Two Terminal Charge Tunneling: A Sanity Check for Majorana zero modes

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ICTS, Bangalore, July, 2017

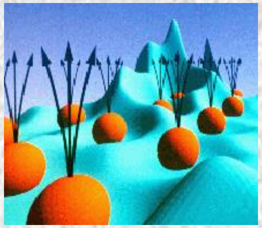
I: **Majorana zero modes: Introduction**

II: **Majorana bound states in non-homogenous systems (position-dependent potentials)**

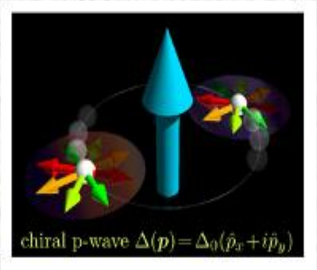
III: **Two terminal charge tunneling: A sanity check**

I
Majorana zero modes: Introduction

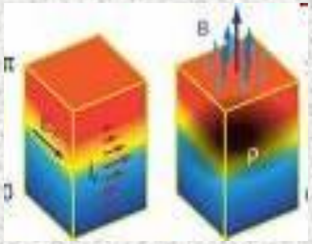
Majorana zero modes: increasing simplicity



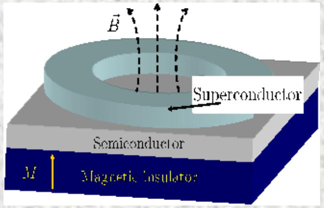
Fractional Quantum Hall
Moore and Read, PRB (1991)



Chiral p-wave SC/superfluids
Read and Green, PRB (2000)
Kitaev, Phy. Uspekhi (2001)



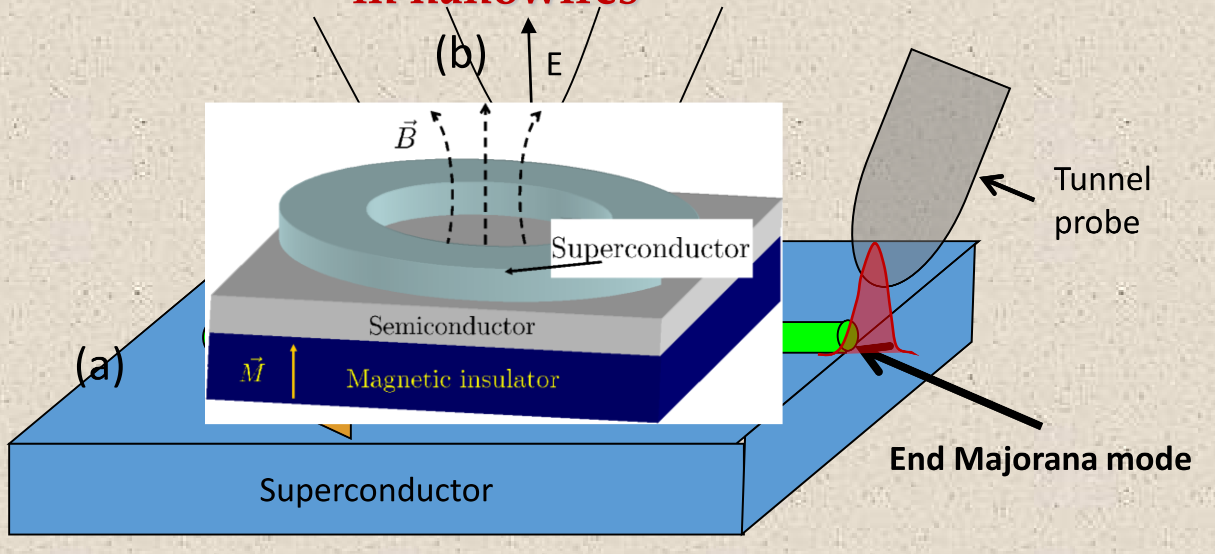
Topological insulator/
s-wave superconductor
Fu and Kane, PRL (2008)



Semiconductor/superconductor
Sau et al, PRL (2010)
Tewari et al Ann. of Physics (2010)
Oreg et al PRL (2010)
Lutchyn et al PRL (2010)

Majorana Fermions

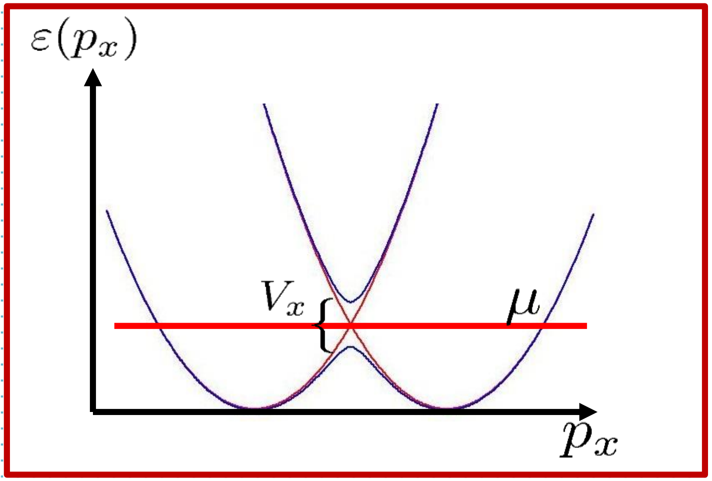
Tunneling detection of Majorana Fermions in nanowires



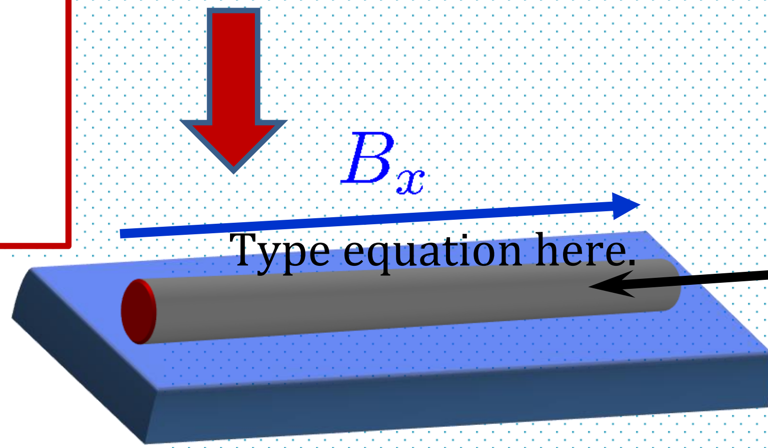
Majorana condition: $V_Z > \sqrt{\Delta^2 + \mu^2}$

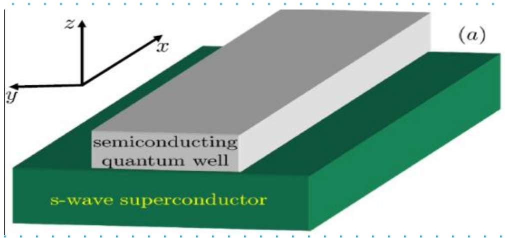
Sau, Lutchyn, Tewari, Das Sarma, PRL (2010); Tewari, Sau, Das Sarma, Annals of Physics (2010) Lutchyn, Sau, Das Sarma, PRL(2010), Oreg, Refael, von-Oppen, PRL(2010), Sau, Tewari, et al, PRB (2010)

Practical route to realizing Majorana zero modes



2D spinless p-wave SC: Moore, Read, PRB 2000
1D spinless p-wave SC, Kitaev, Phys Uspekhi, 01





$V_Z > \sqrt{\Delta^2 + \mu^2}$

Single-channel nanowire

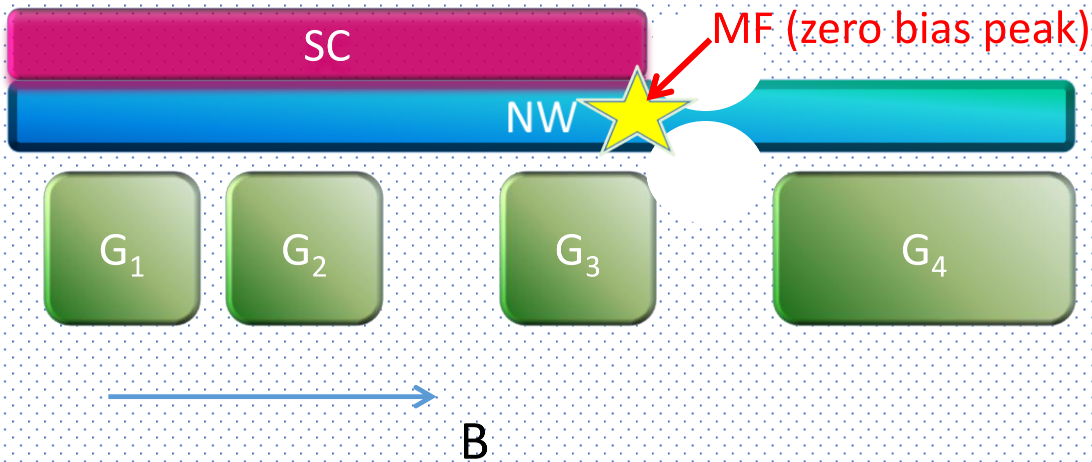
$$H_{\rm MW} = \int_{-L}^{L} dx \left[\psi^\dagger_\sigma \left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x \right)_{\sigma\sigma'} \psi_{\sigma'} + \Delta_0^*\psi_\uparrow\psi_\downarrow + \Delta_0 \psi^\dagger_\downarrow\psi^\dagger_\uparrow \right]$$

Spin-orbit coupling

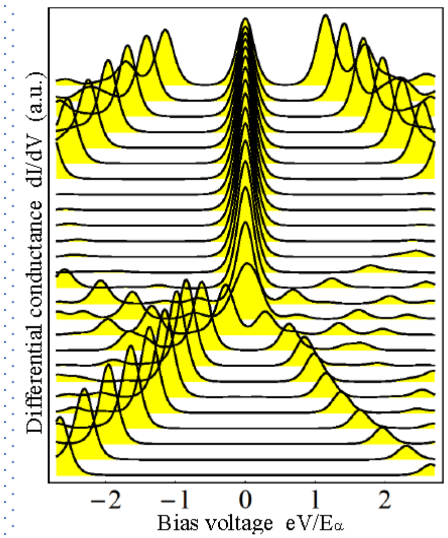
Zeeman splitting

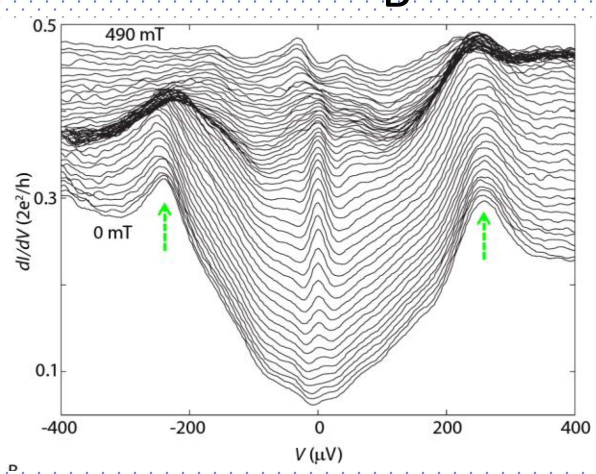
Proximity-induced superconductivity

Has the 'Quest for quirky quantum particles struck gold?' (Stanescu, Tewari, invited topical review, Journal of Physics CM (2013))



Theory



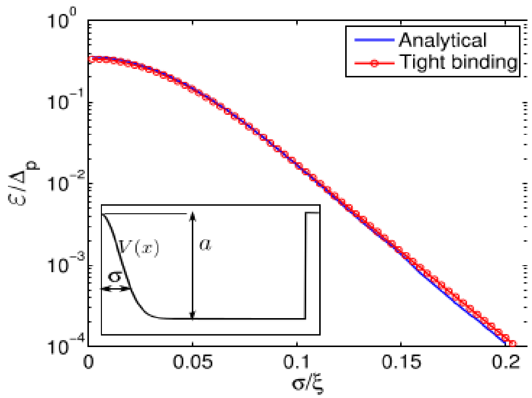


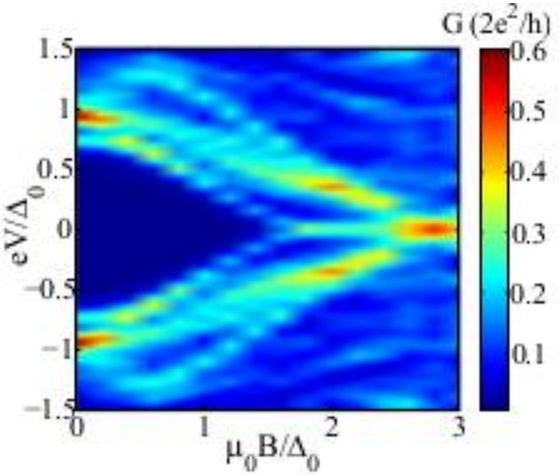
Experiment

- Fractional $(0.1)(2e^2/h)$ and sharp zero-bias peak, clearly separated from bulk gap
- Peak only above finite B
- Peak robust against increase of B (consistent with TS phase)

Mourik et al, Nature (2012), Das et al, Nature Physics (2012), Lund, Purdue, Harvard, Albrecht et al, Nature (2016), Deng et al Science (2016)

Zero modes from non-topological effects





- Soft boundary can lead to zero-modes

  (Kells, Meidan, Brouwer, Phys. Rev. B (2012)
  Roy, Bondyopadhyay, Tewari, PRB (2013))

- Anti-localization peaks

  (Pikulin, Dahlhaus, Wimmer, Schomerus, Beenakker, NJP (2012))

- Impurity induced zero modes

  (Liu, Potter, Law, P. A. Lee PRL (2012))

Disentangling the alternative scenarios from MF ZBP: Lin, Sau, Das Sarma PRB (2012);
Stanescu, Tewari PRB (2012)

Comparison of experiment with theory

Review: Stanescu, Tewari, J. Phys.: Condens. Matter 25, 233201 (2013)

Experiment

- Zero-bias conductance peak (ZBCP) above critical value $V_Z \sim \Delta$
- ZBCP goes away for critical angles of field
- No convincing features for gap-closure at TQPT

Theory

- Consistent with theory if $\mu \sim 0$

$$V_Z > \sqrt{\Delta^2 + \mu^2}$$

- Consistent with theory
- Consistent only if $\mu \sim 0$

Smoking Guns: Closing of bulk gap + ZBCP at same B ✖
Interference using Majorana non-locality ✖

Topological Degeneracy of MZMs

$$\gamma_i^\dagger = \gamma_i, \quad \gamma_i^2 = 1, \quad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

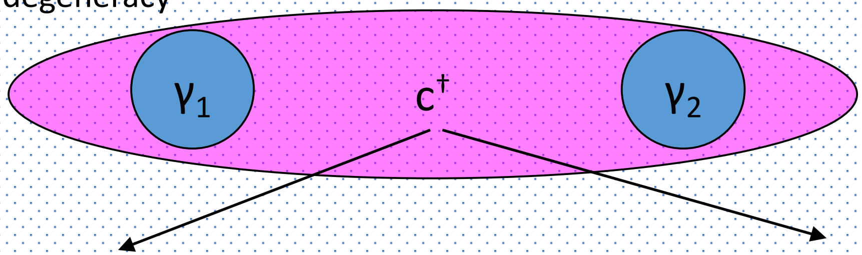
$$\gamma_1 = \hat{c}^\dagger + \hat{c}, \quad \gamma_2 = i(\hat{c}^\dagger - \hat{c}).$$

$$\hat{c} = (\gamma_1 + i\gamma_2)/2, \quad \hat{c}^\dagger = (\gamma_1 - i\gamma_2)/2,$$

Dirac fermion

Topological degeneracy

$c^\dagger = \gamma_1 + i\gamma_2, c = \gamma_1 - i\gamma_2$



Vacuum (empty)

$$c^\dagger c = \frac{1 + i\gamma_1\gamma_2}{2} = 0$$

Quasiparticle (filled)

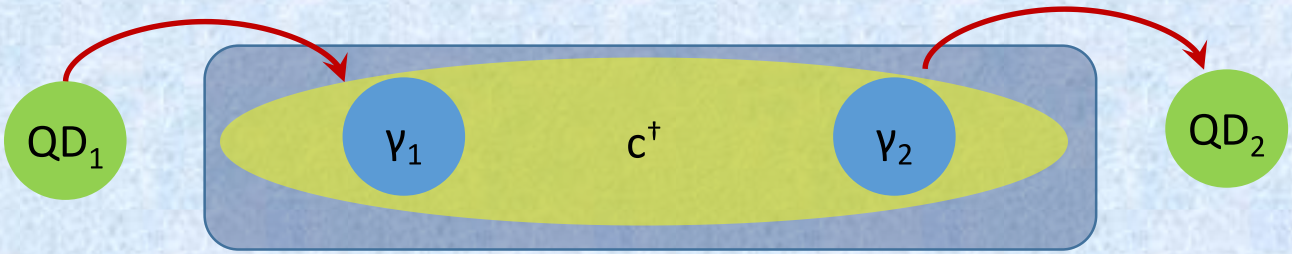
$$c^\dagger c = \frac{1 + i\gamma_1\gamma_2}{2} = 1$$

- Additionally there are electrons in the Cooper pair condensate

- Degenerate states of Majorana zero modes are defined by parity (even or odd) quasiparticles

Kitaev, Annals of Physics (2003)

"Teleportation" signature of MZMs

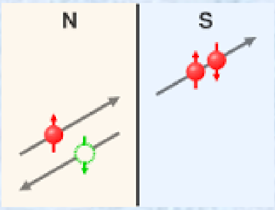


Bolech, Demler, PRL (2007); Tewari, Zhang, Nayak, Das Sarma, Lee, PRL (2008); L. Fu PRL (2010)

Three Processes for current

Local Andreev Reflection
Majorana Assisted Electron Transfer (Teleportation)
Crossed Andreev Reflection

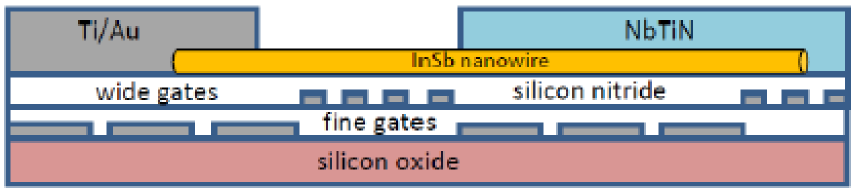
AR

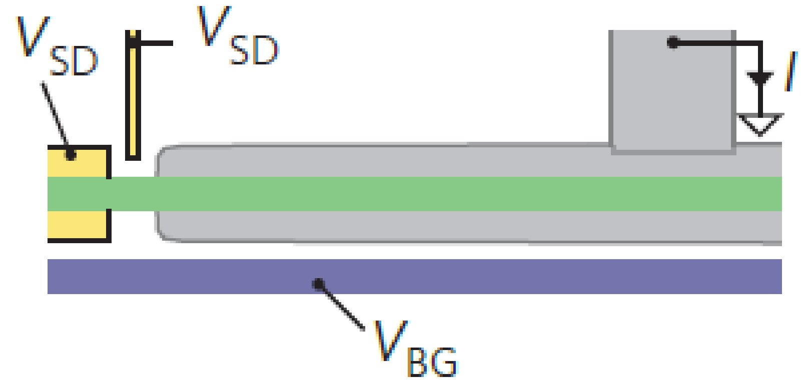


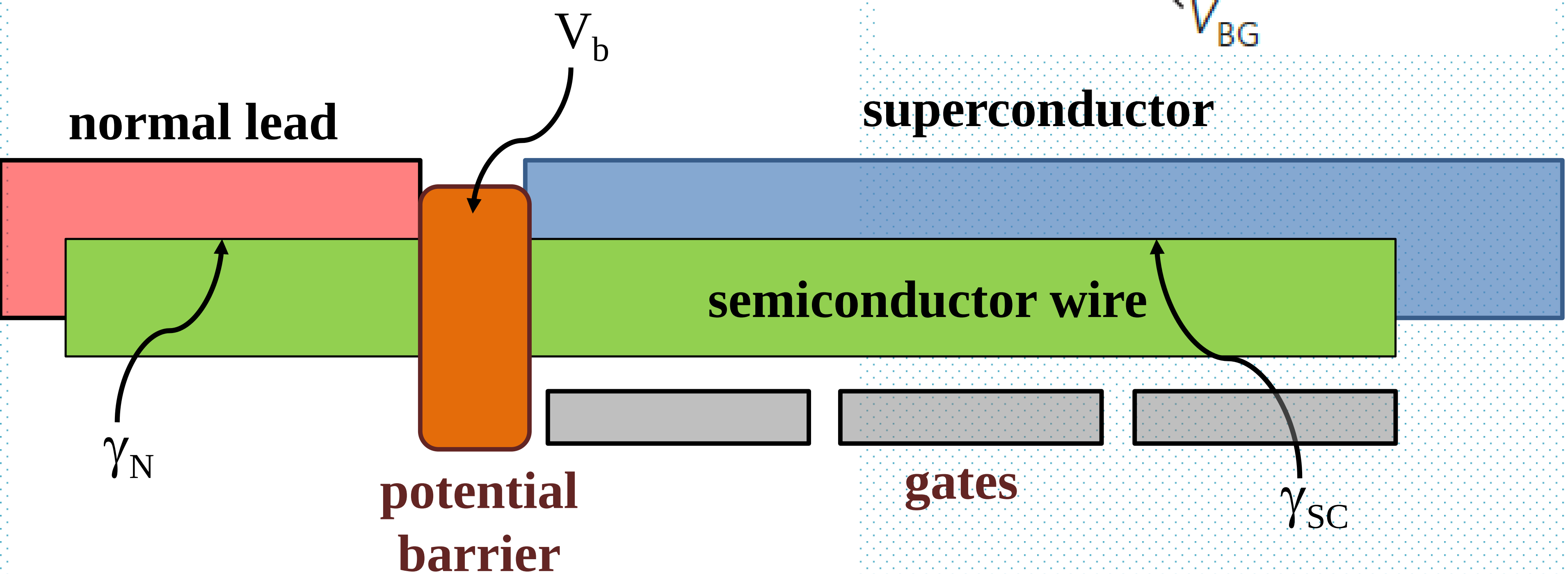
Need charging energy to suppress Andreev processes that increase electron number by two

L. Fu, PRL 2010,
Sau, Swingle, Tewari, PRB (R) (2015)

Schematic hybrid structure



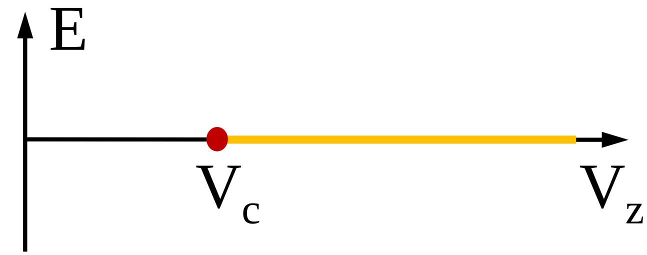




II
Inhomogeneous background potentials

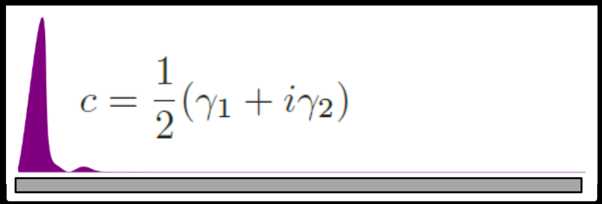
Zero-energy modes (ideal world)

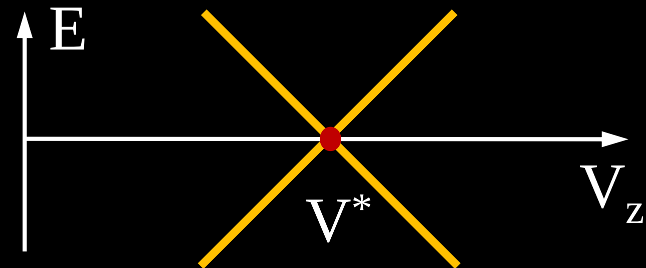
Majorana zero modes



Topological SC phase

Andreev bound states

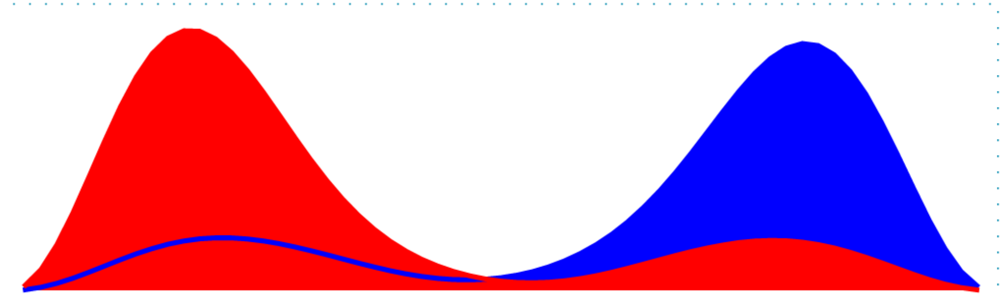




Trivial SC phase

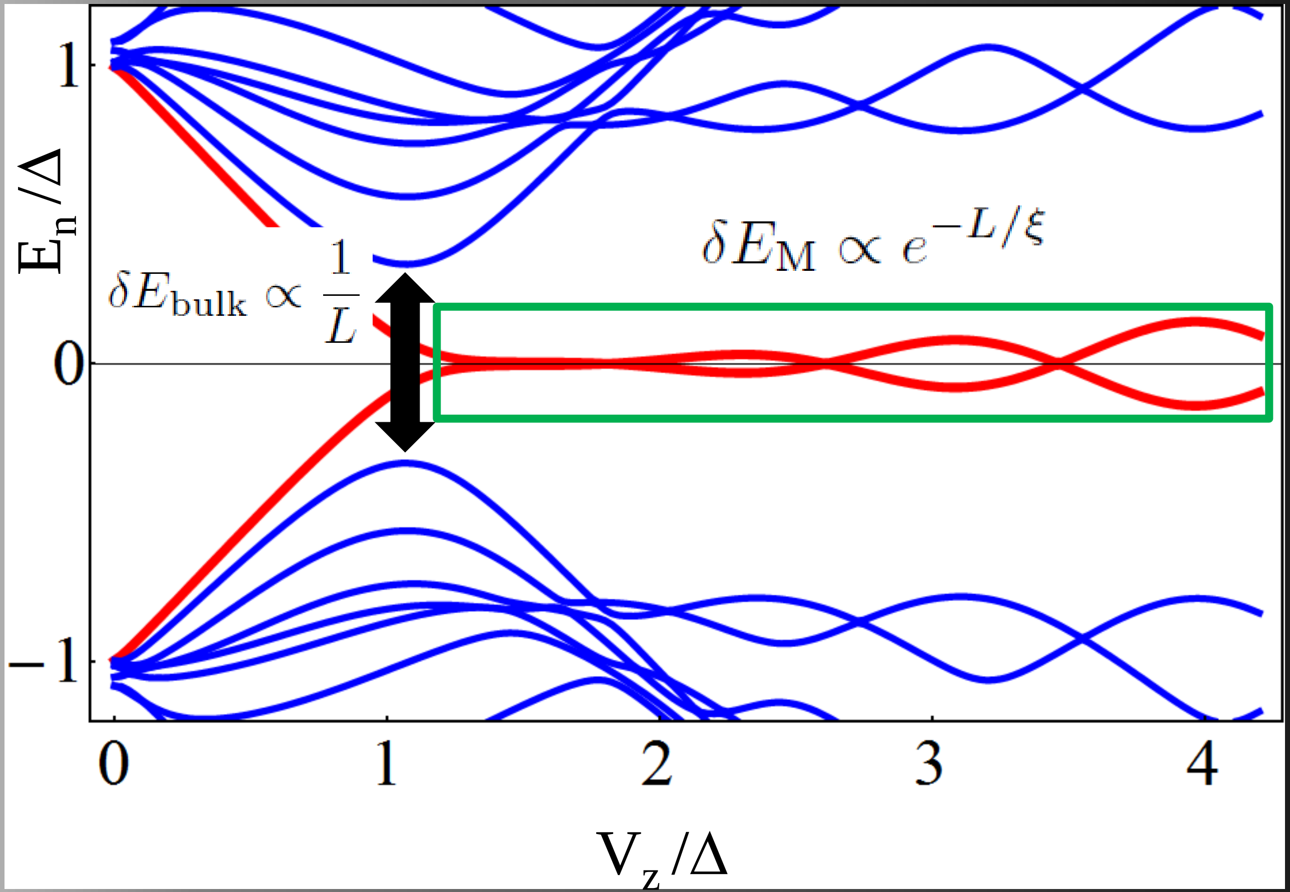
Zero-energy modes (real world)

Partially overlapping Majorana bound states



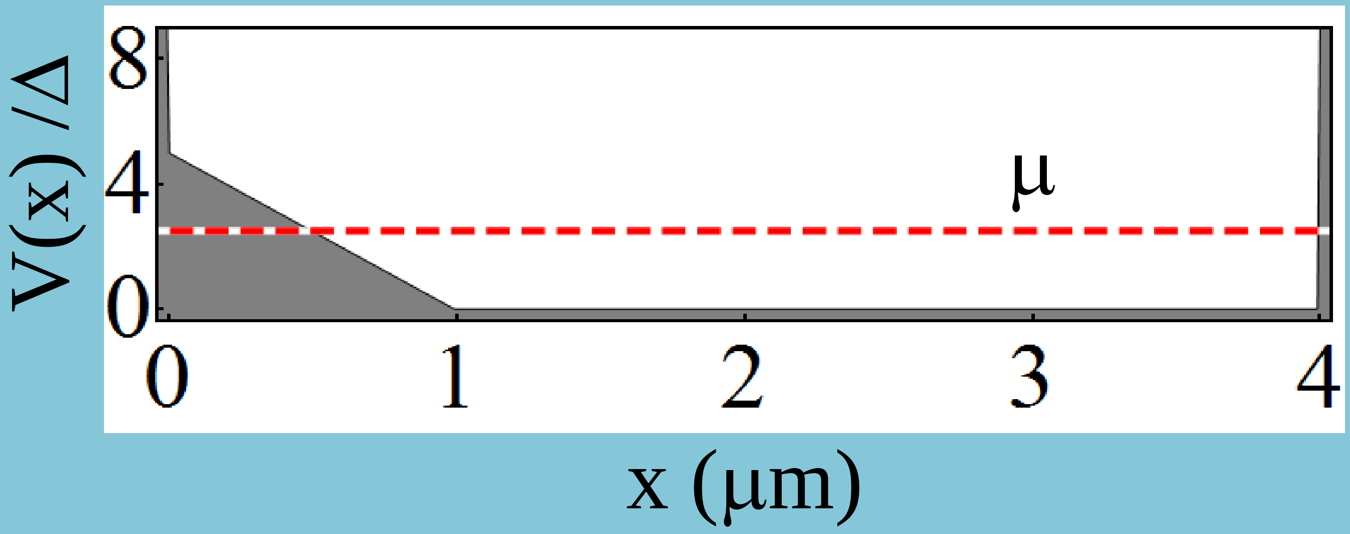
Zero-energy modes (real world)

Partially overlapping Majorana bound states (finite wire)

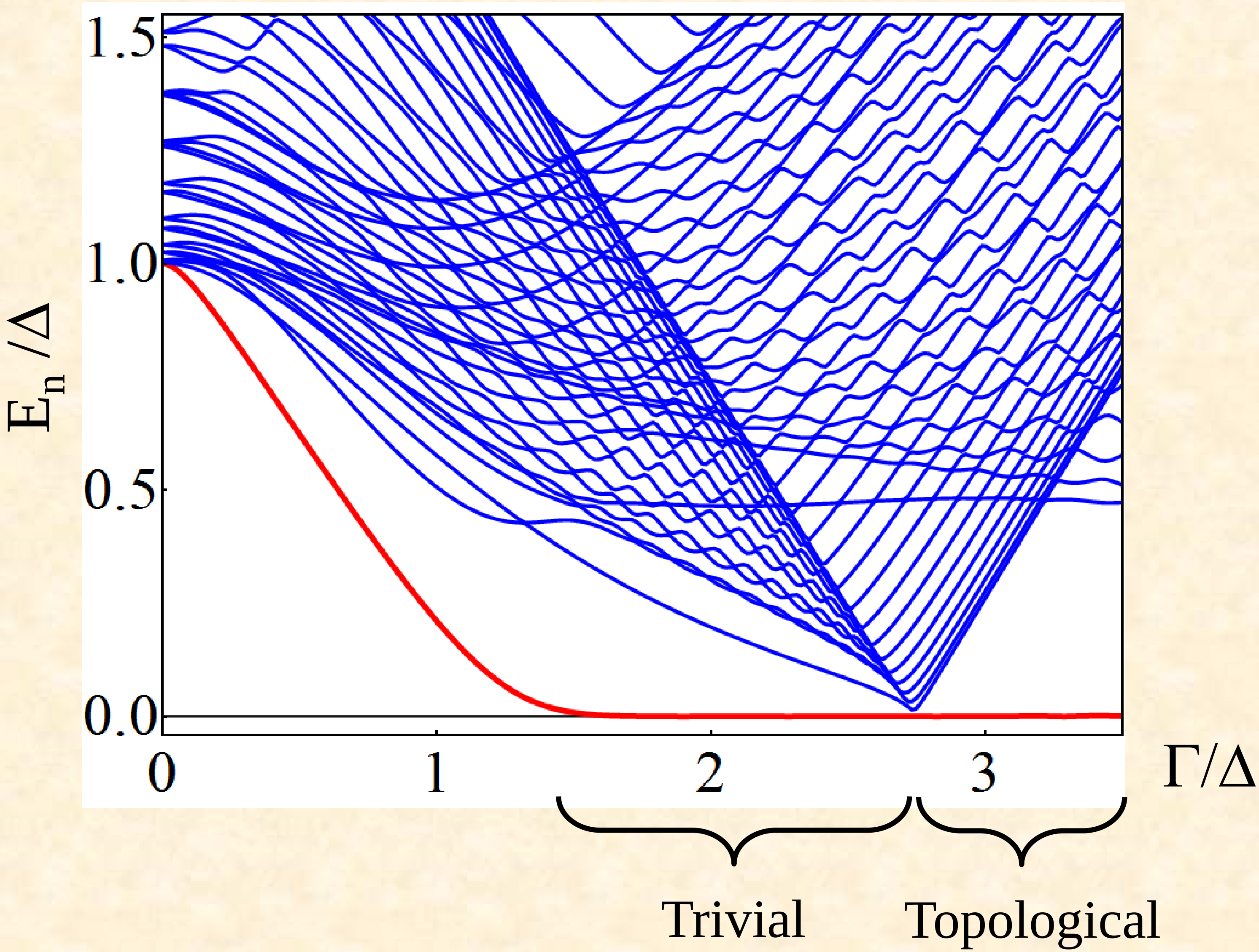


Majorana bound states in non-homogeneous systems

Position-dependent potential

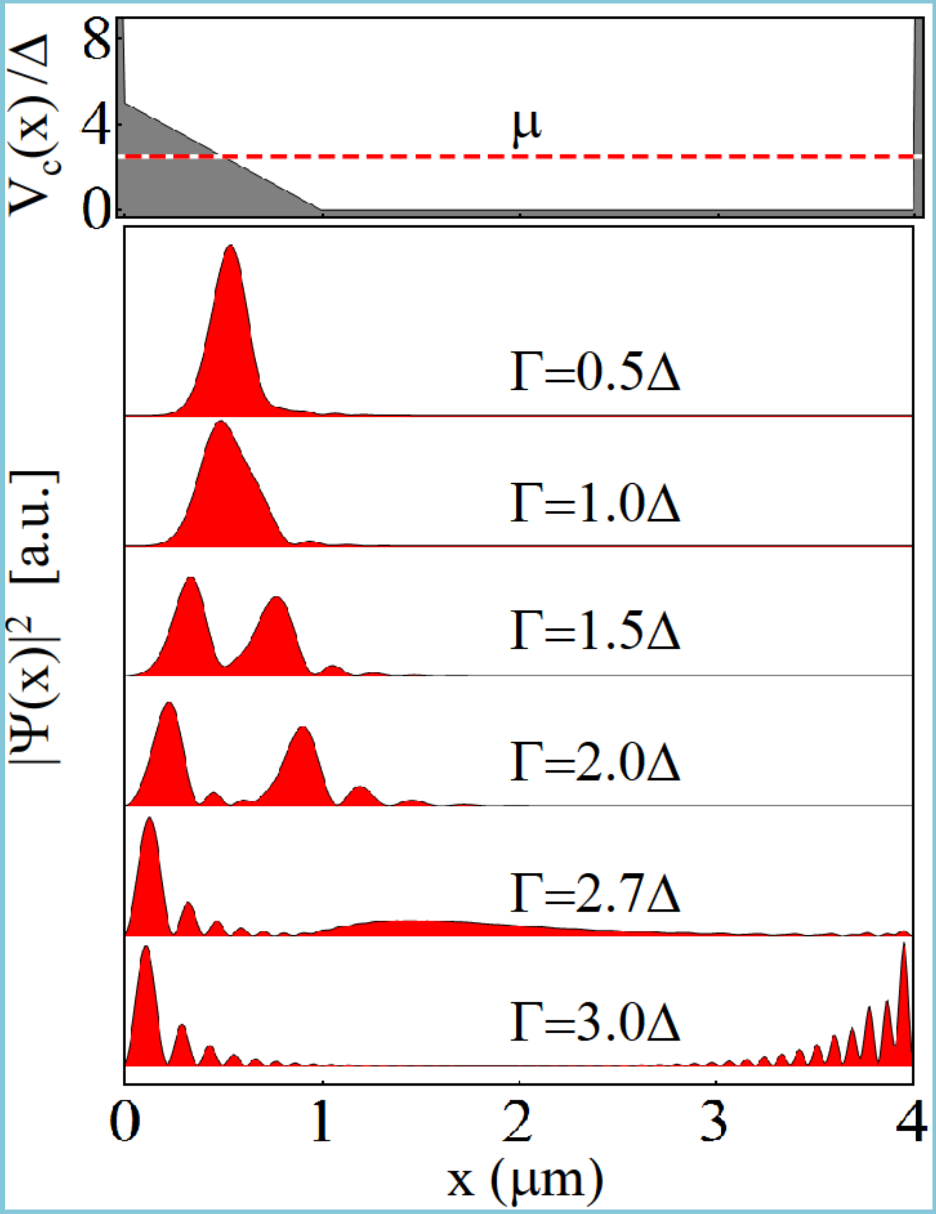


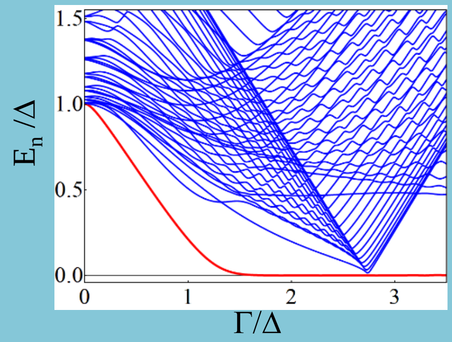
Low-energy spectrum



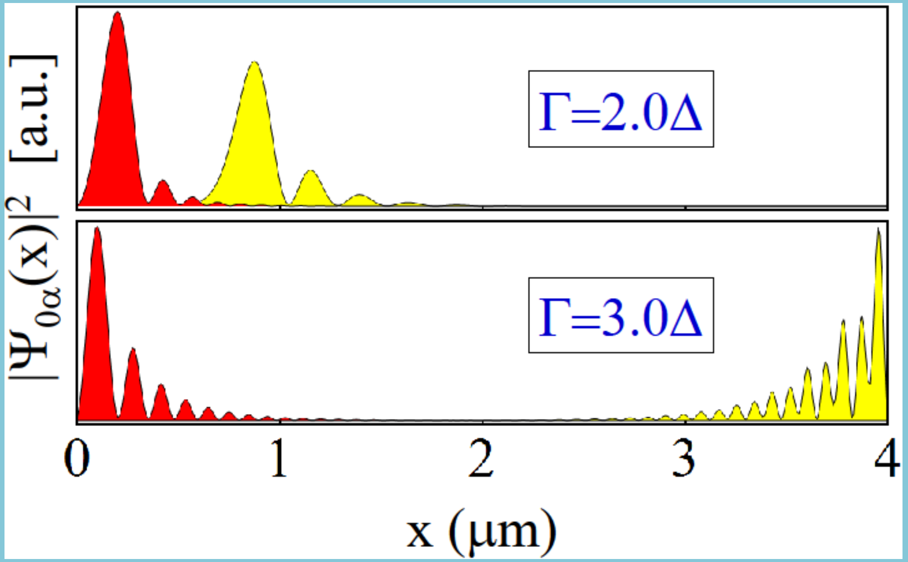
S. Tewari, T. D. Stanescu, Journal of IISC Bangalore, Vol 96, No: 2 (2016)

Wave function (lowest energy mode)

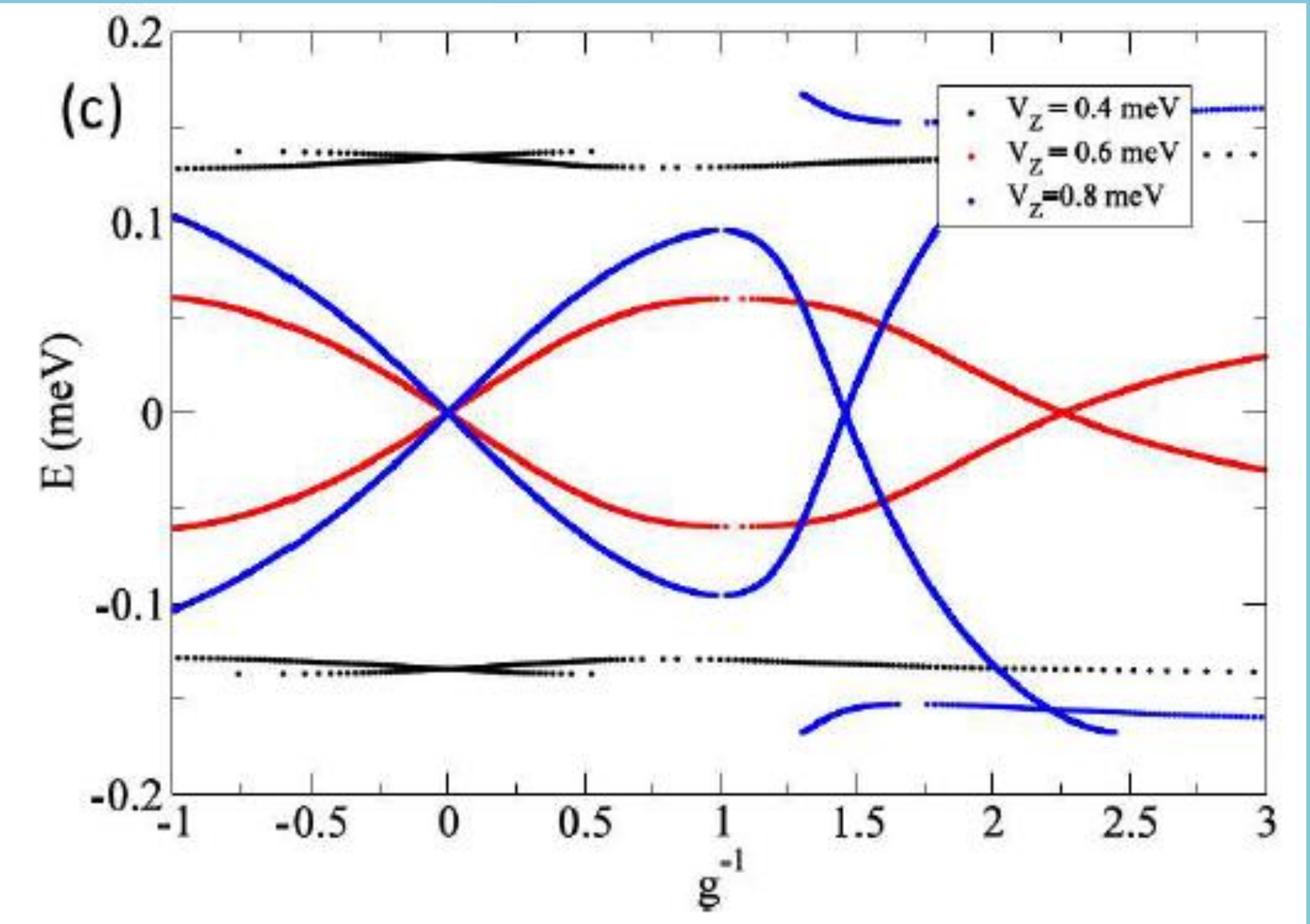




Majorana modes



Bound states induced by a local potential



Earlier result

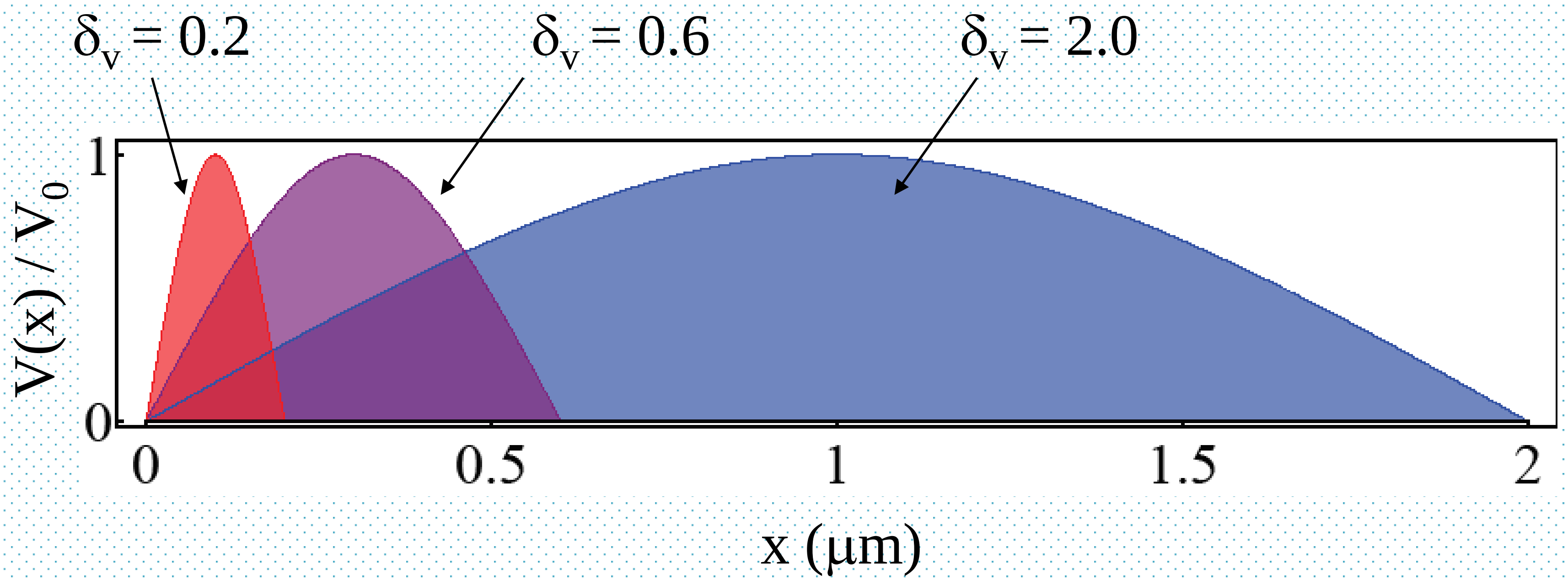
Local potential

$$V(x) = g\delta(x)$$

ABS confined to local potential, At fine tuned field, and only in Topological side.

Sau & Demler, PRB 88, 205402 (2013)

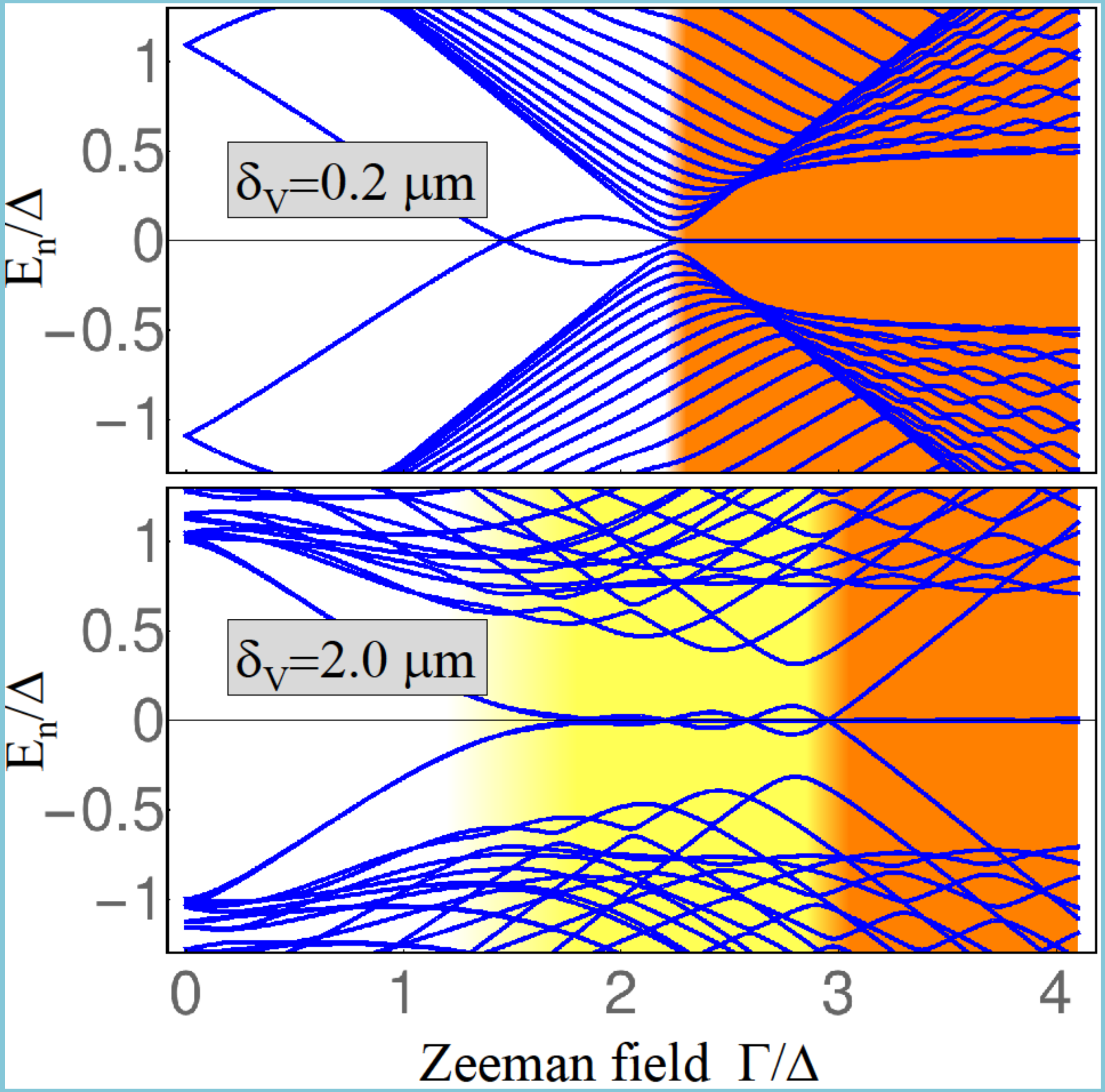
Finite-range potential



C. Moore, T. Stanescu, S. Tewari, arXiv:1611.07058

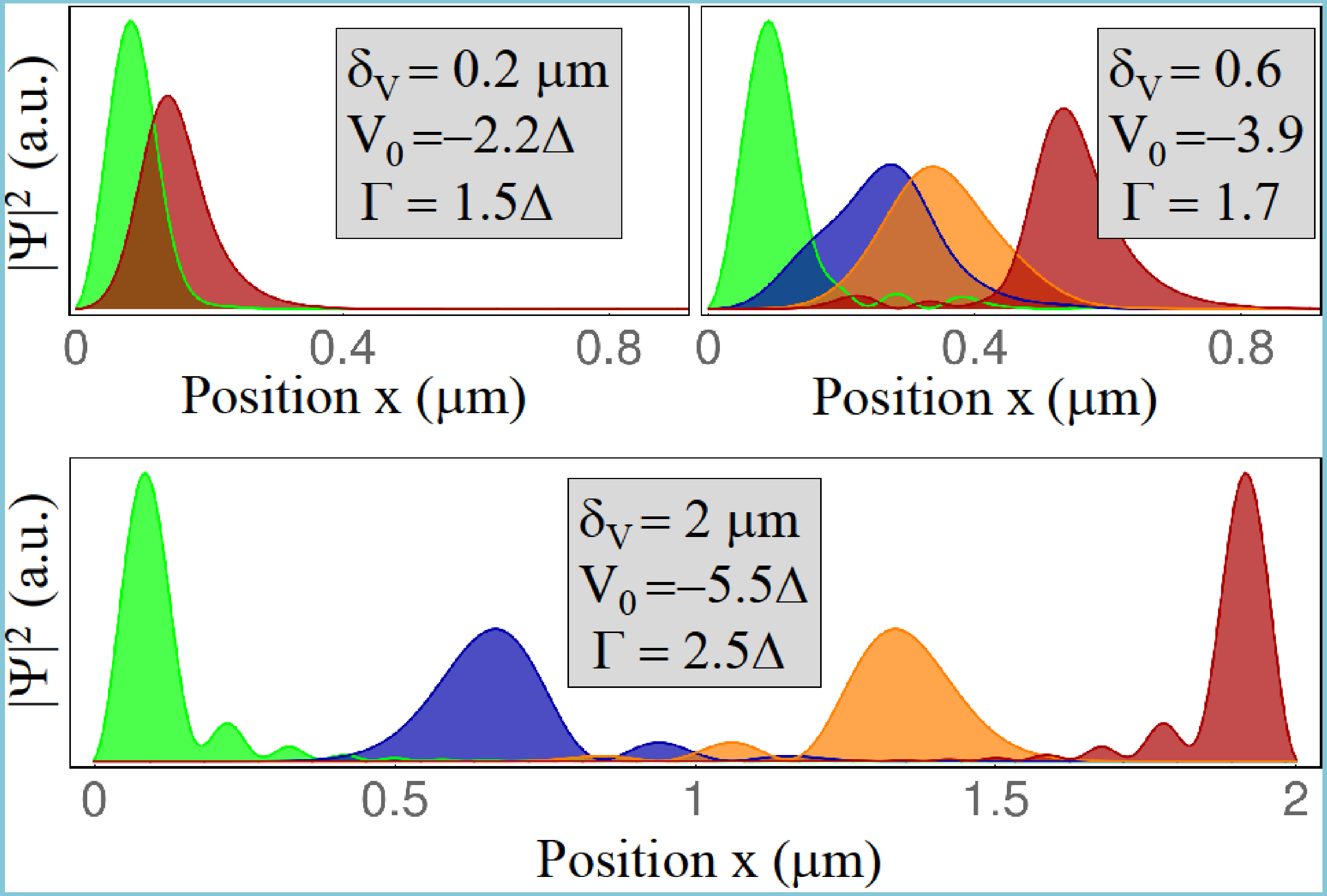
Zeeman field dependence

$\mu = -2\Delta$



C. Moore, T. Stanescu, S. Tewari, arXiv:1611.07058 (2016)

(Partially) overlapping Majorana bound states

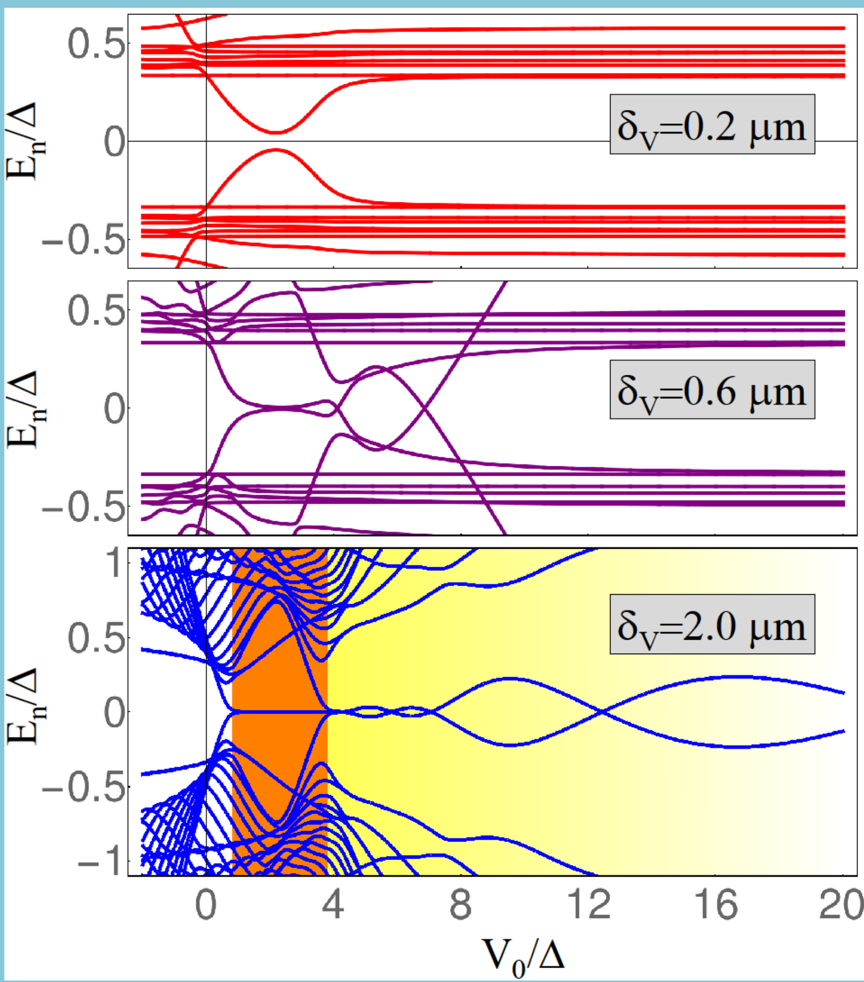


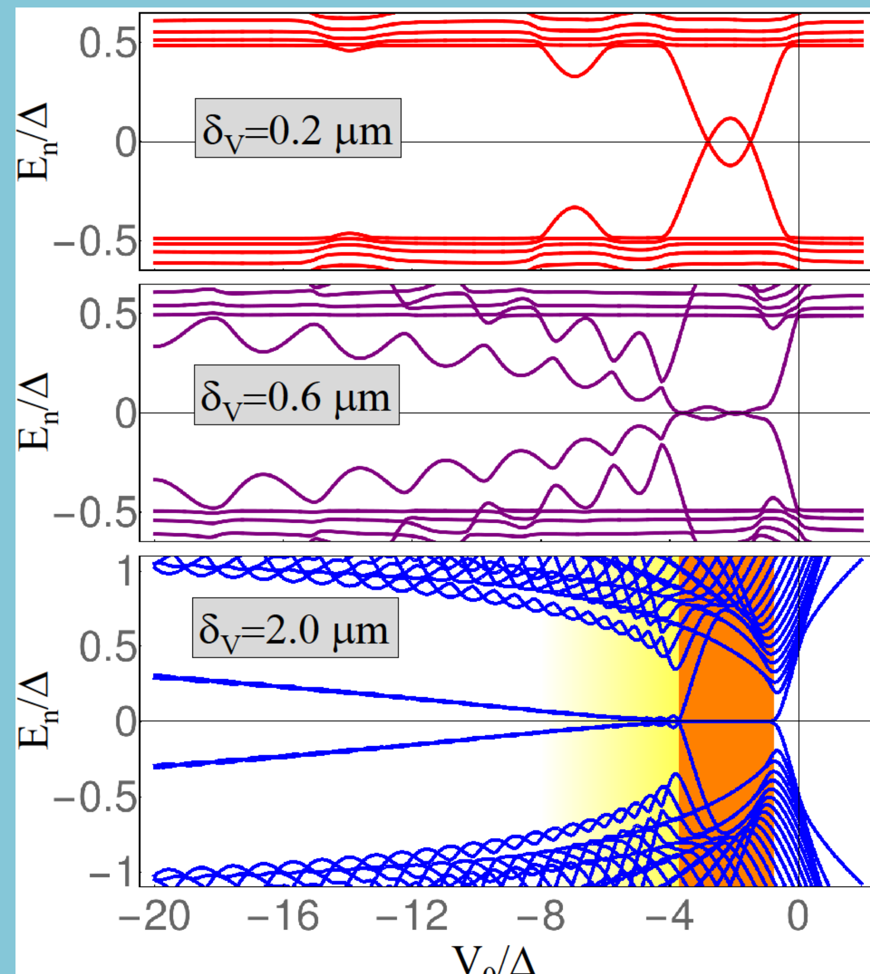
Low-energy modes (dependence on V_0)

$\mu = 2\Delta$
$\Gamma = 1.5\Delta$

$\Gamma < \sqrt{\Delta^2 + \mu^2}$

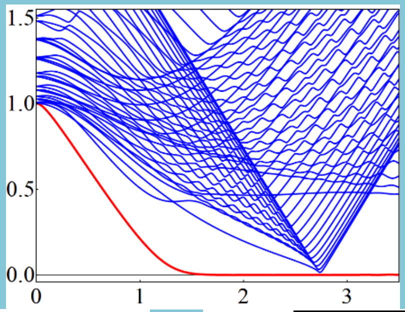
$\mu = -2\Delta$
$\Gamma = 1.5\Delta$

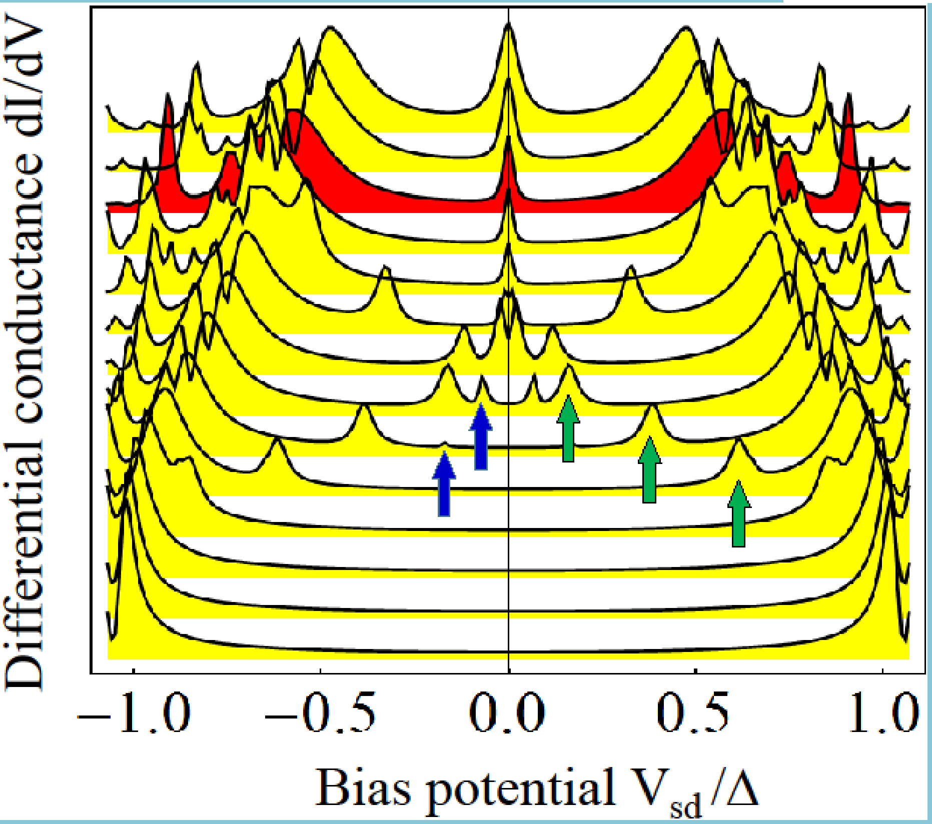


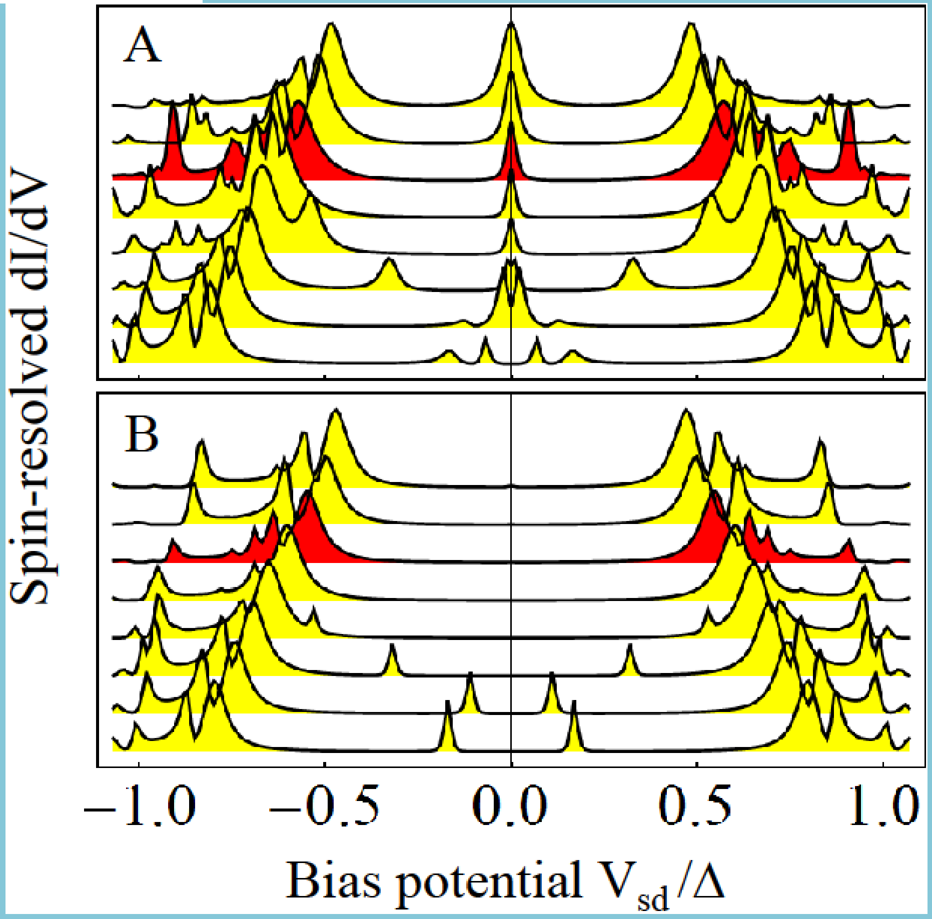


C. Moore, T. Stanescu, ST, arXiv: 1611.07058

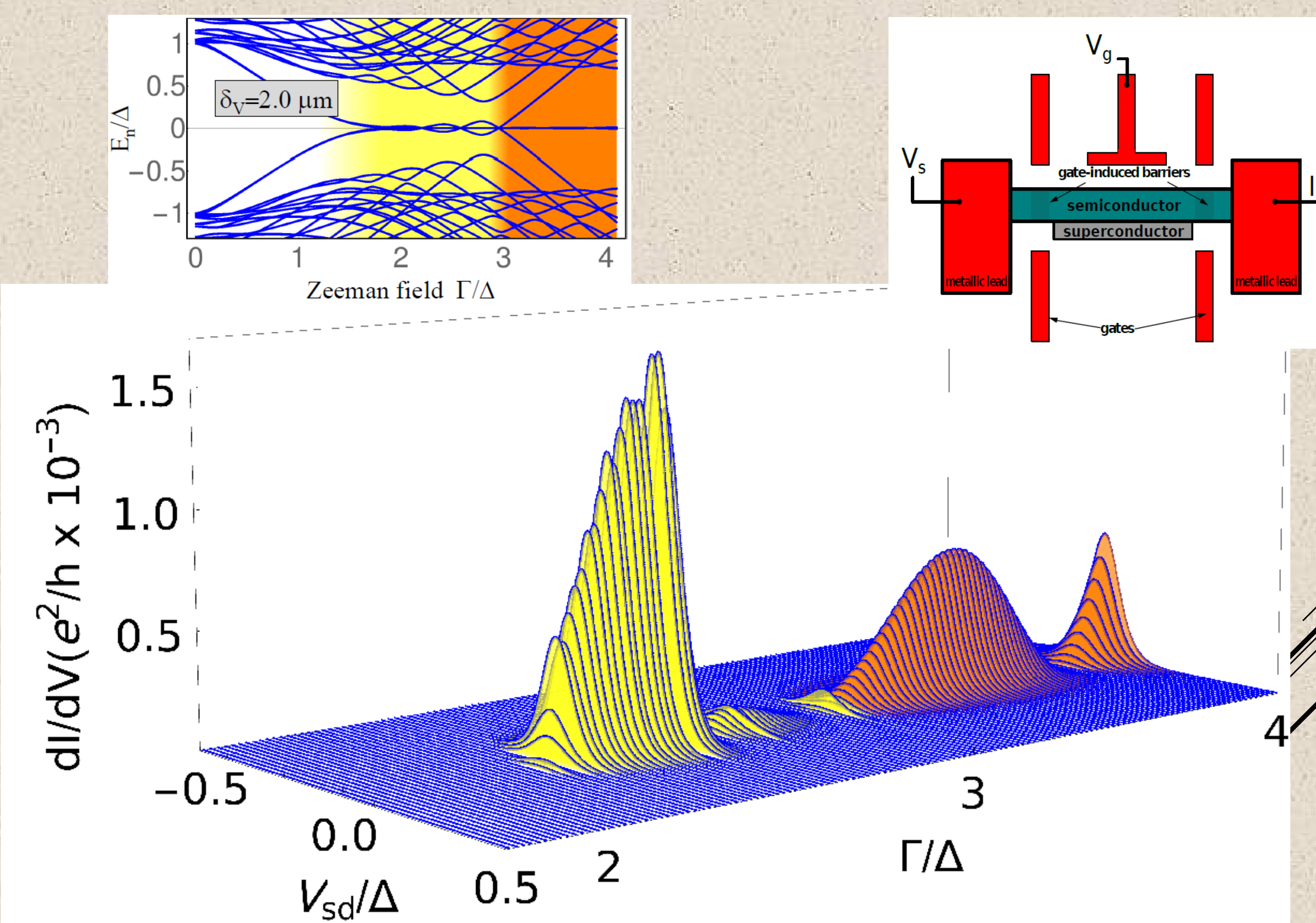
dI/dV and spin-resolved dI/dV







Two-terminal differential conductance with charging energy: "Teleportation"



C. Moore, T. Stanescu, S. Tewari, arXiv:1611.07058 (2016)

III
Two terminal charge tunneling: Sanity check for MZM

Message So Far

1. **With inhomogeneous potentials, low (nearly-zero) energy robust ABSs near the ends are quite generic (Liu, Sau, Stanescu, Das Sarma, arXiv: 1705.02035)**

2. **However, they occur as overlapping pairs of MBSs, confined to segments near both ends of the wire**

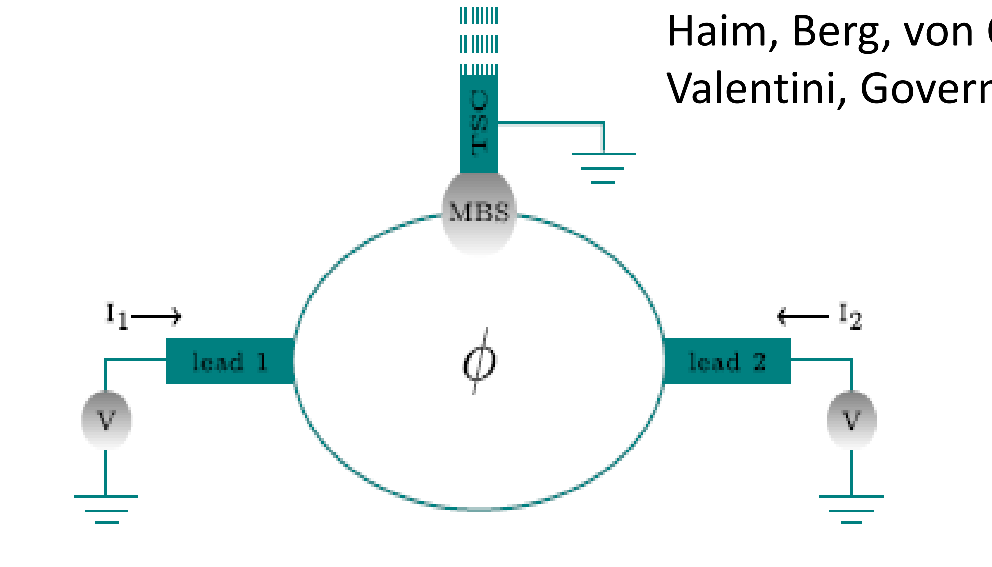
3. **Such partially unfolded ABSs are useless for non-Abelian statistics and TQC**

4. **Need "true" well-separated non-degenerate MZMs localized at the two ends of wire**

5. **No local measurement, e.g., one-terminal charge or spin tunneling, can distinguish between this type of robust ABSs and non-Abelian MZMs**

How about a more non-trivial set up? AB interferometer

Tripathi, Das, Rao, PRL (2016)
Haim, Berg, von Oppen, Oreg, PRB'15
Valentini, Governale et al, Physica E'16



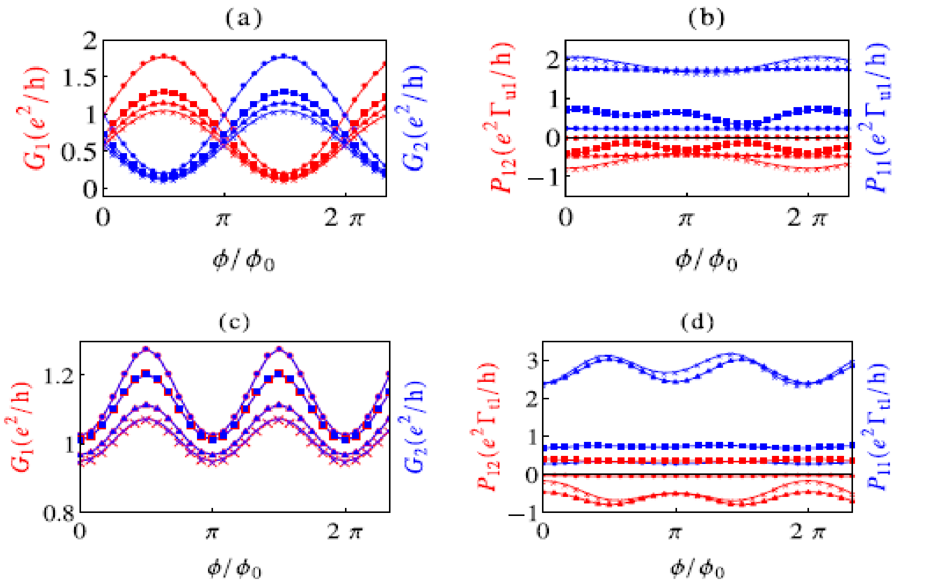
AB ring, two normal leads at voltage V, directly coupled, as well as via a MBS (ABS)

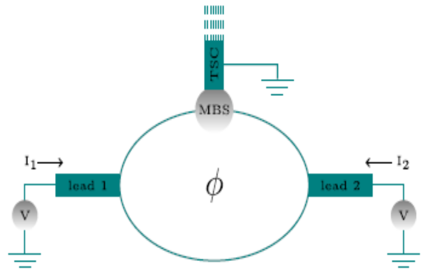
With MBS, with change in AB flux, the currents in the leads are <u>anti-correlated</u>, while currents are correlated with ABS

How about a more non-trivial set up? AB interferometer

MBS

ABS





PRL 116, 166401 (2016) PHYSICAL REVIEW LETTERS week ending 22 APRIL 201

Fingerprints of Majorana Bound States in Aharonov-Bohm Geometry

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(Received 5 October 2015; published 18 April 2016)

We study a ring geometry, coupled to two normal metallic leads, which has a Majorana bound state (MBS) embedded in one of its arms and is threaded by Aharonov-Bohm (AB) flux ϕ. We show that by varying the AB flux, the two leads go through resonance in an anticorrelated fashion while the resonance conductance is quantized to $2e^2/h$. We further show that such anticorrelation is completely absent when the MBS is replaced by an Andreev bound state (ABS). Hence this anti-correlation in conductance when studied as a function of ϕ provides a unique signature of the MBS which cannot be faked by an ABS. We contrast the phase sensitivity of the MBS and ABS in terms of tunneling conductances. We argue that the relative phase between the tunneling amplitude of the electrons and holes from either lead to the level (MBS or ABS), which is constrained to $0, \pi$ for the MBS and unconstrained for the ABS, is responsible for this interesting contrast in the AB effect between the MBS and ABS.

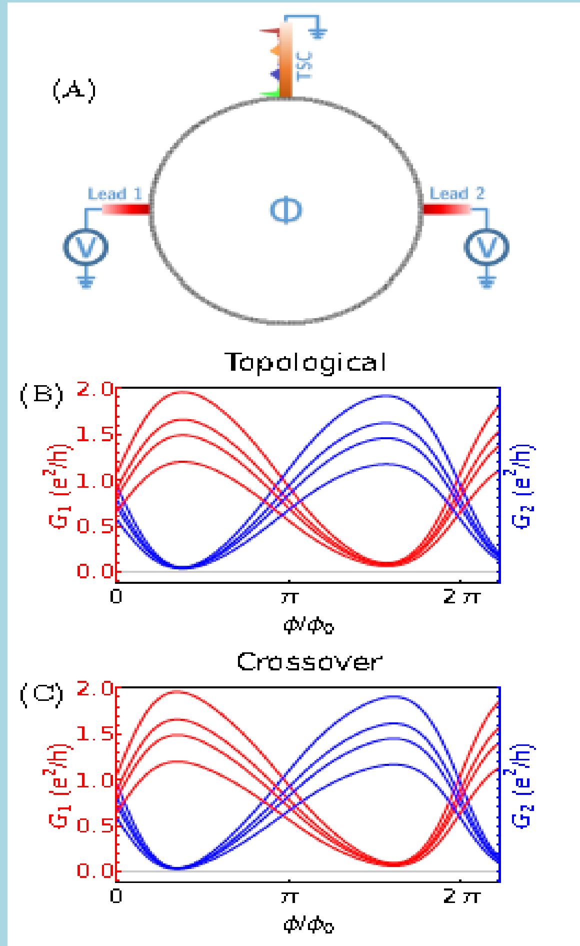
Hence this anti-correlation in conductance, studied as a function of flux, provides a unique signature of MBS which cannot be faked by an ABS

How about a more non-trivial set up? AB interferometer

C. Moore, T. Stanescu, ST
to be published

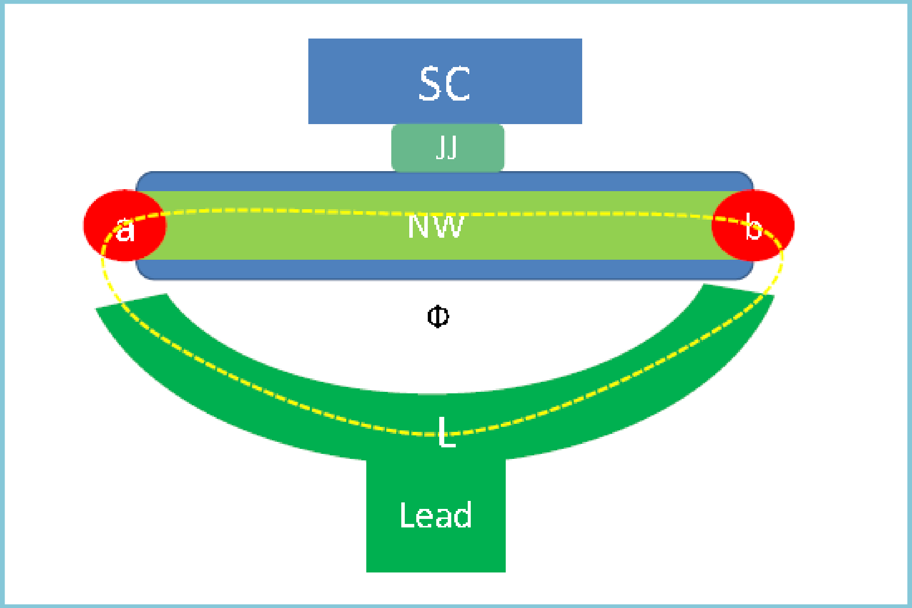
MBS

ABS



AB Interferometer cannot distinguish between topological and trivial (crossover) regimes

Experiments with Quasiparticle interference, fusion, braiding?



Very Hard Experiments

Majorana assisted electron transfer in the nanowire

Similar to Aharonov-Bohm oscillations in a mesoscopic ring, interference can be detected as a $\Phi_0 = \frac{hc}{2e}$ periodicity in the conductance

Sau, Swingle, Tewari, PRB (R) (2015)

Sanity check on MZMs

1. **Interference, braiding, fusion etc are extremely difficult**

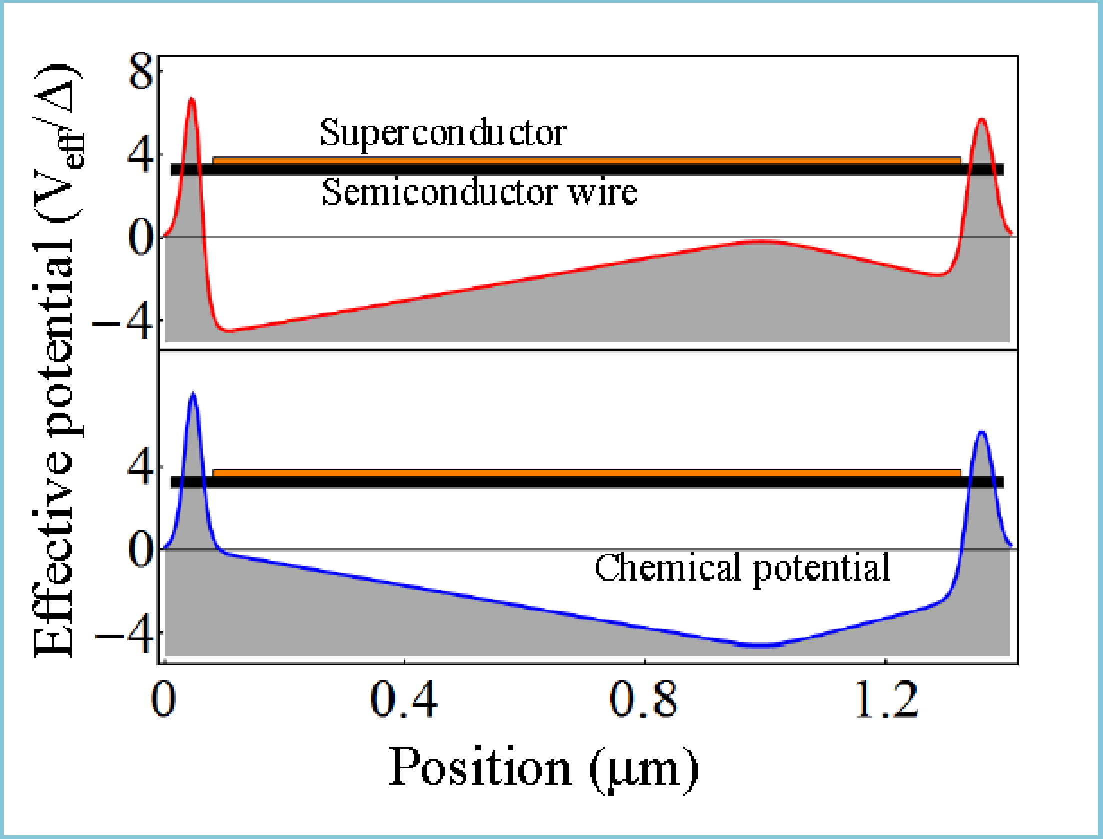
2. **Interpreting such experiments without knowing if the system is homogeneous enough to allow for a single pair of MZMs localized to opposite ends would be a nightmare!**

3. **Crucial to perform simpler tests able to distinguish between topological MZMs and robust partially unfolded ABSs**

Two-terminal charge tunneling measurements

Well within the current experimental capabilities!

Schematics of non-homogeneous potential

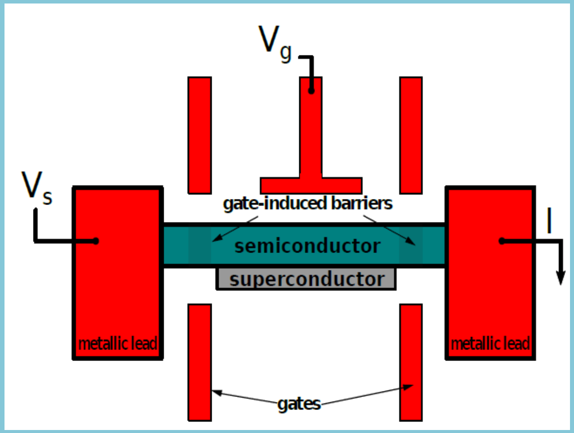


$\Delta \sim 0.25$ meV

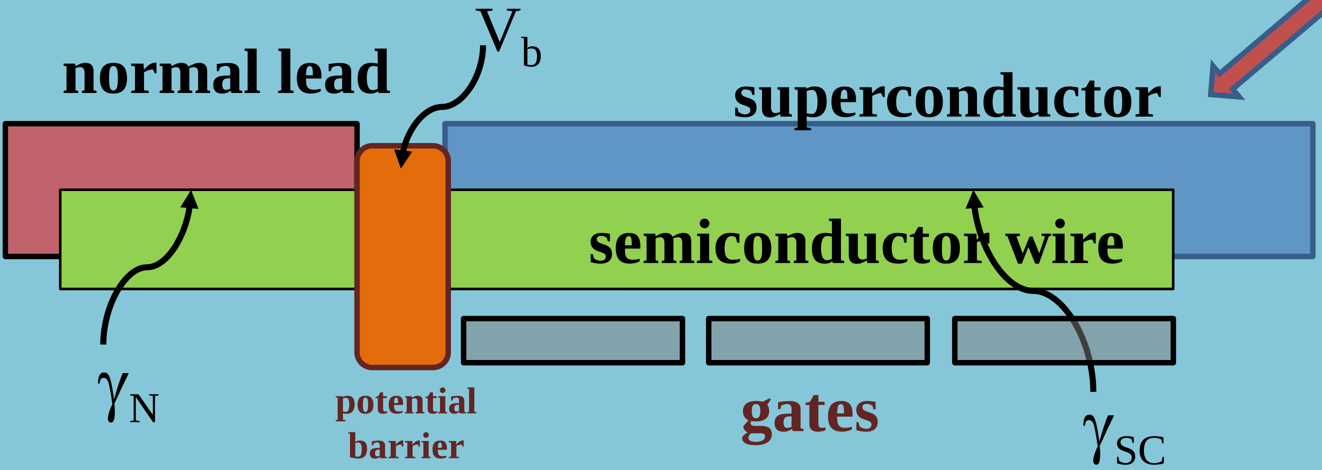
T. Stanescu, C. Moore, S. Tewari, to be published

Schematics of experimental set up

“Teleportation”



Single terminal tunneling



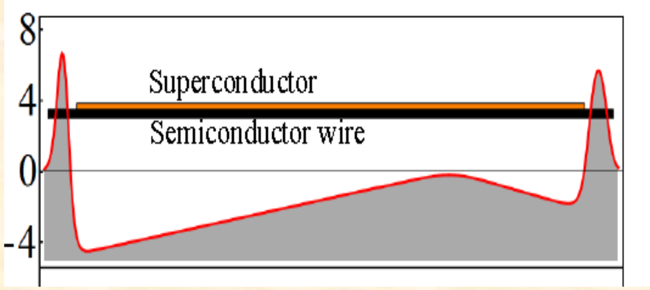
Proposed Third set-up: Collect two sets of data

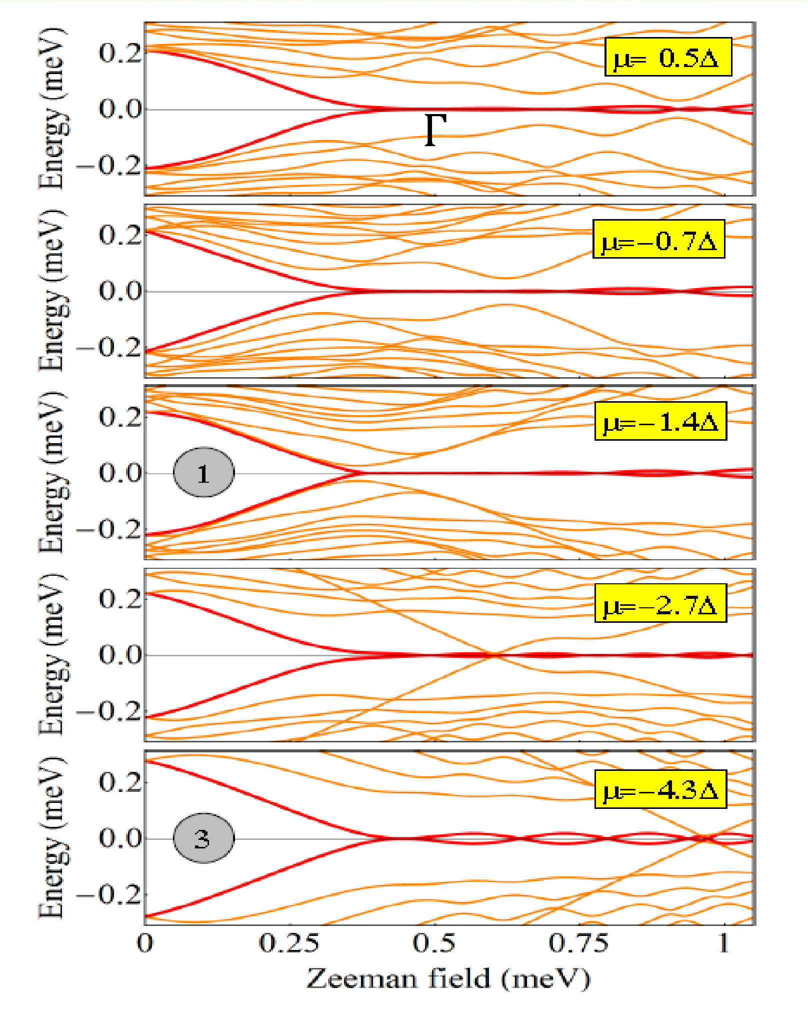
Set 1: Bias potential to left lead, SC grounded, right lead isolated

Set 2: Bias potential to right lead, SC grounded, left lead isolated

Compare and correlate

Low energy spectrum for generic inhomogeneous potential





$\Gamma < \sqrt{\mu^2 + \Delta^2}$

Robust (nearly) zero energy modes appear for Zeeman field ~ 0.4 meV
"Critical" field depends weakly on μ over a range of ~ 1 meV
Clear inconsistency with TQPT in a homogeneous system

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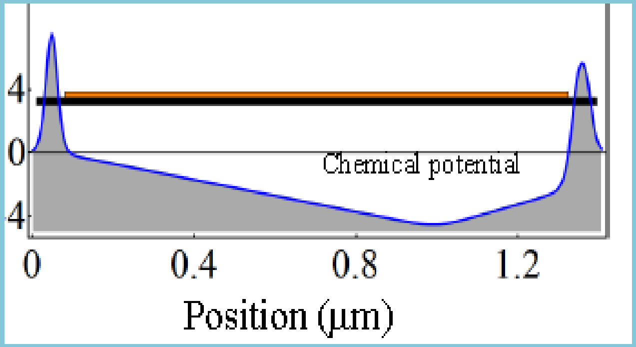

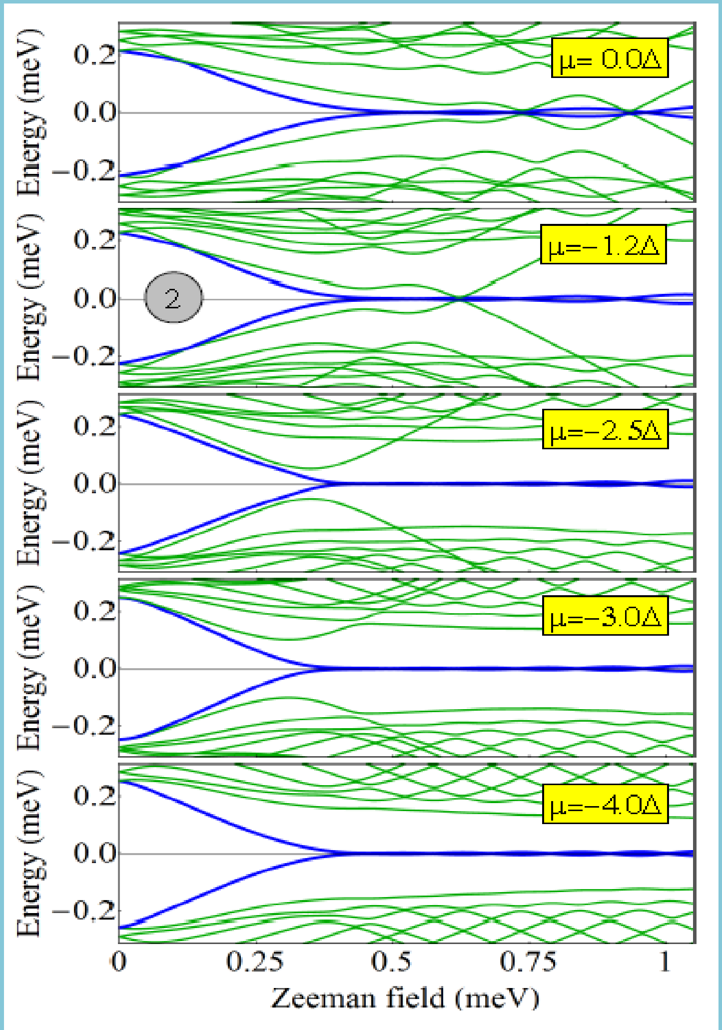

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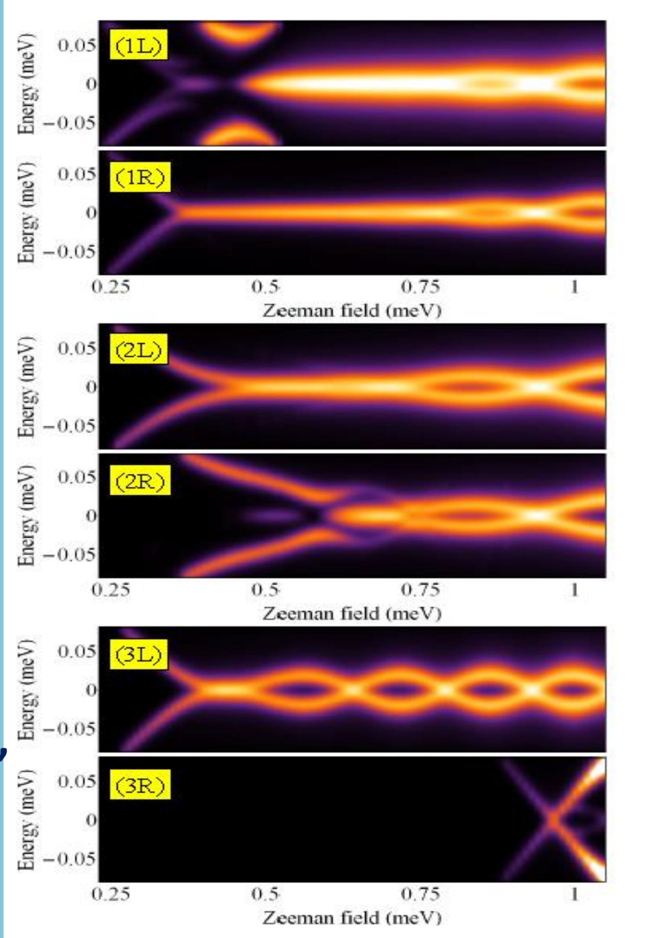
Presence or absence of correlation

Look for correlation in:

(a) Critical Zeeman fields
(b) Splitting oscillations

In crossover regime, NO Correlation for robust ABS

In topological phase, Correlation in the Zeeman and splitting energy scales, NOT in visibility



All three data sets are uncorrelated in for robust ABSs!
Set 1 becomes correlated above Zeeman ~ 0.6 meV
Set 2 becomes correlated above Zeeman ~ 0.8 meV
Set 3 remains uncorrelated!

Conclusions

- ❖ Robust ABSs can be realized (rather generically) in spin-orbit coupled nanowires proximity-coupled to conventional SCs

- ❖ To realize truly non-Abelian, topologically-protected MZMs one has to solve a critical problem:

  - engineer hybrid structures with homogeneous active parts (no defects, no position-dependent external fields, no interface disorder, etc.)
  - In generic inhomogeneous systems, two-terminal charge tunneling is the most obvious “smoking gun” experiment, well within current experimental reach.

Thank you

Schematics of experimental set up