Teaching mathematics using Lean

Patrick Massot

Université Paris-Saclay at Orsay
Hoskinson Center for formal mathematics, Carnegie-Mellon University

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Teaching Lean vs Teaching using Lean

Let's use Lean with our students!

Important question: what is the end goal?

- Lean?
- Logic?
- Math?

General resource

`https://leanprover-community.github.io/teaching/index.html`
contains links to resources

General context of this talk

Use ITP to teach mathematics reasonning to 1st year students

Goals:

- Improve reading and writing of proofs on paper

Non-goals:

- type theory
- abstract logic
- expert use of a proof assistant

Similar goals: Heather Macbeth at Fordham.

Specific goals

- Make sure the proof state is clear: current assumptions and goal.
- Make sure to distinguish what is stated, proven, used.
- Make sure students can react to goal shape.
- Handle bound and free variables.
- Identify risky/irreversible steps.

Experimental context

A course in Orsay

- Title: Computer assisted logic and proofs
- 60 good students enrolled in first year of double-major Math and Computer Science
- 12 times 2 hours of computer lab, 3 groups of 20 students
- Second semester, after one semester of calculus (with ε and δ by teacher but not in exam)
- Main math focus: ε and δ proofs

Challenges

Spent two years using normal Lean syntax as input.

Students liked it, but:

- sometimes Lean was too smart or not enough smart;
- skill transfer to paper was disappointing.

Verbose Lean: use controlled natural language input as front-end and dedicated teaching tactics as back-end.

After four years using Lean 3, make use of Lean 4 flexibility.

Very similar goals are shared by Rocq waterproof

Important note

The goal is not to make input easier.

The goal is to improve skill transfer to paper proofs.

Automation

Automation is crucial: don't want to ask for details we wouldn't want to see on paper.

Automation in Verbose Lean is built on top of Mathlib tactics, and customizable.

Verbose Lean supports several proof styles interpolating between standard Lean proofs and paper proofs.

Demo

Let's see it.

Challenges

- Slight cognitive dissonance introduced by controlled natural language
- Balance expressivity of tactics and requirements on student memory
- Hard to specify the expected automation level
- Some rare difficulties with foundations (the coercion of $\mathbb{N}$ to $\mathbb{R}$ is hard to avoid completely)

Traditional teaching

This experiment reveals things we do wrong at the blackboard. We could:

- Banish truth tables
- Make sure to distinguish:
  - Constructing a statement
  - Proving a statement
  - Using a statement

Examples

- Use the $\Rightarrow$ symbol (and the word "imply") only for implication and not for modus ponens which uses "therefore" or "hence".
- Don't start a *proof* of $\forall x \in \mathbb{R},\ P(x)$ by "For all real number x, ..." but use "Let x be a real number."
- Don't mix up stating the existence of something and extracting a witness: "By assumption on f, there exists δ such that ..." vs "The assumption on f gives δ such that ...".
- Do not put back useless quantifiers.