Elections as a complex system

Margins, voter turnouts and universality

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- Acknowledgement : PMRF

- *Phys. Rev. Lett.* **134,** 017201 (2024)
- arXiv : 2501.01896

Why study elections ?

- Largest examples of collective decision making by humans
- Microscopic interactions leads to macroscopic outcome :
  A non-equilibrium statistical physics problem
- Elections represent a *complex system:*
  Whole ≠ sum of parts, unpredictable,
  complex interactions among agents

TIME (2024)

Year 2024

64 national elections

2 billion people

Elections as a complex system

Maharashtra sees highest turnout in 30 yrs: victory sign, say both alliances

ALOK DESHPANDE & VIKAS PATHAK
MUMBAI, NEW DELHI, NOV 20

VOTERS CAME out in huge numbers in Maharashtra's battle of the alliances Wednesday where the turnout, according to data available at 11.45 pm, crossed 65.1 per cent — the first time since 1995 when the state witnessed a turnout of 71.69 per cent.

The turnout was way above the 61.39 per cent recorded in Maharashtra during the Lok Sabha elections this year, and the 61.4 per cent in the 2019 Assembly elections.

In Jharkhand, where the JMM-led alliance is battling the BJP-led NDA bloc, the turnout was 68.45 per cent.

The rise in voter turnout in Maharashtra is being attributed to the fierce campaign undertaken by the ruling Mahayuti and Opposition Maha Vikas Aghadi (MVA), and could become a key factor in a close contest. During the Lok Sabha elections, the three

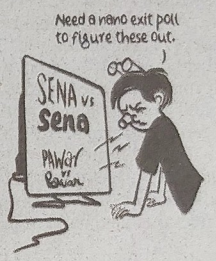
CONTINUED ON PAGE 2

At a poll booth in Ghatkopar West Wednesday. The 10 seats in Mumbai city saw 52% turnout while Mumbai suburban's 26 seats recorded 56% until 11.30 pm. *Sankhadeep Banerjee*

WHAT THE POLLSTERS PREDICT

	Maharashtra (288)			Jharkhand (81)		
Pollster	Mahayuti	MVA	Others	BJP+	JMM-INC	Others
JVC	150-167	107-125	13-14	40-44	30-40	1
Axis MyIndia	-	-	-	17-27	49-59	0-2
Matrize	150-170	110-130	8-10	42-47	25-30	1-4

BUSINESS AS USUAL
BY UNNY



DECISION 2024

VOTER TURNOUT

Maharashtra	
2019	2024
61.4%	65.1%

Jharkhand (Phase 2)	
2019	2024
66.9%	68.4%

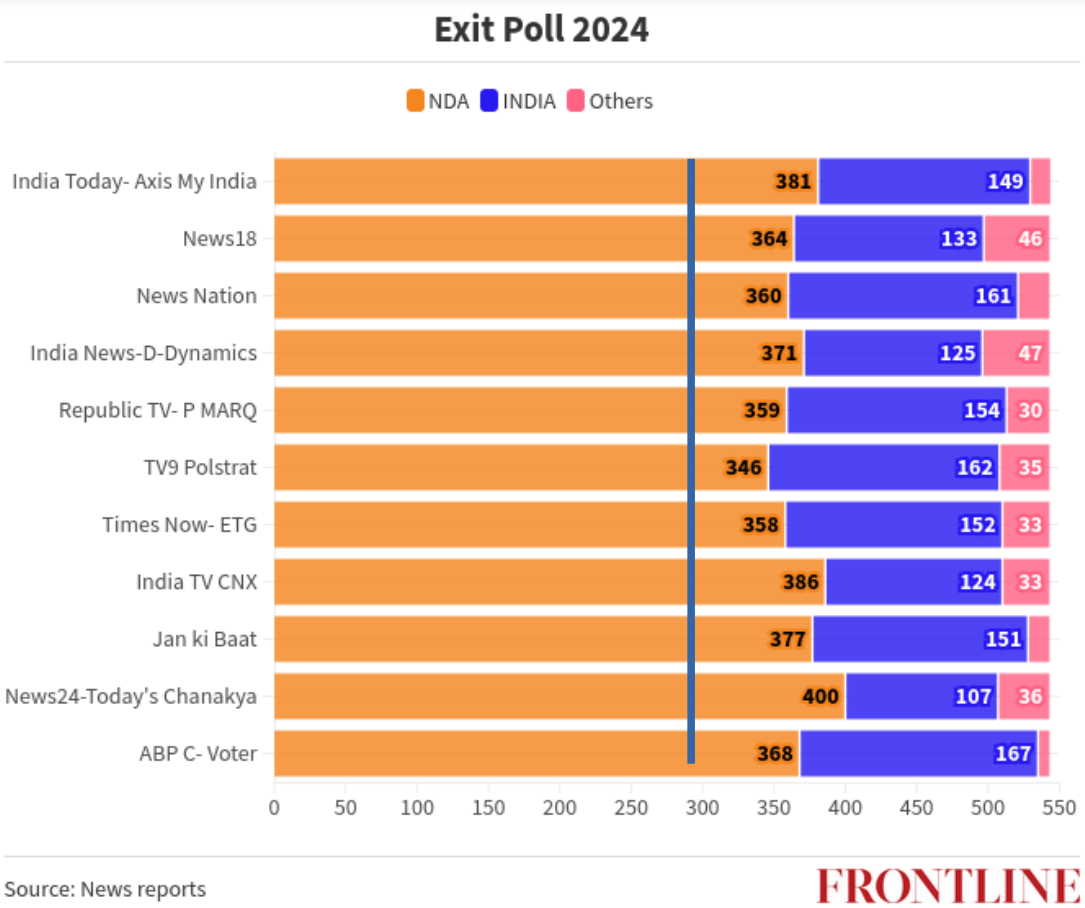
Turnout until 11.45 pm; Source: EC

Indian Express, 21.11.2024

Signs of complexity

whole ≠ sum of parts

Elections as a complex system



US pollsters taking heat – again – for failing to predict Trump triumph

Polling experts called on to explain surveys that showed Trump and Harris deadlocked in a race deemed too close to call

The Guardian Int

6.11.2024

Signs of complexity

Unpredictable

Election Results 2024: Exit polls bite the dust

The performance of the BJP-led NDA was far below the forecast of exit polls, which had given the ruling alliance over 350 seats

Published – June 04, 2024 05:54 pm IST

The Hindu, 4.6.2024

19.4.2019

mint

.29 (0.65%) ↑ | S&P BSE SENSEX 80248.08 +445.29 (0.56%) ↑ | NIFTY 50 24276.05 +144.95 (0.6%) ↑

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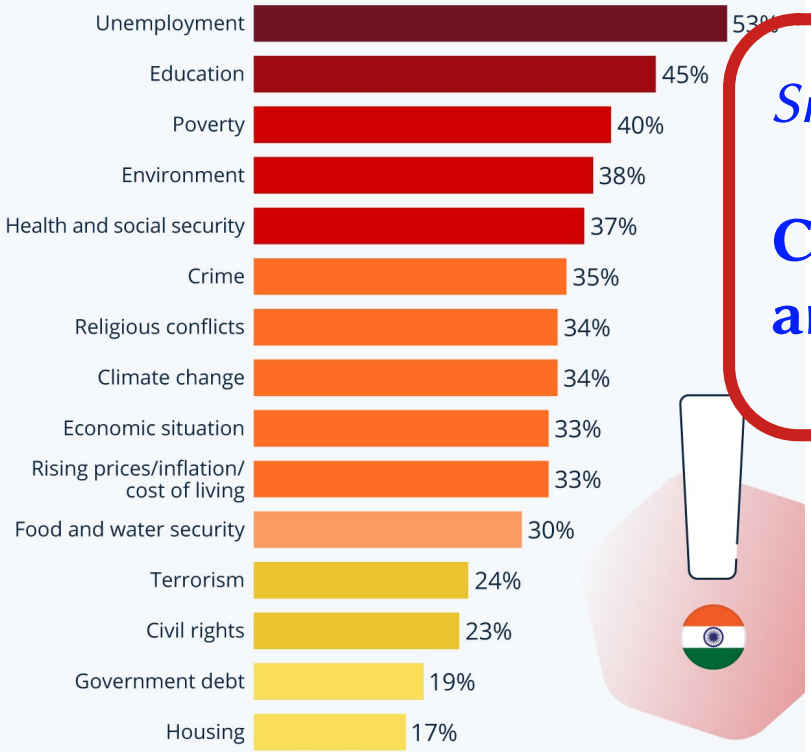
Elections 2019: Caste, coalition equations key factors

Ahead of India's Election, What Do People Care About?

Share of Indian respondents who identified the following issues as among the most important in their country



24,201 respondents (18-54 y/o) surveyed Jan. 2023-Dec. 2023
Source: Statista Consumer Insights

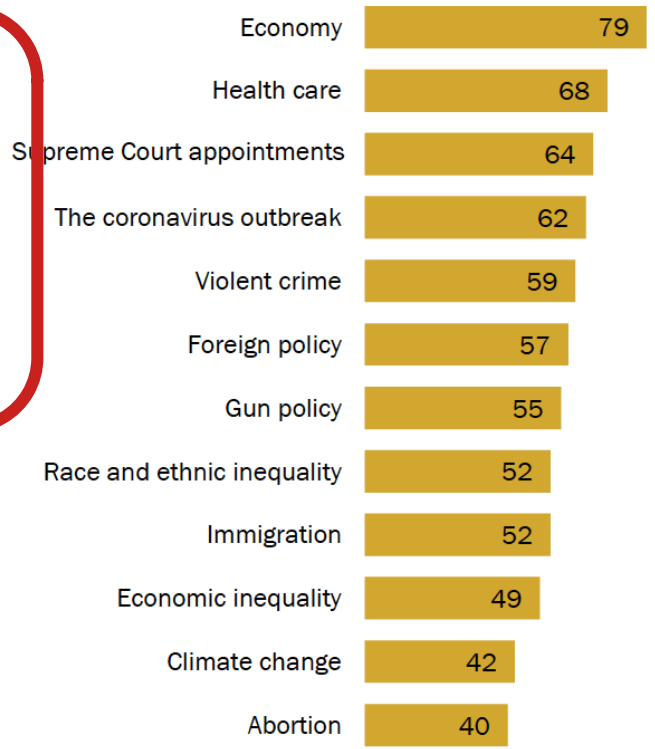
statista

Signs of complexity

Complex interactions among agents

Economy is top issue for voters in the 2020 election

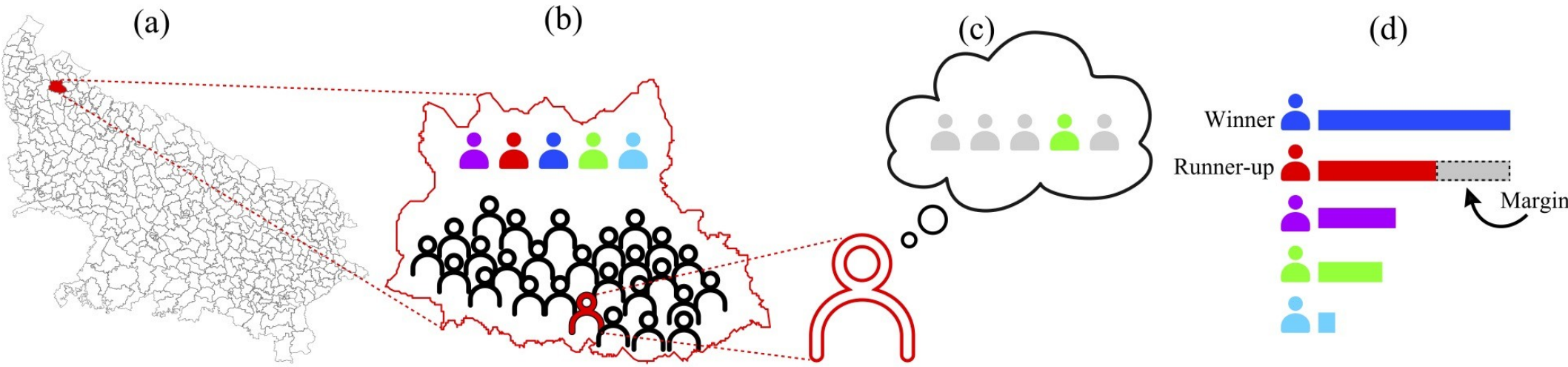
% of registered voters saying each is 'very important' to their vote in the 2020 presidential election



Note: Based on registered voters.
Source: Survey of U.S. adults conducted July 27-Aug. 2, 2020.

PEW RESEARCH CENTER

What is an election ?



N electoral units

Electoral unit

Polling booth
Constituency
County
Precinct

Number of Candidates c_i

Number of voters n_i

Number who actually vote (Turnout) T_i

$i = 1, 2, \ldots N$

A voter can vote for ONLY one candidate

FPTP

Margin M_i

$M_i = v_{i,w} - v_{i,r}$

$0 \leq M_i \leq T_i$

Some motivation from real data :

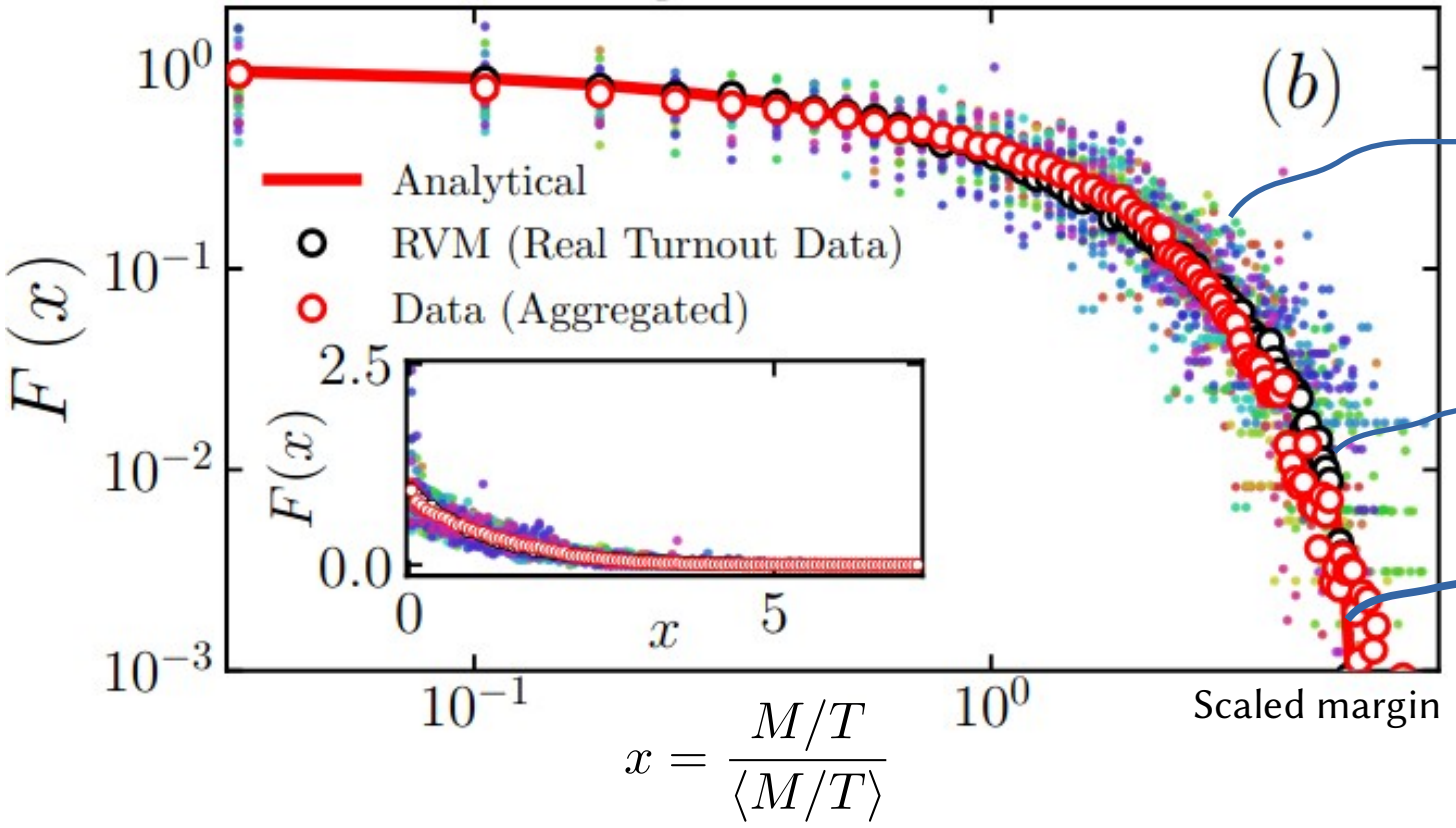
Raichur (2014 GE)		At some polling booth	
▸ Margin	1499	▸ Margin	1499
▸ Turnout	1661606	▸ Turnout	1600
Closely fought election *(winner just scraped through)*		*Landslide election* *(winner dominates)*	

Same margin, but two different scenarios

- **First lesson**

Margin makes sense and is useful only in the context of turnout.

Universality involves both margin and voter turnout



Empirical election data from 34 countries

RandomVoting Model **(RVM)**

Analytical result

Universality observed in data from 34 countries, for elections held over more than a century and for all electoral sizes.

Phys. Rev. Lett. 134, 017201 (2024)

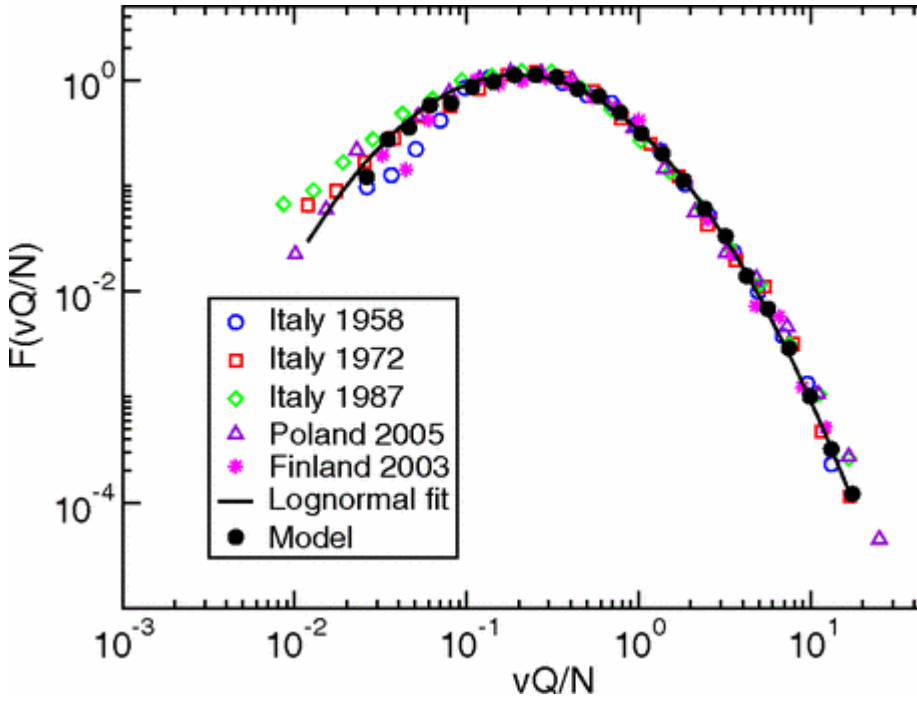
- **Central result**

Distribution of scaled specific margin is *universal* irrespective of when and where elections were held.

Despite attempts, universal behaviour not seen in election data

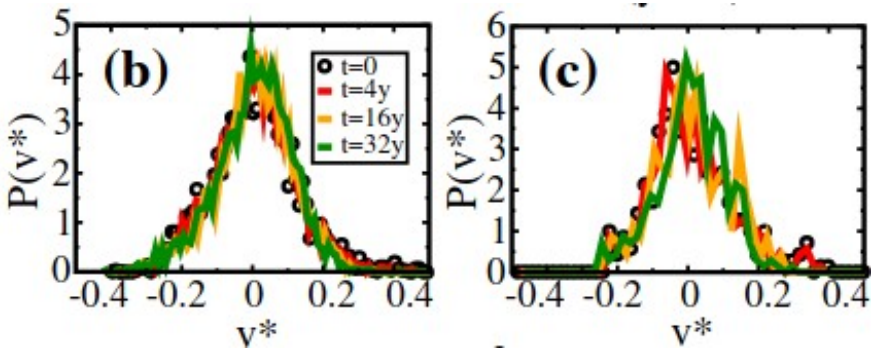
- Scaling and universality in votes obtained by all the candidates in the proportional voting system in Italy, Poland, Brazil and Finland.

  Phys. Rev. Lett. **99**, 138701 (2007)
  Phys. Rev. E **60**, 1067 (1999)
  PLOS One **13**, e0192913 (2018)
  Sci. Rep **3**, 1049 (2013)



- Scaling in scaled vote-share distributions in the US at all scales.

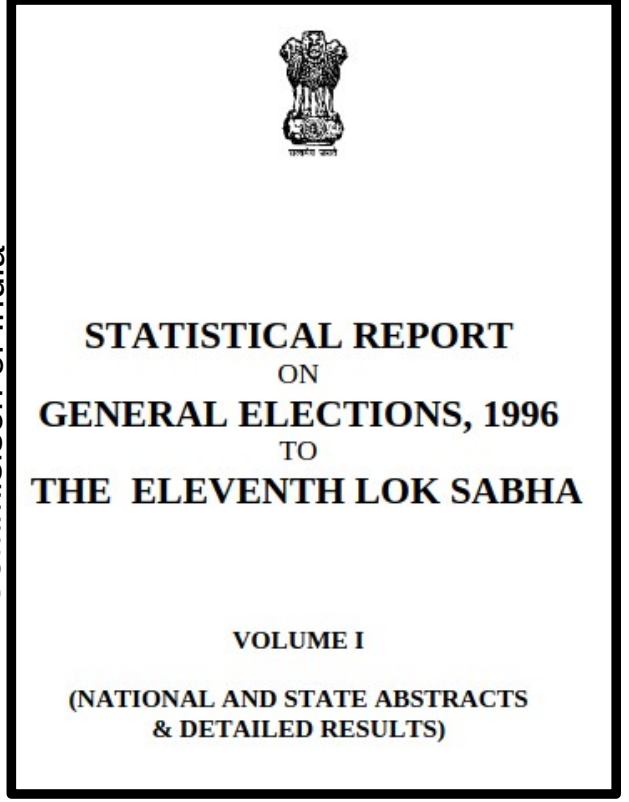
  Phys. Rev. Lett. **112**, 158701 (2014)
  PLOS One **12**, e0177970 (2017)
  Phys. Rev. E **99**, 052307 (2019)



- At best, only limited universal behaviour observed.
- Many more models in social sciences, but not related to universality

Sources of data ?

Election Commission of India www.eci.gov.in
CLEA : www.electiondataarchive.org
MIT Election lab : electionlab.mit.edu
Canada : www.elections.ca



General election : 1996

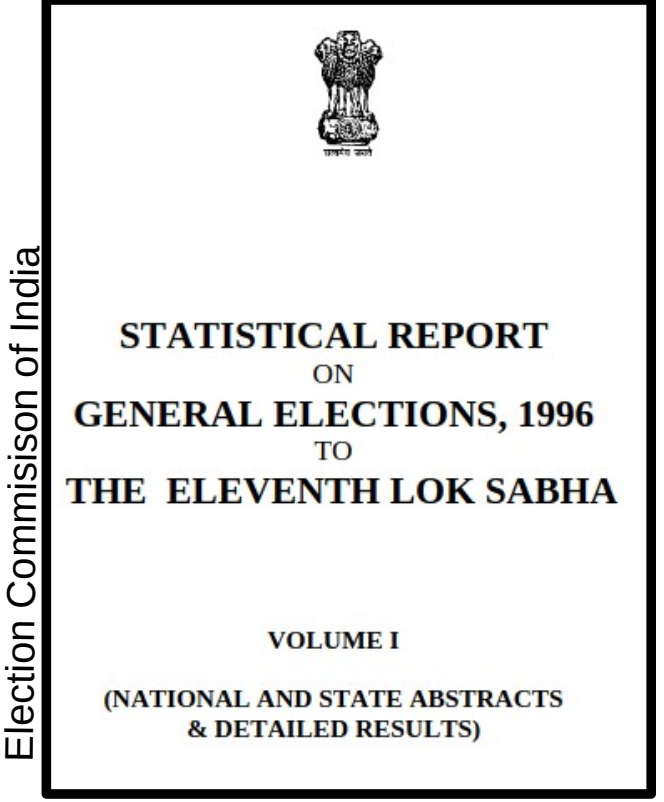
Constituency : **20 . LUCKNOW**

1 .	ATAL BIHARI VAJPAYEE	M	BJP	394865	52.25%
2 .	RAJ BABBAR	M	SP	276194	36.55%
3 .	VED PRAKASH GROVER	M	BSP	42993	5.69%
4 .	OM PATHAK	M	INC	19042	2.52%
5 .	RAMDEV	M	IND	3639	0.48%
6 .	RAJIV JOSHI	M	SMP	2591	0.34%
7 .	ACCHEY LAL BALMIKI	M	AIIC(T)	2316	0.31%
8 .	SIVRAM	M	IND	924	0.12%
9 .	DANESHWAR TIWAR	M	IND	694	0.09%
10 .	CHHEDILAL KURIL	M	IND	642	0.08%

ELECTORS : 1488169	VOTERS : 769886	POLL PERCENTAGE : 51.73%	VALID VOTES : 755746

Sources of data ?

Election Commission of India www.eci.gov.in
CLEA : www.electiondataarchive.org
MIT Election lab : electionlab.mit.edu
Canada : www.elections.ca



General election : 2019

Constituency: 22 . Jadavpur **(Total Electors** 1816857)

SL NO	CANDIDATE NAME	SEX	AGE	CATEGORY	PARTY	Symbol	Votes Secured			% of votes secured	
							GENERAL	POSTAL	TOTAL	Over total elctors in constituency	Over total votes polled in constituency
1	Anupam Hazra	Male	37	general	BJP	Lotus	392610	623	393233	21.64	27.36
2	Bikash Ranjan Bhattacharyya	Male	68	general	CPIM	Hammer, Sickle and Star	301560	704	302264	16.64	21.03
3	Bimal Krishna Mandal	Male	59	sc	BSP	Elephant	5112	2	5114	0.28	0.36
4	Mimi Chakraborty	Female	30	general	AITC	Flowers and Grass	687773	699	688472	37.89	47.9

Random Voting Model (N, n_c, T)

- N: no.of electoral units
- n_c: no.of candidates
- T: Number of voters

- Probability that a *j*-th candidate in *i*-th electoral unit can attract a vote is

$$p_{ij} = \frac{w_{ij}}{\sum_j w_{ij}} \qquad w_{ij} \sim \mathcal{U}(0,1)$$

$$i = 1, 2, \ldots N \qquad j = 1, 2, \ldots, n_c$$

Define how attractive a candidate is to a voter

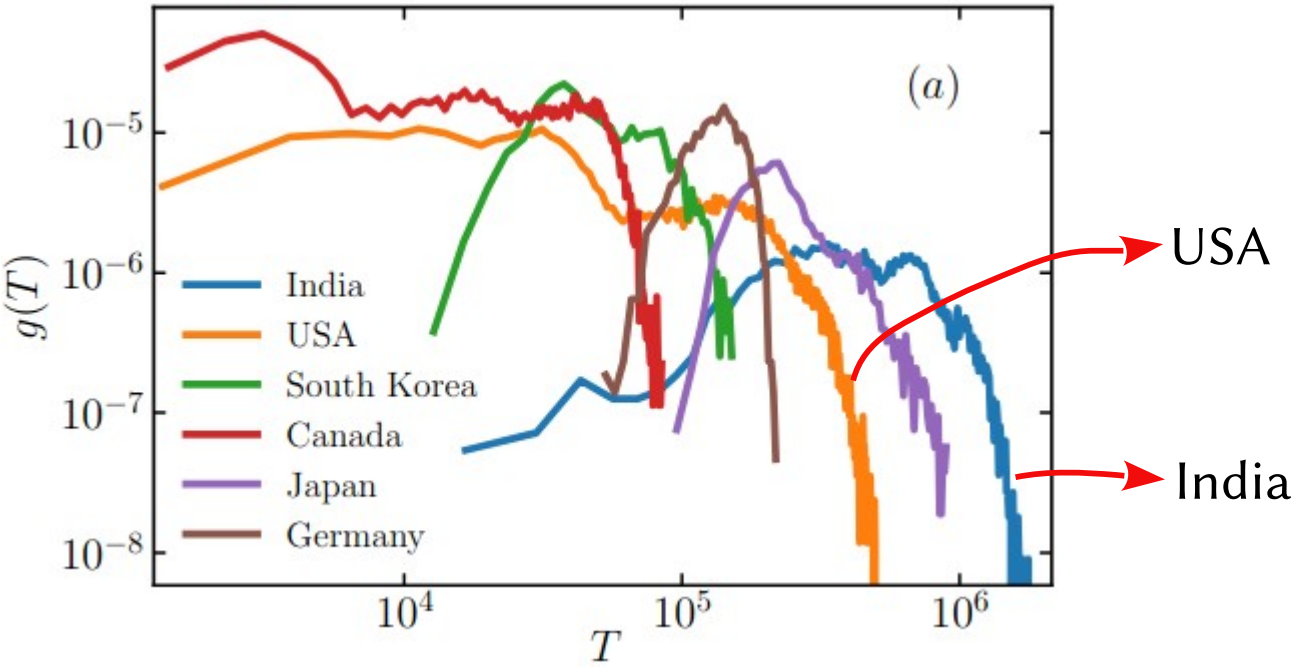
- Each voter votes for *j*-th candidate independently with probability p_{ij}

Each voter makes only one choice

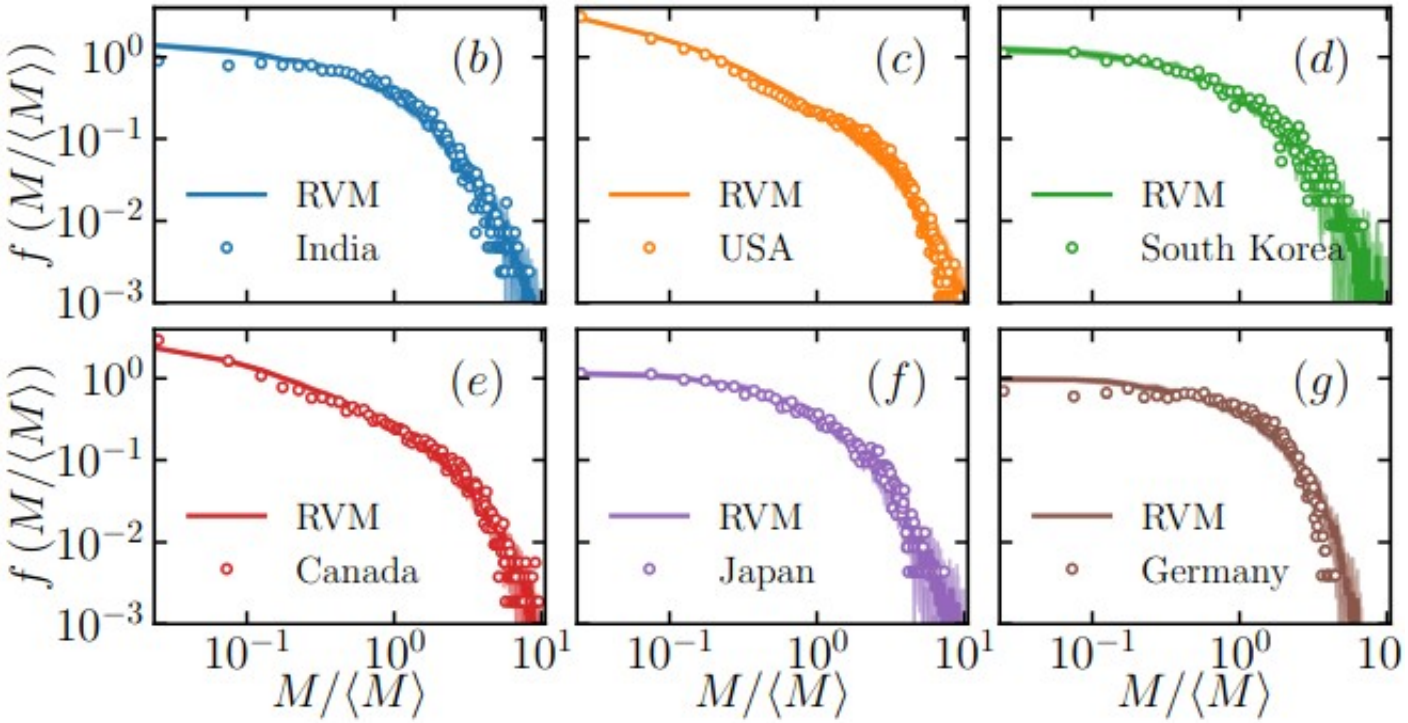
$$M_i = v_{i,w} - v_{i,r}$$

Is this model any good ?

Turnout Distribution



Scaled Margin Distribution



- **INDIA** (1951-2019)

$$\langle T\rangle = 5.69\times10^{5},$$
$$\langle M\rangle = 8.33\times10^{4},$$
$$N = 466$$

- **USA** (1788-2020)

$$\langle T\rangle = 1.14\times10^{5},$$
$$\langle M\rangle = 2.96\times10^{4},$$
$$N = 203$$

- **Germany** (1871-2017)

$$\langle T\rangle = 1.37\times10^{5},$$
$$\langle M\rangle = 2.26\times10^{4},$$
$$N = 268$$

- **Japan** (1947-2017)

$$\langle T\rangle = 2.88\times10^{5},$$
$$\langle M\rangle = 2.35\times10^{4},$$
$$N = 177$$

First lesson

Margin makes sense and is useful only in the context of turnout.

Stronger claim

Turnout distribution drives margin distribution

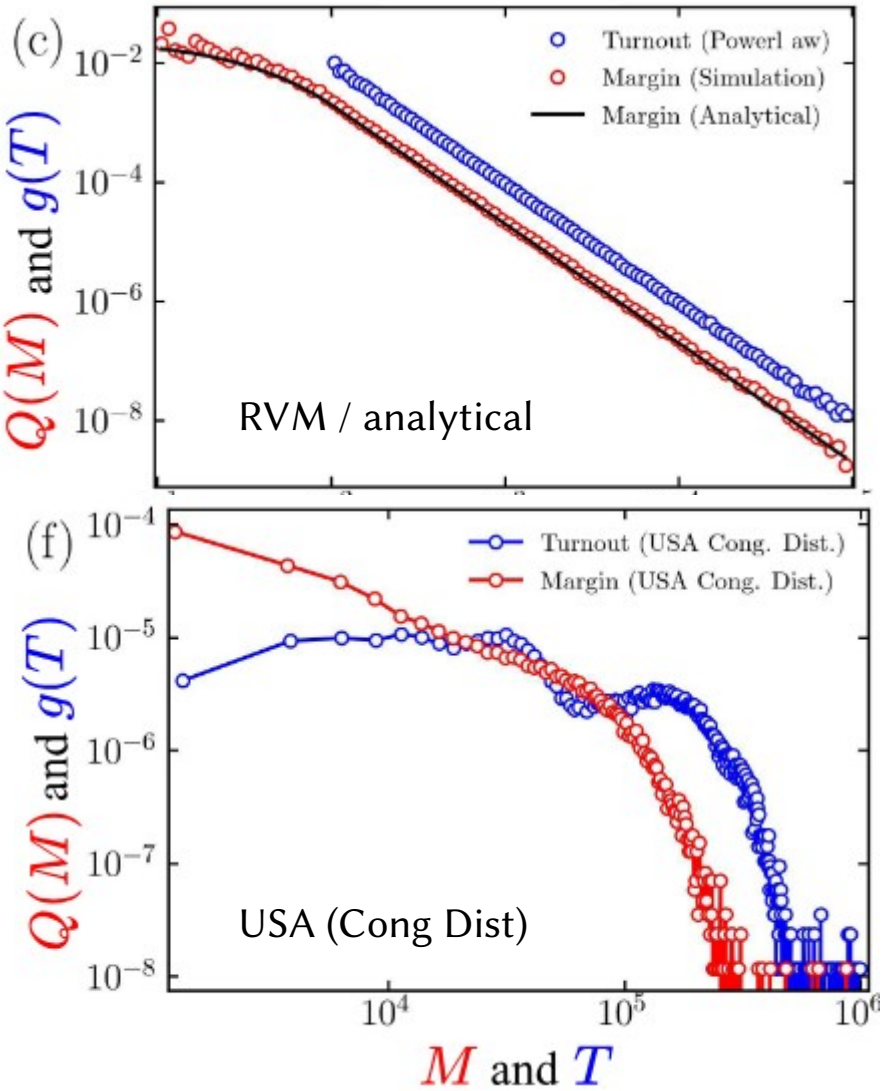
RVM can be analytically solved to show this result in general.

$$g(T) = \frac{\alpha - 1}{T_{\min}^{1-\alpha}} T^{-\alpha}, \qquad \alpha > 1$$

$$Q(M) = C(M)\frac{\alpha - 1}{T_{min}^{1-\alpha}}(M)^{-\alpha},$$

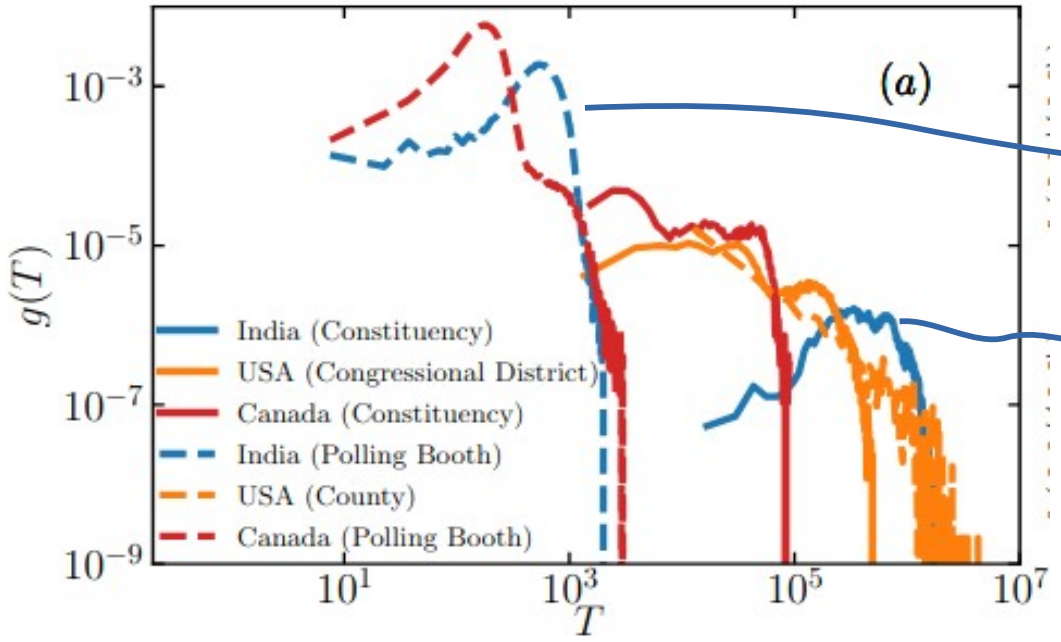
$$C(M) = \begin{cases} I_1(\infty) - I_1(T_{min}/M), \text{if } M \leq T_{min} \\ I_1(\infty) - I_1(1), \text{otherwise,} \end{cases}$$

$$I_1(y) = \int \frac{y^{1-\alpha}(y-1)(5y+7)}{(1+y)^2(2+y)^2} dy,$$



Turnout Distribution

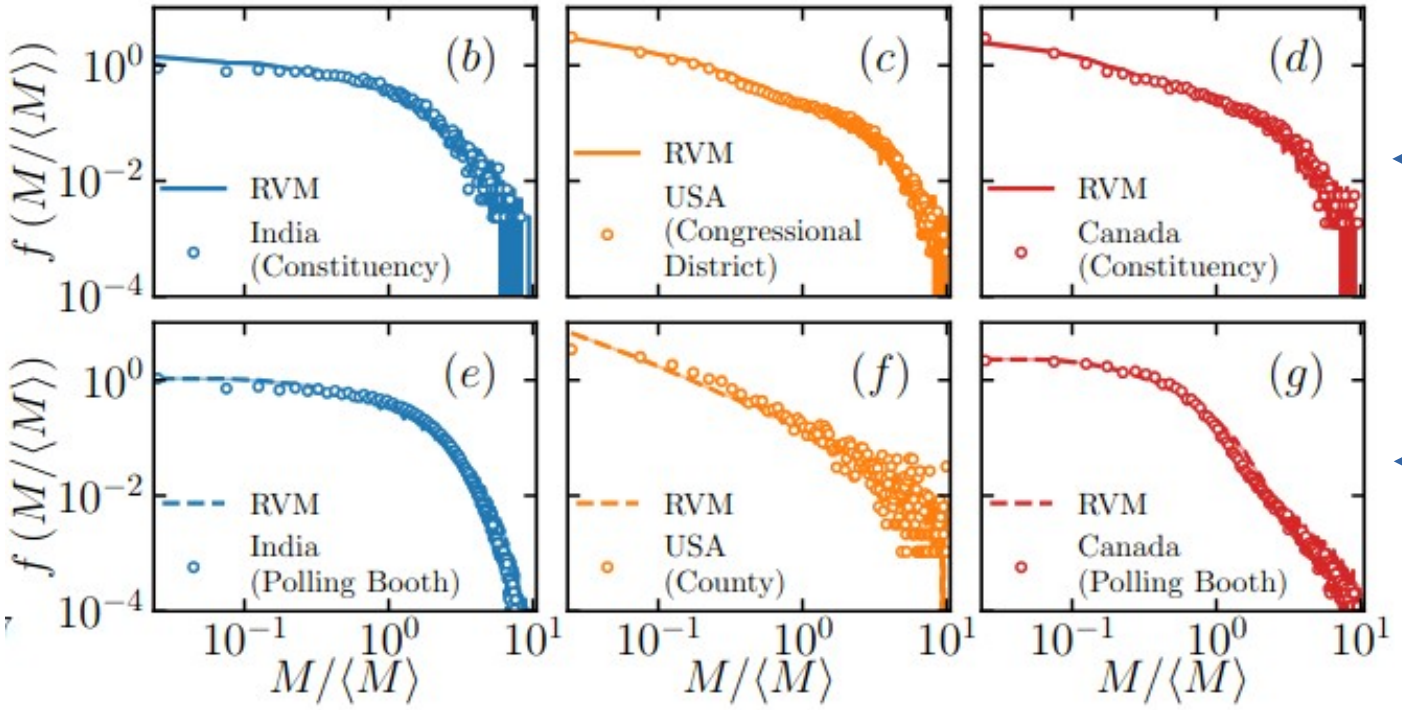
Does this work at all scales ?



At polling booth scale (India)

At constituency scale (India)

Scaled Margin Distribution

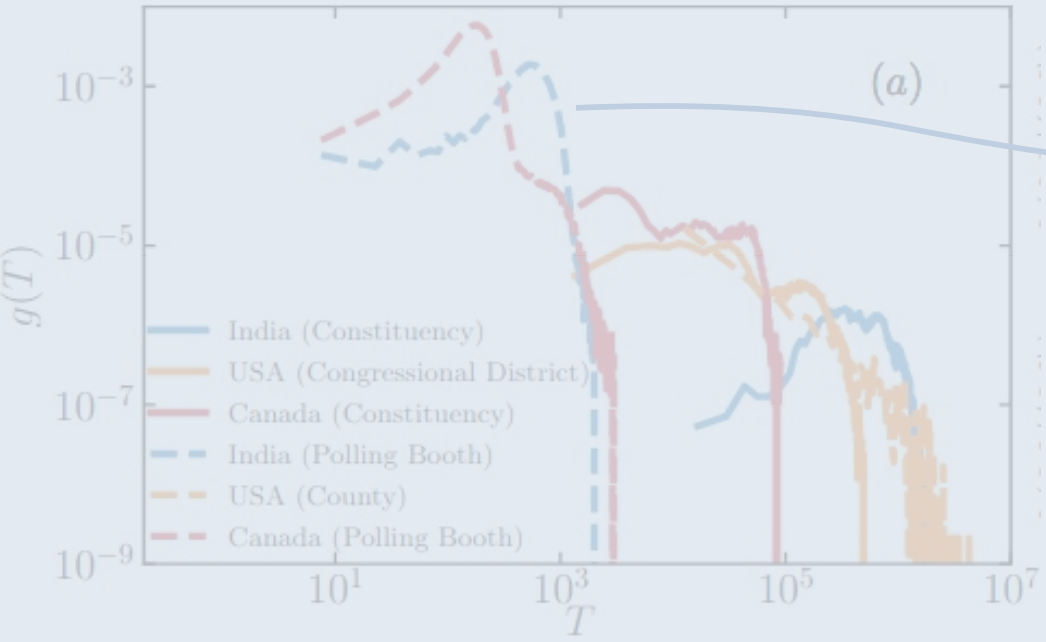


Constituency scale
Size : 10^6

Polling booth scale
Size : 10^3

Turnout Distribution

Does this work at all scales ?

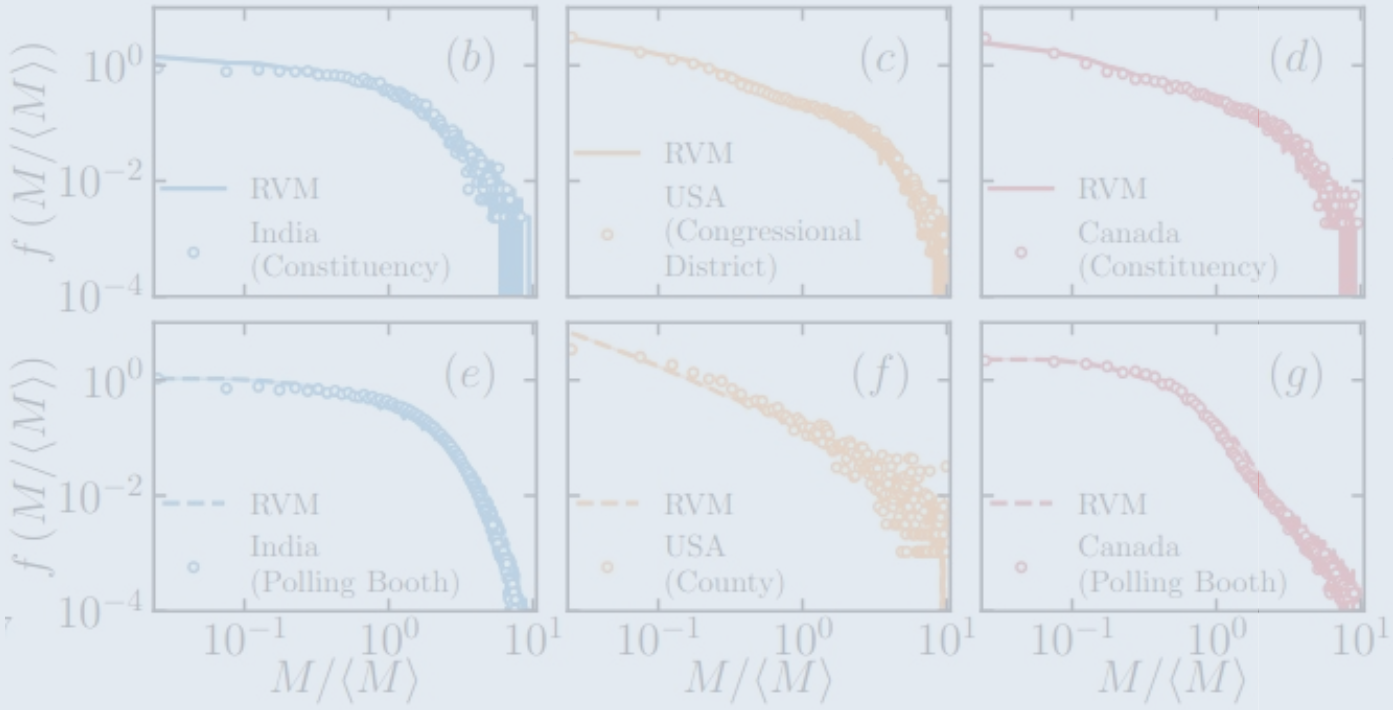


At polling booth scale (India)

More stronger claim

Turnout distribution drives margin distribution *at all electoral scales*

Margin Distribution

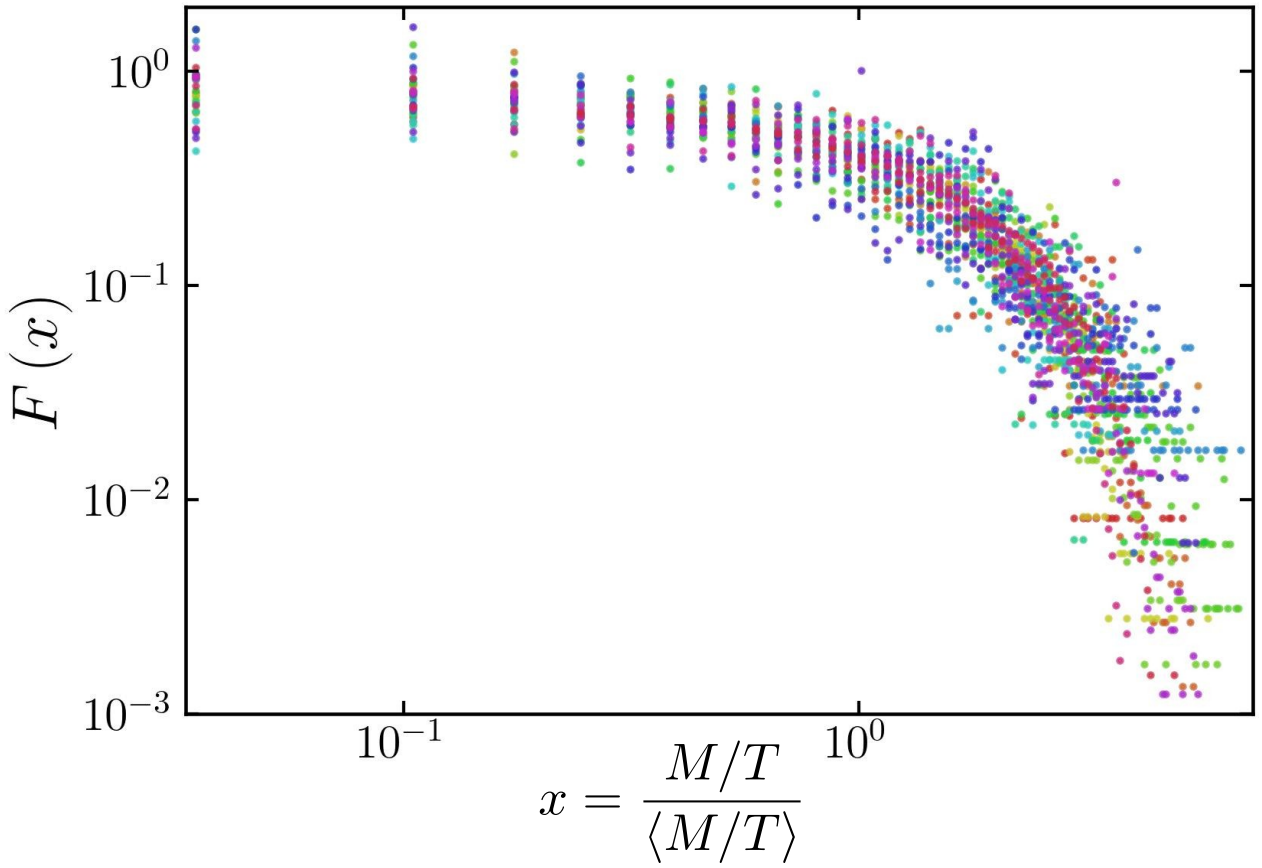


Constituency scale
Size : 10^6

Polling booth scale
Size : 10^3

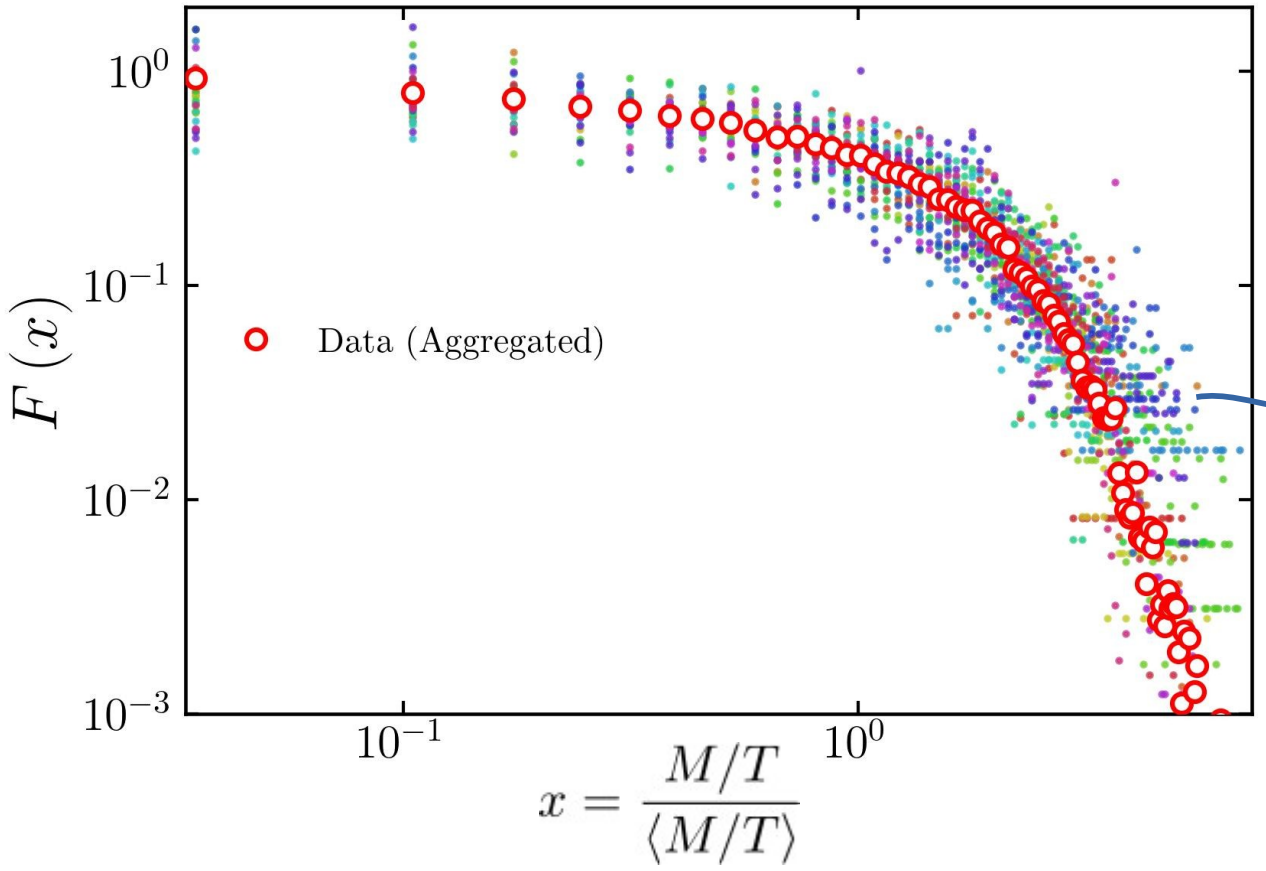
Central result : Scaled margin to turnout ratio is universal

- Empirical election data from 34 countries and over many elections. Each colour represents different country.



Central result : Scaled margin to turnout ratio is universal

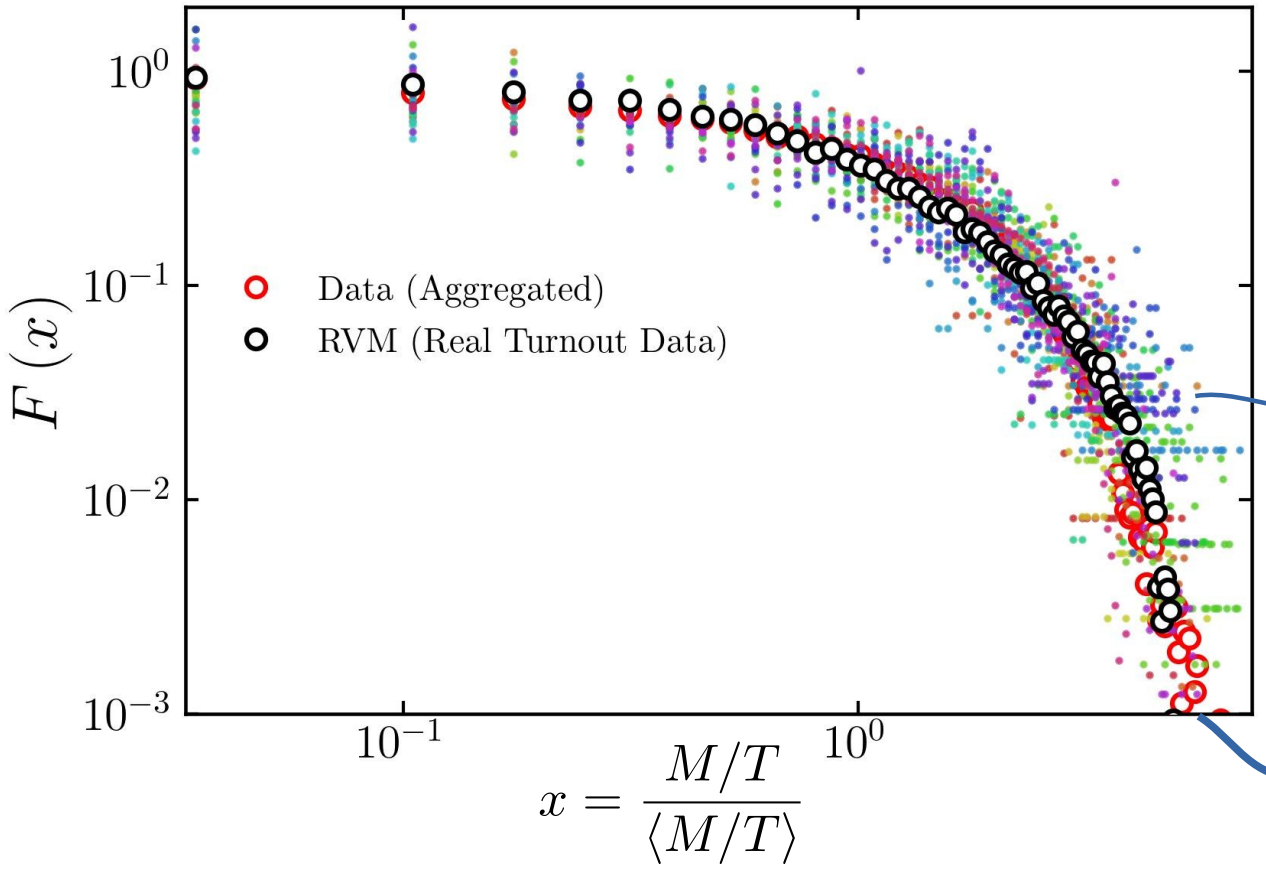
- Aggregated empirical election data from 34 countries.



Empirical election data from 34 countries

Central result : Scaled margin to turnout ratio is universal

- RVM simulations using real turnout data



Empirical election data from 34 countries

Aggregated empirical election data from 34 countries.

Central result : Scaled margin to turnout ratio is universal

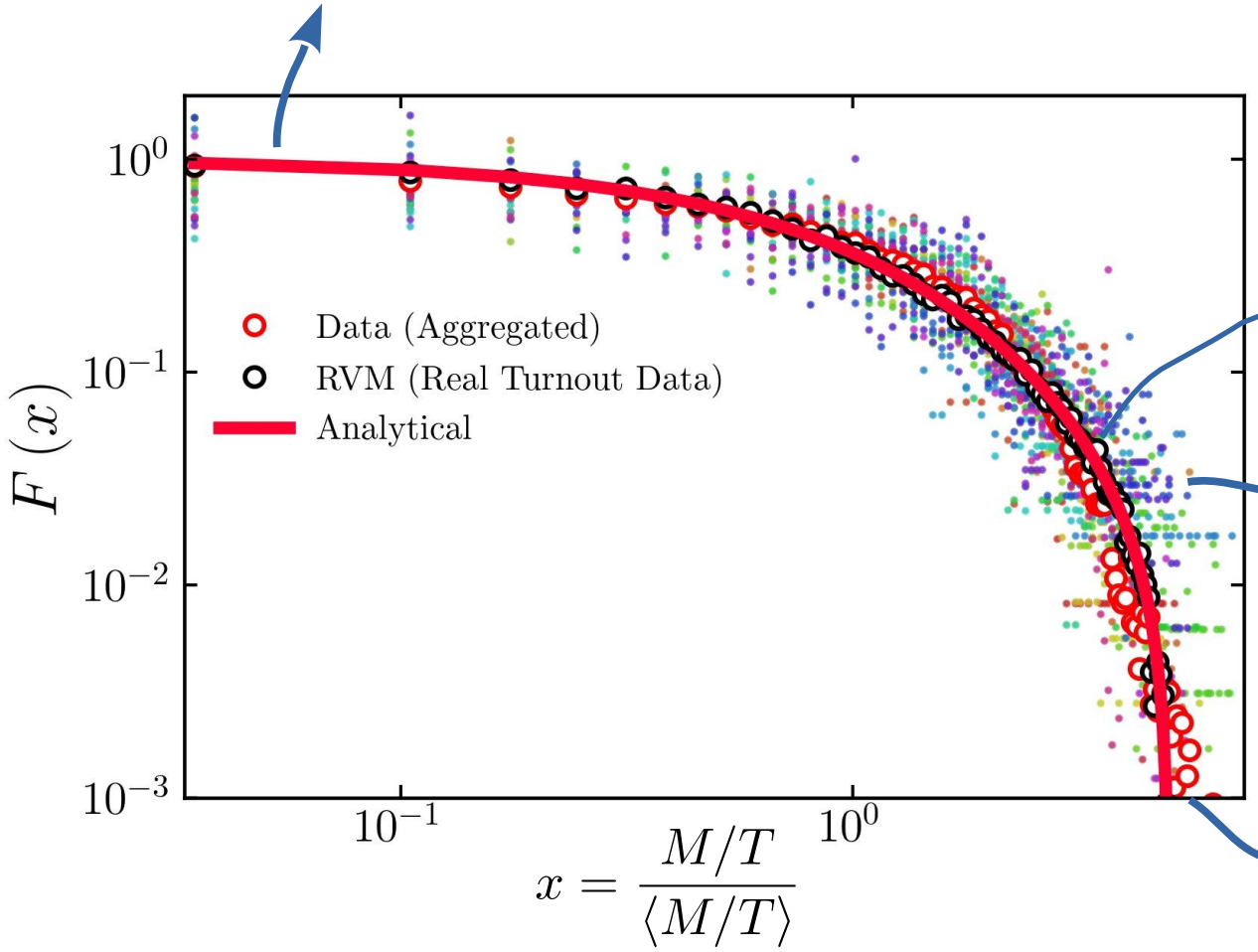
$$\mu = M/T$$

As $T \gg 1$, RVM(2)

$$\begin{cases} F(x) = \langle\mu\rangle P(x\langle\mu\rangle) \\ P(\mu) = \dfrac{(1-\mu)(5+7\mu)}{(1+\mu^2)(1+2\mu)^2} \end{cases}$$

- Analytical scaling function by solving RVM in the limit *T*>>1

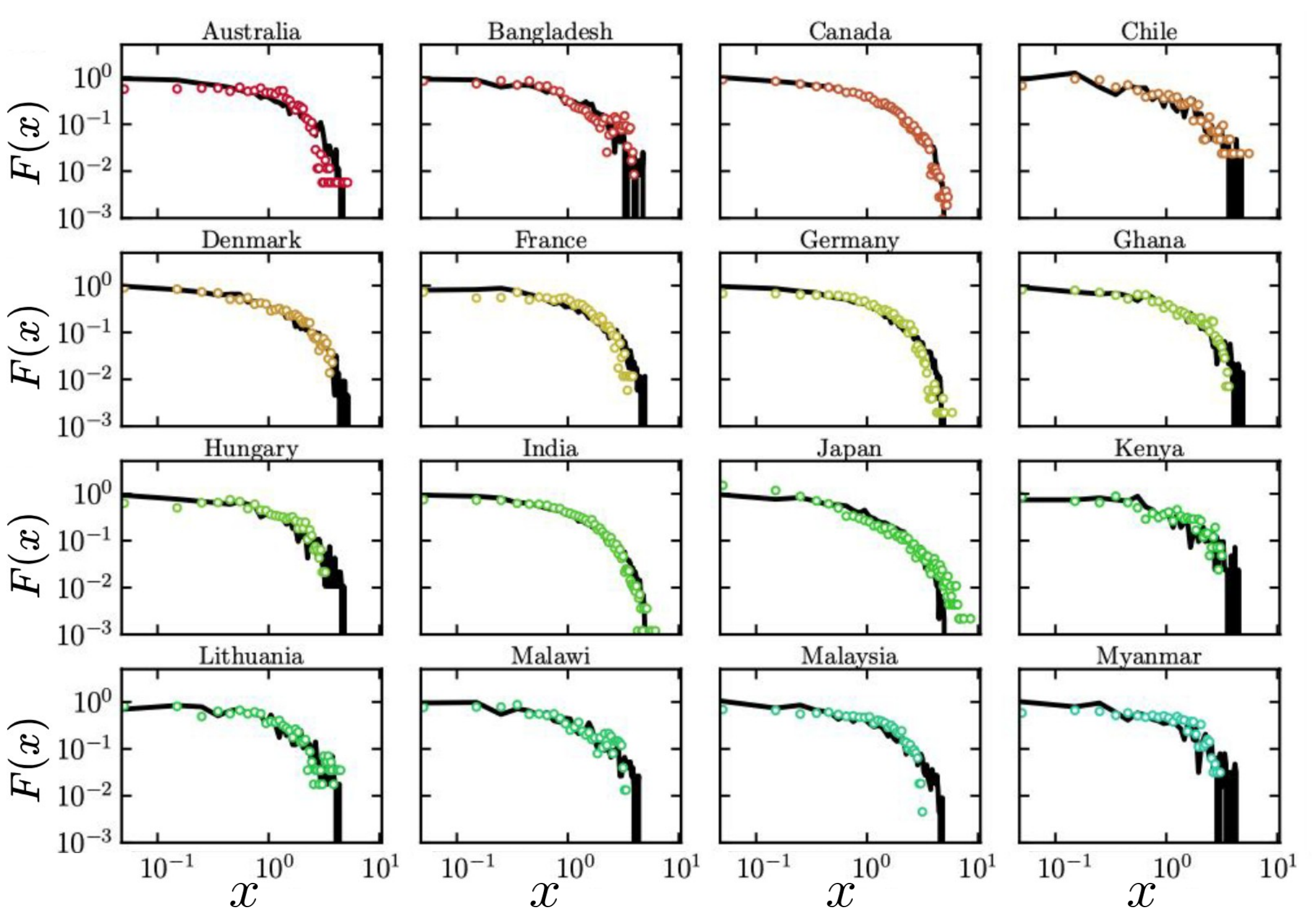
$$\langle\mu\rangle = \frac{1}{2} + \ln\left(\frac{9\times 3^{1/4}}{16}\right)$$



RVM simulations using real turnout data

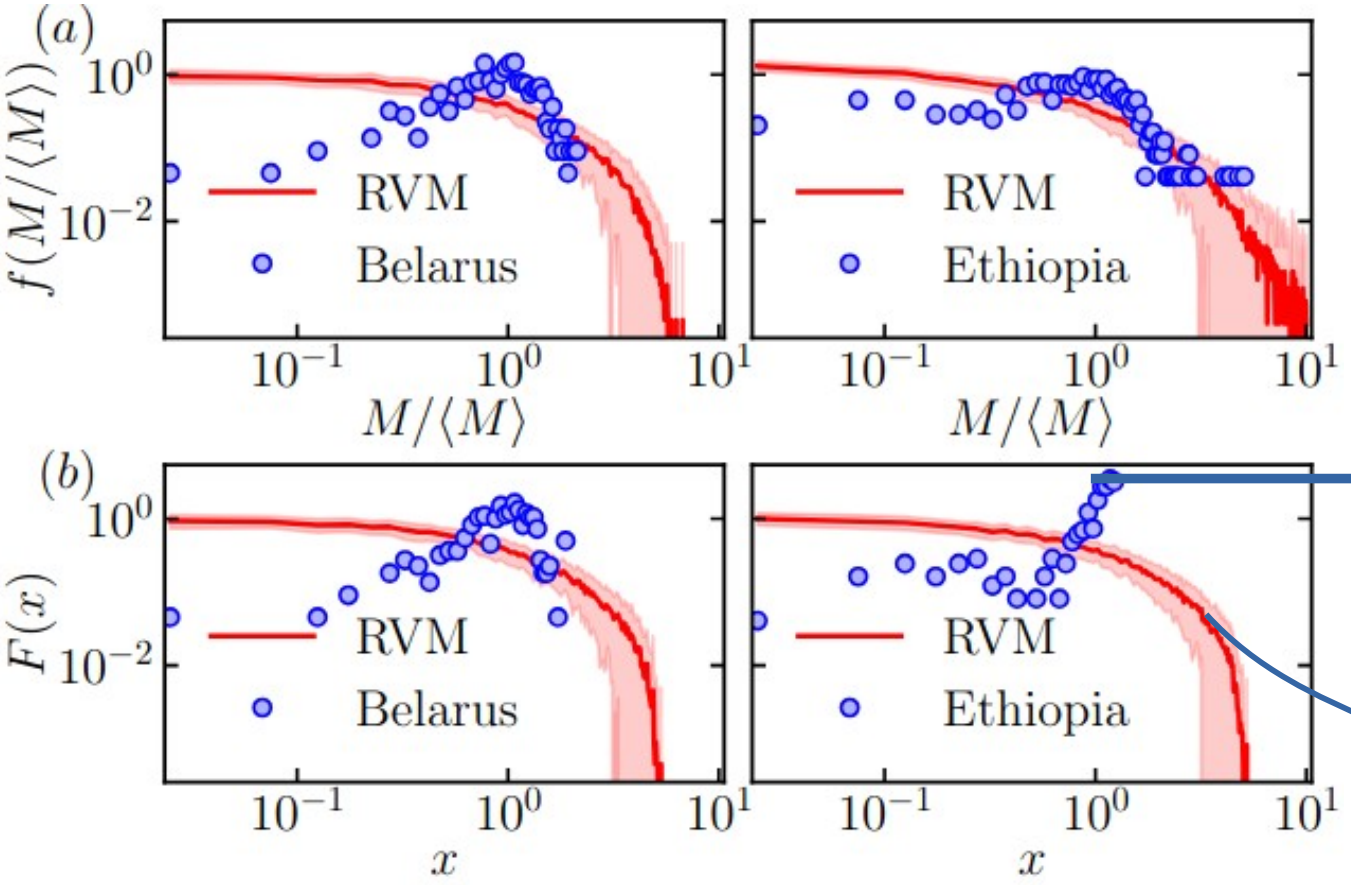
Empirical election data from 34 countries

Aggregated empirical election data from 34 countries.


Australia
Bangladesh
Canada
Chile
Denmark
France
Germany
Ghana
Hungary
India
Japan
Kenya
Lithuania
Malawi
Malaysia
Myanmar
$F(x)$
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-1}
10^{0}
10^{1}
x

What can we do with universality ?

- Deviations from universality indicate large-scale electoral malpractice



Empirical universal curve from election data

Expected universality based on reported voter turnout

BBC

11.8.2020

Belarus election: Opposition disputes Lukashenko landslide win

11 August 2020

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13.8.2020

Leaders | Fraud in Russia's near-abroad

Belarus's election was a sham.

CNN Politics SCOTUS Congress Facts First 2024 Elections

9.8.2023

Biden administration takes action against Lukashenko regime on third anniversary of fraudulent election

By Jennifer Hansler, CNN

3 minute read · Updated 4:46 PM EDT, Wed August 9, 2023

EU observers say Ethiopia election 'falls short'

25 May 2010

25.5.2010

25.5.2010

The Guardian Int

Ethiopia election marred by intimidation, say rights group

Human Rights Watch says poll, won by Ethiopian People's Revolutionary Democratic Front, was corrupted by threats

RESEARCH HIGHLIGHT | 10 January 2025

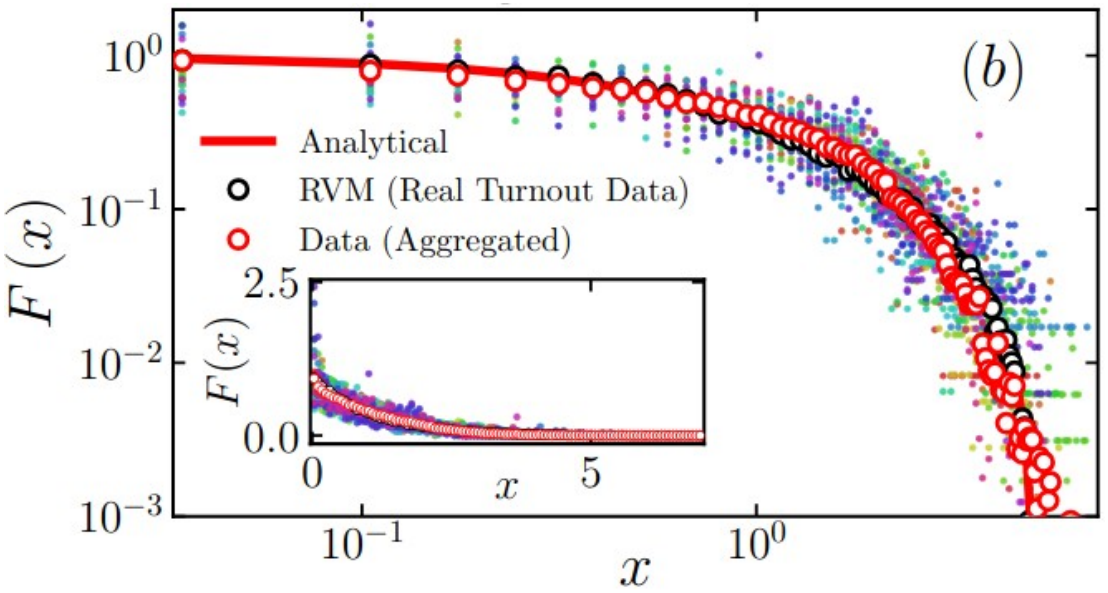
Voter turnout drives margins of victory — if elections are fair

Model that predicts the spread of winning margins could be used to detect electoral interference.

IISER's global study unveils universal poll pattern, may aid in fraud detection

Summary

- Despite the noise associated with elections across the world, there is underlying order *in fair elections*.
- Given the raw turnout data, Random Voting Model captures the margin distribution
- The observed universality must be regarded as stylized fact of elections. Can be used to flag large-scale election malpractices.



For lot more details, visit

electioninsights.in