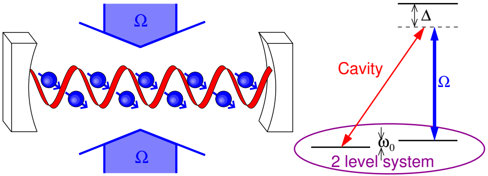
Quantum Many-Body Physics with Multimode Cavity QED

Jonathan Keeling

Open Quantum Systems, ICTS-TIFR, July 2017

Synthetic cavity QED: Raman driving

- Tunable coupling via Raman



$$H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} \left(\sigma_n^+ a + \text{H.c.}\right)$$

- Real systems: loss $\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[a, \rho] + \ldots$
- To balance loss, counter-rotating:

$$H_{\text{eff}} = \ldots \frac{\Omega g}{\Delta} \sigma_n^x (a + a^\dagger)$$

[Dimer *et al.* PRA '07]

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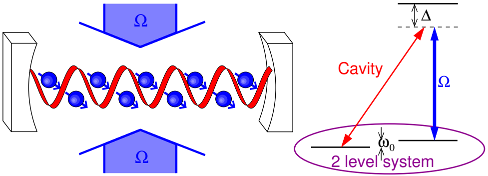

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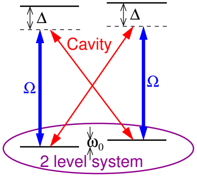

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(Multimode) cavity QED

$$H = \sum_k \omega_k a_k^\dagger a_k + \sum_n \omega_0 \sigma_n^+ \sigma_n^- + \sum_{n,k} g_{k,n} (a_k^\dagger + a_{-k})(\sigma_n^+ + \sigma_n^-)$$

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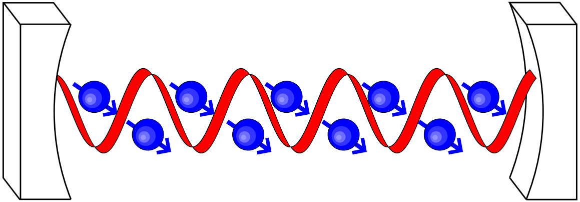

- Compare g (or $g\sqrt{N}$) vs:
  - κ, γ

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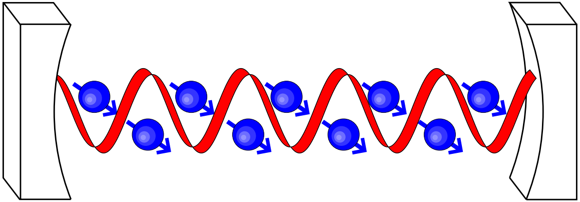

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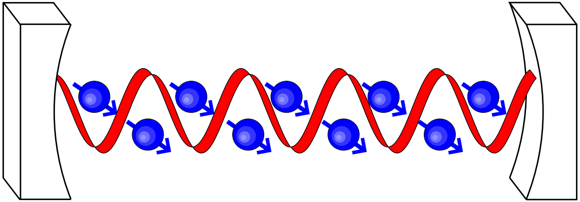
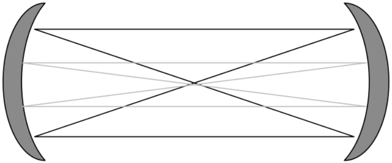
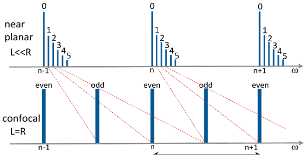


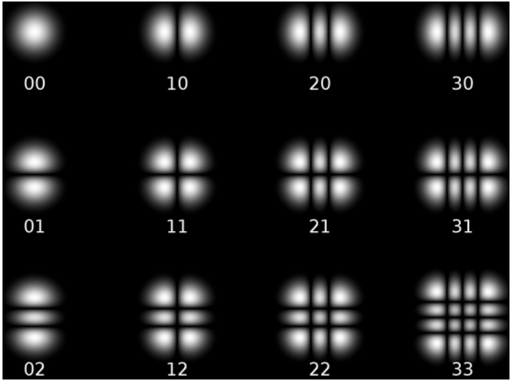
Multimode cavities

Confocal cavity $L = R$:



- Modes $\Xi_{l,m}(\mathbf{r}) = H_l(x)H_m(y)$, $l + m$ fixed parity





- Tune between degenerate vs non-degenerate

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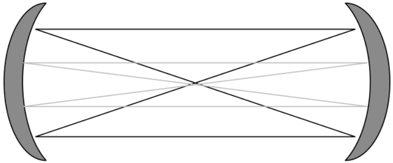

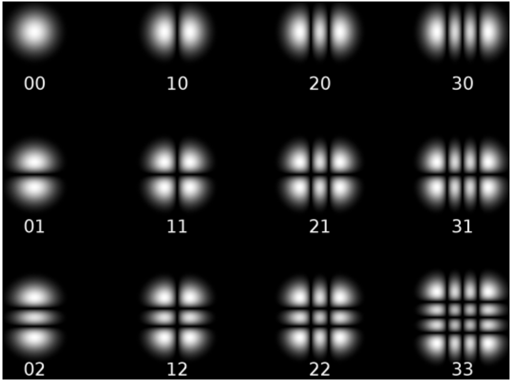

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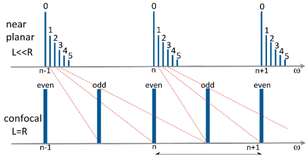
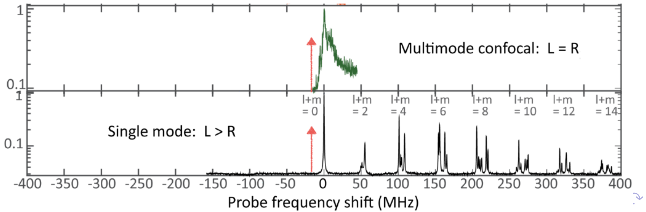
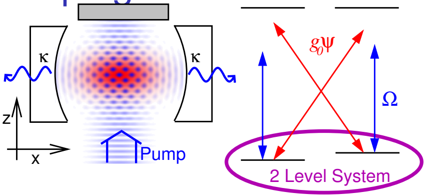




- Tune between degenerate *vs* non-degenerate

1. Introduction: Tunable multimode Cavity QED
2. Density wave polaritons
   - Superradiance transition
   - Supermode density wave polariton condensation
   - Degenerate limit
3. Spin wave polaritons
   - Spin glass
   - Effects of loss
4. Meissner-like effect

Mapping transverse pumping to Dicke model



- Atomic states: $\psi(\mathbf{r}) = \psi_{\Downarrow} + \psi_{\Uparrow} \cos(qx)\cos(qz)$

$$H_{\text{eff}} = \underbrace{(\omega_c - \omega_P)}_{-\Delta_c} a^\dagger a + \sum_n \frac{\omega_0}{2}\sigma_n^z + \underbrace{\frac{\Omega g_0}{\Delta}}_{g_{\text{eff}}} \sigma_n^x (a + a^\dagger)$$

- Extra "feedback" term U, cavity loss κ
- $\mathbb{Z}_2$ symmetry
- 1. Phase of light.
  2. Atom checquerboard

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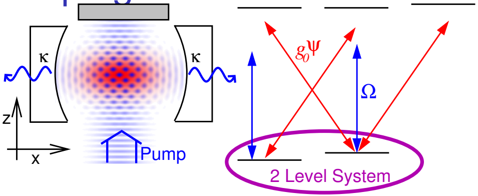

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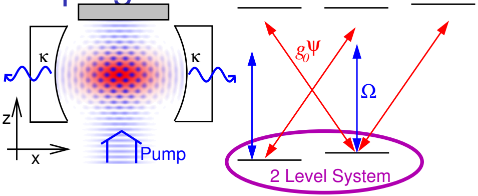

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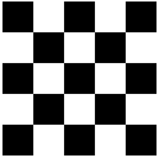
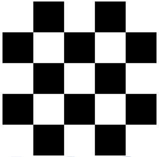
Atoms:  vs 

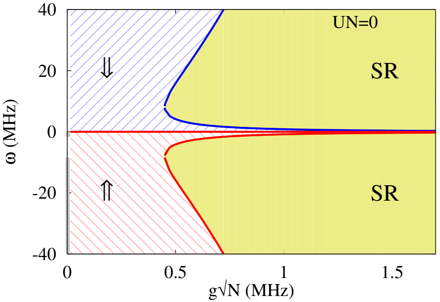
Classical dynamics

Changing U:

$U = 0$

$U < 0$

$U > 0$



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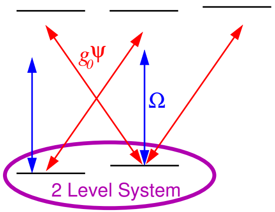
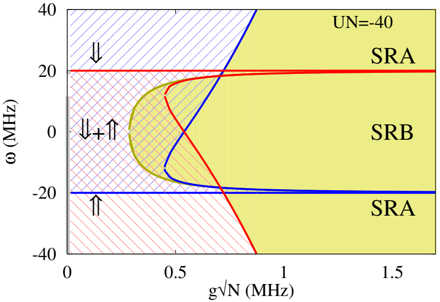


$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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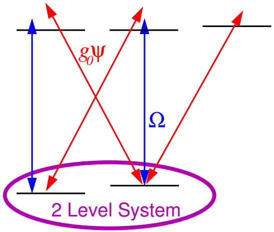

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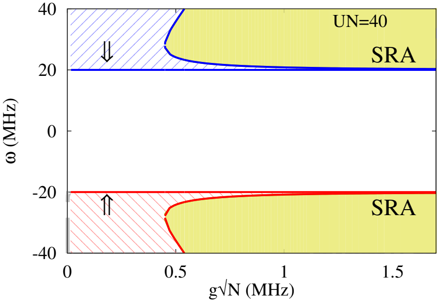

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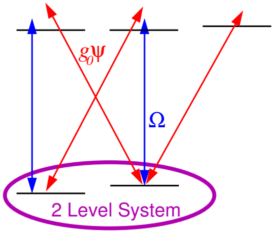

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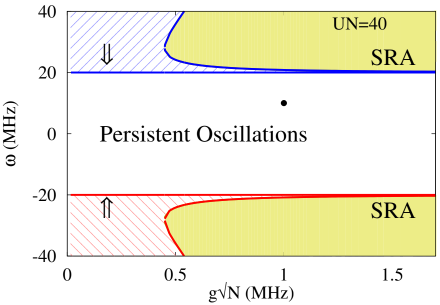
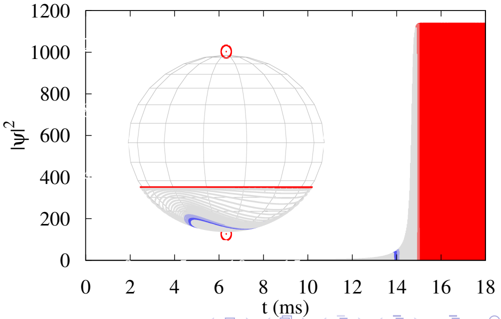
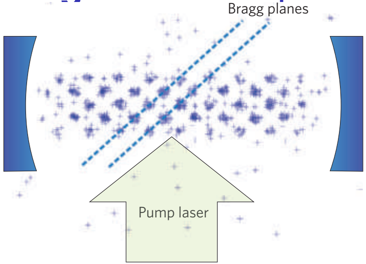




Single mode experiments



Ritsch *et al.* PRL '02

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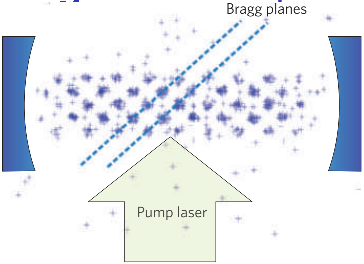
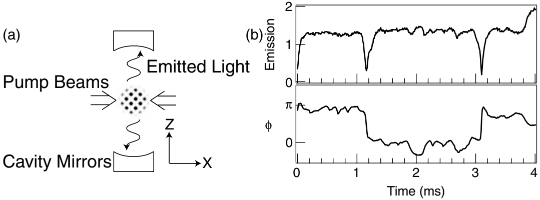


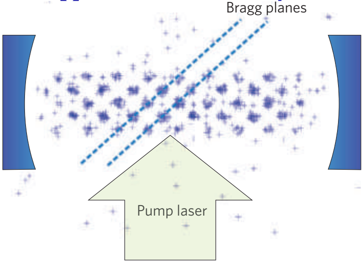
Ritsch *et al.* PRL ’02

Thermal atoms, momentum state



Vuletic *et al.* PRL ’03 (MIT)

Single mode experiments



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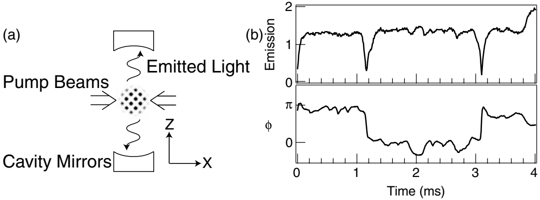
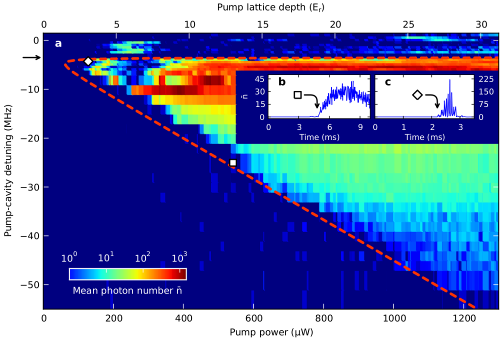


Vuletic *et al.* PRL '03 (MIT)

BEC, momentum state



Baumann *et al.* Nature '10 (ETH)

Kinder *et al.* PRL '15 (Hamburg)

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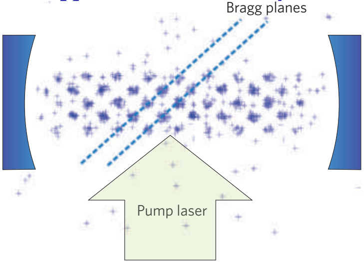

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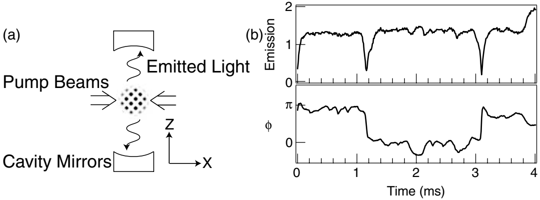

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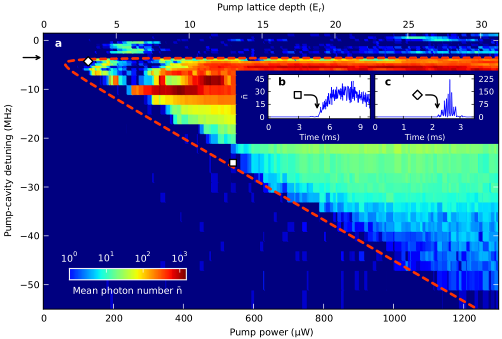

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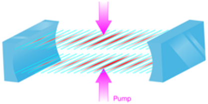
BEC, hyperfine states

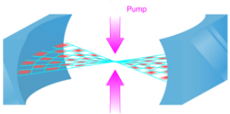
Baden *et al.* PRL '14 (Singapore)

Synthetic cQED Possibilities

- Single mode *vs* **multimode**



Credit: Alan Stonebreaker, Physics 3, 88 (2010)



- Momentum state *vs* hyperfine state
- XY *vs* Ising
- Thermal gas *vs* BEC *vs* disorder localised

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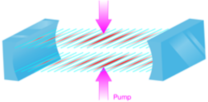

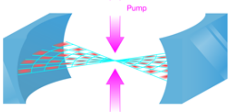

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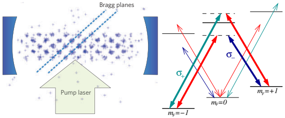

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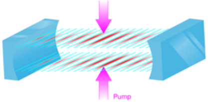

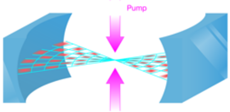

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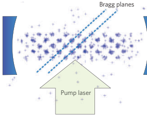

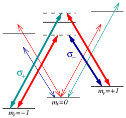

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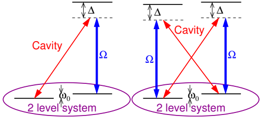

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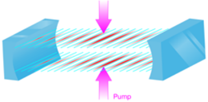

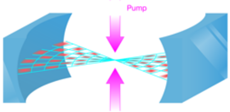

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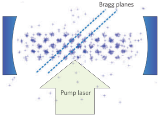

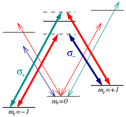

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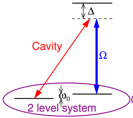

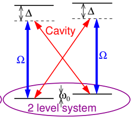

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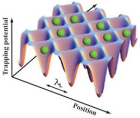

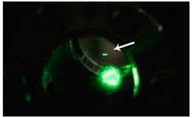

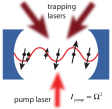

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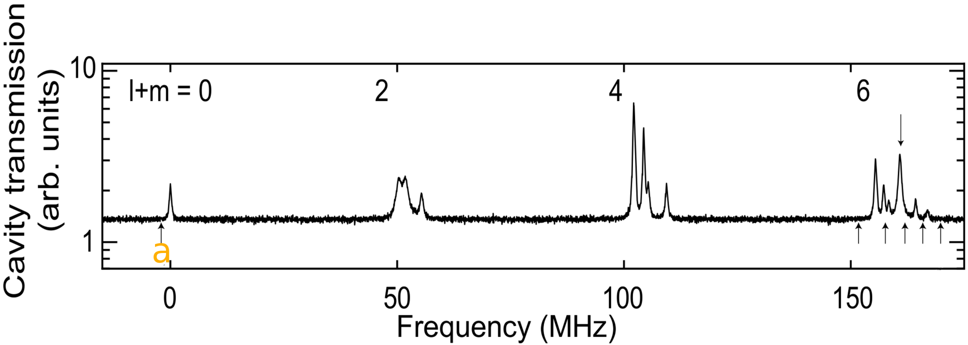
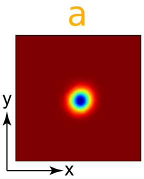
Superradiance in multimode cavity: Even family





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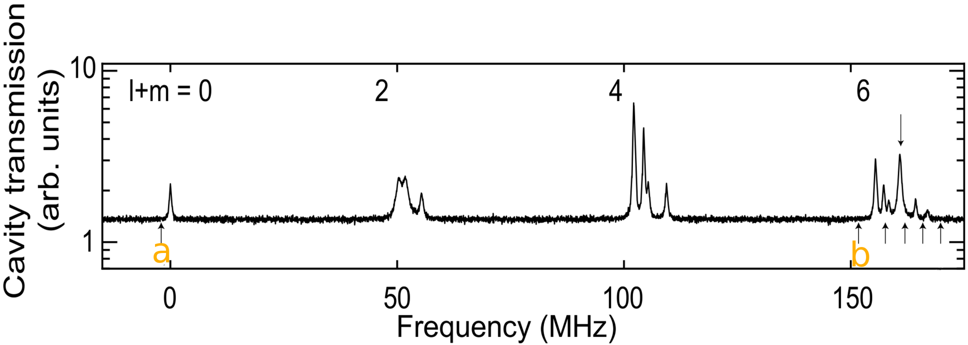

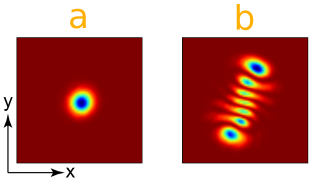

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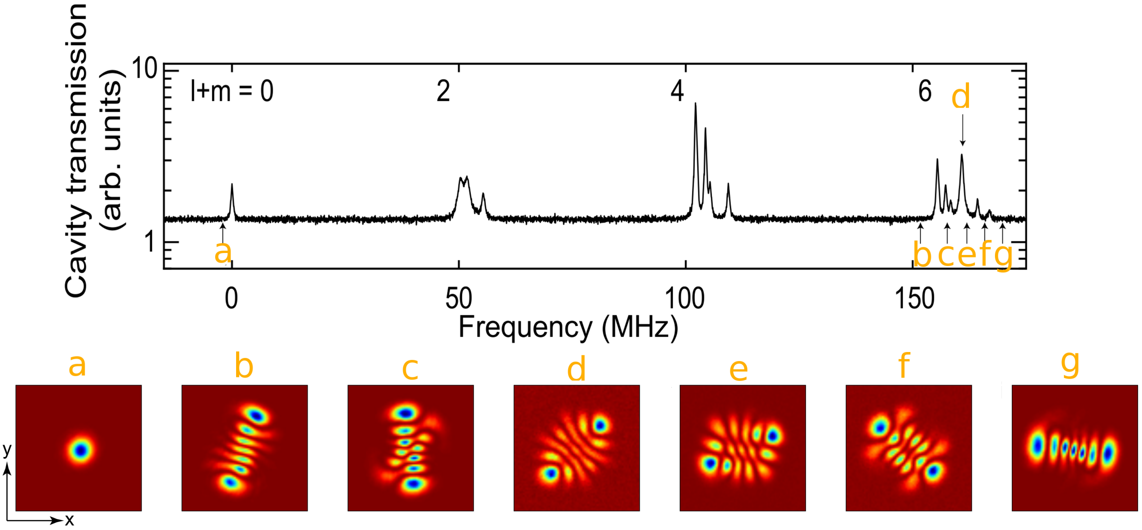
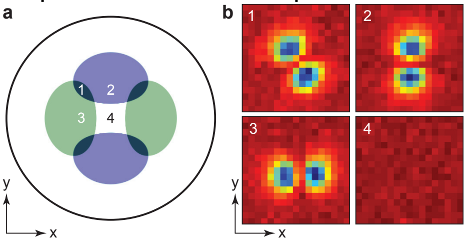


Superradiance in multimode cavity: Odd family

- Dependence on cloud position



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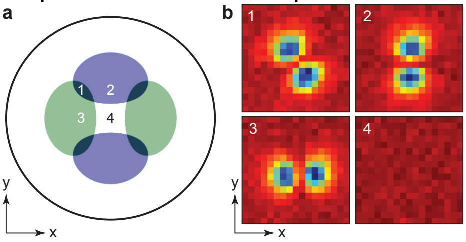

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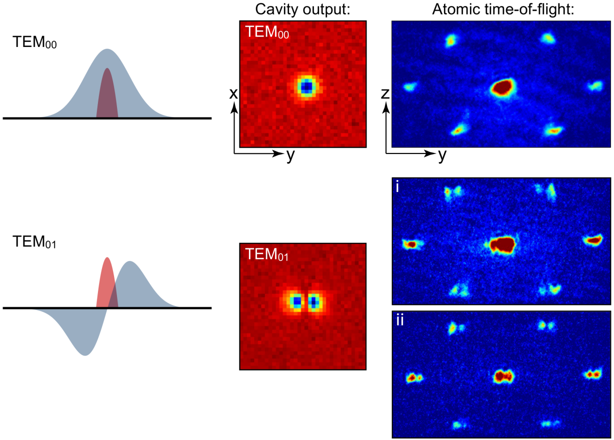

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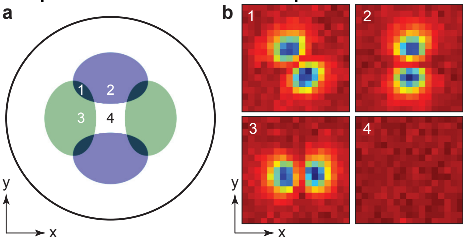

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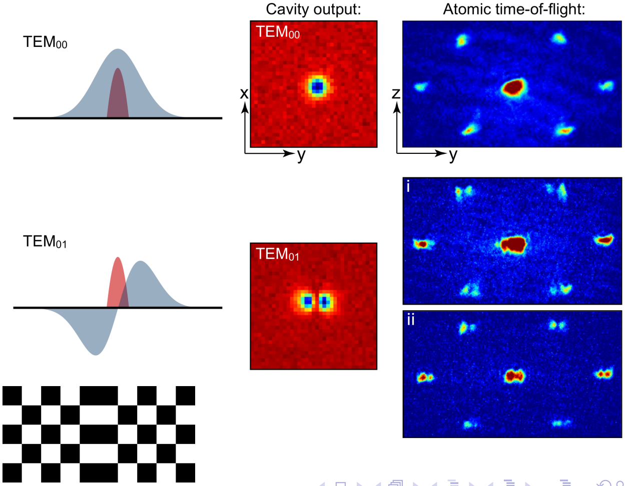
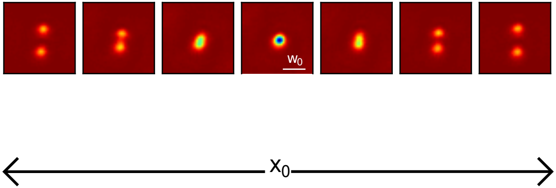


Density wave polaritons

Degenerate cavity limit

- Multimode cavity — light follows atom cloud



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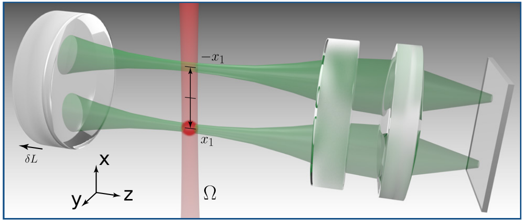

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[Vaidya, Guo, Kroeze, Ballantine, Kollar, JK, Lev, in preparation]

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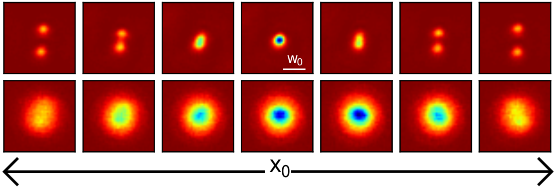

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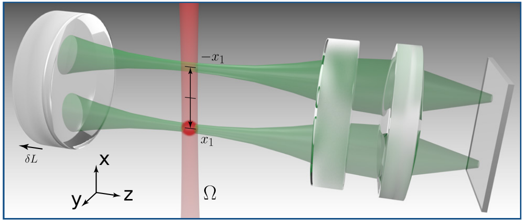

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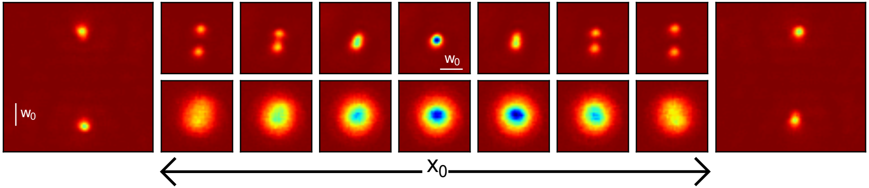

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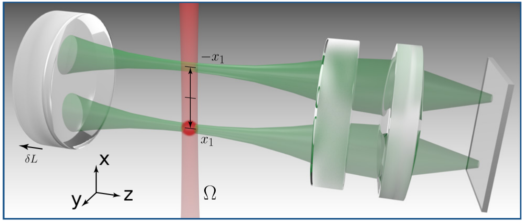

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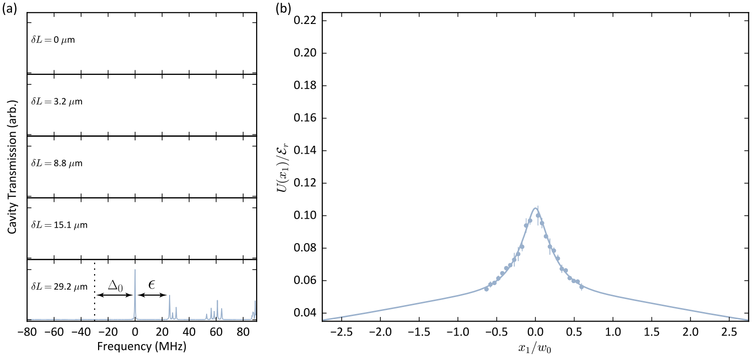
Measuring atom-image interaction

- Threshold measures $U(x_0, -x_0)$, cavity Green's function



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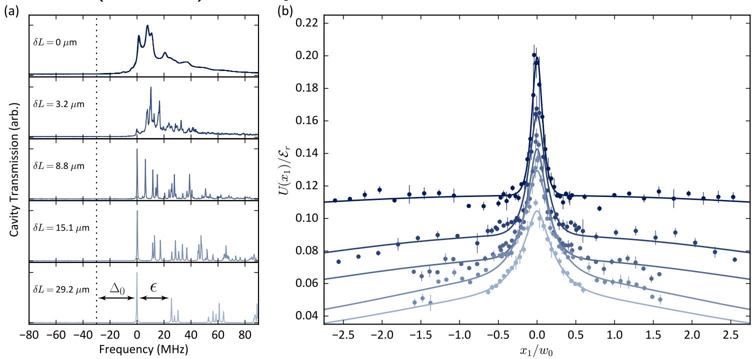

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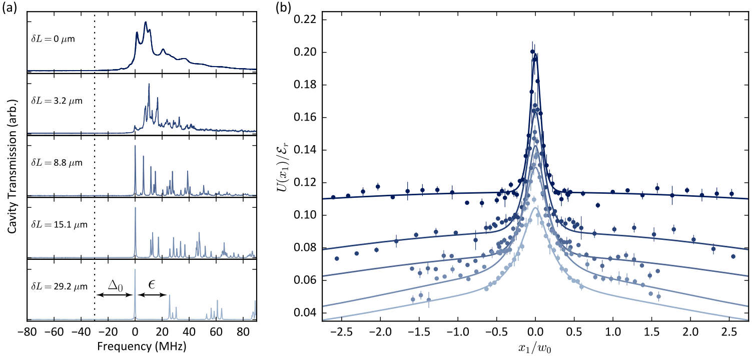

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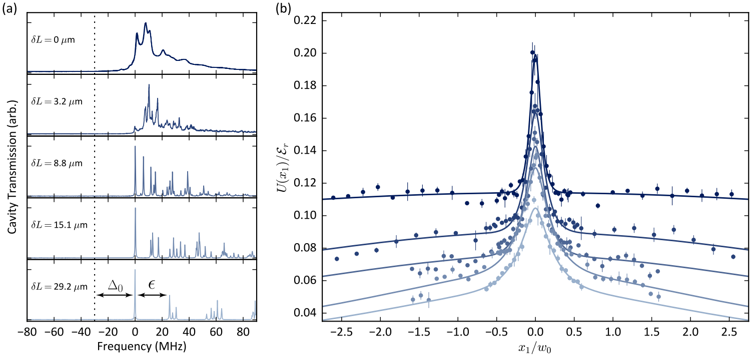

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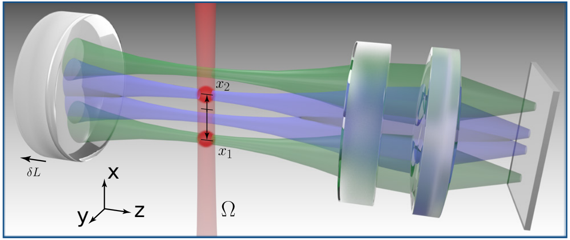
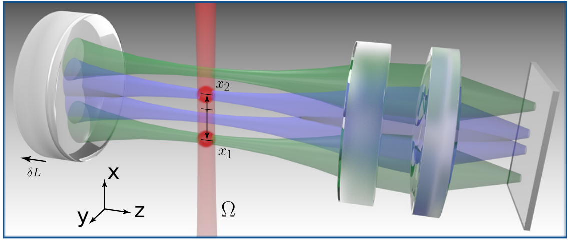
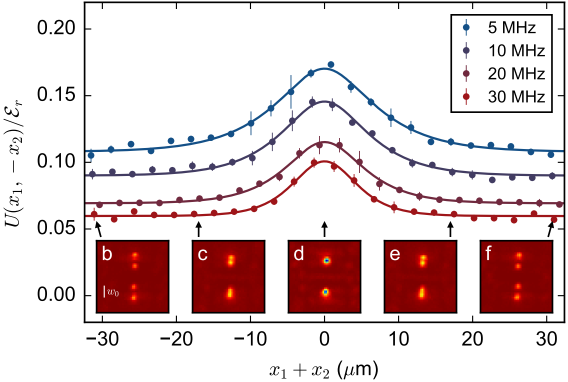


Measuring atom-atom interaction

- Two BECs, vary x_1, x_2





Long-range part of interaction

- Atom-atom Green's function at $\epsilon \to 0$:

$$U(x, x') = \frac{1}{4}\left(\delta(x - x') + \delta(x + x') + \frac{1}{\pi}\cos(xx')\right)$$

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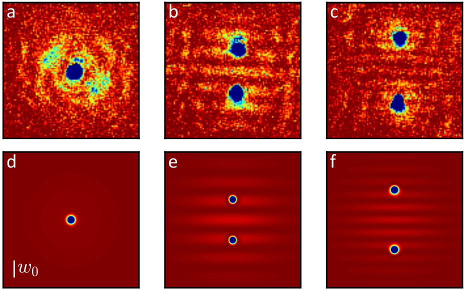

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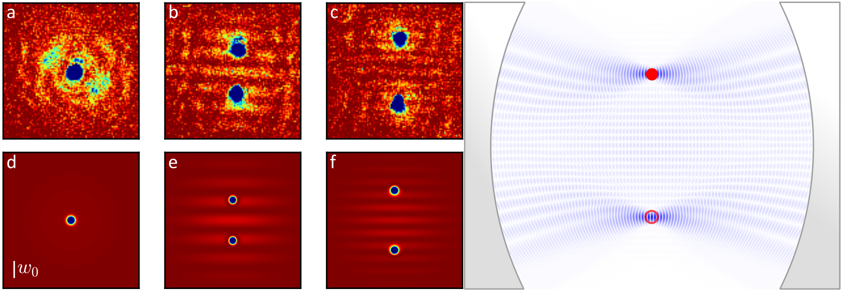

Spin wave polaritons

Disordered atoms

- Multimode cavity, Hyperfine states,

$$H_{\text{eff}} = -\sum_{\mu} \Delta_\mu a^\dagger_\mu a_\mu + \sum_n \frac{\omega_0}{2}\sigma^z_n + \frac{\Omega g_0}{\Delta} \sum_\mu \Xi_\mu(\mathbf{r}_n)\sigma^x_n (a_\mu + a^\dagger_\mu)$$

- Random atom positions – quenched disorder

- Effective XY/Ising spin glass

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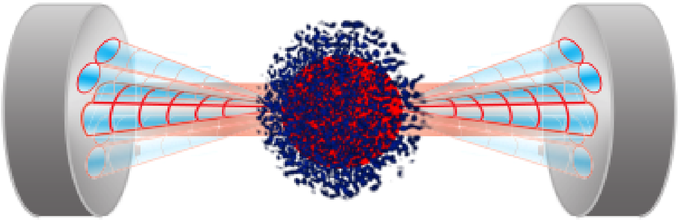

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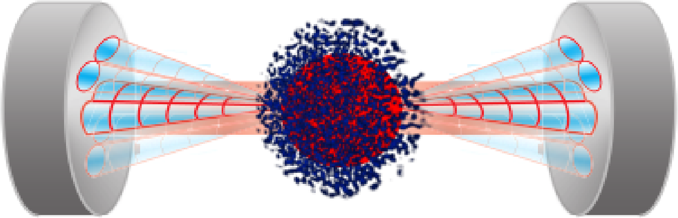

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Internal states: Effect of particle losses

- Dicke Hamiltonian: $H = \omega a^\dagger a + \sum_i \omega_0 \sigma_i^+ \sigma_i^- + g\sigma_i^x (a^\dagger + a)$
- Adding other loss terms

$$\partial_t \rho = -i[H, \rho] + \kappa \mathcal{L}[\hat{a}] + \sum_i \Gamma_\downarrow \mathcal{L}[\sigma_i^-] + \Gamma_\phi \mathcal{L}[\sigma_i^z]$$

$$\mathcal{L}[X] = X\rho X^\dagger - (X^\dagger X \rho + \rho X^\dagger X)/2$$

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[Dalla Torre *et al.* , PRA (Rapid) 2016, Kirton & JK, PRL 2017]

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Effect of particle losses

- Wigner function $W(\hat{a} = \hat{x} + i\hat{p})$

- Γ_{ϕ} only: MFT $\rightarrow$ no SR
- Asymptotic scaling

- $N = 30$: no symmetry breaking

[Kirton & JK, PRL 2017]

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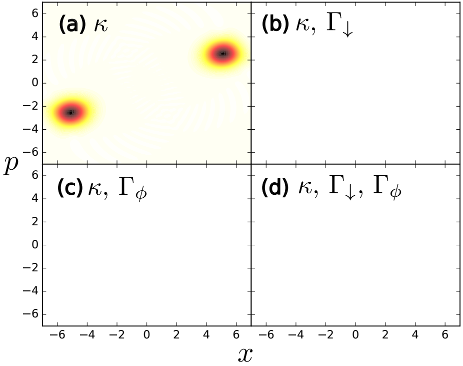

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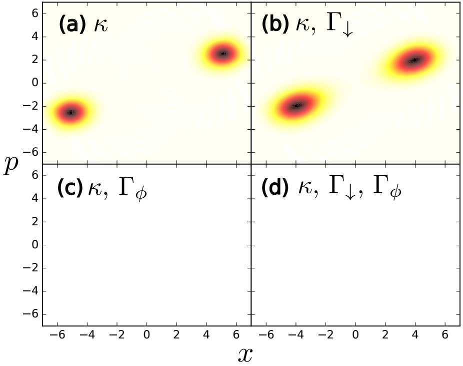

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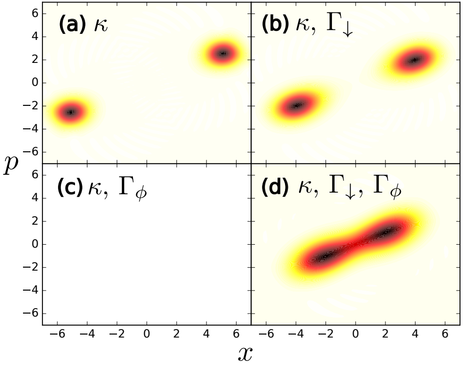

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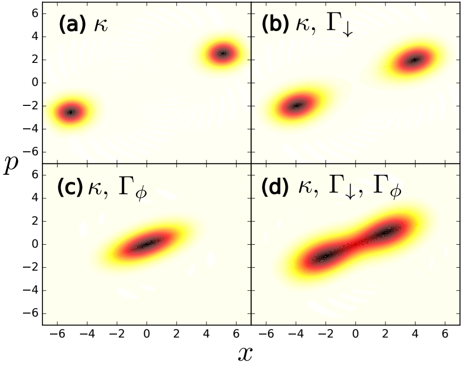

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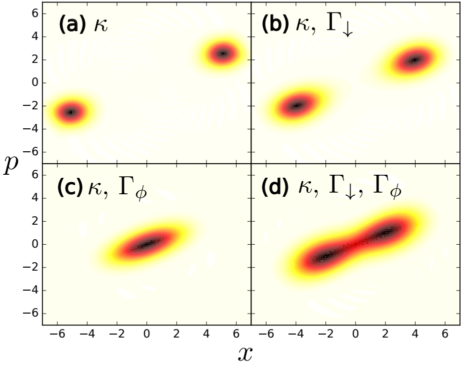

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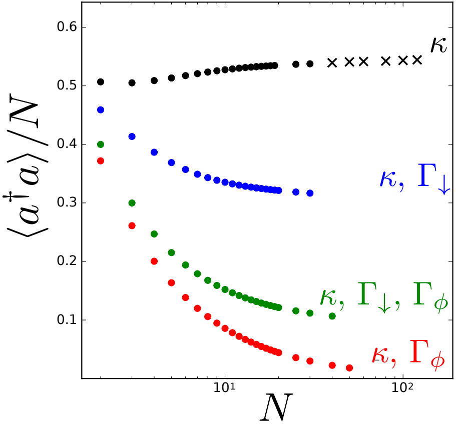

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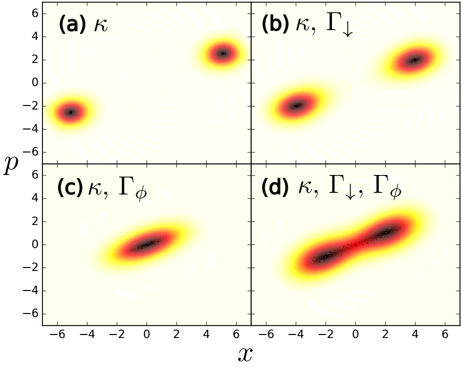

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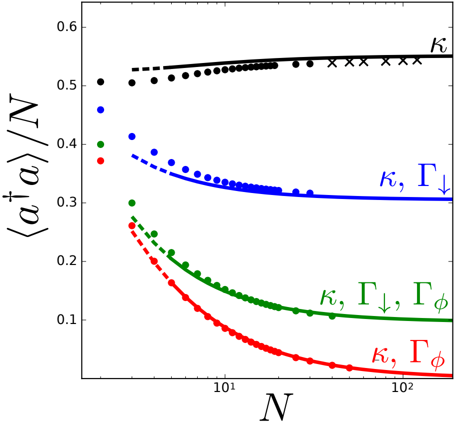

[Kirton & JK, PRL 2017]

Meissner-like effect

Cavity QED and synthetic gauge fields

- [Spielman, PRA '09] scheme, hyperfine states A, B

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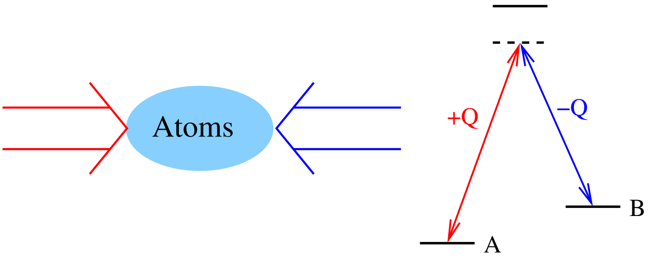

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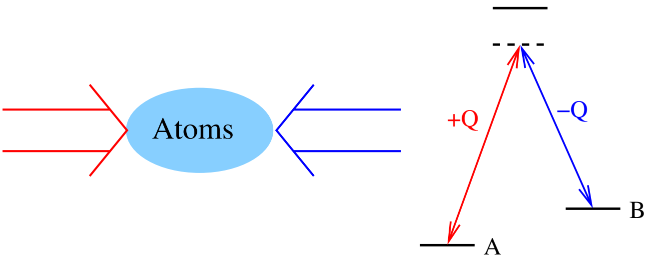

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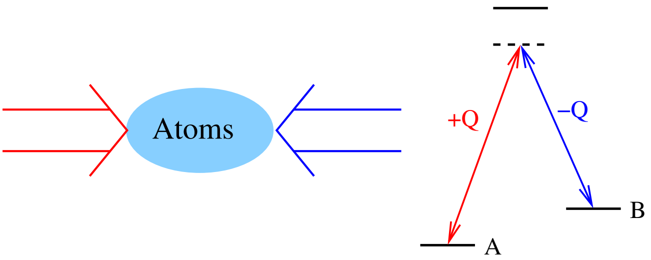

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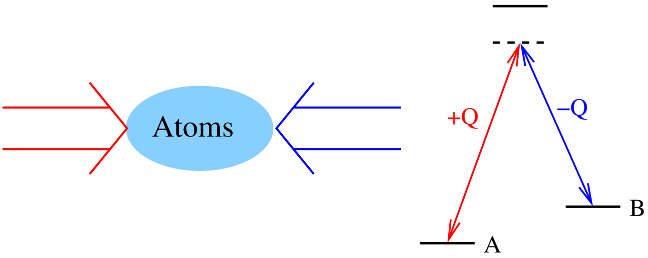
Cavity QED and synthetic gauge fields

- [Spielman, PRA '09] scheme, hyperfine states A, B

$$H = \begin{pmatrix}\psi_A & \psi_B\end{pmatrix} \begin{pmatrix} E_A + (\nabla - Q\hat{x})^2 & \Omega/2 \\ \Omega/2 & E_B + (\nabla + Q\hat{x})^2 \end{pmatrix} \begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix}$$



Ground state:

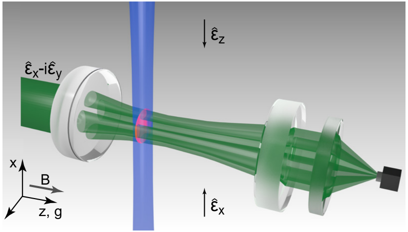
$$E_-(\mathbf{k}) \simeq \frac{(\mathbf{k} - \frac{(E_B - E_A)Q\hat{\mathbf{x}}}{\Omega})^2}{2m^*}$$

- Feedback
  - Why?
    - ★ Meissner effect, Anderson-Higgs mechanism, confinement-deconfinement transition.
  - How?
    - ★ Multimode cavity QED

Meissner-like physics: idea

- Follow Spielman scheme

$$\begin{pmatrix} E_A + (\nabla - Q\hat{x})^2 & \Omega/2 \\ \Omega/2 & E_B + (\nabla + Q\hat{x})^2 \end{pmatrix}$$



[Ballantine *et al.* PRL 2017]

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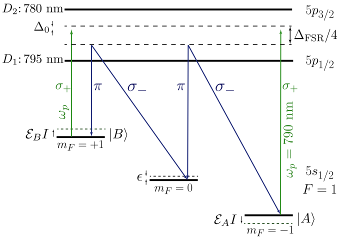
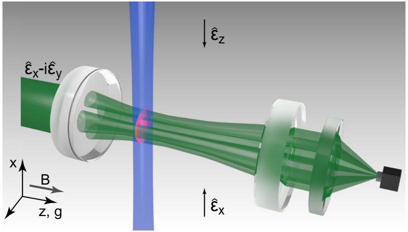
- $E_A, E_B \propto |\varphi|^2$ from cavity Stark shift
- Ground state $E_-(\mathbf{k}) \propto (\mathbf{k} - Q\hat{x}|\varphi|^2)^2$



[Ballantine *et al.* PRL 2017]



- Multimode cQED → local matter-light coupling
- Variable profile synthetic gauge field?
- Reciprocity: matter affects field

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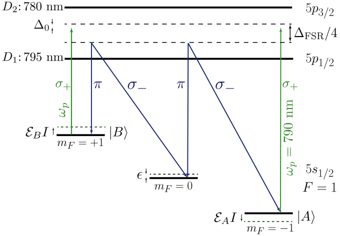
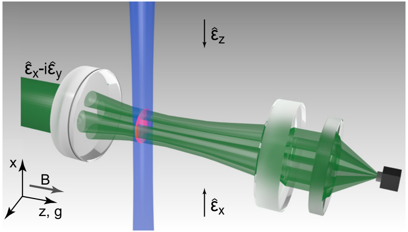


[Ballantine *et al.* PRL 2017]



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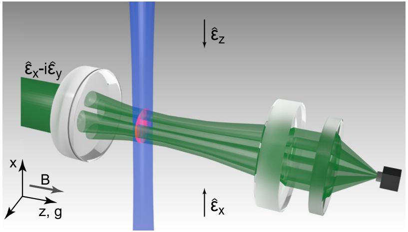

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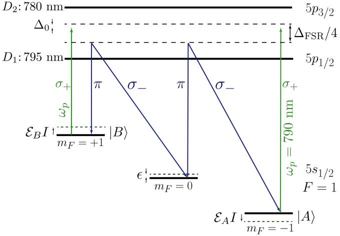

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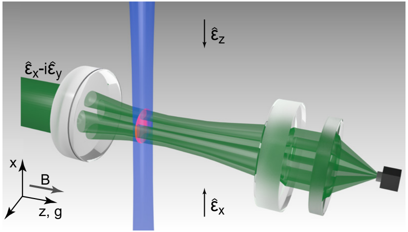

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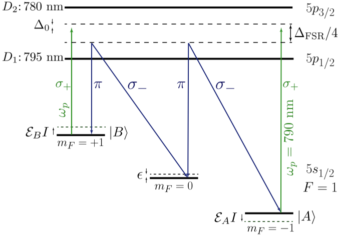

[Ballantine *et al.* PRL 2017]

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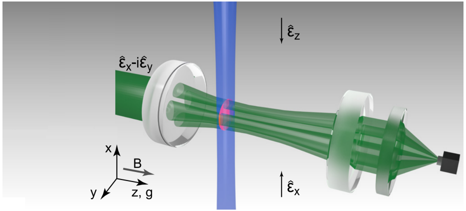
Meissner-like physics: setup



- Atoms: $$i\partial_t \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \left[-\frac{\nabla^2}{2m} + \begin{pmatrix} -\mathcal{E}_\Delta |\varphi|^2 + i\frac{q}{m}\partial_x & \Omega/2 \\ \Omega/2 & \mathcal{E}_\Delta |\varphi|^2 - i\frac{q}{m}\partial_x \end{pmatrix} + \ldots \right] \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}.$$

[Ballantine *et al.* PRL 2017]

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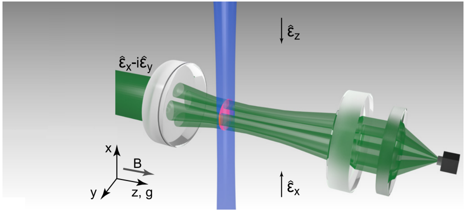

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[Ballantine *et al.* PRL 2017]

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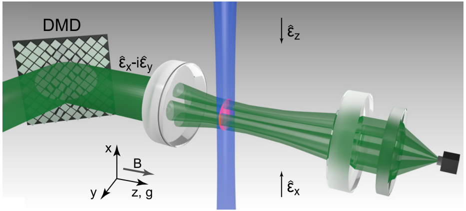

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[Ballantine *et al.* PRL 2017]

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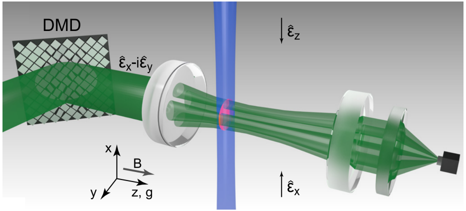

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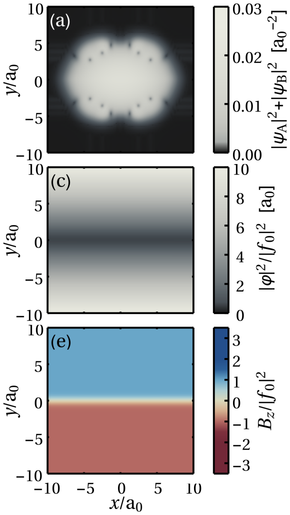
[Ballantine *et al.* PRL 2017]

Meissner-like physics: numerical simulations

Atoms

Cavity light

Synthetic field



- Consider $f(\mathbf{r})$ such that $|\varphi|^2 \propto y$.
- Without feedback ($\mathcal{E}_\Delta = 0$) for field
- With feedback

[Ballantine *et al.* PRL 2017]

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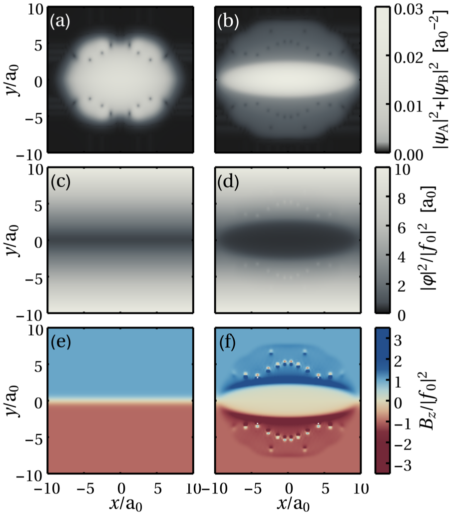

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[Ballantine *et al.* PRL 2017]

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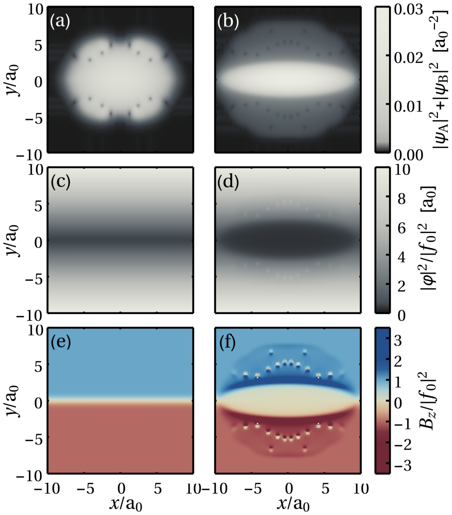

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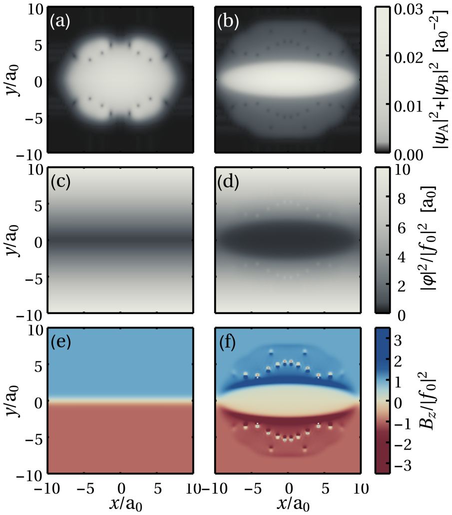

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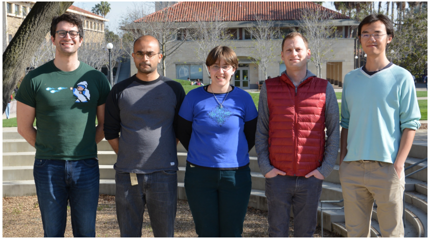
Acknowledgments

Experiment (Stanford):
Benjamin Lev



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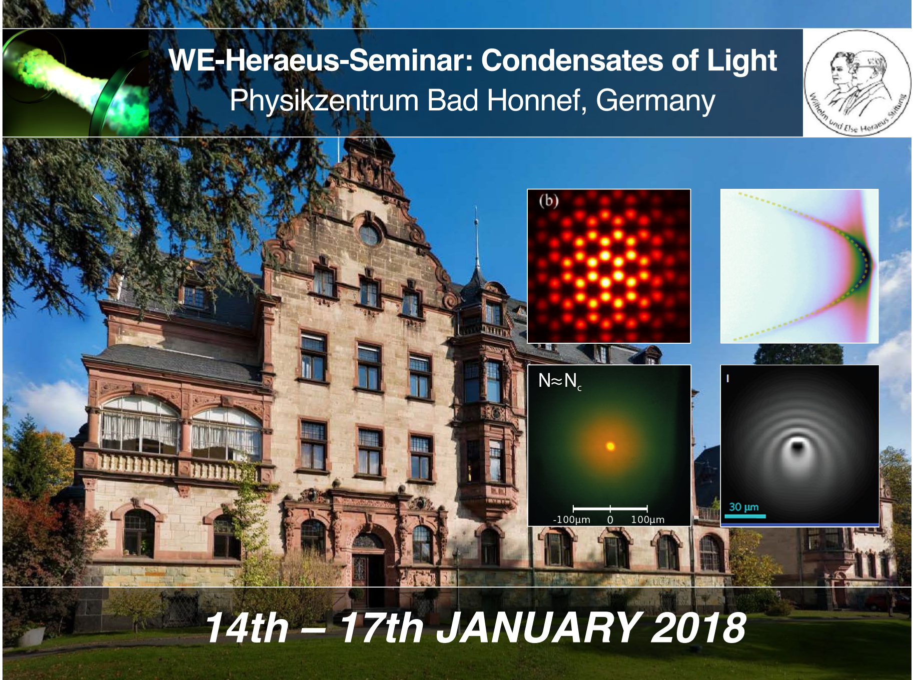
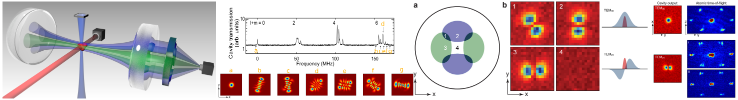
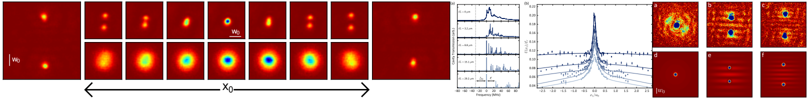

WE-Heraeus-Seminar: Condensates of Light
Physikzentrum Bad Honnef, Germany
Wilhelm und Else Heraeus Stiftung
(b)
N≈Nc
-100µm 0 100µm
30 µm
14th – 17th JANUARY 2018

Summary

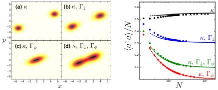
- Supermode polariton condensation [Kollár *et al.* Nat. Comms. 2017]



- Variable range atom-atom interaction [Vaidya *et al.* in preparation]



- Open Dicke model, $\kappa, \Gamma_\phi, \Gamma_\downarrow$ [Kirton & JK, PRL 2017]



- Meissner like effect [Ballantine *et al.* PRL 2017]