Variable Range Hopping in a Non-equilibrium Steady State

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Outline

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Coulomb Glass Model

- A **Coulomb glass** is a disordered insulating system where electrons are localized and interact via long-range Coulomb repulsion.
- The system is modeled by the Hamiltonian:

$$H = \sum_i \phi_i n_i + \frac{1}{2} \sum_{i \neq j} \frac{n_i n_j}{r_{ij}}, \quad \text{with } n_i \in \{0, 1\}$$

- ϕ_i: random on-site energies from disorder; n_i: occupancy number.
- The competition between random potential (ϕ_i) and interactions causes frustration and a rugged energy landscape.
- This leads to many nearly-degenerate metastable states and extremely slow relaxation — **glass-like behavior**.

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Coulomb Gap and Transport behavior

- A key feature of a Coulomb glass is the formation of a **Coulomb gap** — a soft suppression in the density of states (DOS) near the Fermi level:

$$g(\varepsilon) \propto |\varepsilon - \mu|^{d-1}, \quad \text{in } d \text{ dimensions}$$

- Due to this gap, variable-range hopping (VRH) conductivity deviates from Mott's law.
- Instead, the **Efros-Shklovskii (ES) law** governs low-temperature transport:

$$\sigma(T) \propto \exp\left[-\left(\frac{T_0}{T}\right)^{1/2}\right]$$

- This behavior has been observed in doped semiconductors, amorphous films, and granular metals.

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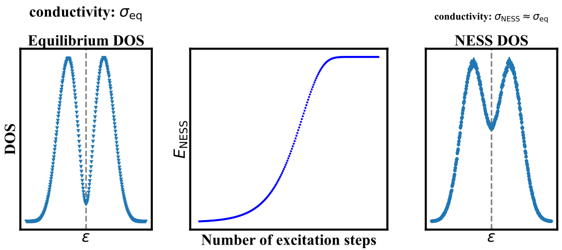
Motivation

From Equilibrium to Nonequilibrium Steady State (NESS)

- At **equilibrium**, a well-defined Coulomb gap forms in the DOS.
- Under repeated excitations, the system reaches a **nonequilibrium steady state (NESS)**.
- In NESS:
  - **DOS:** The Coulomb gap is significantly filled.
  - **Conductivity:** Changes by only about **2.8%**.



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Simulation Setup

- The system is first annealed to equilibrium using the Metropolis Monte Carlo algorithm.
- NESS is created by periodically alternating:
  - **Excitation:** Random spin (occupation) swaps that increase the system's energy ($\Delta E > 0$), mimicking photon absorption by an electron.
  - **Relaxation:** Kinetic Monte Carlo for x steps
- Conductivity is measured by applying a small electric field: $F = T/10$
- After the system reaches a steady energy plateau, time and disorder averages are taken to evaluate observables.

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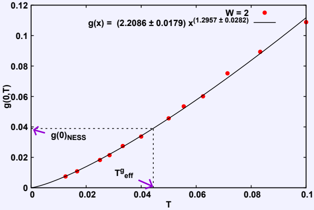
In the NESS regime, effective temperatures are inferred from different observables:

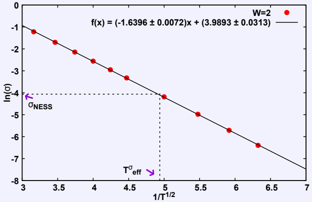
From DOS

$$g(0) = c\left(T_{\text{eff}}^{g(0)}\right)^{\alpha}$$



From Conductivity

$$\sigma_{\text{NESS}} = \sigma_0 \exp\left[-\left(\frac{T_0}{T_{\text{eff}}^{\sigma}}\right)^{1/2}\right]$$



Key Takeaway

$T_{\text{eff}}^{g(0)} \neq T_{\text{eff}}^{\sigma} \Rightarrow$ NESS is not characterized by a single temperature.

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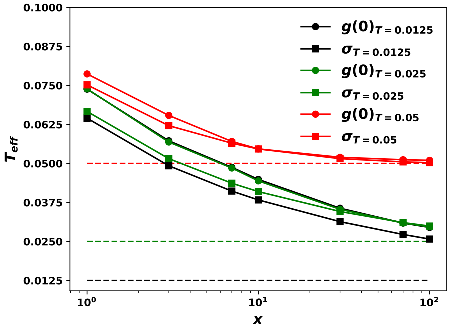
Numerical vs Experimental Results



- Our results are qualitatively consistent with experiment, where

$$T_{\text{eff}}^{\sigma} < T_{\text{eff}}^{g(0)}$$

- Quantitative differences may arise because the experimental memory dip is only proportional to the DOS reduction, not a direct measure.
- Another possible reason: quantum effects not captured in this classical simulation.

(P. Bhandari, V. Malik, and M. Schechter, Phys. Rev. B ***108****, 024203 (2023))*

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Hot Electron Model (HEM)

What is the Hot Electron Model?

The Hot Electron Model (HEM) describes how a system can behave as if it's at a higher "effective temperature" T_{eff}, even when the surrounding bath (like the lattice) is cold.

Core Idea

- System is **externally driven** (e.g. light, voltage)
- Electrons absorb energy and thermalize *among themselves*

Transport Mechanism

$$\sigma(T_{\text{eff}}) = \sigma_0 \exp\left[-\left(\frac{T_0}{T_{\text{eff}}}\right)^{1/2}\right]$$

Analogy

It's like heating the electrons without heating the room — they feel hot, move faster, but the rest of the material stays cold.

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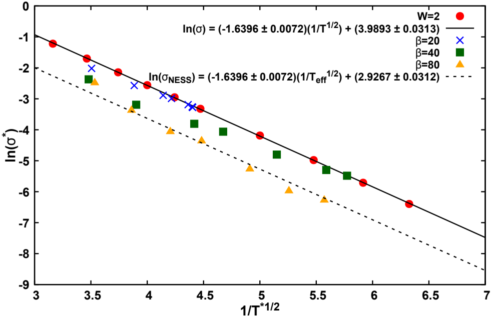

Efros-Shklovskii (ES) Fit

$$\ln \sigma = -A \left(\frac{1}{T_{\text{eff}}^{1/2}} \right) + \text{const.}$$

- Both NESS and equilibrium follow ES law
- Intercept drops with increasing β $\rightarrow$ smaller σ_0
- Slope unchanged $\rightarrow$ same VRH mechanism

P. Bhandari, V. Malik, and M. Schechter, Phys. Rev. B ***108****, 024203 (2023)*

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Phononless vs. Phonon-Assisted Hopping

- In equilibrium, hopping is phonon-assisted — electrons use phonons to overcome energy barriers.
- In the NESS regime:
  - For moderate β (e.g. $\beta = 20, 40$), σ_0 decreases moderately: phonon involvement reduced but still present.
  - At large β (e.g. $\beta = 80$), $\sigma_0 \approx \sigma_0^{\text{eq}}/2 \rightarrow$ hopping becomes nearly **phononless**.
- This transition reflects how energy-lowering transitions dominate in far-from-equilibrium conditions, breaking detailed balance.
- A key signature of nonthermal transport in disordered systems.

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Conclusion

Summary of Findings

- We studied variable range hopping (VRH) transport in a Coulomb glass driven into a nonequilibrium steady state (NESS).
- The system shows observable-specific effective temperatures:

$$T_{\text{eff}}^{\sigma} < T_{\text{eff}}^{g(0)}$$

  indicating that NESS cannot be described by a single thermal parameter.
- The Hot Electron Model (HEM) provides a useful phenomenological framework for describing transport under NESS.

Key Insight

Although the hopping mechanism (slope) remains unchanged, the reduced prefactor σ_0 under NESS indicates a transition to **phononless hopping** — a distinct signature of far-from-equilibrium transport.

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Thank You!