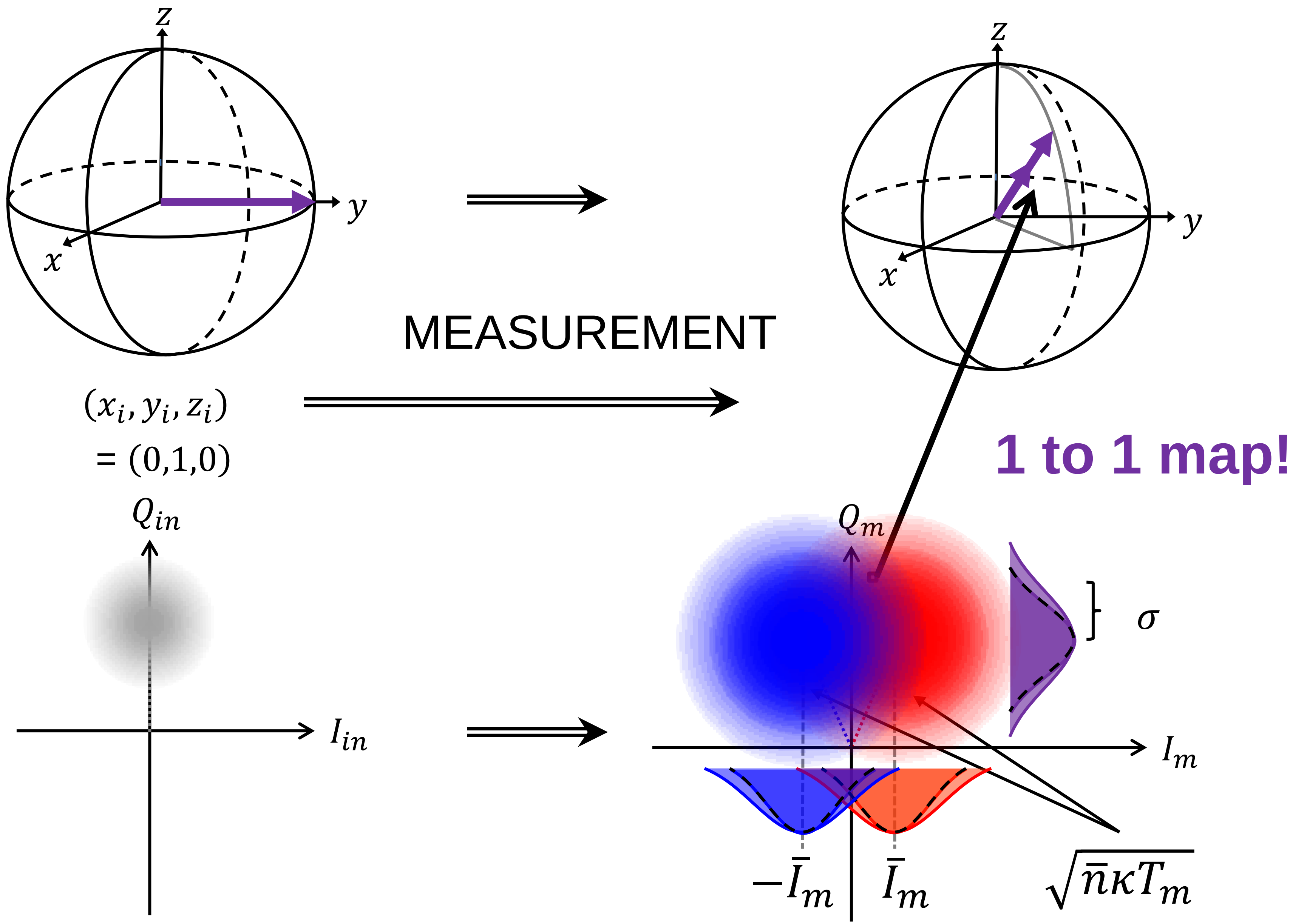
How ideal is this operation?



Fidelity=0.994!

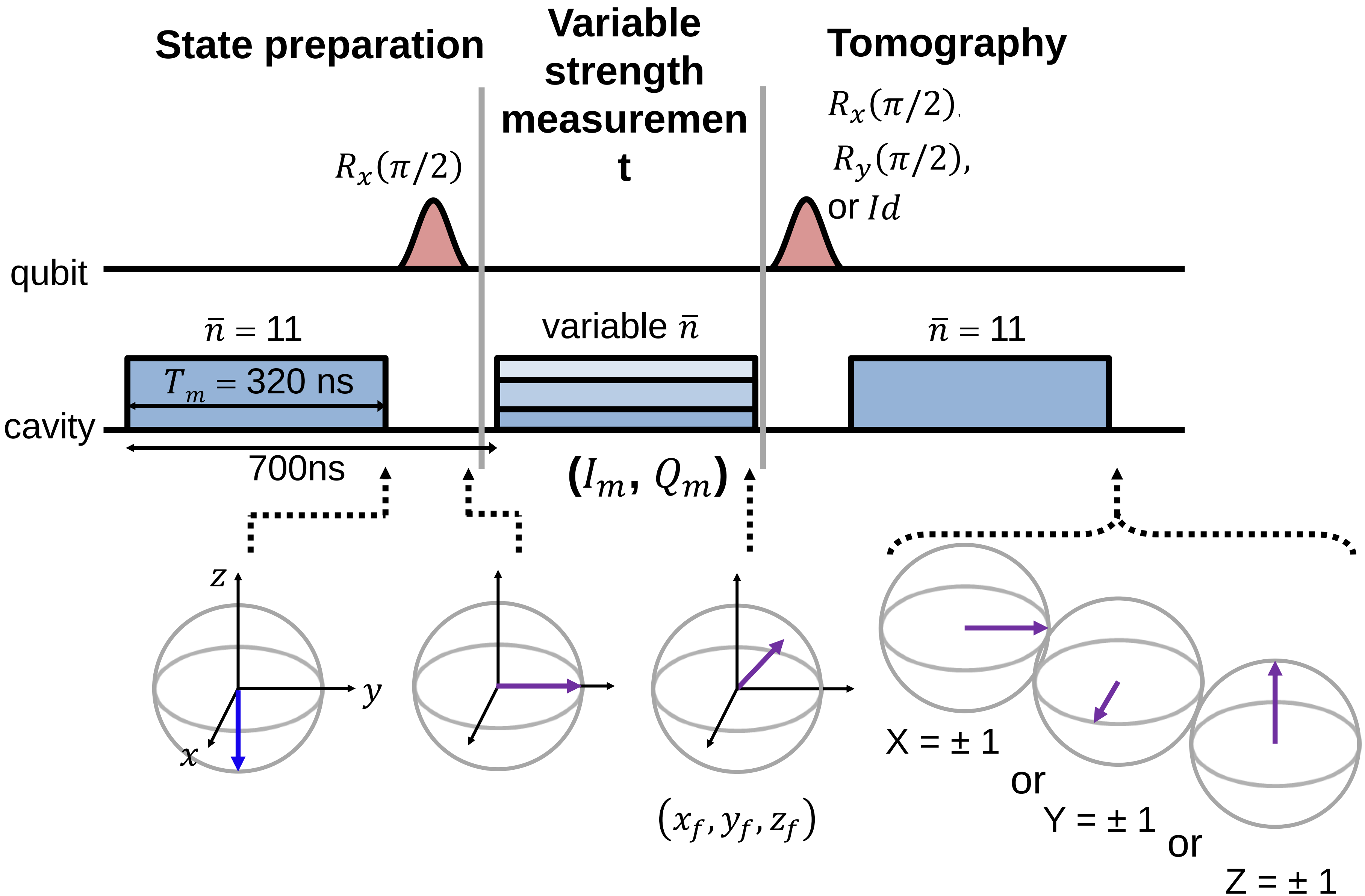
Strong measurements allow rapid, high-fidelity state preparation and tomography

Back-action of partial measurement*

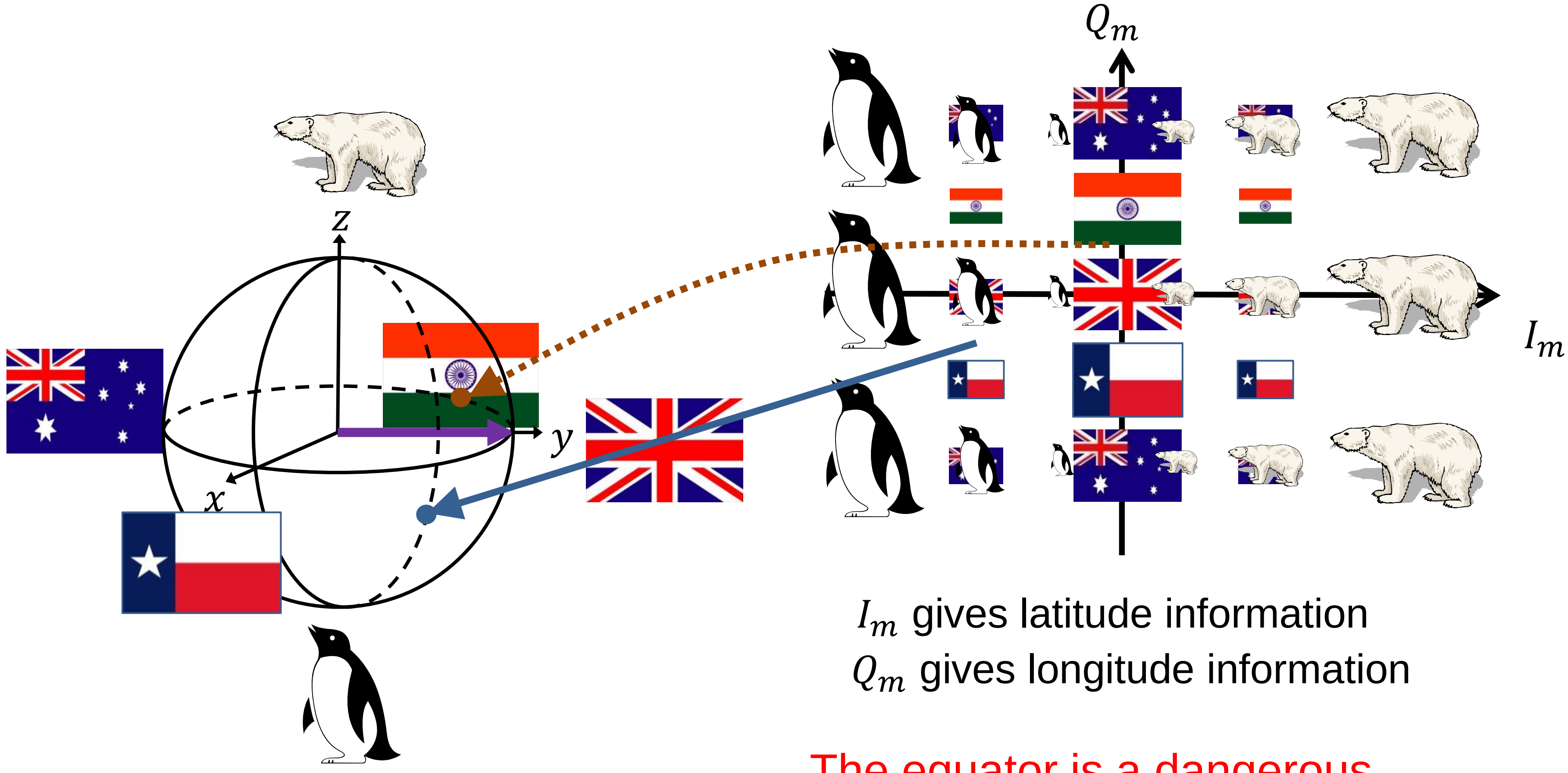


*Gambetta, *et al* PRA (2008); Korotkov/Girvin, Les Houches (2011); M. Hatridge *et al* Science (2013)

Back-action characterization protocol



A picture is worth a thousand math symbols * : Mapping (I_m, Q_m) to the Bloch vector



I_m gives latitude information
Q_m gives longitude information

The equator is a dangerous place: lost information pulls trajectory towards the z-axis

*Gambetta, *et al* PRA (2008); Korotkov/Girvin, Les Houches (2011); M. Hatridge *et al* Science (2013)

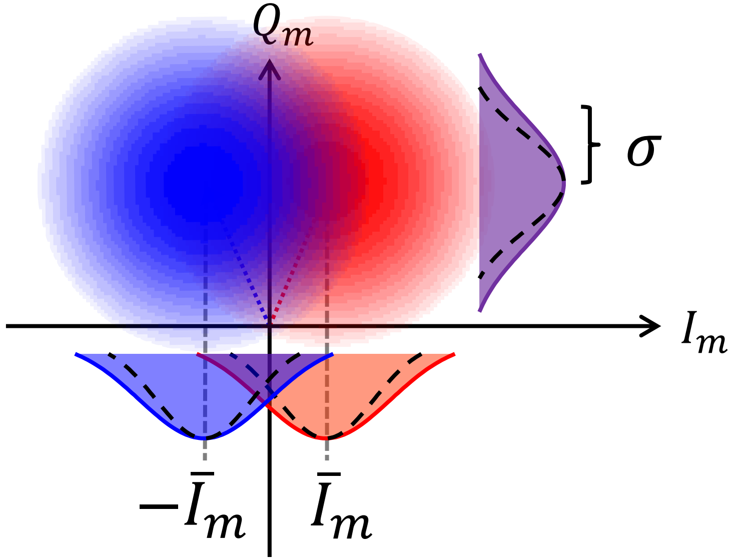
Determining efficiency with partial measurement

Convergence to poles (I_m) | **Rotation (Q_m)** | **Measurement induced dephasing**

$$x_f = \operatorname{sech}\left(\frac{I_m \bar{I}_m}{\sigma^2}\right) \sin\left(\frac{Q_m \bar{I}_m}{\sigma^2}\right) \exp\left\{-\left(\frac{\bar{I}_m}{\sigma}\right)^2 \frac{1-\eta}{\eta}\right\}$$

$$y_f = \operatorname{sech}\left(\frac{I_m \bar{I}_m}{\sigma^2}\right) \cos\left(\frac{Q_m \bar{I}_m}{\sigma^2}\right) \exp\left\{-\left(\frac{\bar{I}_m}{\sigma}\right)^2 \frac{1-\eta}{\eta}\right\}$$

$$z_f = \tanh\left(\frac{I_m \bar{I}_m}{\sigma^2}\right)$$



I_m gives latitude information

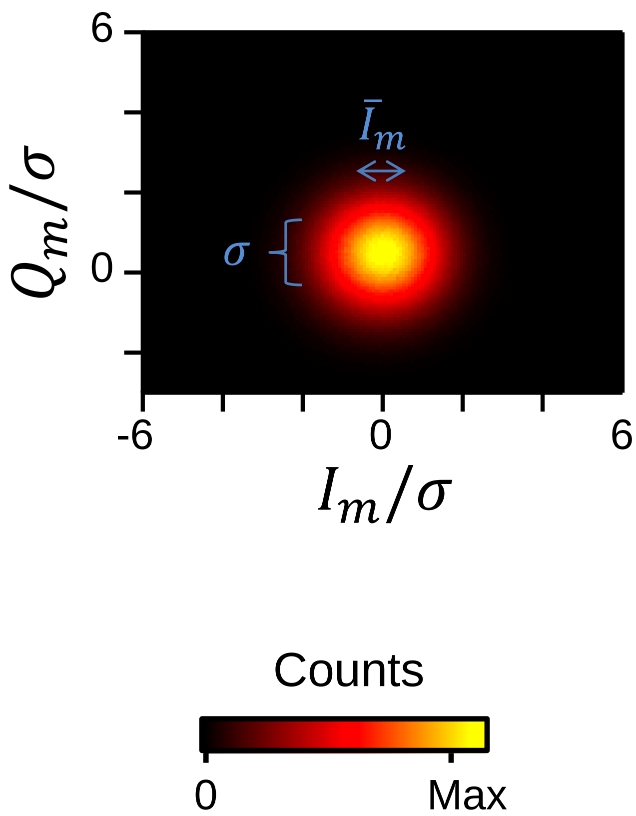
Q_m gives longitude information

Efficiency $\eta = \left({}^{\sigma}/_{\sigma_{ideal}}\right)^2$

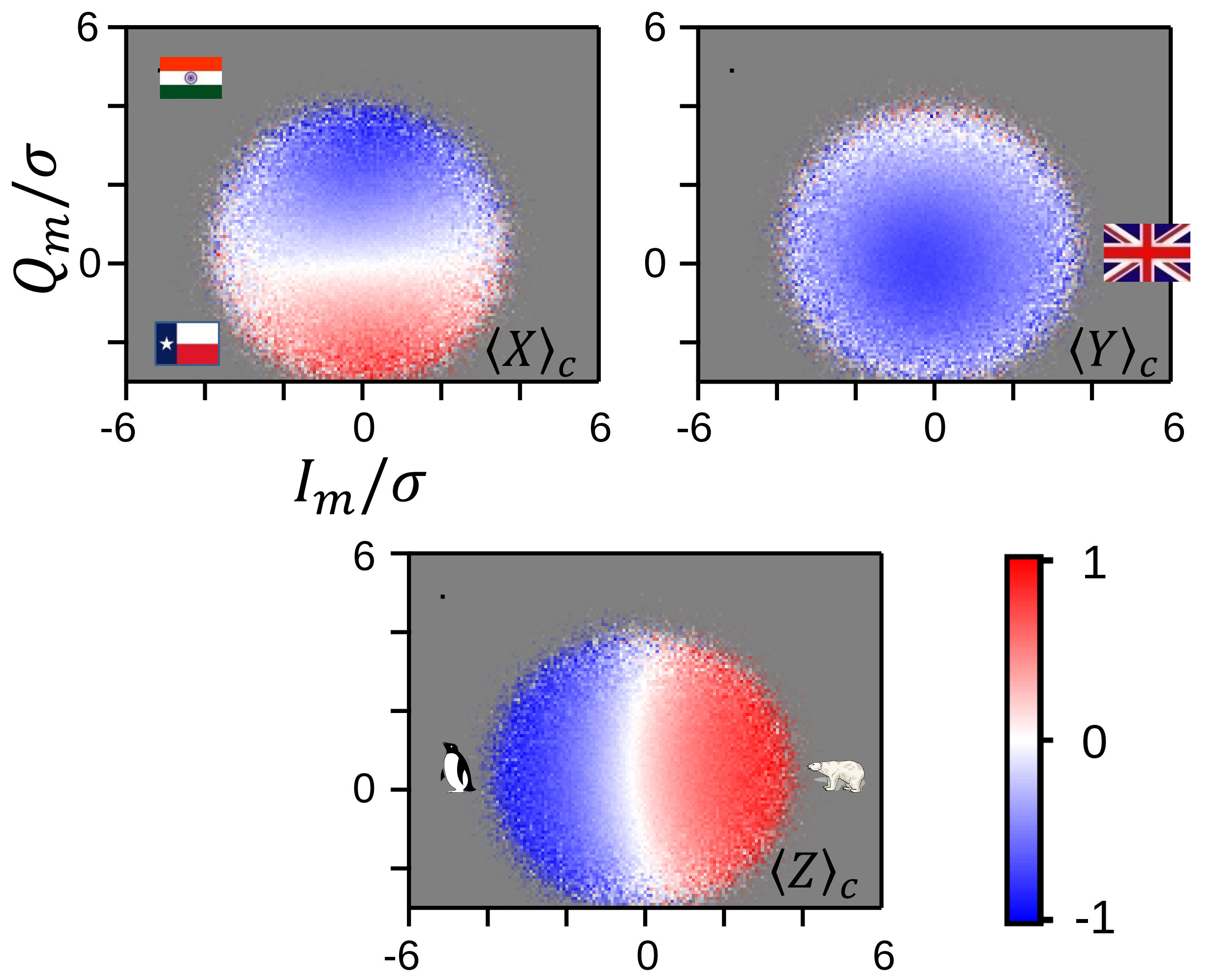
See Hatridge *et al.*, Science 2013

Measurement with $\bar{I}_m/\sigma = 0.4$

histogram of measurement after π/2 pulse

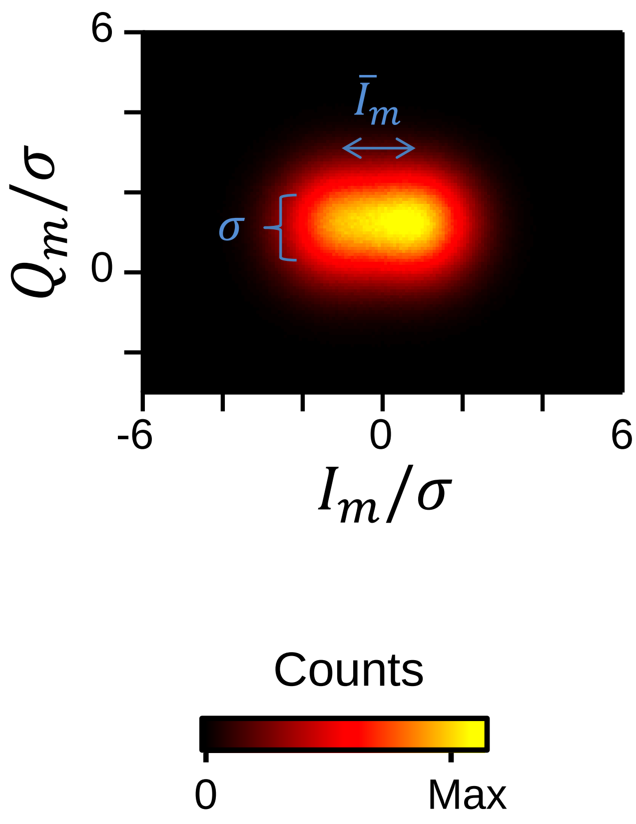


tomography along X, Y and Z after measurement

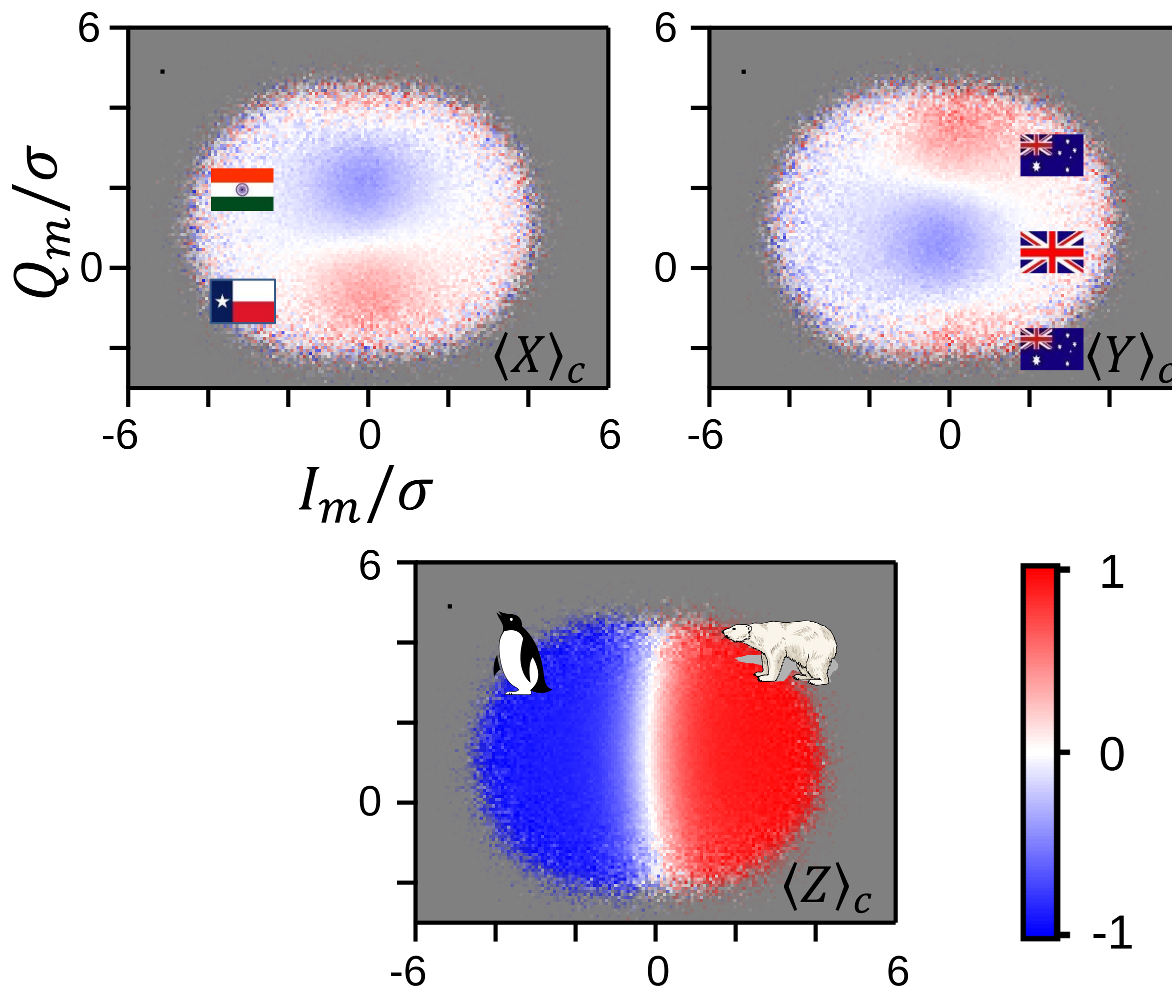


Measurement with $\bar{I}_m/\sigma = 1.0$

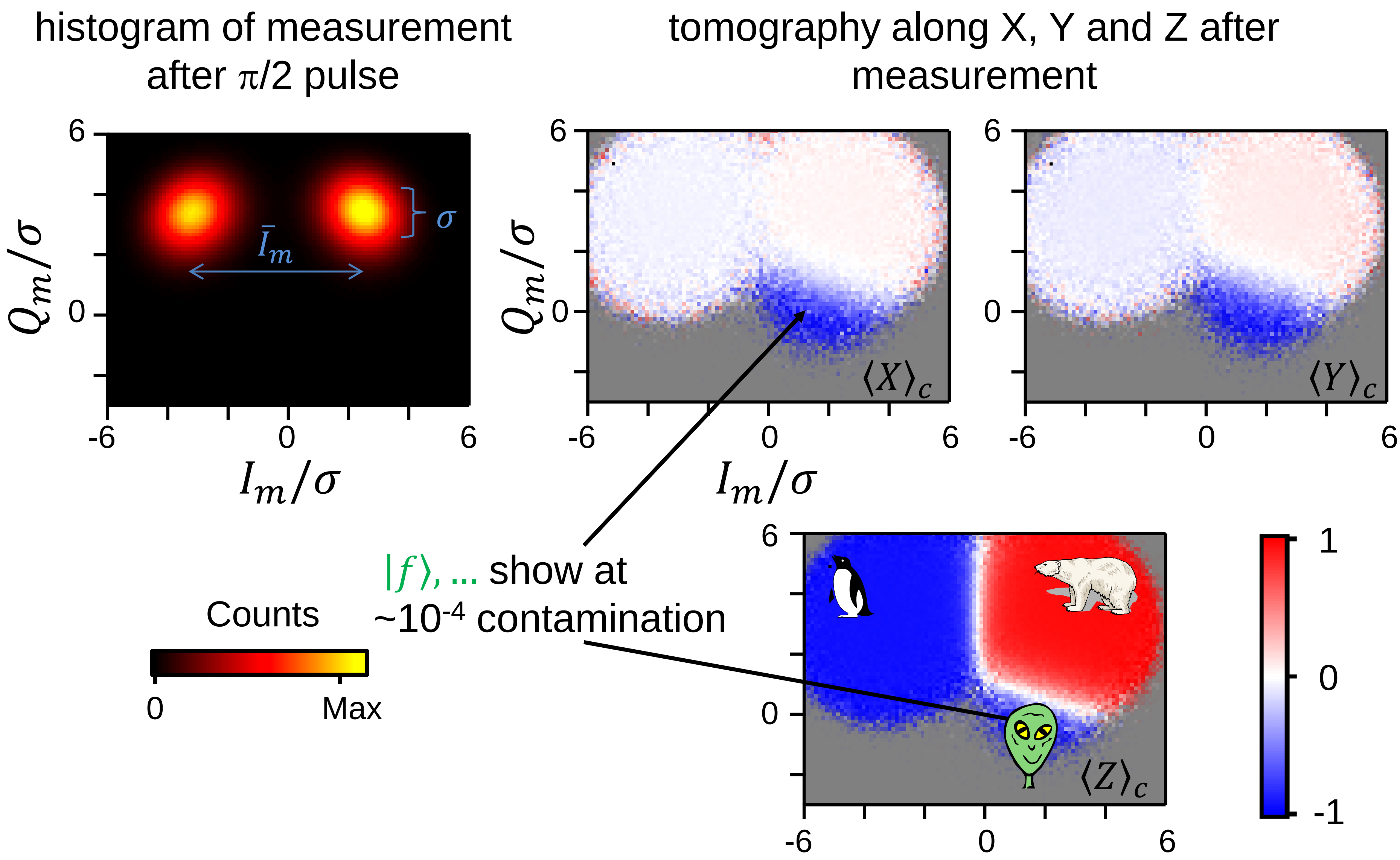
histogram of measurement after π/2 pulse



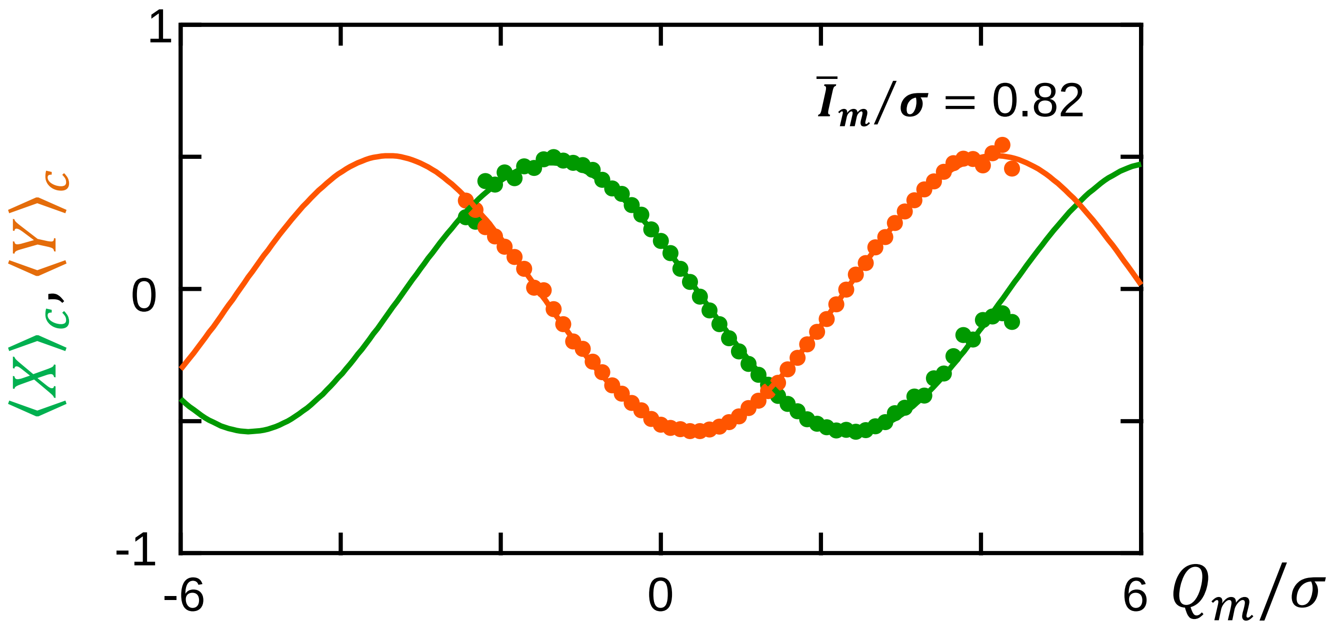
tomography along X, Y and Z after measurement



Measurement with $\bar{I}_m/\sigma = 2.8$



x- and y-component along $\boldsymbol{I_m = 0}$



$$\langle X \rangle_c = \sin\left(\frac{Q_m}{\sigma}\frac{\bar{I}_m}{\sigma} + \theta\right) \exp\left\{-\left(\frac{\bar{I}_m}{\sigma}\right)^2 \frac{1-\eta}{\eta}\right\}$$

$$\langle Y \rangle_c = \cos\left(\frac{Q_m}{\sigma}\frac{\bar{I}_m}{\sigma} + \theta\right) \exp\left\{-\left(\frac{\bar{I}_m}{\sigma}\right)^2 \frac{1-\eta}{\eta}\right\}$$

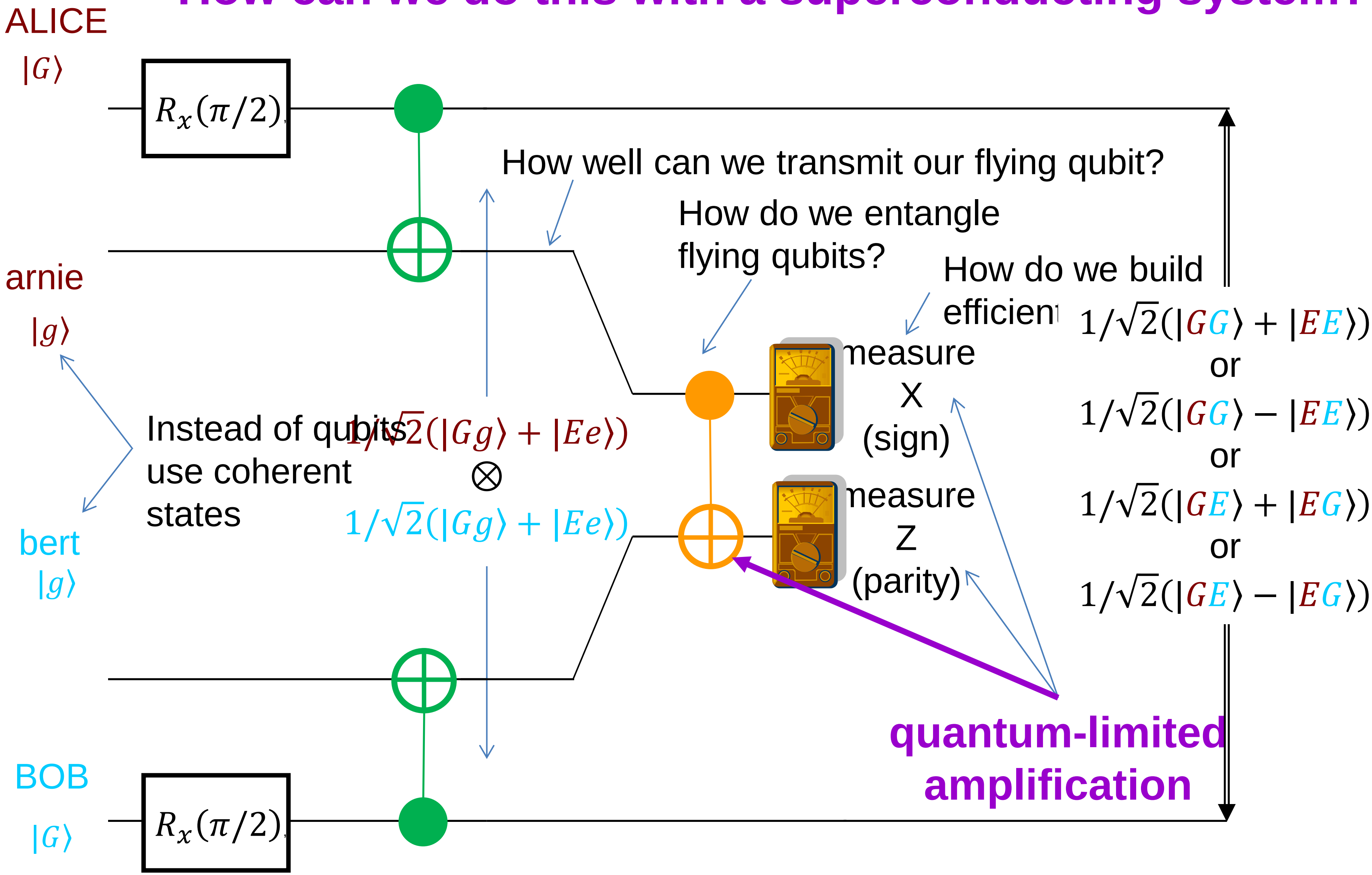
Amplitude determined by one fit parameter: $\boldsymbol{\eta}$ = 0.57 ± 0.02

$\boldsymbol{\eta \geq 0.5 \rightarrow}$ 3 body entanglement (qubit, signal, idler)

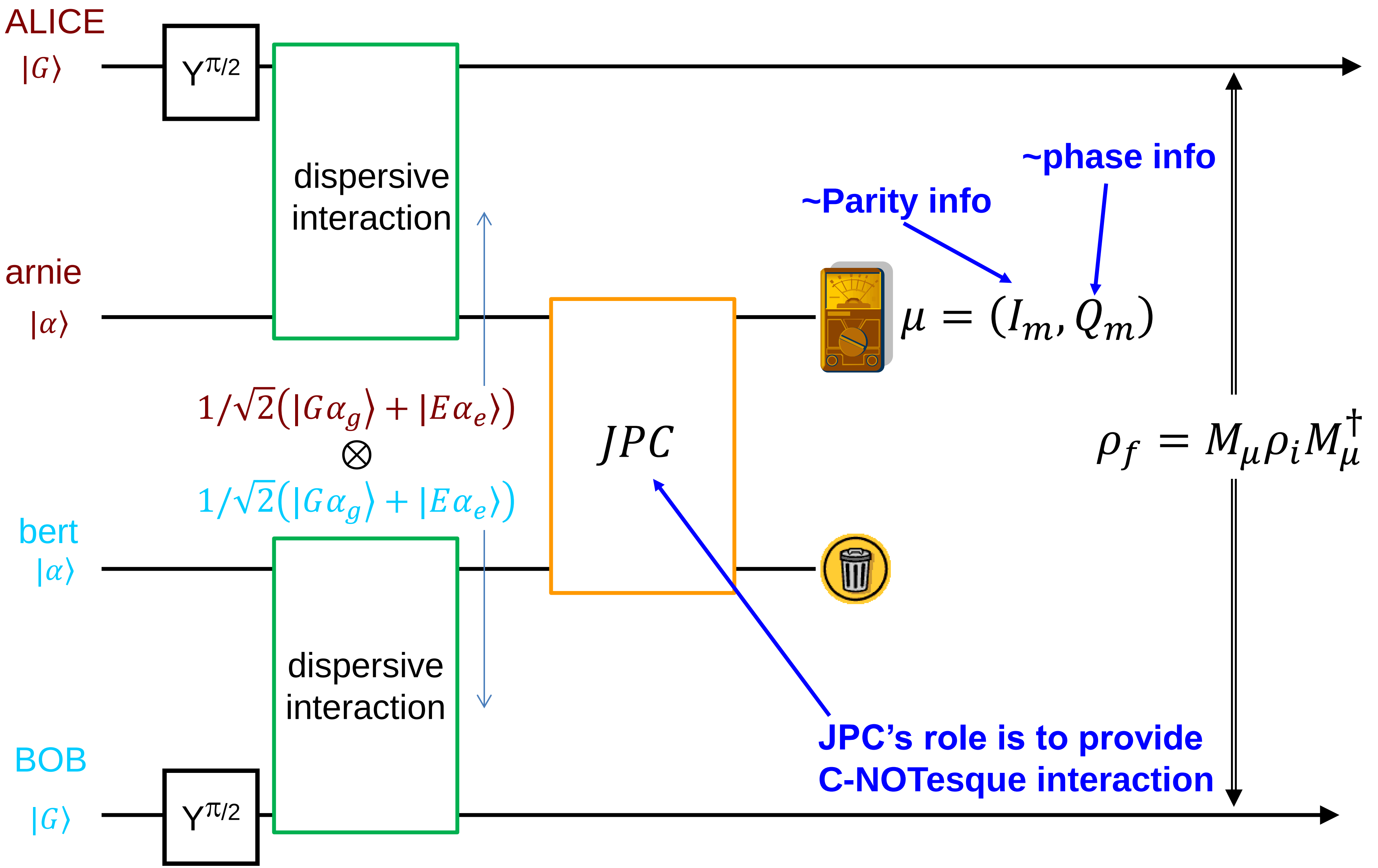
Part 3A: Remote Entanglement via joint measurement (two mode squeezing)

Remote entanglement with flying qubits

How can we do this with a superconducting system?

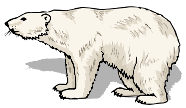


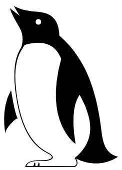
Remote entanglement with transmon and JPC

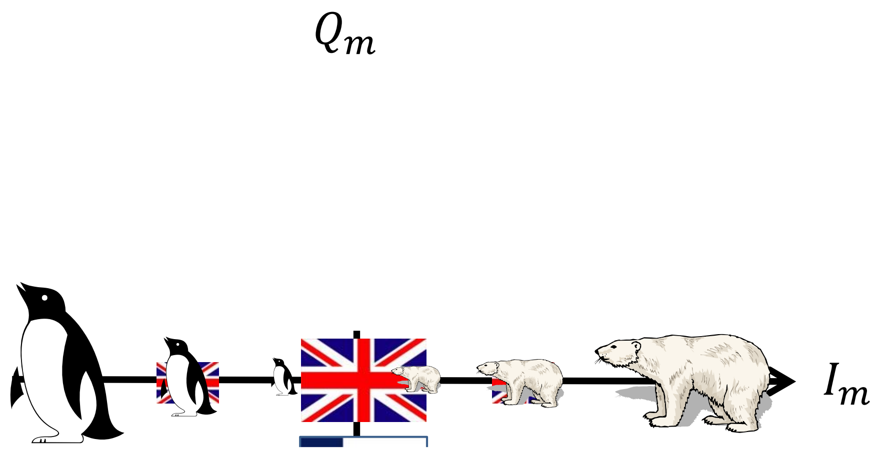


Back action of two qubit msmt creates entanglement

Even parity states:

 $= |gg\rangle$

 $= |ee\rangle$

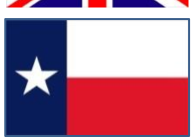


Odd parity states:

 $= |ge\rangle - |eg\rangle$

 $= |ge\rangle + i|eg\rangle$

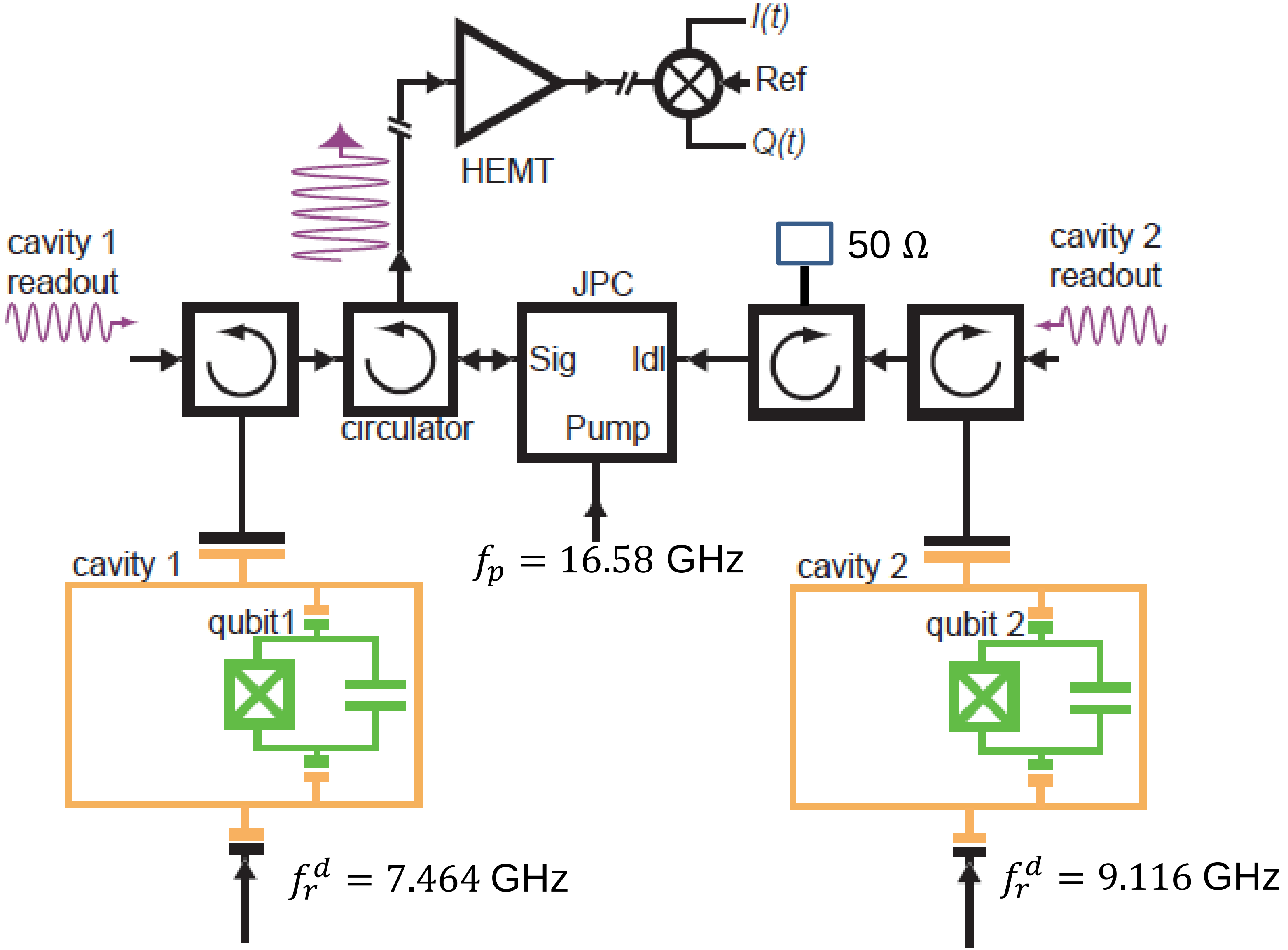
 $= |ge\rangle + |eg\rangle$

 $= |ge\rangle - i|eg\rangle$

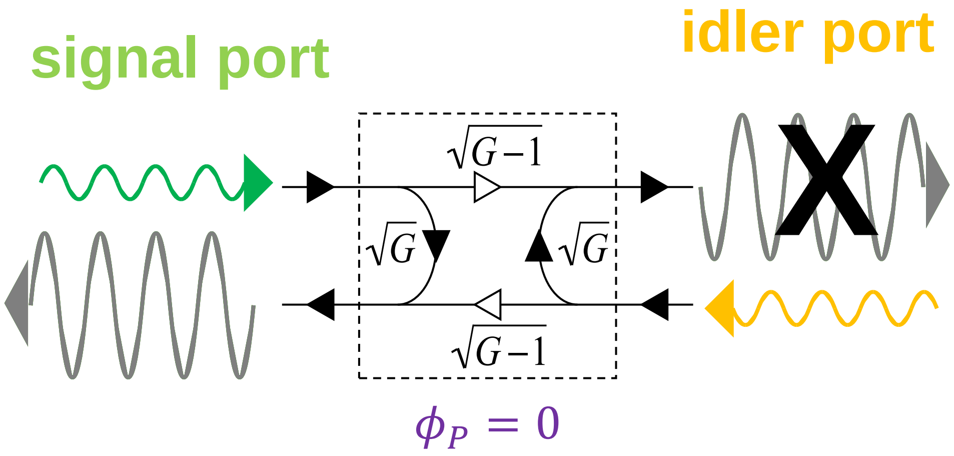
I_m gives info on even vs. odd parity (a bit too much, actually)

Q_m gives sign info for odd parity states

Two qubit readout schematic



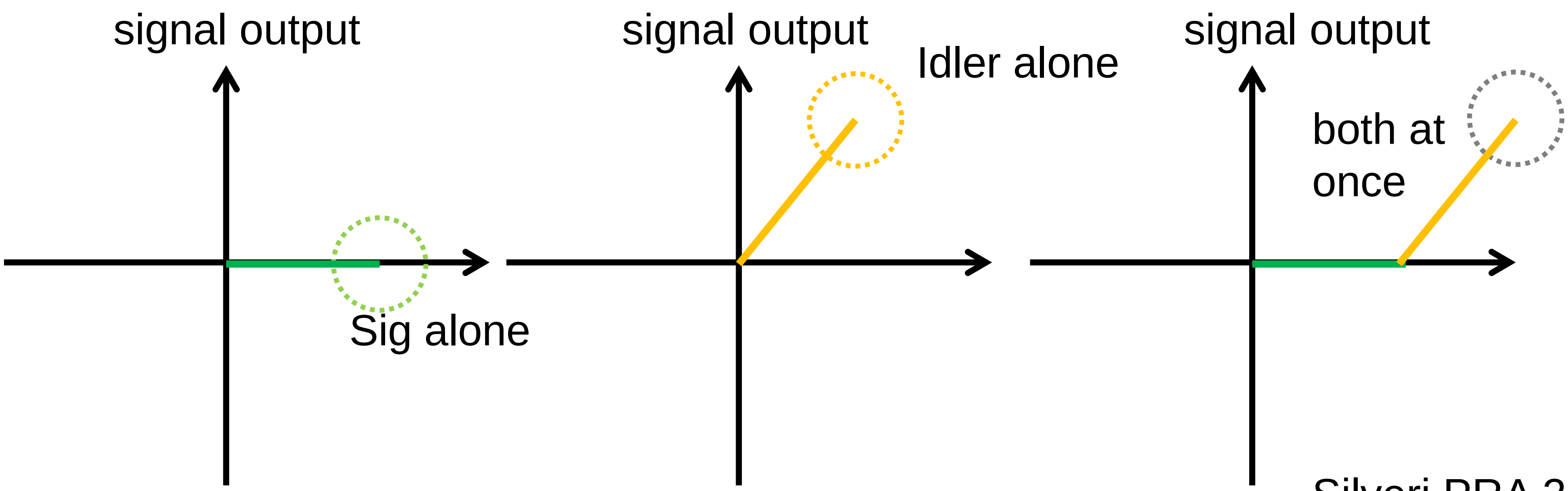
How do coherent states at different frequencies interact?



recall our pump relationship

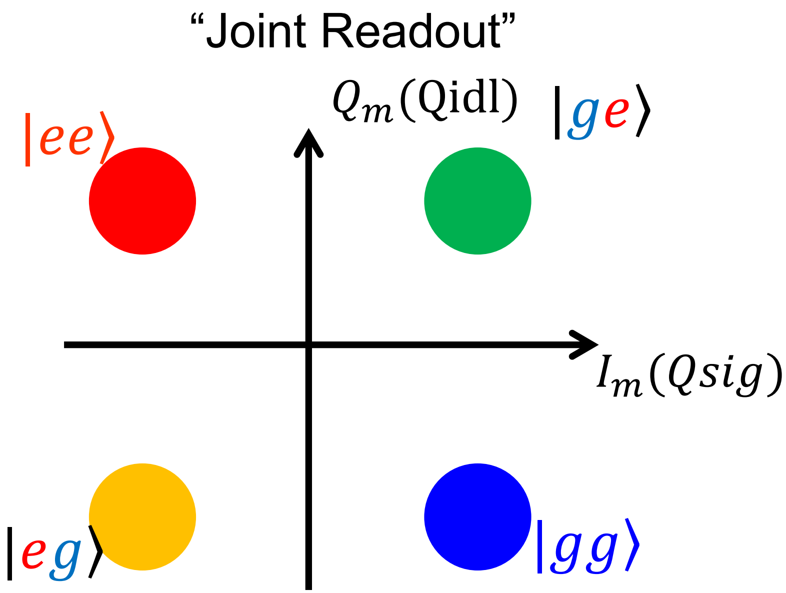
$$\Omega_P = \omega_S + \omega_I$$

Trans gain converts frequency coherently, e.q. $\omega_I \rightarrow \omega_S = \omega_I - \Omega_P$



Silveri PRA 2015

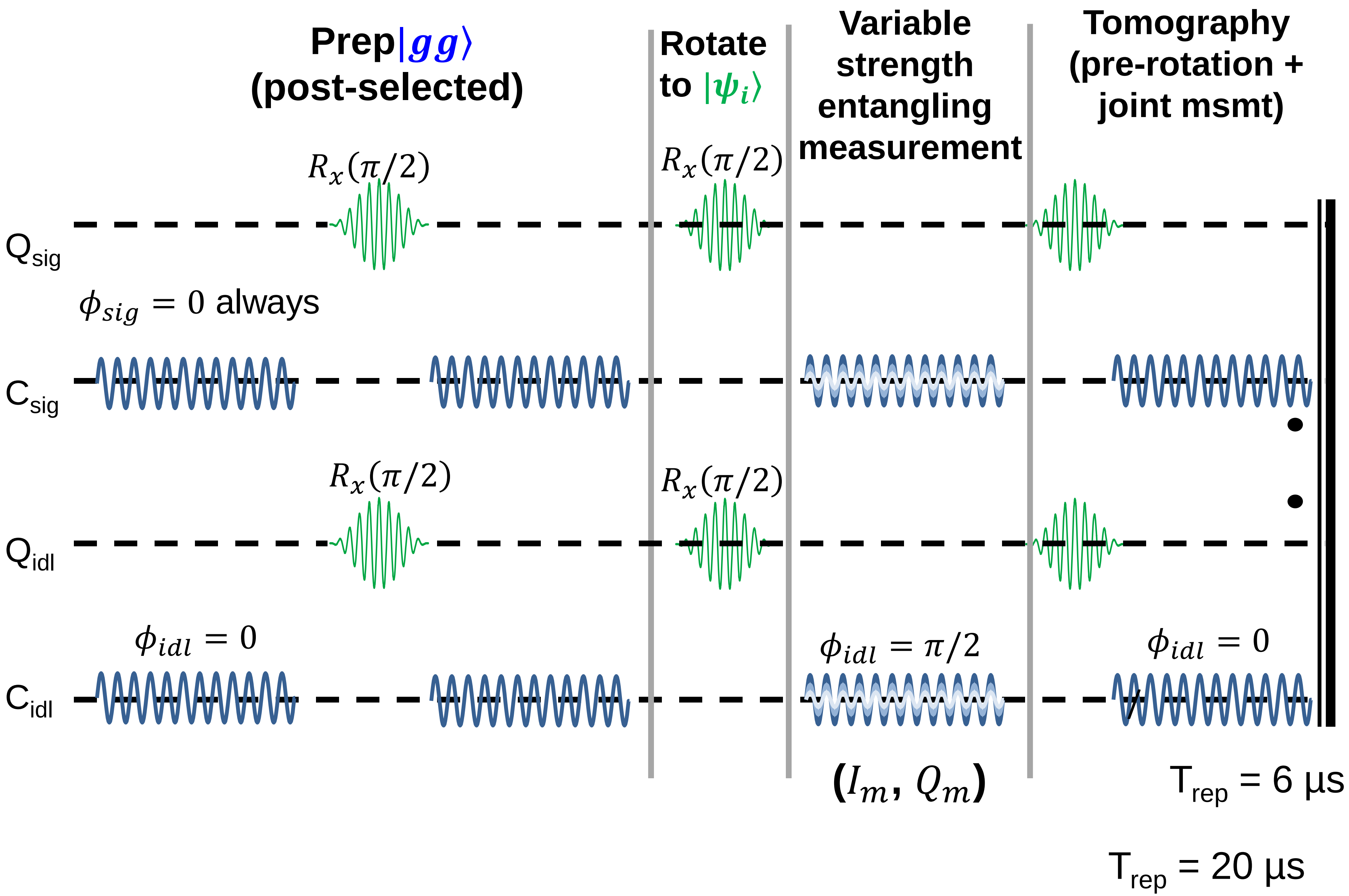
Tomography with full 2-bit joint readout



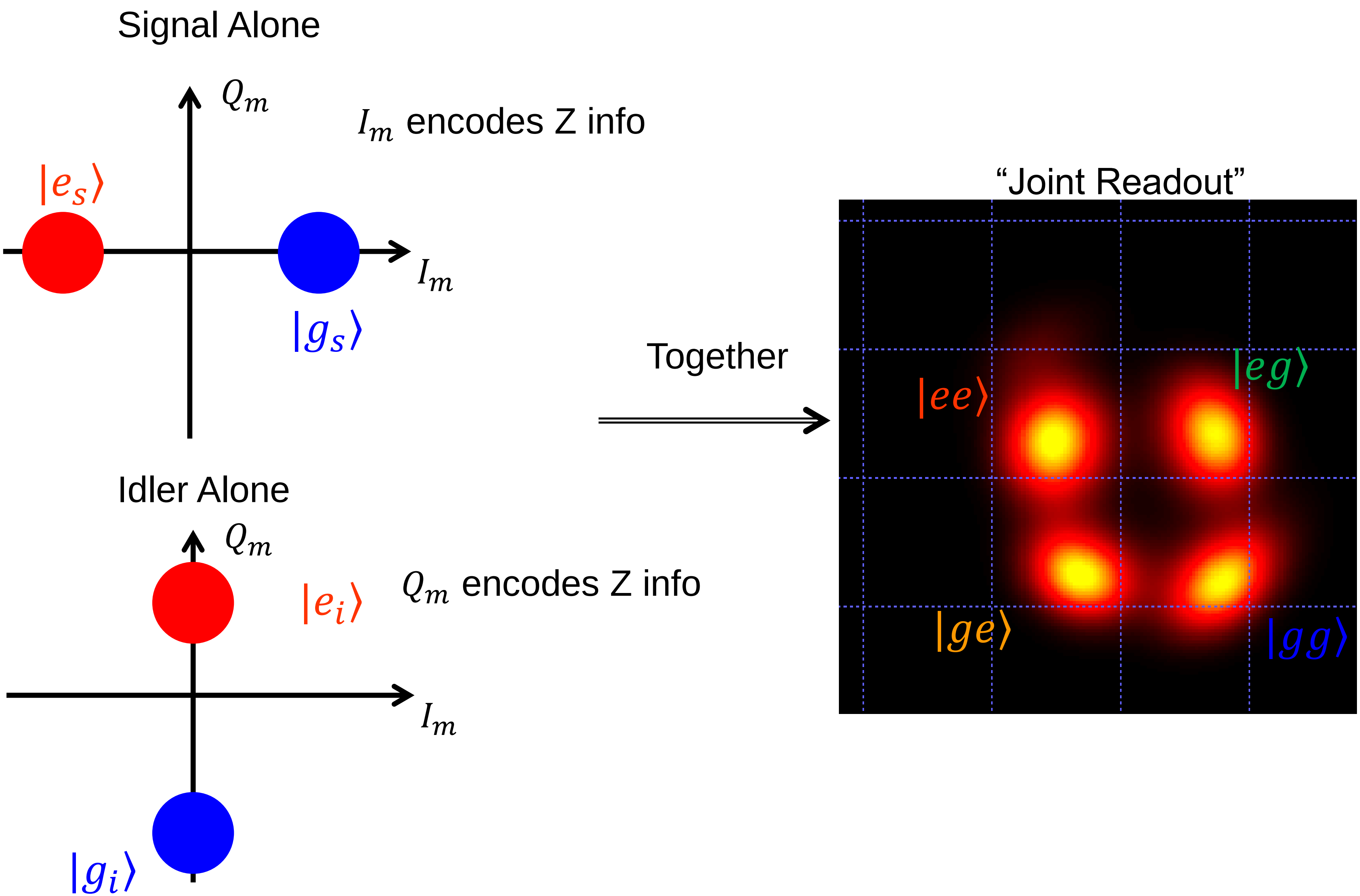
- From sign of I_m we learn $ZI = \pm 1$
- From sign of Q_m we learn $IZ = \pm 1$
- From these, we get ZZ for free
- 9 pre-rotations gives us all two-qubit corelators, and each single qubit component 3x

- Because we're paranoid we also do single qubit readout and compare the answers

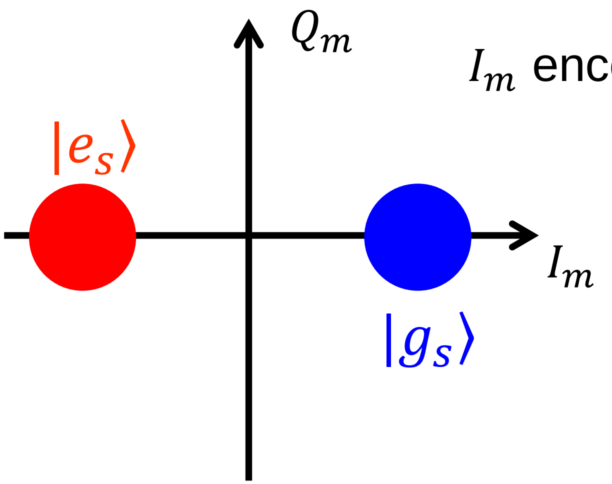
Remote entanglement pulse sequence



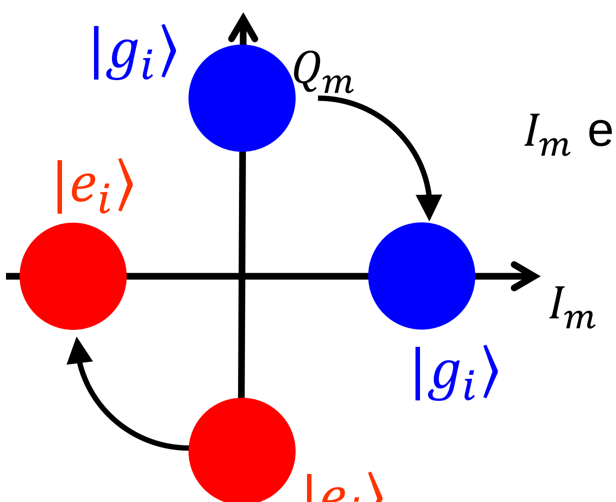
Simultaneous readout of two qubits



How to perform “entangling readout”



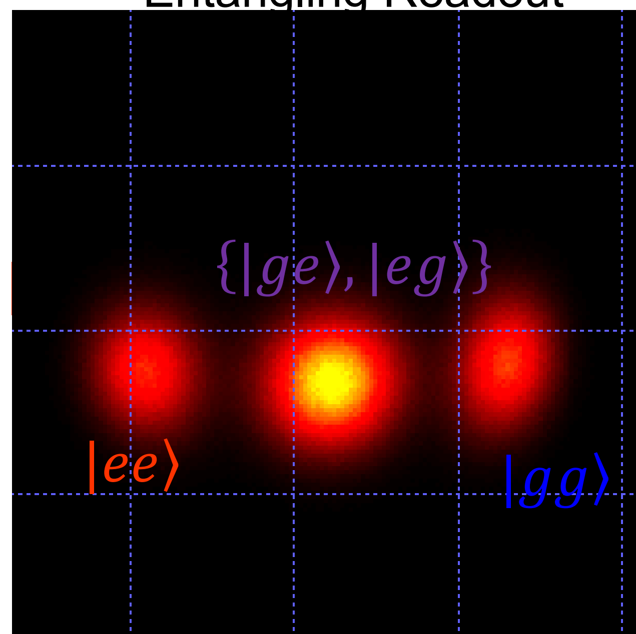
I_m encodes Z info



I_m encodes Z info

Together

“Entangling Readout”



ity)

- I_m is now blind to contents of $\{|ge\rangle, |eg\rangle\}$
- With/ appropriate initial state, outcome is Bell state w/ 50 % success rate
- Q_m encodes phase of Bell state

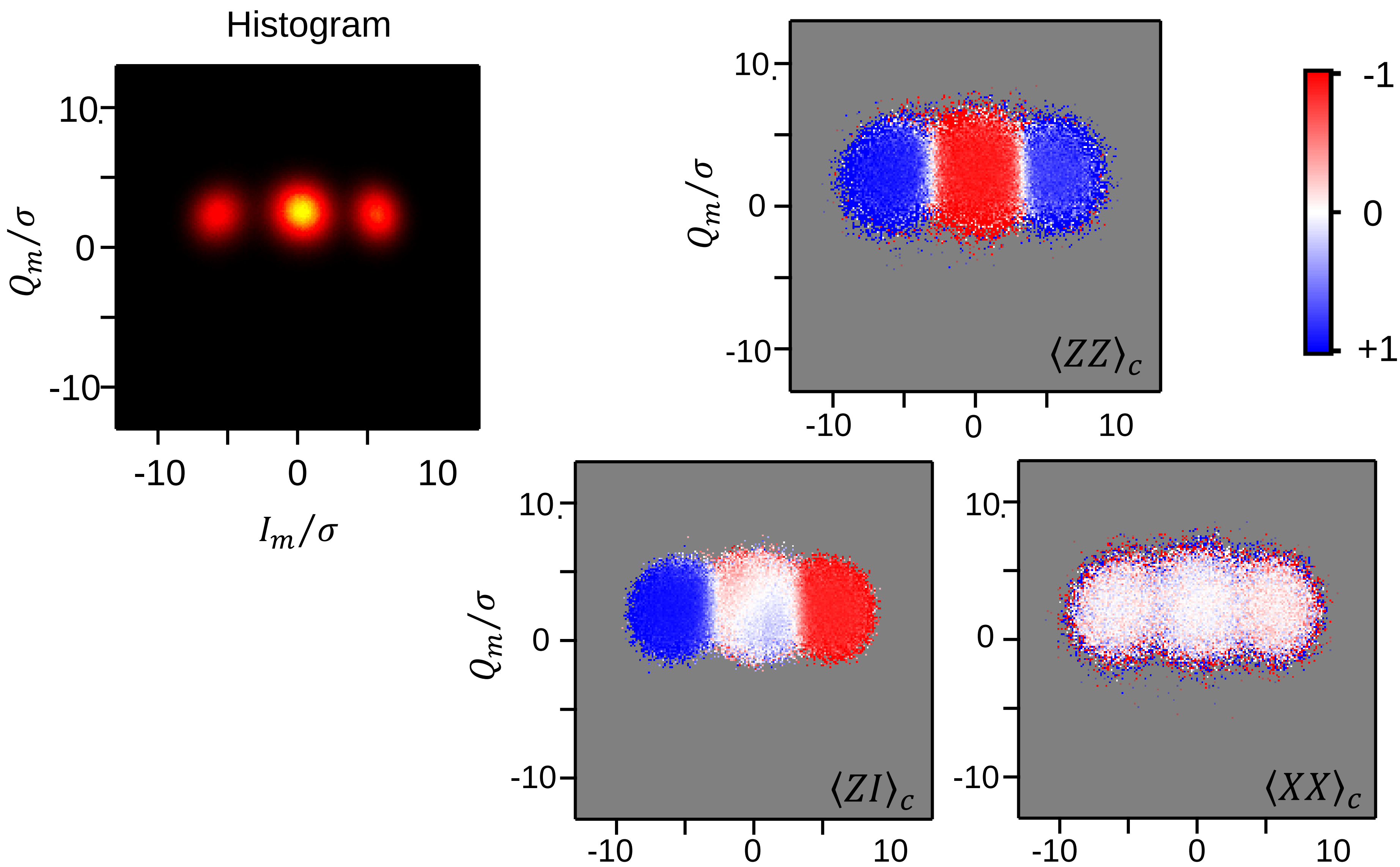
An aside on tomography and Bloch vectors

	Two qubit Bloch vector:	$\lvert -Y-Y\rangle$	$\lvert ge\rangle + \lvert eg\rangle$	$\lvert ge\rangle - \lvert eg\rangle$	$\lvert gg\rangle + i\lvert ee\rangle$	$\lvert gg\rangle - i\lvert ee\rangle$	
sig	ZI						
	XI						
	YI	-1					
idl	IZ						
	IX						
	IY	-1					
coherences	ZZ		-1	-1	+1	+1	Parity
	ZX						
	ZY						
	XZ						
	XX		+1	-1	+1	-1	
	XY						Sign
	YZ						
	YX						
	YY	+1	+1	-1	-1	+1	

The last four columns are headed "Bell Basis".

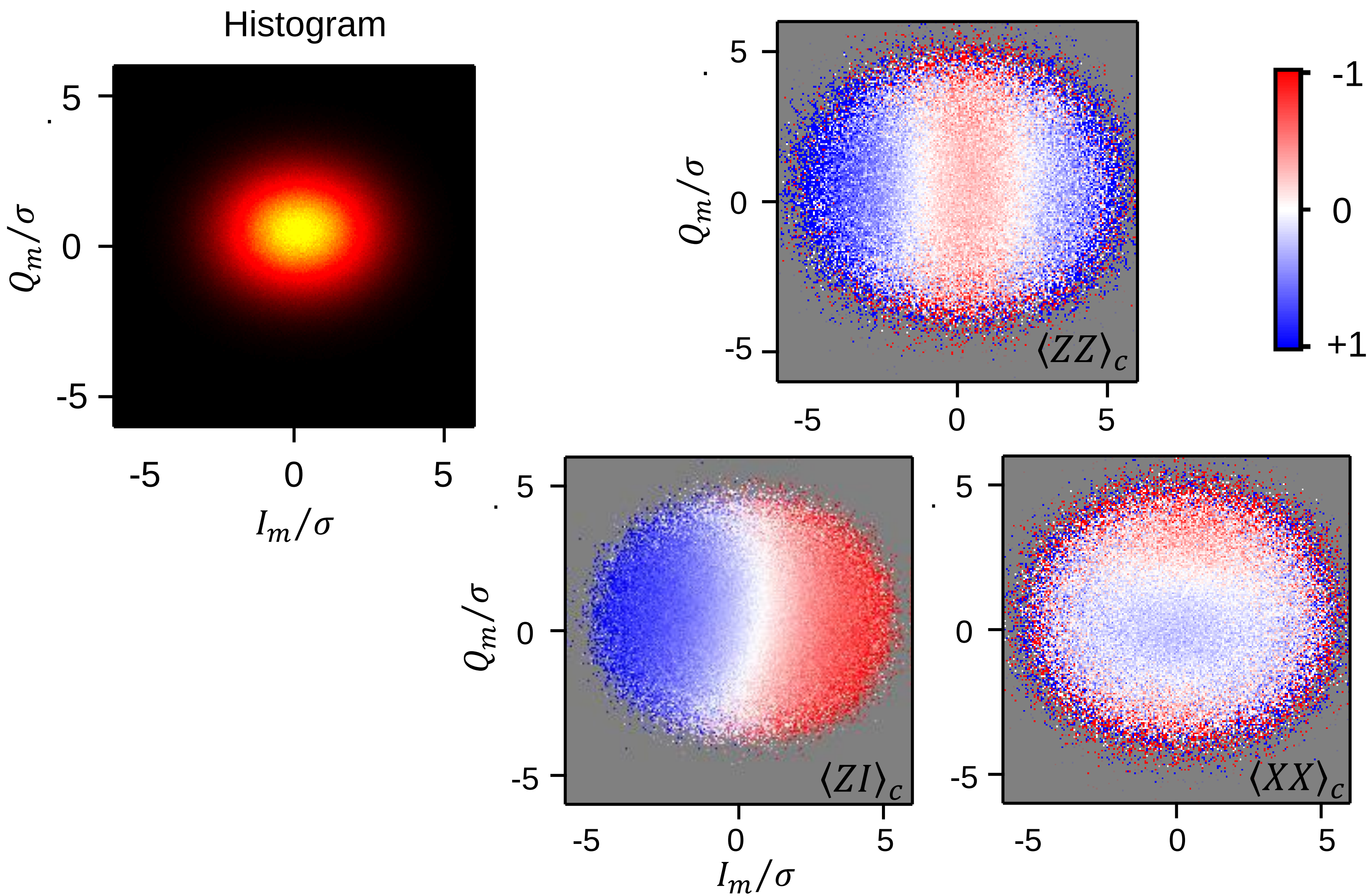
Note: All 15 numbers NOT linearly independent, and pure states sum to 3

Tomography of strong entangling msmt

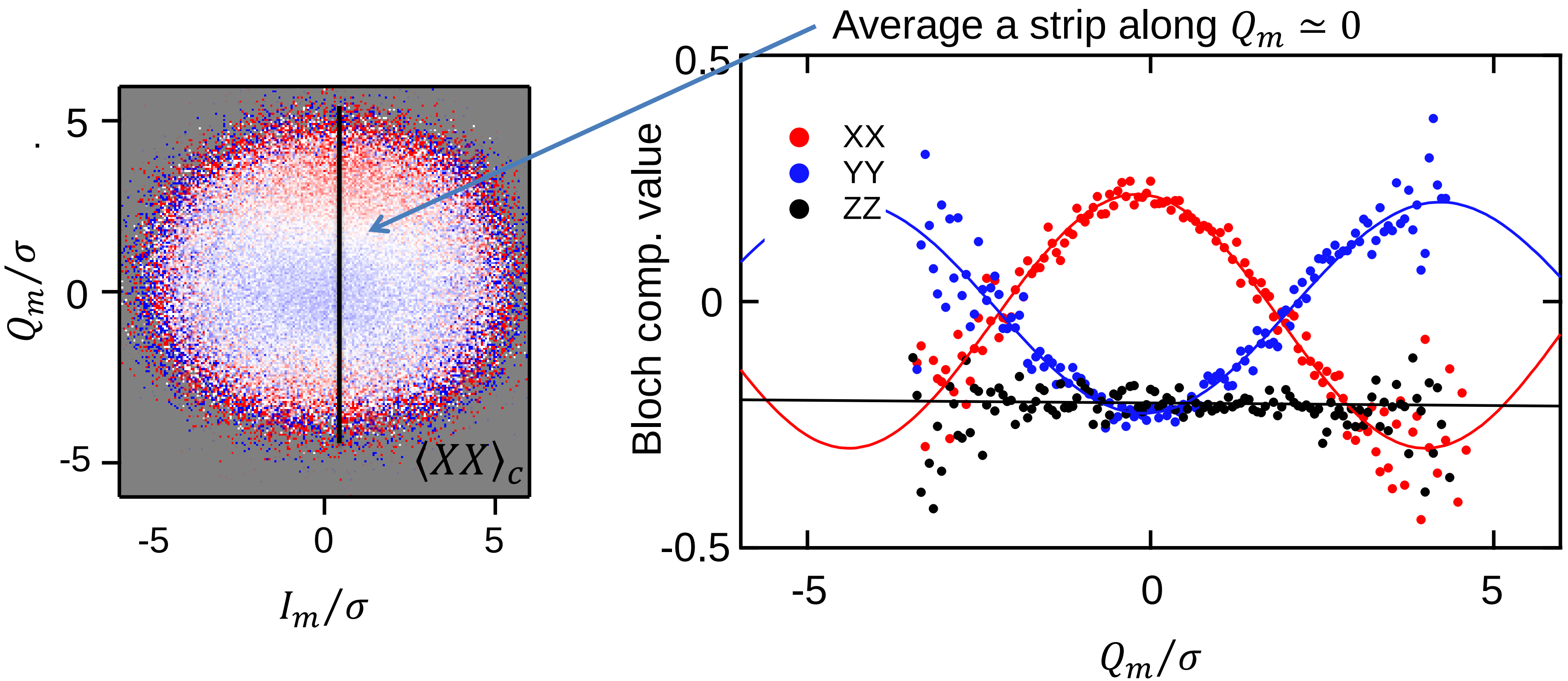


See Zalys-Gellar arxiv:1803.01275 (2018)

Tomography of weak entangling msmt

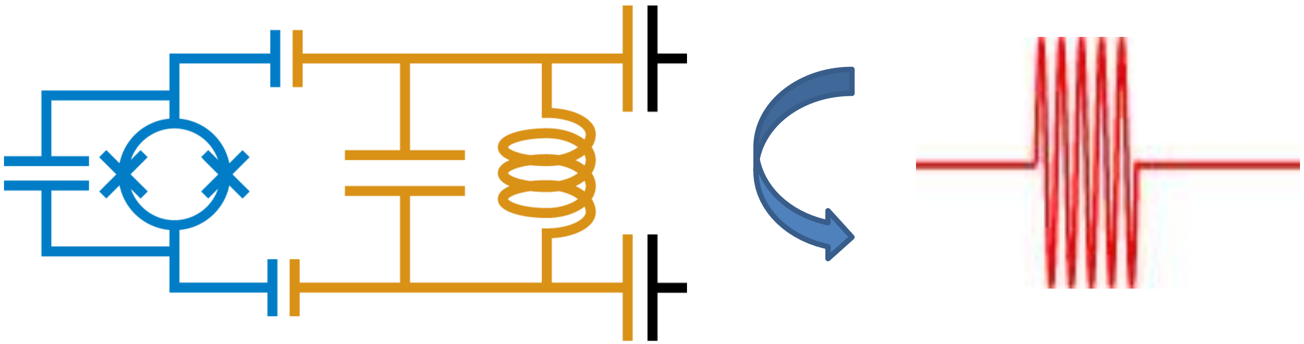


Signature of entangling operation



Part 3B: Remote Entanglement via sequential measurement (bounce-bounce)

Measurement based entanglement



Can one use the entanglement between the qubit and the microwave field to entangle two qubits ?

$$\Psi = |00\rangle + |11\rangle$$

Ristè et al., Nature **502 ,** 350-354 (2013)

Deterministic entanglement with parity measurement and feedback

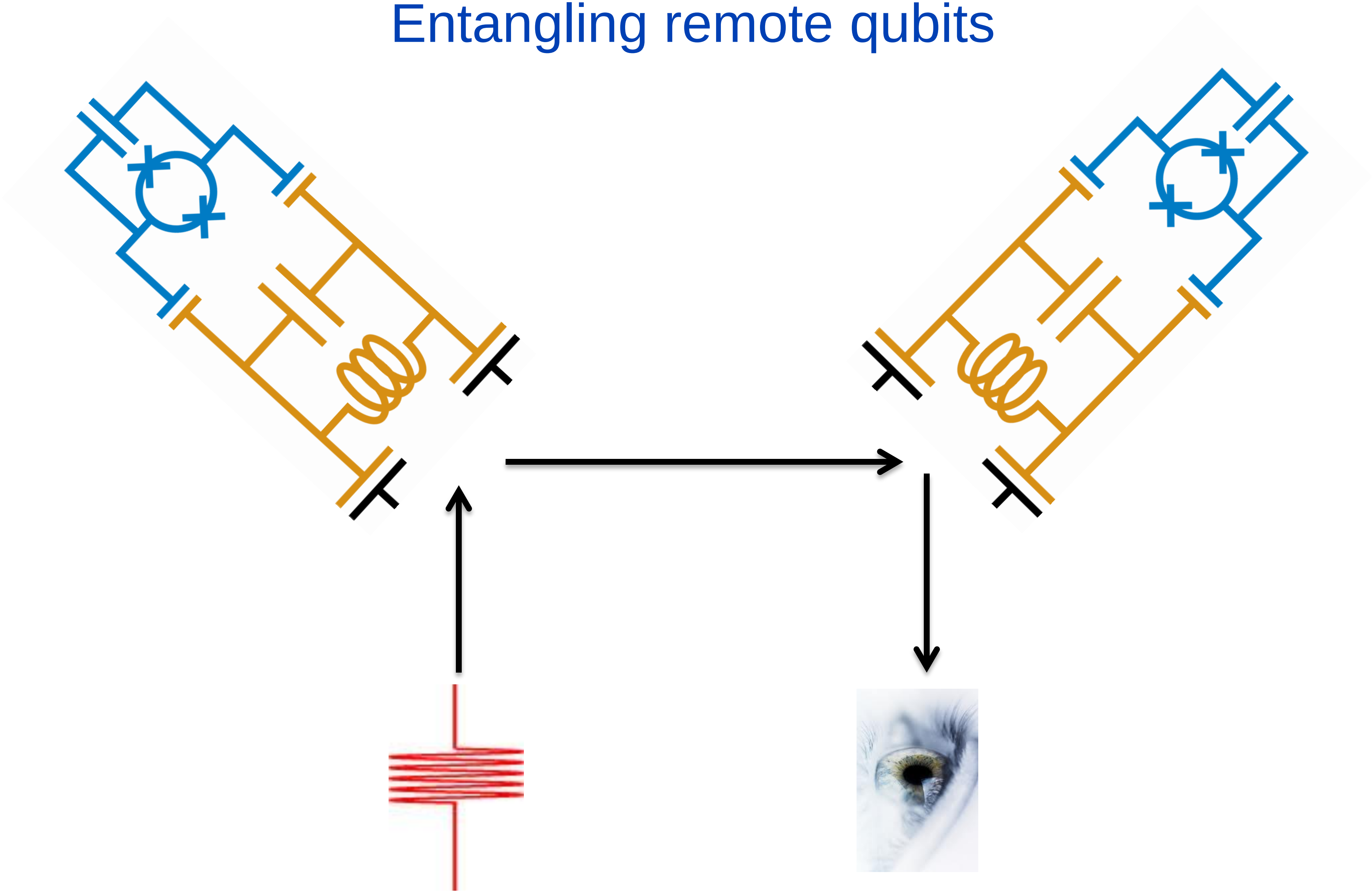
Shankar et al. , *Nature* **504**, 419–422 (2013)

Lehtas et al. , **Phys. Rev. A 88, 023849 (2013)**

Stabilizing entanglement by dissipation engineering

Roch et al. , Phys. Rev. Lett. 112, 170501 (2014) **, Entanglement in remote qubits**

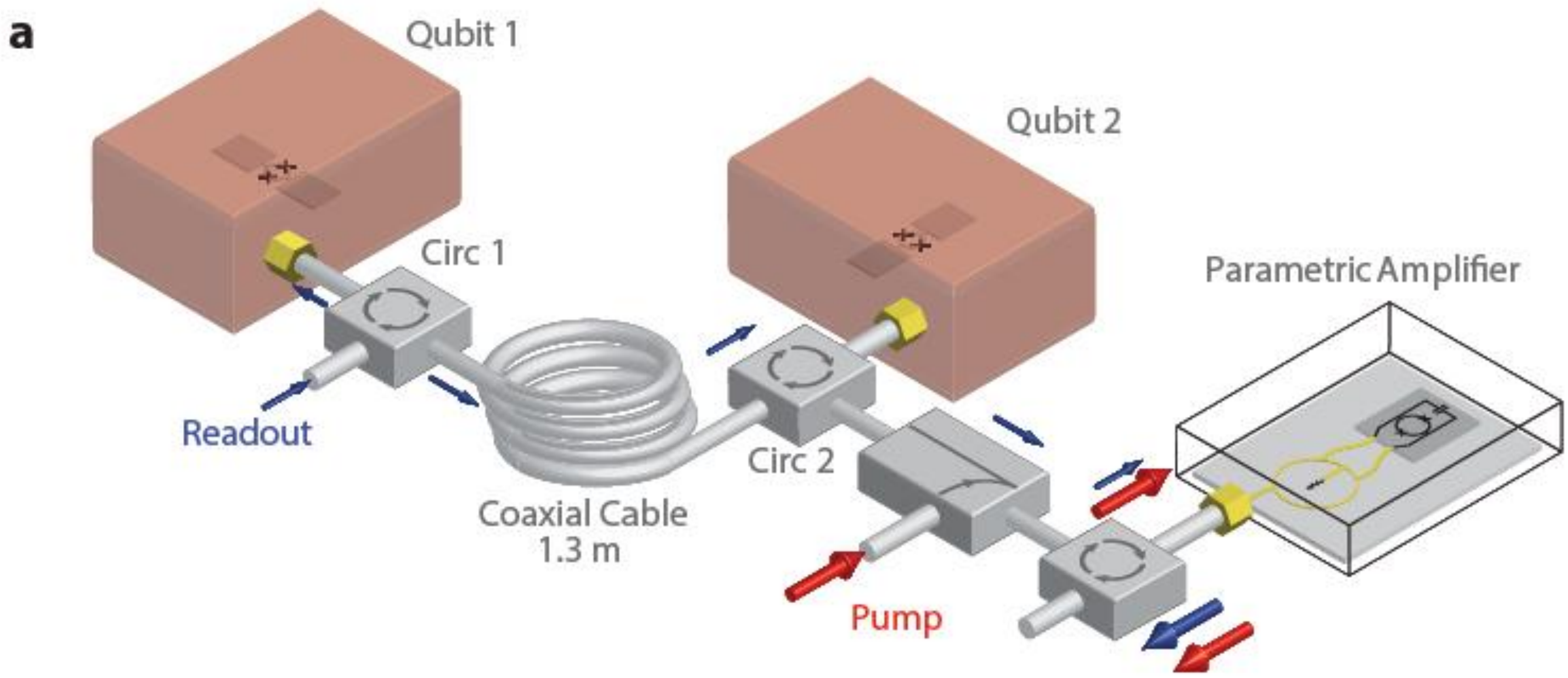
Entangling remote qubits



Kerckhoff, Bouten, Silberfarb & Mabuchi, **Phys Rev A** (2009)

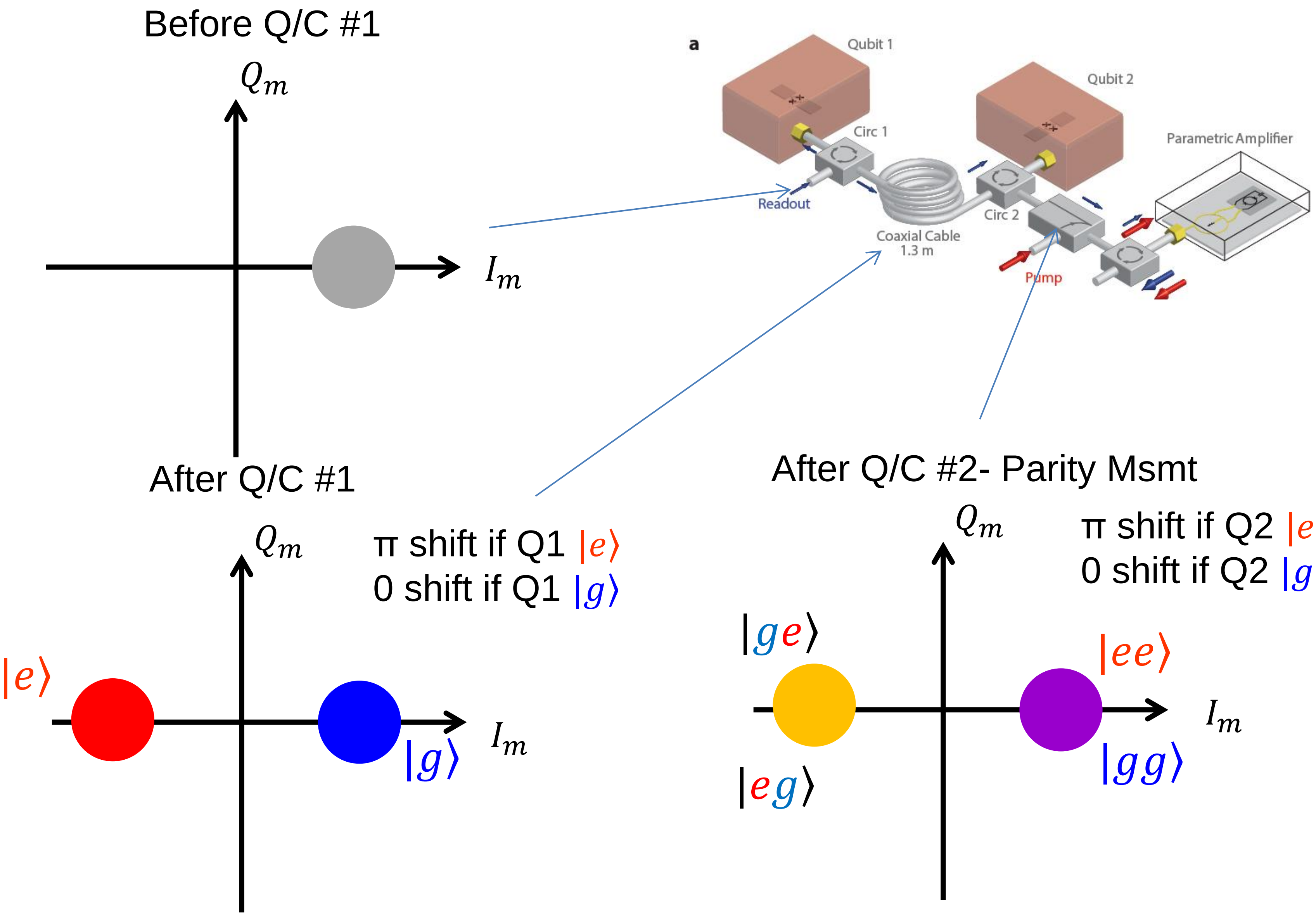
Roch et al. , Phys. Rev. Lett. 112, 170501 (2014)

cQED Setup

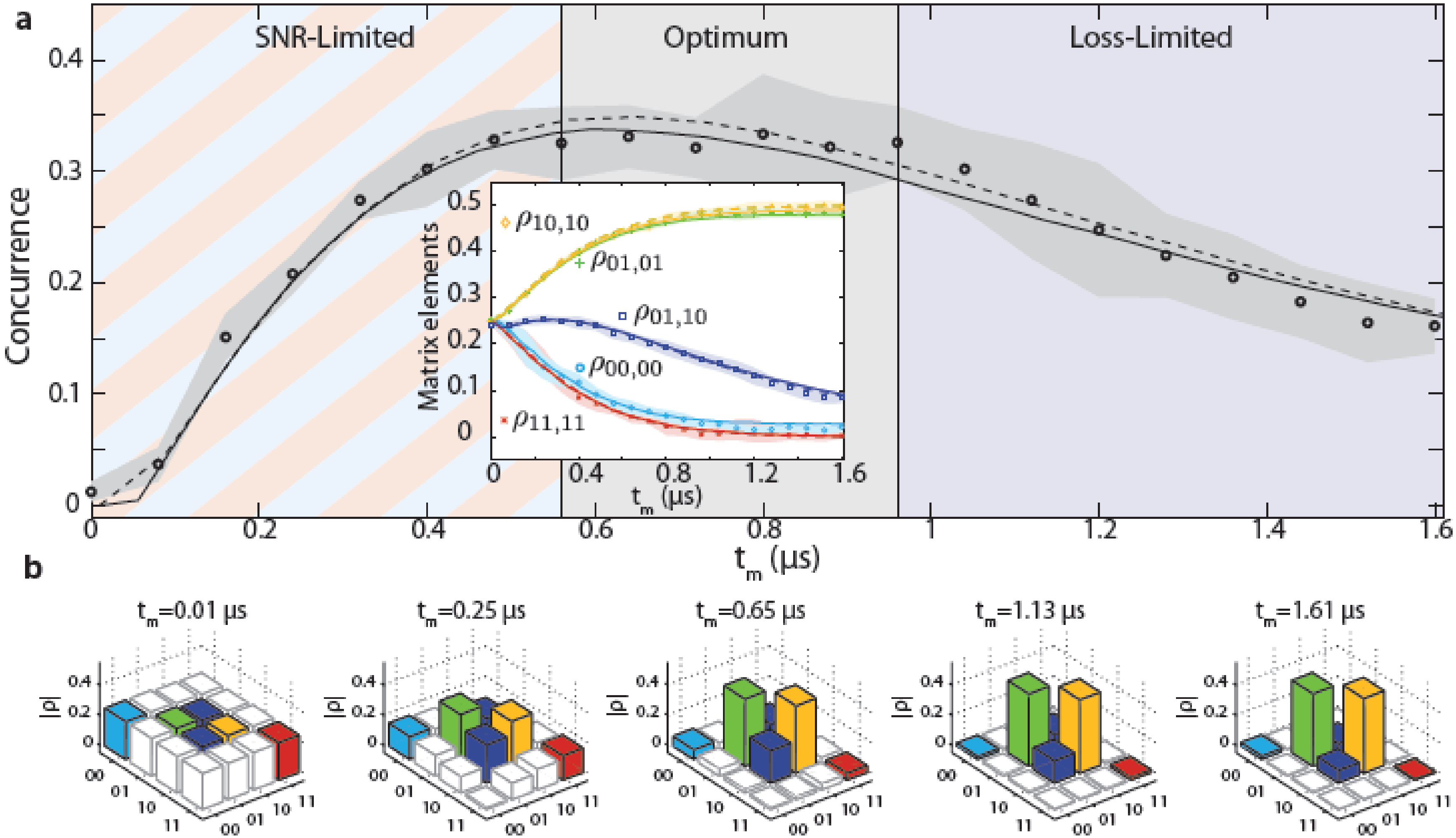


Roch et al. , Phys. Rev. Lett. 112, 170501 (2014)

Pointer state encoding



Remote Entanglement



Roch et al. , Phys. Rev. Lett. 112, 170501 (2014)

Outlook

- Parametric amplifiers are near quantum-limited and enable rapid, single-shot, QND measurements required for quantum information
- We can keep quantum light coherent in our systems long enough for basic manipulations– results: exotic measurement operators, remote qubit entanglement
- They have several shortcomings which we are learning to eliminate via Hamiltonian engineering and more sophisticated (multiple) parametric couplings