The Influence of Noise and Monitoring On Symmetry Breaking and Chaos

Adolfo del Campo

UNIVERSITÉ DU LUXEMBOURG

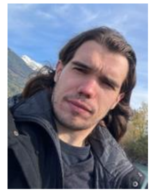
ICTS Bengaluru, Feb 5th 2025

Group at Luxembourg

Current members

OPEN
PHD
POSITION
APPLY NOW


K. Takahashi


A. Grabarits


S. Shinn

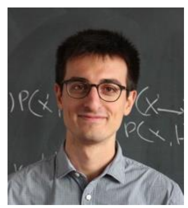

K. R. Swain

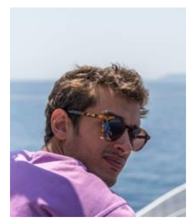

M. Massaro

Past recent members


Jing Yang
Nordita


M. Thudiyangal
MAHE


F. Balducci
MPI-PKS


A. Matsoukas-Roubeas
U. of Cambridge


L. Dupays
King's College

Collaborator Aurelia Chenu

AURELIA'S QUANTUM LAB

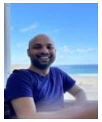
uni.lu UNIVERSITÉ DU LUXEMBOURG

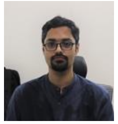
Grateful for collaborations and friendships








…

AdC, M. G. Boshier, **A. Saxena**, Sci. Rep. 4, 5274 (2014)

AdC, **K. Sengupt**a, Eur. Phys. J. Special Topics 224, 189 (2015)

G. Watanabe, **B. Prasanna Venkatesh**, P. Talkner, AdC PRL. 118, 050601 (2017)

G Watanabe, **B. Prasanna Venkatesh**, P Talkner, MJ Hwang, AdC PRL. 124, 210603 (2020)

Revathy B. S., Victor Mukherjee, Uma Divakaran, AdC PRR 2, 043247 (2020)

P. Chandarana, N. N. Hegade, K. Paul et al. PRR 4, 013141 (2022)



P Martinez-Azcona, **Aritra Kundu, A. Saxena**, ADC, A Chenu, arXiv:2407.09316; PRL. 131, 160202 (2023)

Mithun Thudiyangal, AdC PRR. 6, 033152 (2024)

P Nandy et al arXiv:2405.09628

Pranav Chandarana, Koushik Paul, Kasturi Ranjan Swain, Xi Chen, AdC arXiv:2409.12525

Budhaditya Bhattacharjee, K. Takahashi, AdC, ongoing

Preethi Gopalakrishnan, starting …

Open Quantum Systems

System of interest embedded in an environment: composite system-environment

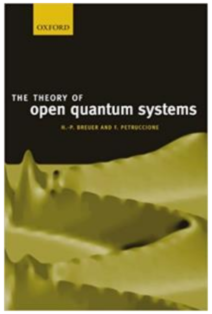
$$\rho(t) = \hat{U}_{SE}(t,0)\rho_S(0) \otimes \rho_E \hat{U}_{SE}(t,0)^\dagger$$

Reduced dynamics via master equation

$$\frac{d}{dt}\rho_S = \frac{-i}{\hbar}\left[\hat{H}_S, \rho_S\right] + \mathcal{D}(\rho_S)$$

Markovian limit: Universal Lindblad form

$$\frac{d}{dt}\rho_S = \frac{-i}{\hbar}\left[\hat{H}_S, \rho_S\right] + \sum_\alpha \gamma_\alpha \left[L_\alpha \rho_S L_\alpha^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger L_\alpha, \rho_S\right\}\right]$$



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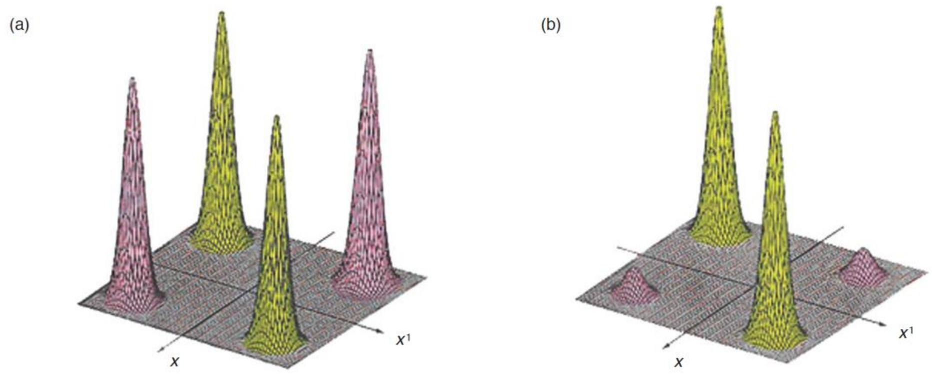
Open Quantum Systems: Decoherence

Decay of coherences of density matrix, e.g. of a Schrodinger cat state



Quantum Brownian Motion

$$\frac{d}{dt}\rho_S(t) = \frac{1}{i\hbar}[H,\rho_S(t)] - \frac{i\gamma}{\hbar}[x,\{p,\rho_S(t)\}] - \frac{2m\gamma k_B T}{\hbar^2}[x,[x,\rho_S(t)]]$$

Decoherence time in the high-temperature limit

$$\rho_S(x,x';t) \approx \rho_S(x,x';0)e^{-t2m\gamma k_B T(x-x')^2/\hbar^2} \qquad \tau_D = \frac{\lambda_\beta^2}{2\gamma\Delta x^2}$$

e.g. Zurek, Physics Today

Decoherence from Quantum Decay: Purity

Purity

$$P_t = \mathrm{tr}\rho_S^2 \in [1/d, 1]$$

Master equation

$$\frac{d}{dt}\rho_S = \frac{-i}{\hbar}\left[\hat{H}_S, \rho_S\right] + \sum_\alpha \gamma_\alpha \left[L_\alpha \rho_S L_\alpha^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger L_\alpha, \rho_S\right\}\right]$$

Short time decay

$$P_t = P_0[1 - Dt + \mathcal{O}(t^2)]$$

Universal decoherence time & rate

$$D = \frac{2}{P_0}\frac{1}{\tau_D} \qquad \tau_D = \frac{1}{\sum_\alpha \gamma_\alpha \mathrm{Cov}\left(L_\alpha, L_\alpha^\dagger\right)}$$

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M. Beau, J. Kiukas, I. L. Egusquiza, AdC, Phys. Rev. Lett. 119, 130401 (2017)

Stochastic Hamiltonians

Full system

Deterministic part + stochastic part with real Gaussian process

$$H(t) = H_0(t) + \gamma(t)V$$

Stochastic Schrodinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = [H_0(t) + \gamma(t)V]\,|\psi(t)\rangle$$

Extensive literature

G. J. Milburn, PRA 44, 5401 (1991)
H. Moya-Cessa, V. Bužek, M. S. Kim, and P. L. Knight, PRA 48, 3900 (1993)
A. Budini, PRA 64, 052110 (2001)
[…]
A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

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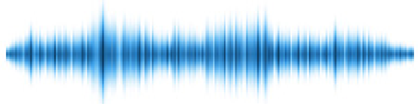
Exact Noise-Averaged dynamics

Density matrix averaged over realizations



$$\rho(t) = \langle \rho_{\rm st}(t) \rangle = \Big\langle |\psi(t)\rangle\langle\psi(t)| \Big\rangle$$

Master equation requires stochastic unravelling

$$\frac{d}{dt}\rho(t) = -i[H_0(t),\rho] - \int_0^t ds \langle \gamma(t)\gamma(s)\rangle \Big[V, \langle [\hat{U}_{\rm st}(t,s) V \hat{U}_{\rm st}^\dagger(t,s), \rho_{\rm st}(t)]\rangle \Big]$$

Simplified via Novikov's theorem for white noise $\langle \gamma(t)\gamma(t')\rangle = W^2 \delta(t-t')$

$$\frac{d}{dt}\rho(t) = -i[H_0(t),\rho(t)] - \frac{W^2}{2}[V,[V,\rho(t)]]$$

For any W

A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

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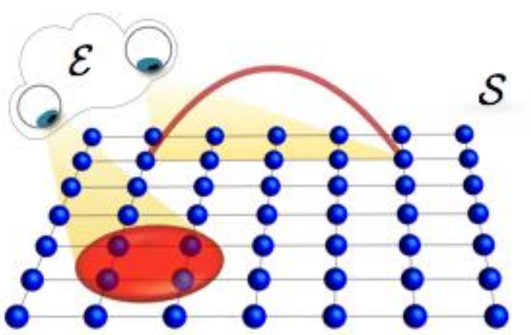
Experimental tests?

Quantum simulation of open systems with many-body dissipators

Target Hamiltonian: Ising chain

$$\hat{H}_I = -\sum_{i<j} J_{ij}\, \sigma_i^z \sigma_j^z - h \sum_{i=1}^{N} \sigma_i^x .$$



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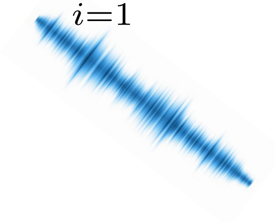

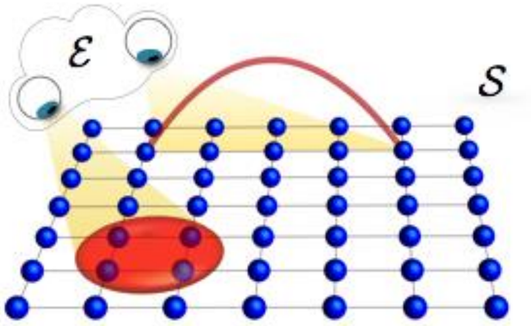

A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

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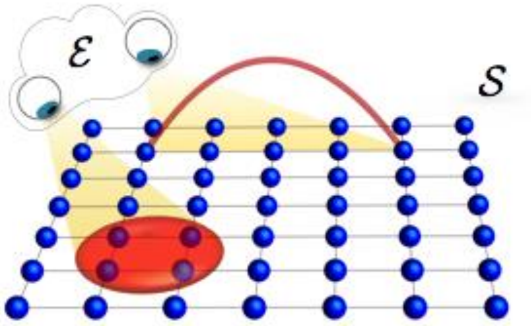
Modulating magnetic field

$$\mathcal{D}[\rho(t)] = -\gamma \sum_{ij} [\sigma_i^x [\sigma_j^x, \rho(t)]]$$



Nonlocal "2-body" dissipator

$$\tau_D \sim 1/N^2$$

A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

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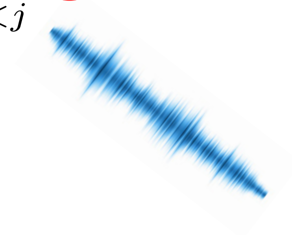
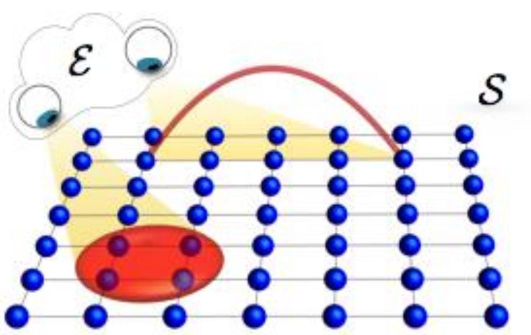




A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

Experimental tests?

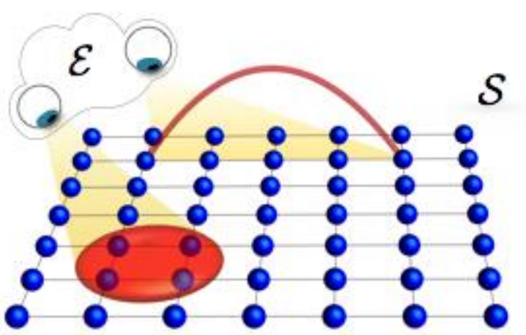
Quantum simulation of open systems with many-body dissipators

Target Hamiltonian: Ising chain

$$\hat{H}_I = -\sum_{i<j} J_{ij}\, \sigma_i^z \sigma_j^z - h \sum_{i=1}^{N} \sigma_i^x .$$

Modulating ferromagnetic couplings

$$\mathcal{D}[\rho(t)] = -\gamma \sum_{i<j} \sum_{i'<j'} \left[\sigma_i^z \sigma_j^z, \left[\sigma_{i'}^z \sigma_{j'}^z, \rho(t)\right]\right]$$



Nonlocal “4-body” dissipator

$$\tau_D \sim 1/N^4$$

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A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

Stochastic k-body Hamiltonians

Stochastic Hamiltonian

$$\hat{H}_S(t) = \hat{H}_T(t) + \sum_\alpha \lambda_\alpha(t)\,\hat{L}_\alpha$$

k-body operators

$$\hat{L}_\alpha = \sum_{i_1<\cdots<i_k} \mathbb{L}^{(\alpha,k)}_{i_1,\ldots,i_k}$$

“2k-body” dissipators

$$\mathcal{D}(\rho) = -\sum_\alpha \sum_{i_1<\cdots<i_k} \sum_{i'_1<\cdots<i'_k} \frac{\gamma_\alpha}{2}[\mathbb{L}^{(\alpha,k)}_{i_1,\ldots,i_k}, [\mathbb{L}^{(\alpha,k)}_{i'_1,\ldots,i'_k}, \rho]]$$

Double sum over indices vs usual single sum ~ correlated environment

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A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

Stochastic k-body Hamiltonians

Stochastic Hamiltonian

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k-body operators

$$\hat{L}_\alpha = \sum_{i_1<\cdots<i_k} \mathbb{L}^{(\alpha,k)}_{i_1,\ldots,i_k}$$

"2k-body" dissipators

$$\tau_D \sim 1/N^{2k}$$

Polynomial scaling

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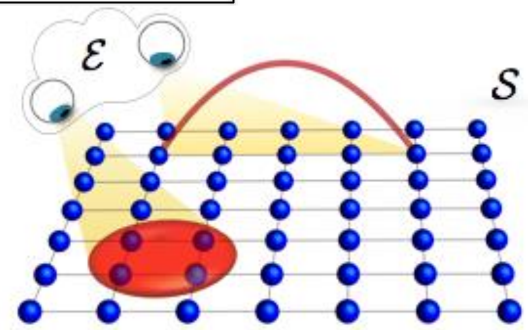
A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

Dephasing dynamics

Master equation with Hermitian Lindblad op

$$\frac{d}{dt}\rho(t) = -i[H_0(t), \rho(t)] - \frac{W^2}{2}[V, [V, \rho(t)]]$$

Decoherence rate scaling for k-body Lindblad op



$$\tau_D \sim 1/N^{2k}$$

Polynomial scaling in system size

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A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

Dephasing dynamics

Master equation with Hermitian Lindblad op

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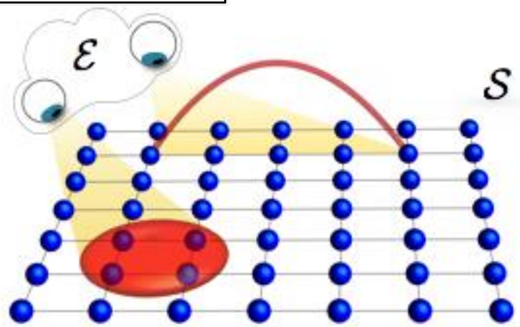
Decoherence rate scaling for k-body Lindblad op



What are the ultimate bounds to the decoherence rates?

A. Chenu, M. Beau, J. Cao, AdC, Phys. Rev. Lett. 118, 140403 (2017)

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Decoherence rate in RMT: GUE

Noise-averaged master equation

$$\frac{d}{dt}\rho(t) = \frac{1}{i\hbar}[\hat{H}_T, \rho(t)] - \sum_\alpha \lambda_\alpha(t)[\hat{L}_\alpha, [\hat{L}_\alpha, \rho(t)]]$$

Decoherence rate

$$D = \frac{2}{P_0}\frac{1}{\tau_D} = \sum_\alpha \lambda_\alpha(t)\Delta\hat{L}_\alpha^2$$

Ensemble average →

$$\hat{L}_\alpha \in \mathrm{GUE}(d)$$

$$D_{\mathrm{GUE}} \sim \Gamma d \sim 2^N$$

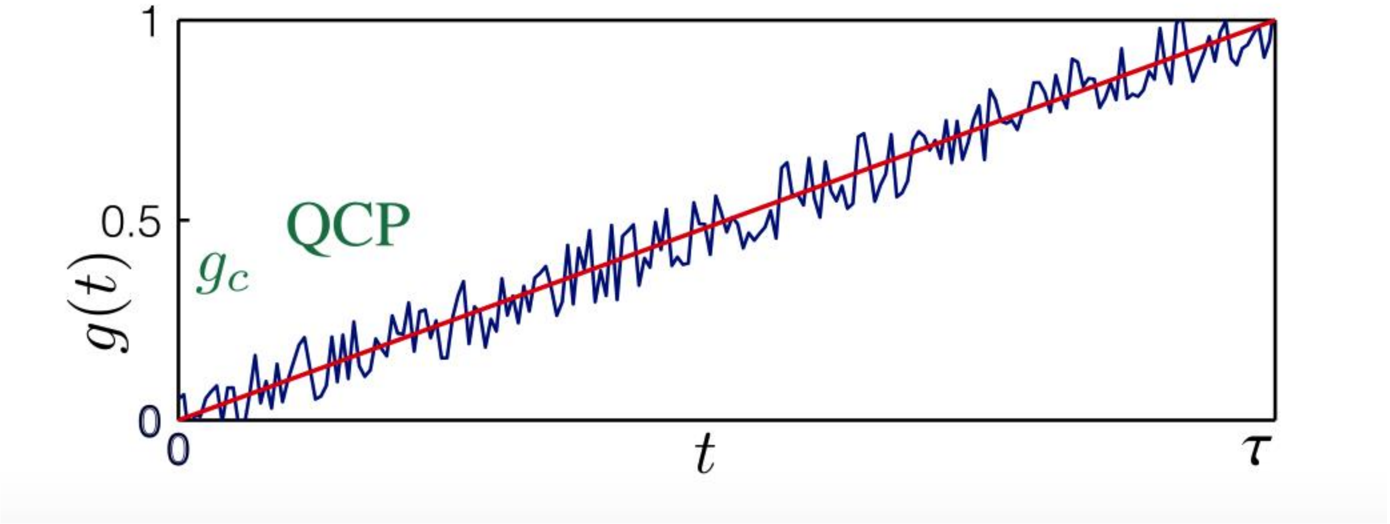
$$\Gamma = \sum_\alpha \lambda_\alpha$$

Exponential dependence on particle number!

Extreme decoherence

Z. Xu, L. P. García-Pintos, A. Chenu, and AdC, PRL 122, 014103 (2019)

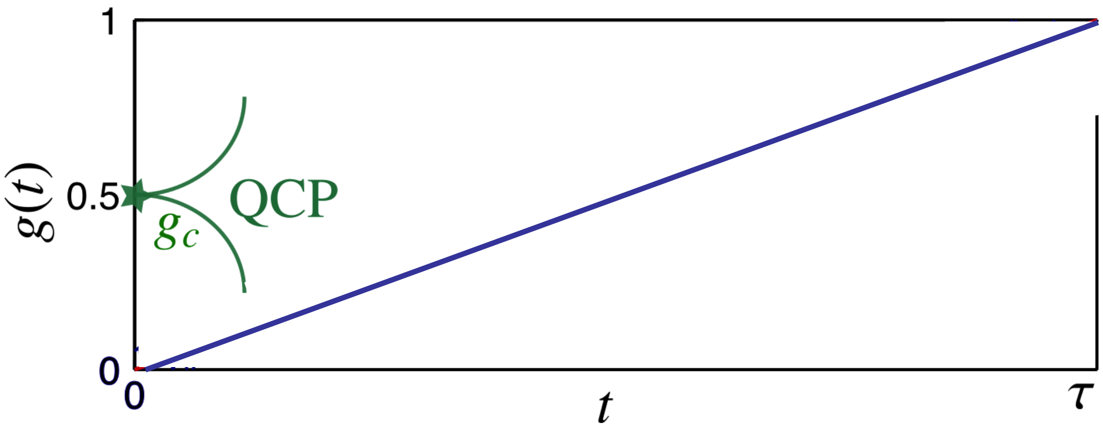
Noise-induced anti-Kibble-Zurek scaling



Quantum Annealing Protocol

Example

$$H = -\Lambda \sum_{n=1}^{N} \left\{ [1 - g(t)]\, \hat{\sigma}_n^x + g(t) \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z \right\}$$



$$H = -\Lambda \sum_{n=1}^{N} \hat{\sigma}_n^x$$

$$H = -\Lambda \sum_{n=1}^{N} \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z$$

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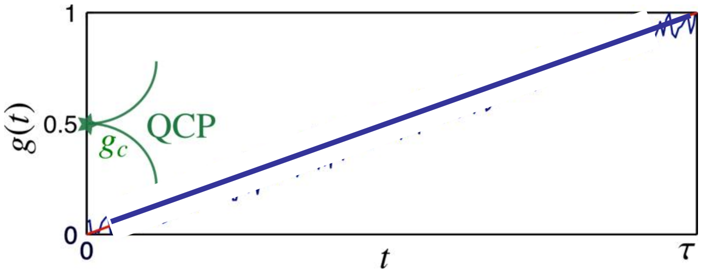
Dutta, Rahmani, AdC PRL 117, 080402 (2016)

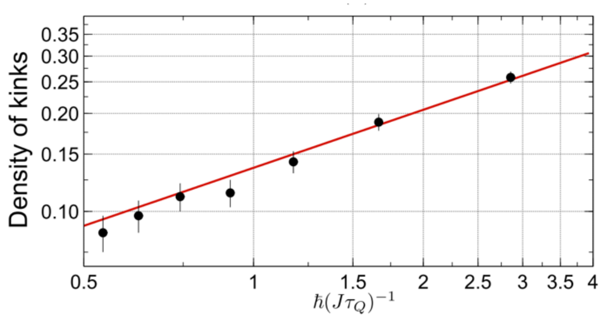
Defect density obeys Kibble-Zurek scaling

$$n_{\text{defects}} \propto \tau^{-\frac{d\nu}{1+z\nu}}$$

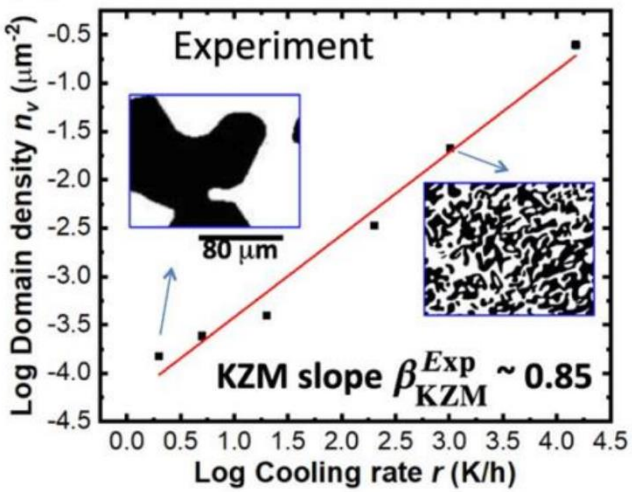


2005
Polkovnikov
Damski
Dziarmaga
Zurek, Dorner, Zoller

1D Ising [Sci. Rep. 6, 33381 (2016)]



3D Ising [Du et al. Nature Physics 19, 1495 (2023)]



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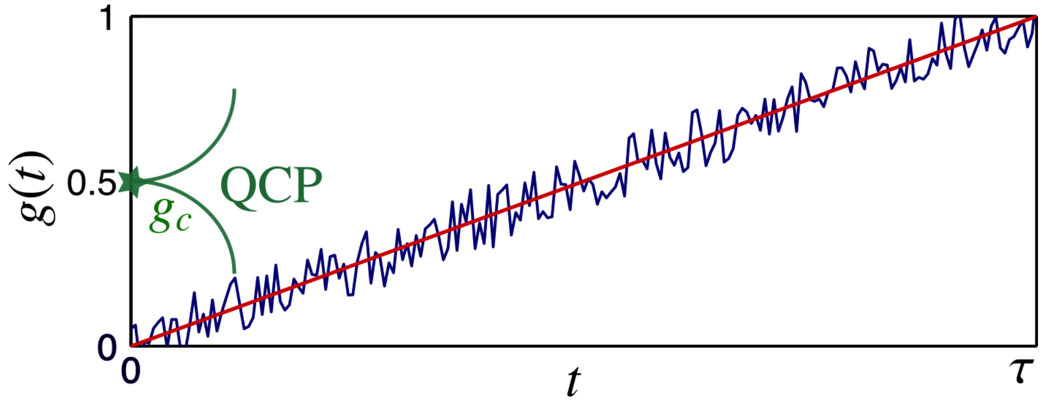
Noise in the control fields



Noisy control field

$$g(t) = t/\tau + \gamma(t), \quad 0 < t < \tau$$

$$\langle \gamma(t)\gamma(t')\rangle = W^2\delta(t-t')$$

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Dutta, Rahmani, AdC PRL 117, 080402 (2016)

Stochastic many-body Hamiltonians

Full system

$$H(t) = H_0(t) + \gamma(t)V$$

Deterministic and stochastic parts

$$H_0(t) = -\sum_{n=1}^{N} \left\{ \left[1 - g_0(t)\right] \hat{\sigma}_n^x + g_0(t) \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z \right\}$$

$$V = -\sum_{n=1}^{N} \left(-\hat{\sigma}_n^x + \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z \right)$$

Stochastic Schrodinger equation

$$i \frac{d}{dt} |\psi(t)\rangle = \left[H_0(t) + \gamma(t)V\right] |\psi(t)\rangle$$

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Dutta, Rahmani, AdC PRL 117, 080402 (2016)

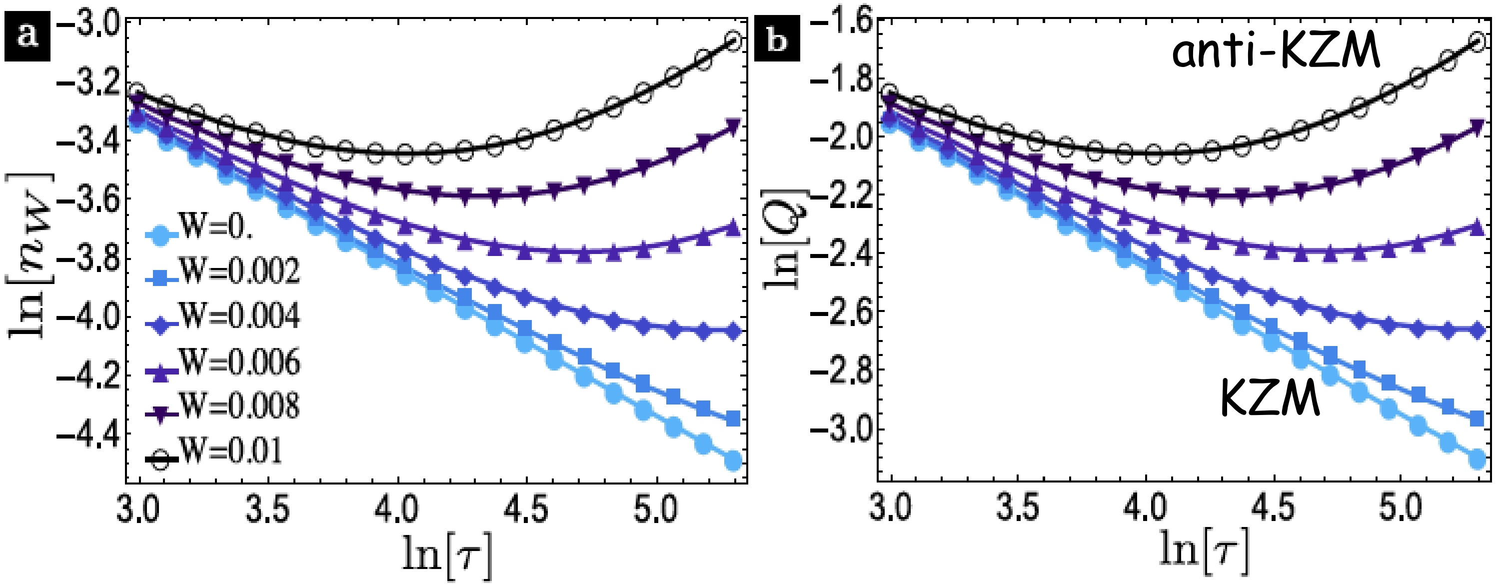
Annealing dynamics

Density of excitations

$$n_W = 1 - \frac{1}{N}\sum_{k>0}\langle G_k(\tau)|\varrho_k(t)|G_k(\tau)\rangle$$

Residual energy

$$Q = [E(\tau) - E_{GS}(\tau)]/N$$



Noise-induced breakdown of adiabatic protocols
Anti-Kibble-Zurek behavior

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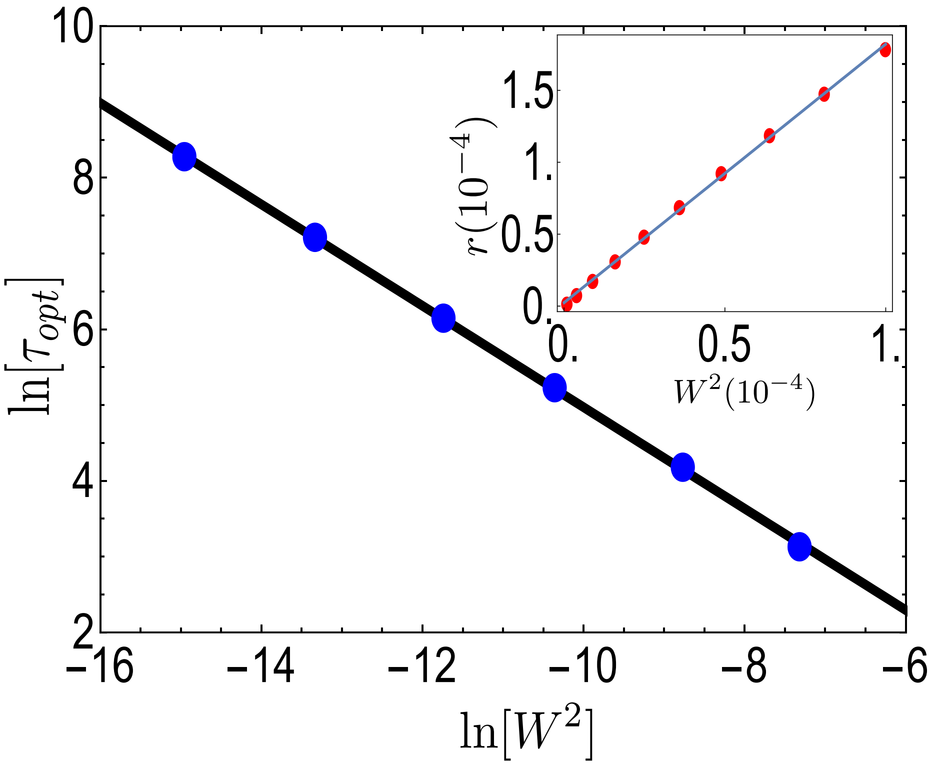
Dutta, Rahmani, AdC PRL 117, 080402 (2016)

Universality of optimal annealing time



Density of excitations

$$n_W \approx r\tau + c\tau^{-\beta}$$

Kibble-Zurek exponent

$$\beta = \frac{d\nu}{1 + z\nu}$$

Optimal annealing time

$$\tau_{\rm opt} \propto (W^2)^{-1/(\beta+1)}$$

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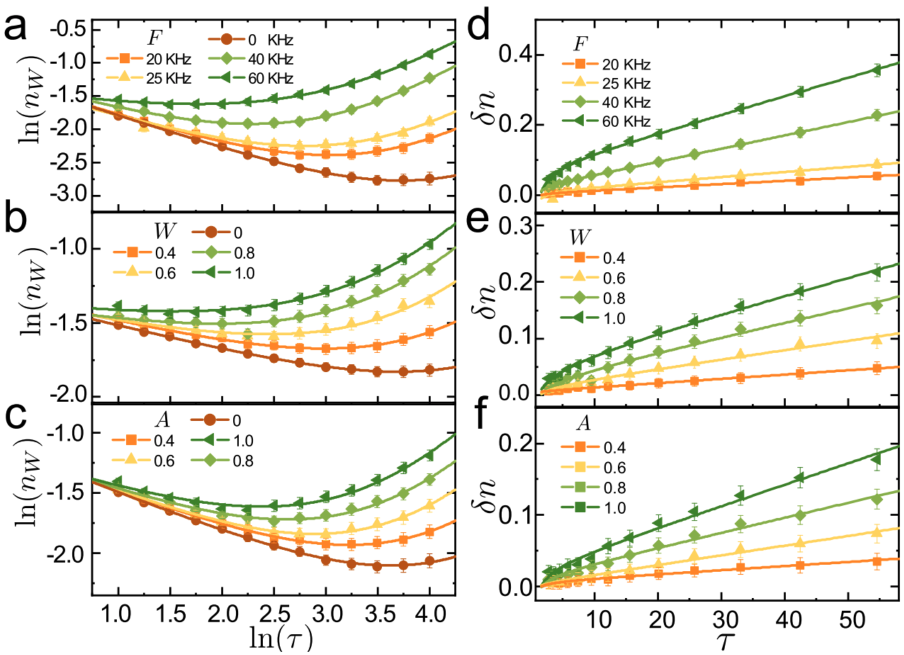
Dutta, Rahmani, AdC PRL 117, 080402 (2016)

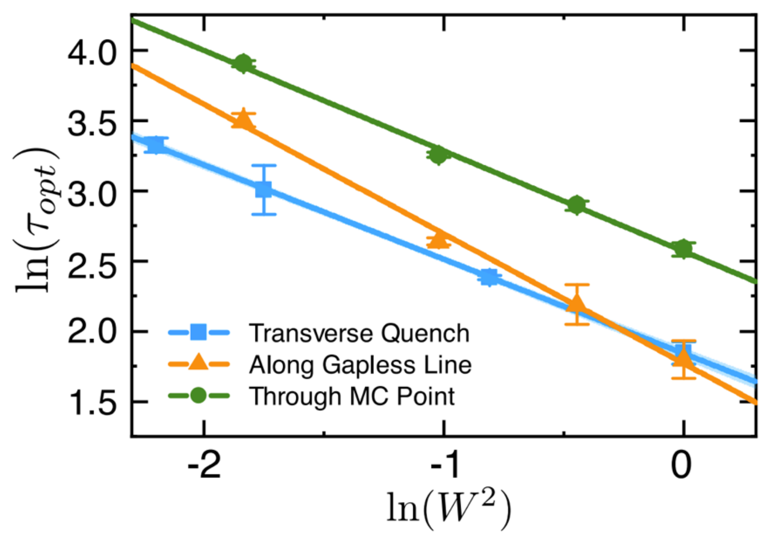
Experimental verification

KZ/Anti-KZ

Linear excess



Universal optimal driving time



Single trapped $^{171}Yb^{+}$ ion simulator of transverse-field XY chain

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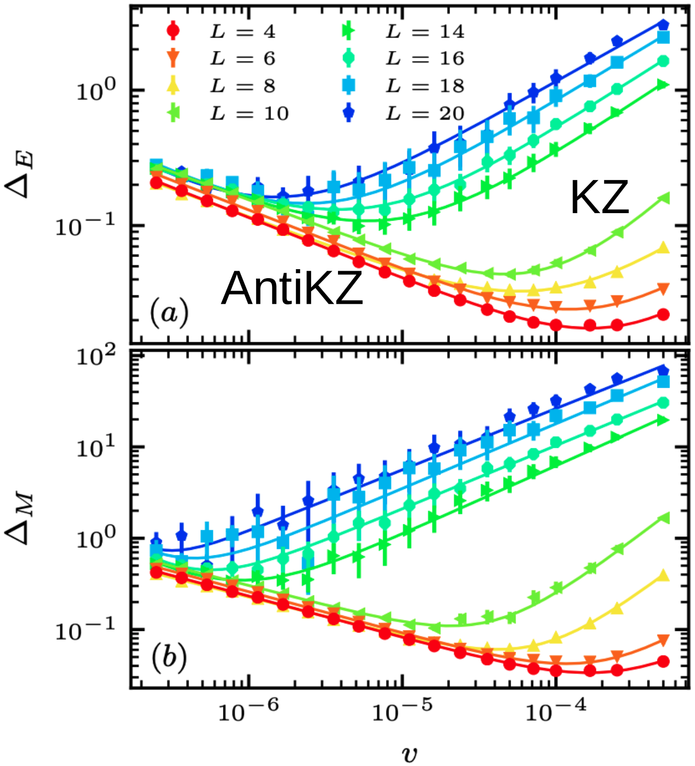
Ai et al. PRA 103, 012608 (2021)

Noise-induced Anti-KZM in QC

Analog Quantum Simulation

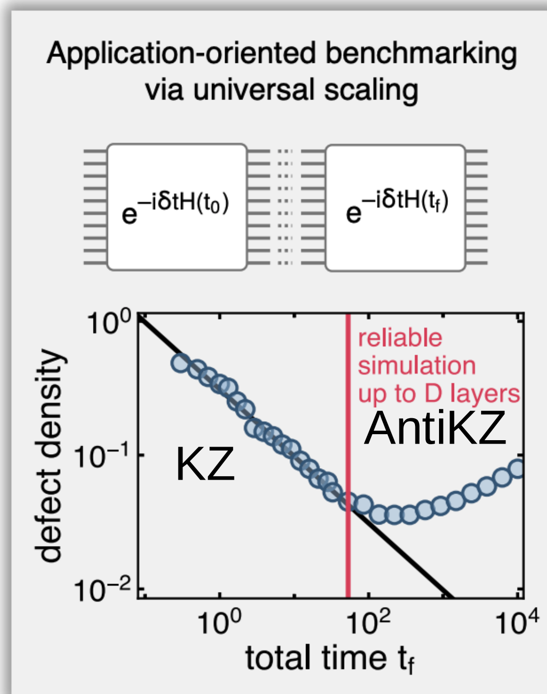
2D Quantum Ising model (Dwave)

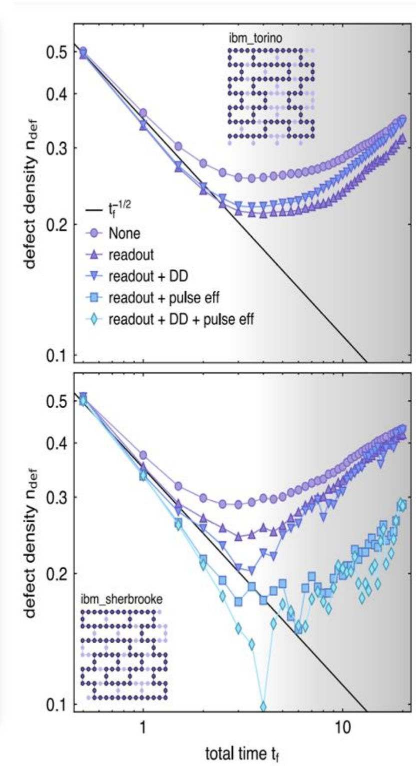


Weinberg et al. PRL 124, 090502 (2020)

Digital Quantum Simulation

Anti-KZ induced by digitized errors (IBM)





Miessen et al PRX Quantum 5, 040320 (2024)

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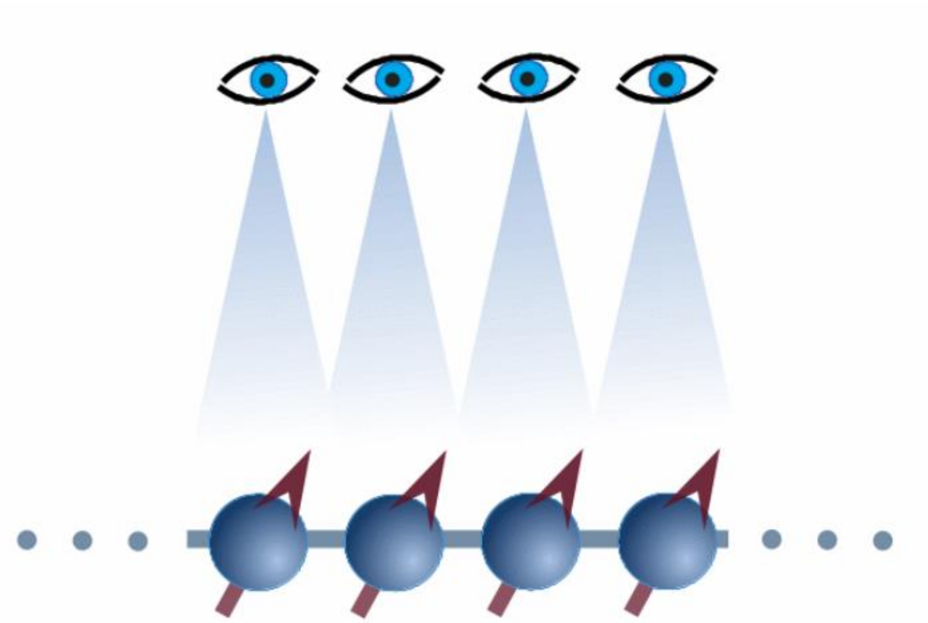
Symmetry breaking induced by monitoring



García-Pintos et al. PRL 123, 090403 (2019)

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Continuous Quantum Measurement: Dynamics

Equation of motion

$$d\rho_t = L\left[\rho_t\right] dt + \sum_\alpha I_\alpha\left[\rho_t\right] dW_t^\alpha$$

$$L[\rho_t] = -i\left[H, \rho_t\right] - \sum_\alpha \frac{1}{8\tau_m^\alpha}\left[A_\alpha, \left[A_\alpha, \rho_t\right]\right]$$

Nonlinear innovation term conditioning to a measurement record

$$I_\alpha\left[\rho_t\right] = \sqrt{\frac{1}{4\tau_m^\alpha}}\left(\{A_\alpha, \rho_t\} - 2\mathrm{Tr}(A_\alpha \rho_t)\rho_t\right)$$

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García-Pintos et al. PRL 123, 090403 (2019)

Continuous Measurement of Ising Model

Paramagnet to ferromagnet

$$H(t)/\Lambda = -\left(1-\frac{t}{\tau_Q}\right)\sum_{j=1}^{N}\sigma_j^x - \frac{t}{\tau_Q}\sum_{j=1}^{N}\sigma_j^z\sigma_{j+1}^z$$

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García-Pintos et al. PRL 123, 090403 (2019)

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Magnetization conserved on average

$$|\Psi(0)\rangle = \bigotimes_{j=1}^{N}|\rightarrow\rangle_j \quad \langle\Psi(0)|M|\Psi(0)\rangle = 0 \qquad \mathbb{E}[\langle\Psi(t)|M|\Psi(t)\rangle] = 0$$

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García-Pintos et al. PRL 123, 090403 (2019)

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Symmetry breaking induced by measurement feedback

$$\begin{aligned} d\langle\sigma_j^z\rangle(t) &= -\mathrm{Tr}\left(I\left[\rho_t\right]\sigma_j^z\right)dW_t^j \\ &= -\sqrt{\frac{1}{\tau_m}\Delta^2_{\sigma_j^z}(t)}\,dW_t^j \end{aligned}$$

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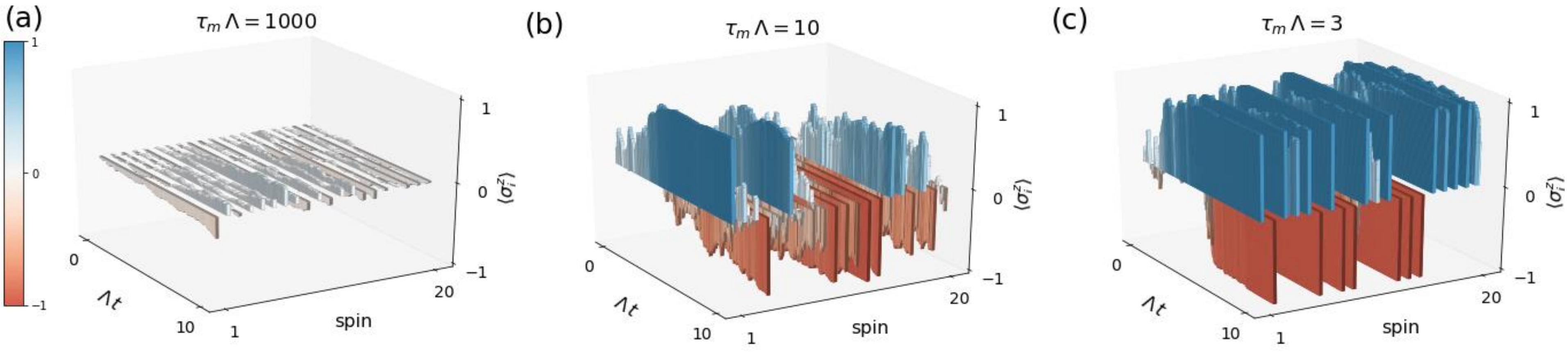
Increasing measurement strength



Symmetry breaking induced by monitoring

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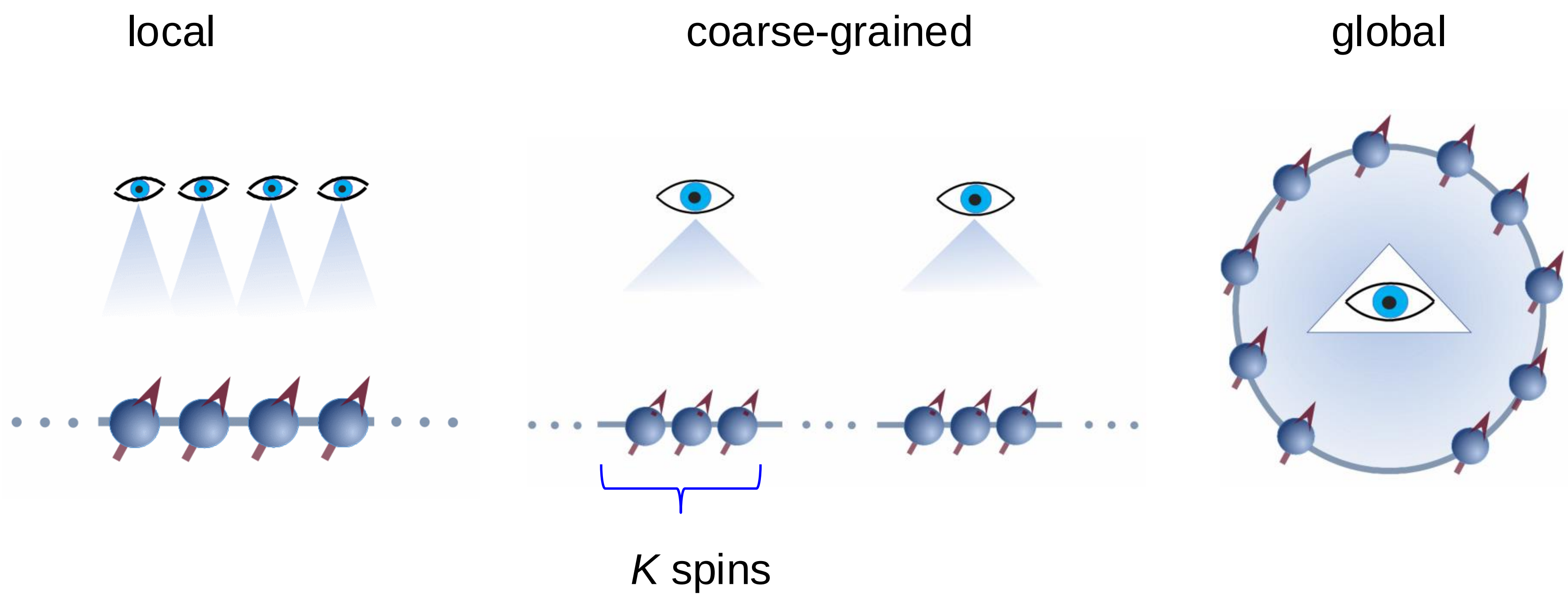
García-Pintos et al. PRL 123, 090403 (2019)

Locality of monitoring



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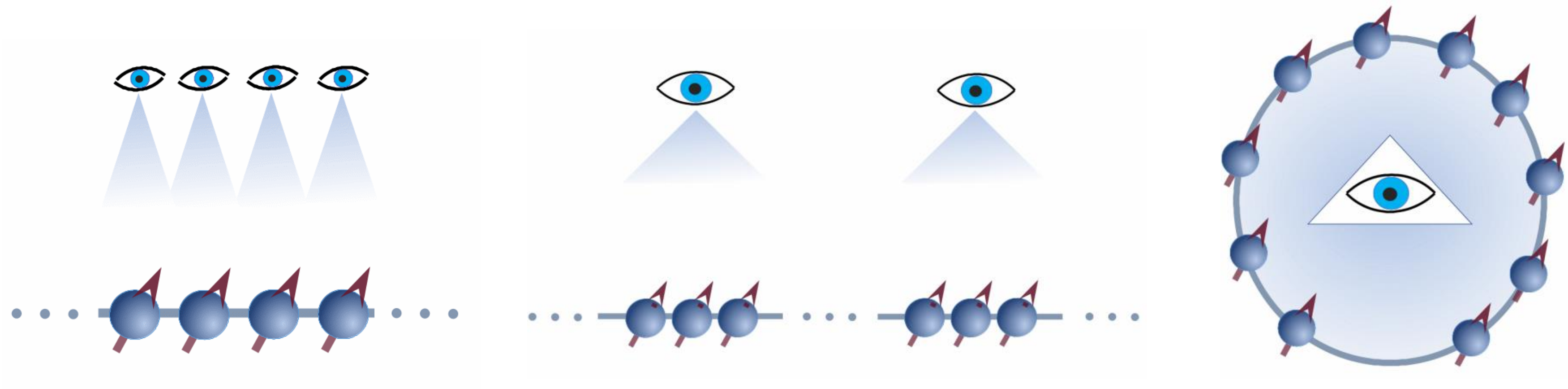
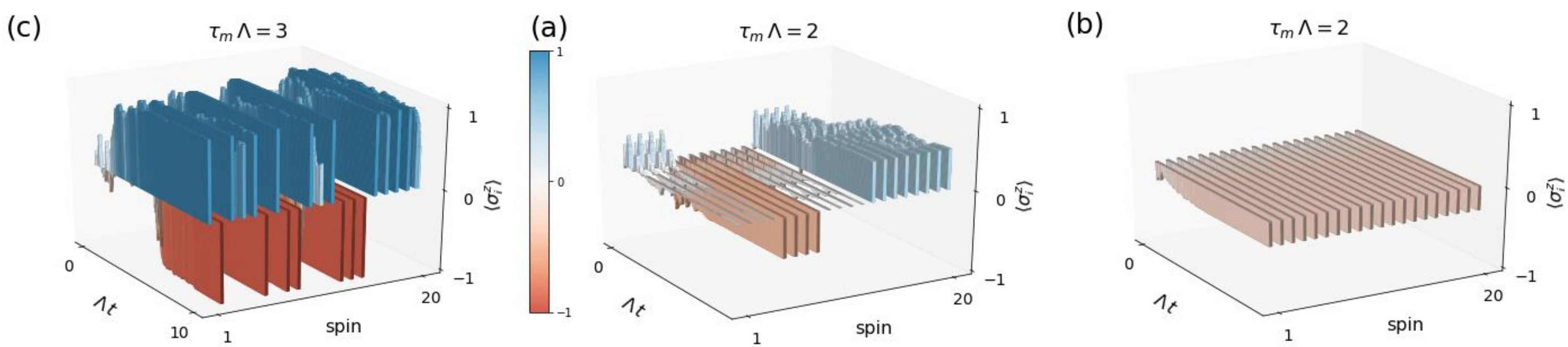
García-Pintos et al. PRL 123, 090403 (2019)

Locality of monitoring





$$\{-K, -K+1, \dots, K-1, K\}$$

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García-Pintos et al. PRL 123, 090403 (2019)

Classification of topological defects

Using the vacuum manifold of the system via homotopy theory
[Kibble '76, Mermin '79]

$$\mathcal{M} \simeq G/H \qquad \pi_n(\mathcal{M})$$

$$\mathcal{M} \simeq \mathbb{Z}_2/e \simeq \mathbb{Z}_2$$

Monitoring agent alters the pattern of symmetry-breaking

$$\mathcal{M} \simeq \mathbb{Z}_2 \Longrightarrow \mathcal{M} \simeq \mathbb{Z}_{K+1}$$

Need for classification framework including monitoring

García-Pintos et al. PRL 123, 090403 (2019)

Quantum Chaos in Open Quantum Systems

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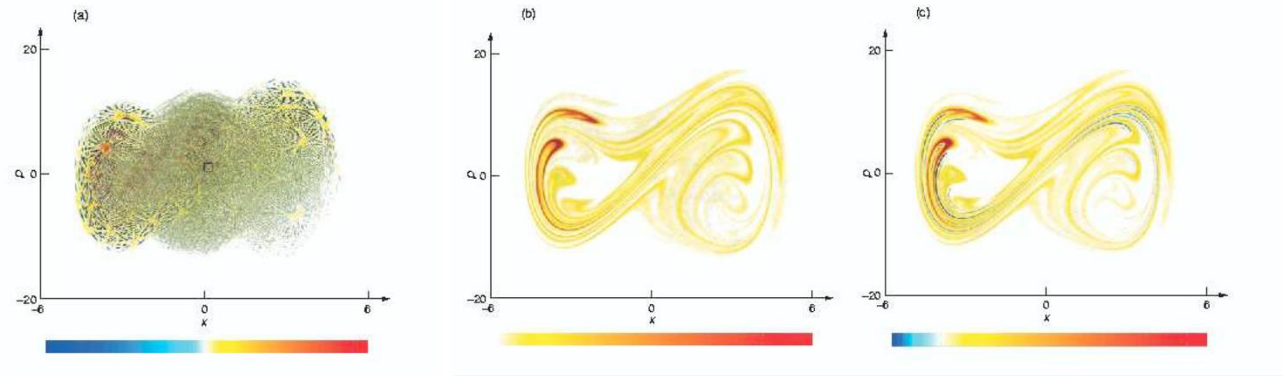
Z. Xu, A. Chenu, T. Prosen, and AdC, PRB 103, 064309 (2021)

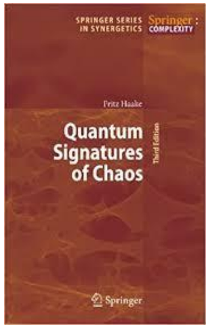
Adolfo del Campo: adolfo.delcampo@umb.edu

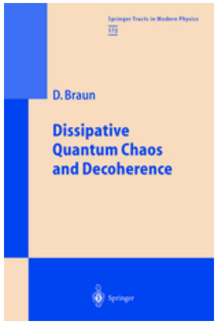
QChaos vs Decoherence: Long-standing problem

Seminal results …



Books …





We focus on spectral properties & introduce analogue of Spectral Form Factor

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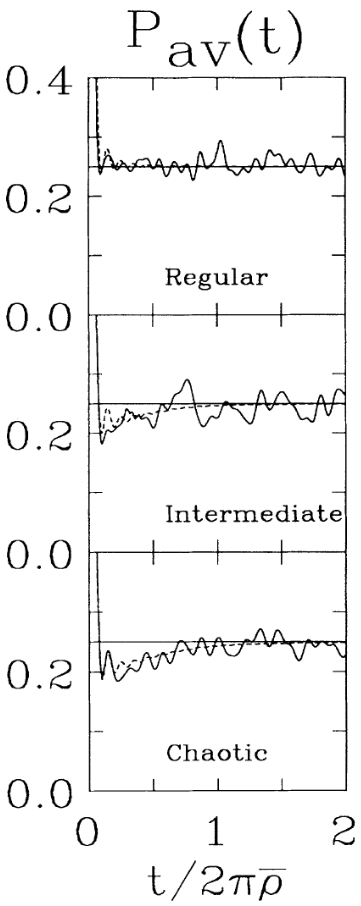
Z. Xu, A. Chenu, T. Prosen, and AdC, PRB 103, 064309 (2021)

SFF as Fourier transform of the spectrum

Leviander et al. PRL 56, 2449 (1986)
Wilkie Brumer, PRL 67, 1185 (1991)
Alhassid, Levine, PRA 46, 4650 (1992)
Alhassid, Whelan, PRL 70, 572 (1993)

$$\left\langle |\langle\Psi(0)|\Psi(t)\rangle|^2 \right\rangle_{\mathcal{E}}$$



Cotler et al JHEP05(2017)118
Dyer Gur-Ari JHEP08(2017)075
AdC Molina-Vilaplana Sonner PRD 95, 126008 (2017)

$$\left\langle \left| \frac{Z(\beta+it)}{Z(\beta)} \right|^2 \right\rangle_{\mathcal{E}(H)}$$

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SFF as Fourier transform of the spectrum

Survival amplitude

$$\langle\Psi(0)|\Psi(t)\rangle = \int dE\rho(E)e^{-iEt} \qquad \rho(E) = \sum_n |\langle E_n|\Psi(0)\rangle|^2\delta(E - E_n)$$

Survival probability/fidelity averaged over ensemble: sum of 3 terms

$$\begin{aligned}\big\langle F(t)\big\rangle_{\mathcal{E}} &= \int dEdE'\big\langle\rho(E)\rho(E')\big\rangle_{\mathcal{E}}e^{-i(E-E')t}\\ &= \left|\int dE\big\langle\rho(E)\big\rangle_{\mathcal{E}}e^{-iEt}\right|^2 + \int dEdE'\big\langle\rho(E)\rho(E')\big\rangle_{\mathcal{E}}^{(c)}e^{-i(E-E')t} + F_\infty\end{aligned}$$

1 2 3

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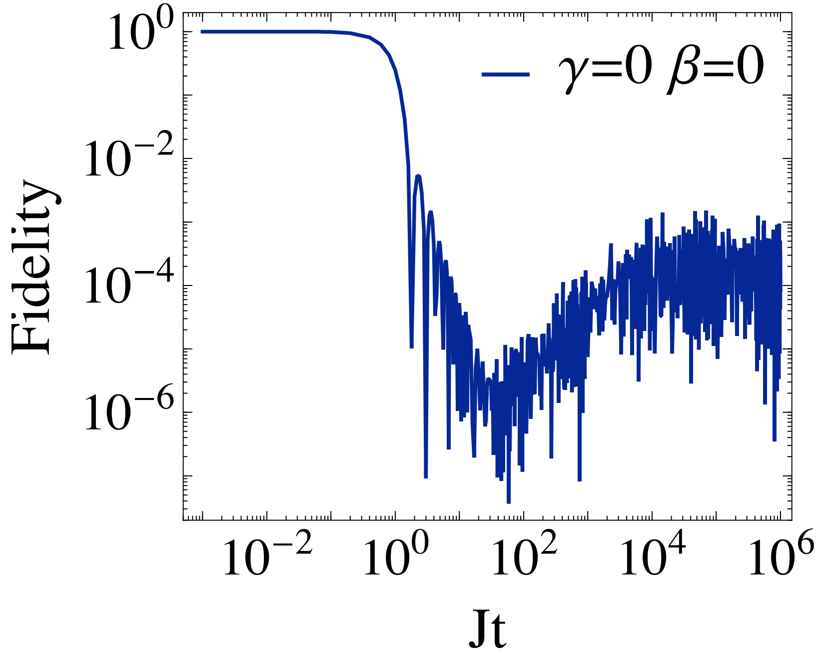
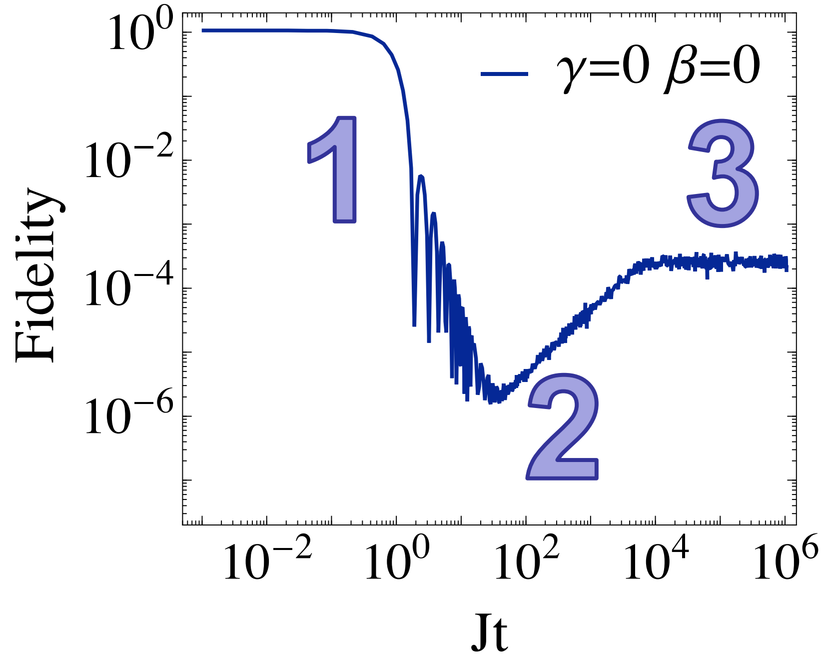
Chaos in isolated systems

Partition function with complex-valued temperature (SYK, N=26)





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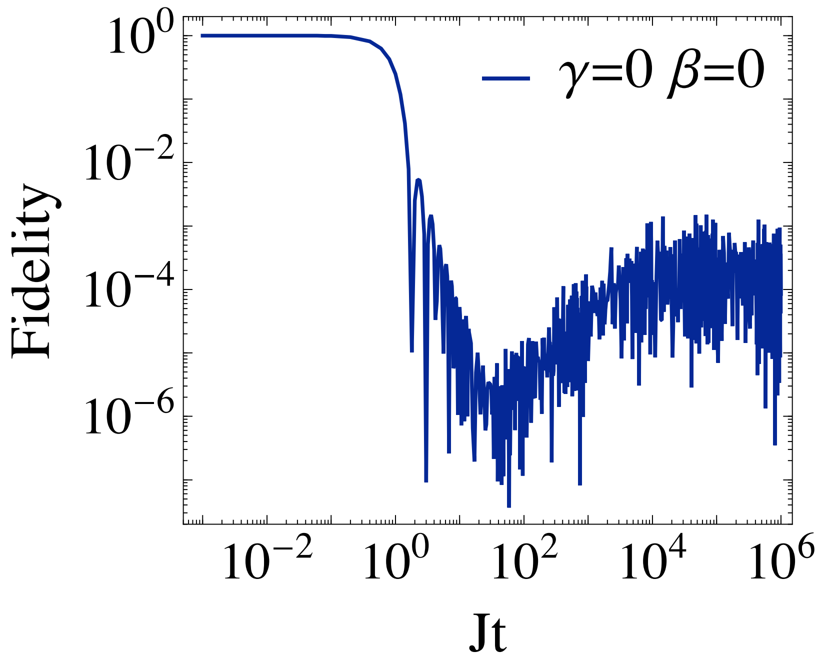
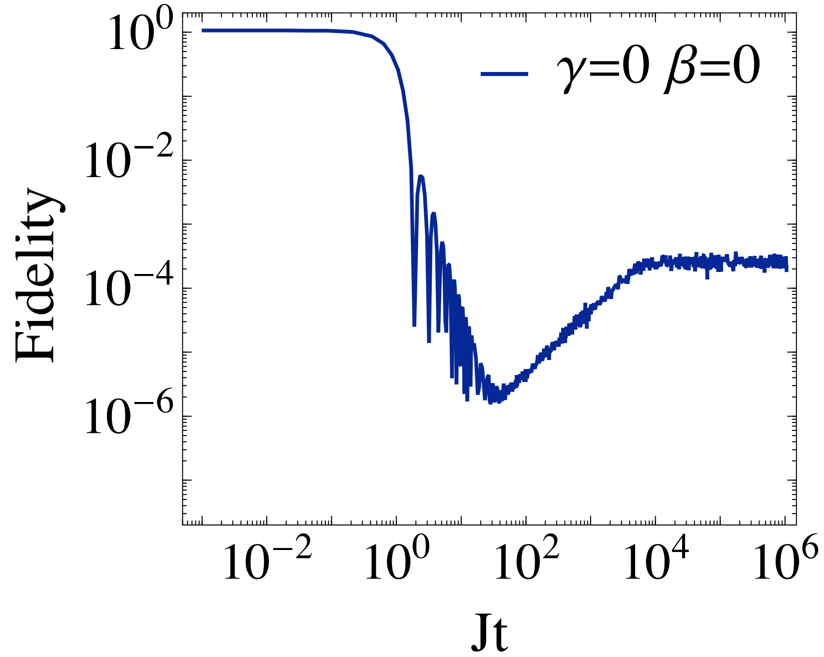
Chaos in isolated systems

Partition function with complex-valued temperature (SYK, N=26)





Quantum noise

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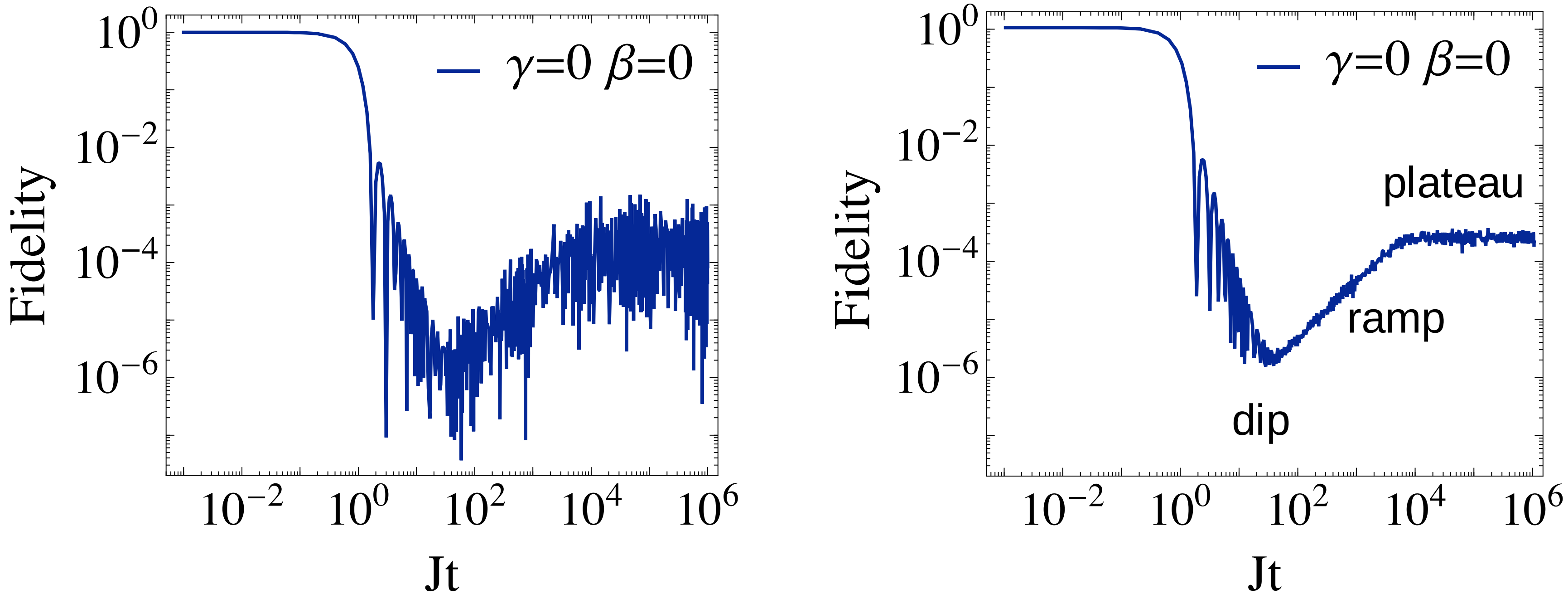
J Barbón & E Rabinovici
Fortschr. Phys. 62, 626 (2014)

Chaos in isolated systems

Partition function with complex-valued temperature (SYK, N=26)



Loschmidt echoes
NMR exps
JHEP

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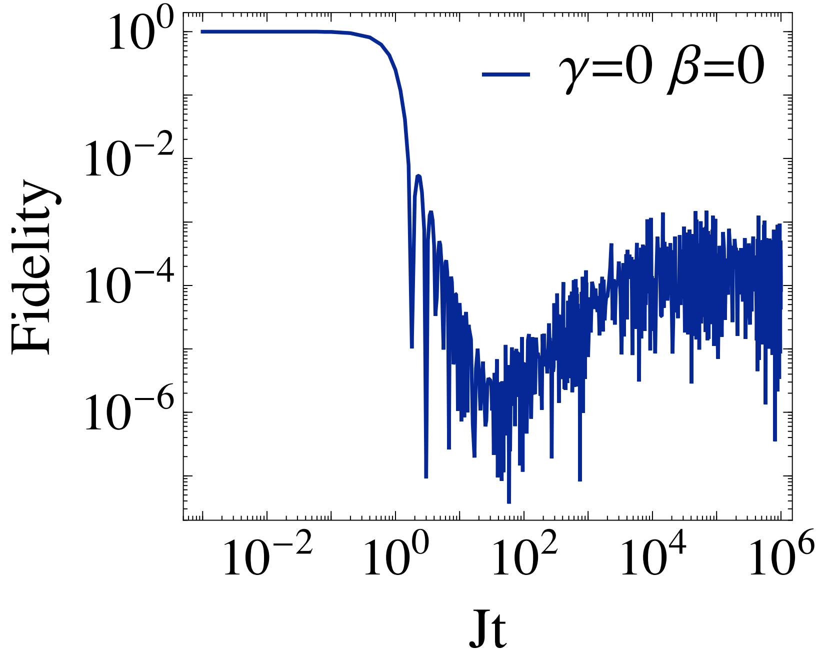
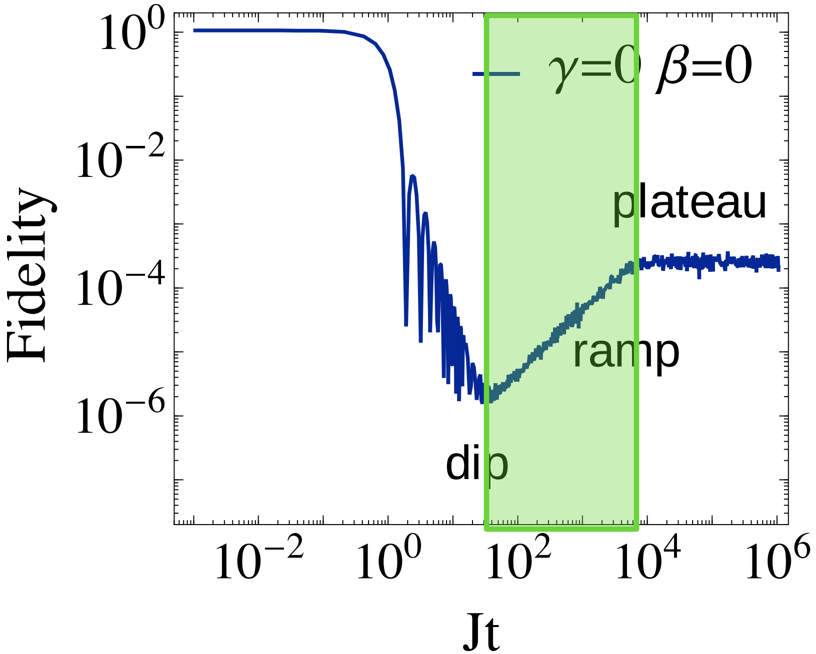
Chaos in isolated systems

Partition function with complex-valued temperature (SYK, N=26)





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J. Šuntajs, J. Bonča, T. Prosen, L. Vidmar, PRE 102, 062144 (2020)

Fidelity-based SFF for open quantum systems

Conventional definition of SFF as Fourier transform of energy spectrum

Open problem: Generalization to Open and Non-Hermitian systems

My view:

Fidelity-based approach provides natural generalization grounded on quantum dynamics and the geometry of quantum states

AdC, Molina-Vilaplana, Sonner, PRD 95, 126008 (2017)
AdC & T Takayanagi, JHEP02(2020)170
Xu, AdC, PRL 122, 160602 (2019)
Z. Xu, A. Chenu, T. Prosen, and AdC, PRB 103, 064309 (2021)
Cornelius et al. PRL 128, 190402 (2022)
Matsoukas-Roubeas et al. JHEP 2023, 60 (2023)
Matsoukas-Roubeas et al. PRA 108, 062201 (2023)
Matsoukas-Roubeas, Prosen, AdC, Quantum 8, 1446 (2024)

Fidelity-based SFF for isolated systems

Coherent Gibbs state (pure)

$$|\psi_\beta\rangle = \sum_n \frac{e^{-\beta E_n/2}}{\sqrt{Z_0(\beta)}}|n\rangle$$

evolves into

$$\rho_t = U(t)|\psi_\beta\rangle\langle\psi_\beta|U(t)^\dagger$$

Fidelity-based SFF

$$\mathrm{SFF} = F_t = \left|\frac{Z(\beta+it)}{Z(\beta)}\right|^2$$

Z. Xu, A. Chenu, T. Prosen, and AdC, PRB 103, 064309 (2021)

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Fidelity-based SFF for open quantum systems

Coherent Gibbs state (pure) $$|\psi_\beta\rangle = \sum_n \frac{e^{-\beta E_n/2}}{\sqrt{Z_0(\beta)}}|n\rangle$$

evolves into $$\rho_t = \Lambda_t\left[|\psi_\beta\rangle\langle\psi_\beta|\right]$$

Fidelity-based SFF $$\mathrm{SFF} = F_t = \langle\psi_\beta|\rho_t|\psi_\beta\rangle$$

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Z. Xu, A. Chenu, T. Prosen, and AdC, PRB 103, 064309 (2021)

Chaos in **open** systems: Energy dephasing

Survival probability after Hubbard-Stratonovich transformation

$$F_t = \frac{1}{2\sqrt{\pi\gamma t}} \int_{-\infty}^{+\infty} d\tau e^{-\left(\frac{\tau-2t}{2\sqrt{\gamma t}}\right)^2} g_\beta(\tau)$$

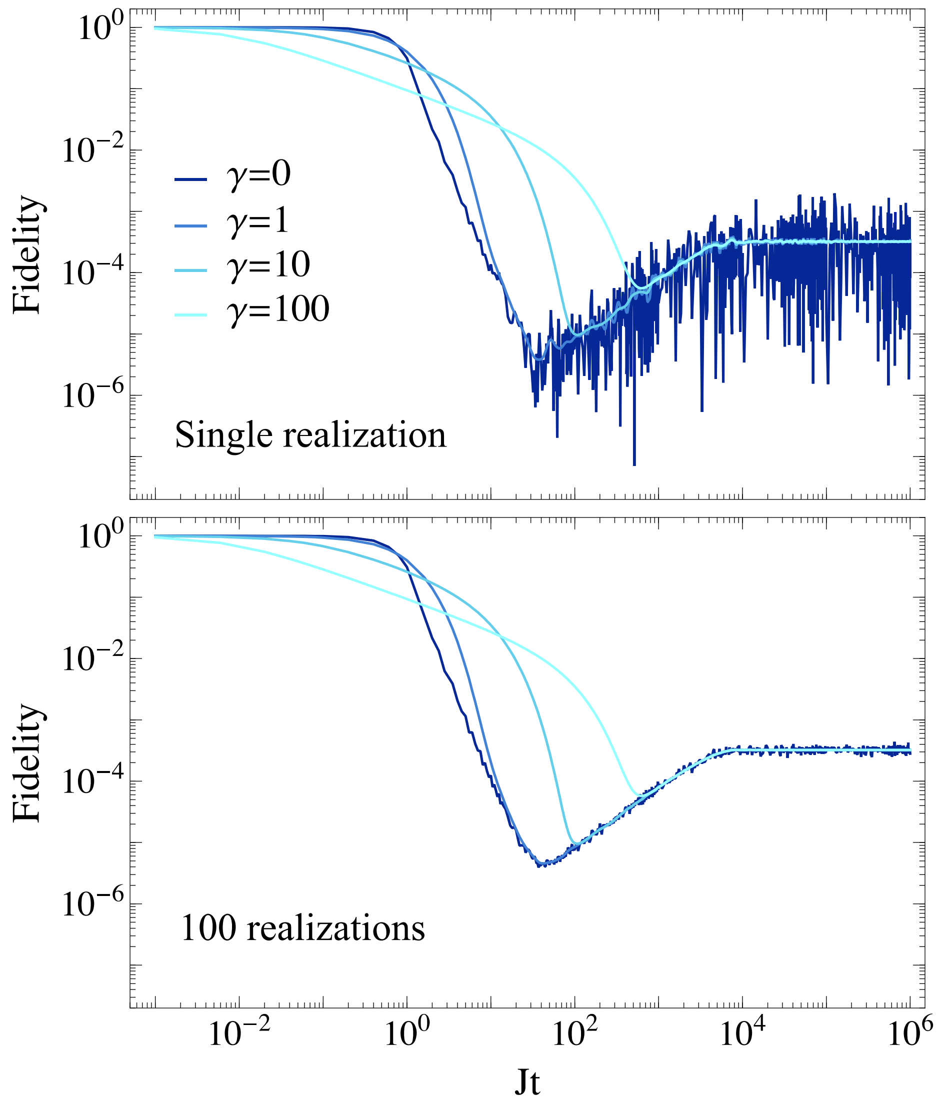
Spectral Form Factor for isolated quantum system

$$g_\beta(\tau) = \frac{|Z(\beta+i\tau)|^2}{Z^2(\beta)}$$

Energy dephasing equivalent to time averaging with given kernel

Z. Xu, A. Chenu, T. Prosen, and AdC, PRB 103, 064309 (2021)

TFD Fidelity as SFF for open systems



Dip shallower and delayed

"Quantum noise" suppressed

Ramp shortened, otherwise robust

Plateau robust

Z. Xu et al. PRB 103, 064309 (2021)

Competition of Time scales

Most quantities are monotonic, ruled by decoherence time

Fidelity sensitive to

- Decoherence time $$\frac{1}{\tau_D} = 4\gamma \frac{\mathrm{d}^2}{\mathrm{d}\beta^2} \ln Z(\beta) \rightarrow \tau_D = \frac{1}{\gamma N}$$
- Dip/Thouless time $$t_d \sim \left(\frac{\sqrt{\pi} \exp\left(-N\beta^2/4\right)}{c_N^{3/2}\sqrt{2N}} \right)^{1/4} \sqrt{d}$$
- Plateau/Heisenberg time $$t_p \simeq \alpha \sqrt{\frac{2\pi}{N}} d$$

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Z. Xu, A. Chenu, T. Prosen, and AdC, PRB 103, 064309 (2021)

Spectral filtering in Non-Hermitian dynamics balanced gain and loss

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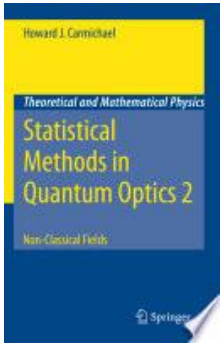
J. Cornelius et al. PRL 128, 190402 (2022)

Adolfo del Campo: adolfo.delcampo@umb.edu

Conditioning & Balanced gain and loss



Rewriting the Master equation

$$
\begin{aligned}
d_t\rho &= -i[H_0,\rho] + \sum_\alpha \gamma_\alpha \left(K_\alpha \rho K_\alpha^\dagger - \frac{1}{2}\{K_\alpha^\dagger K_\alpha, \rho\} \right) \\
&= -i\left(H_{\rm eff}\rho - \rho H_{\rm eff}^\dagger \right) + J(\rho) \\
H_{\rm eff} &= H_0 - \frac{i}{2}\sum_\alpha \gamma_\alpha K_\alpha^\dagger K_\alpha \\
J(\rho) &= \sum_\alpha \gamma_\alpha K_\alpha \rho K_\alpha^\dagger
\end{aligned}
$$

Null-measurement conditioning (no jumps) yields “balanced gain and loss”

$$
d_t\rho = -i\left(H_{\rm eff}\rho - \rho H_{\rm eff}^\dagger \right) + \frac{1}{2}{\rm tr}\left[\left(H_{\rm eff}^\dagger - H_{\rm eff} \right)\rho \right]\rho
$$

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BGL and energy dephasing

BGL Equations of motion: Non-Hermitian and Non-Linear

$$d_t\rho = -i[H_0,\rho] - \gamma[H_0,[H_0,\rho]] \quad \text{energy dephasing}$$

$$d_t\rho = -i\left(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger\right) + \frac{1}{2}\text{tr}\left[\left(H_{\text{eff}}^\dagger - H_{\text{eff}}\right)\rho\right]\rho \quad \text{no jumps}$$

$$H_{\text{eff}} = H_0 - i\gamma H_0^2$$

Quantum state evolution

$$\rho(t) = \frac{\sum_{nm}\rho_{nm}(0)e^{-i(E_n-E_m)t-\gamma t(E_n^2+E_m^2)}}{\sum_n \rho_{nn}(0)e^{-2t\gamma E_n^2}}|n\rangle\langle m|$$

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J. Cornelius et al. PRL 128, 190402 (2022)

Fidelity-based SFF for open quantum systems

Coherent Gibbs state (pure)

$$|\psi_\beta\rangle = \sum_n \frac{e^{-\beta E_n/2}}{\sqrt{Z_0(\beta)}}|n\rangle$$

Uhlmann fidelity

$$\mathrm{SFF} = F_t = \langle\psi_\beta|\rho_t|\psi_\beta\rangle$$

SFF under BGL

$$F_t = \frac{\left|\sum_n e^{-(\beta+it)E_n-\gamma t E_n^2}\right|^2}{Z_0(\beta)\sum_j e^{-\beta E_j-2t\gamma E_j^2}}$$

Ensemble average

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Fidelity-based SFF for open quantum systems

Coherent Gibbs state (pure)

$$|\psi_\beta\rangle = \sum_n \frac{e^{-\beta E_n/2}}{\sqrt{Z_0(\beta)}}|n\rangle$$

Uhlmann fidelity

$$\mathrm{SFF} = F_t = \langle\psi_\beta|\rho_t|\psi_\beta\rangle$$

SFF under BGL

$$F_t = \frac{\left|\int_{-\infty}^{\infty} \mathrm{d}s K(t,s) Z_0(\beta + is)\right|^2}{Z_0(\beta)\int_{-\infty}^{\infty} \mathrm{d}s\mathrm{d}s' K(t,s)K(t,s')Z_0[\beta + i(s-s')]}$$

$$K(t,s) = \frac{1}{\sqrt{4\pi\gamma t}} e^{-\frac{(t-s)^2}{4\gamma t}}$$

Ensemble average

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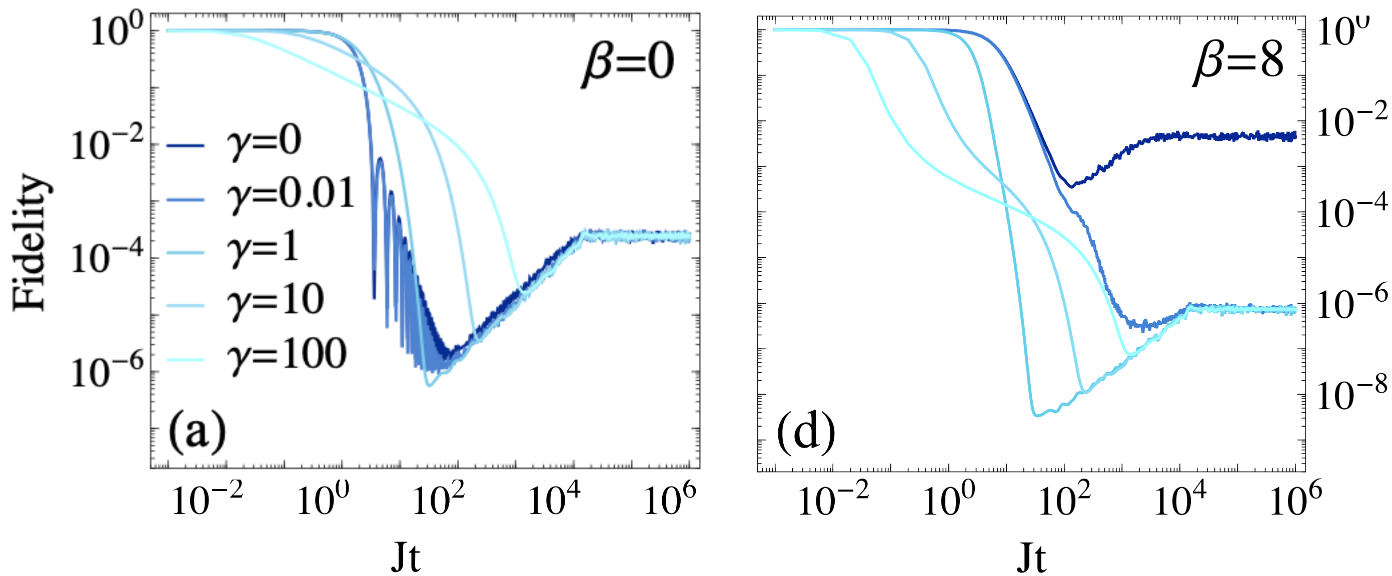
Spectral filtering enhances chaos

$$F_p \sim \frac{\sum_n N_n^2 e^{-2\beta E_n - 2\gamma t E_n^2}}{Z_0(\beta)\sum_n N_n e^{-\beta E_n - 2\gamma t E_n^2}} \geq \frac{1}{Z_0(\beta)}$$



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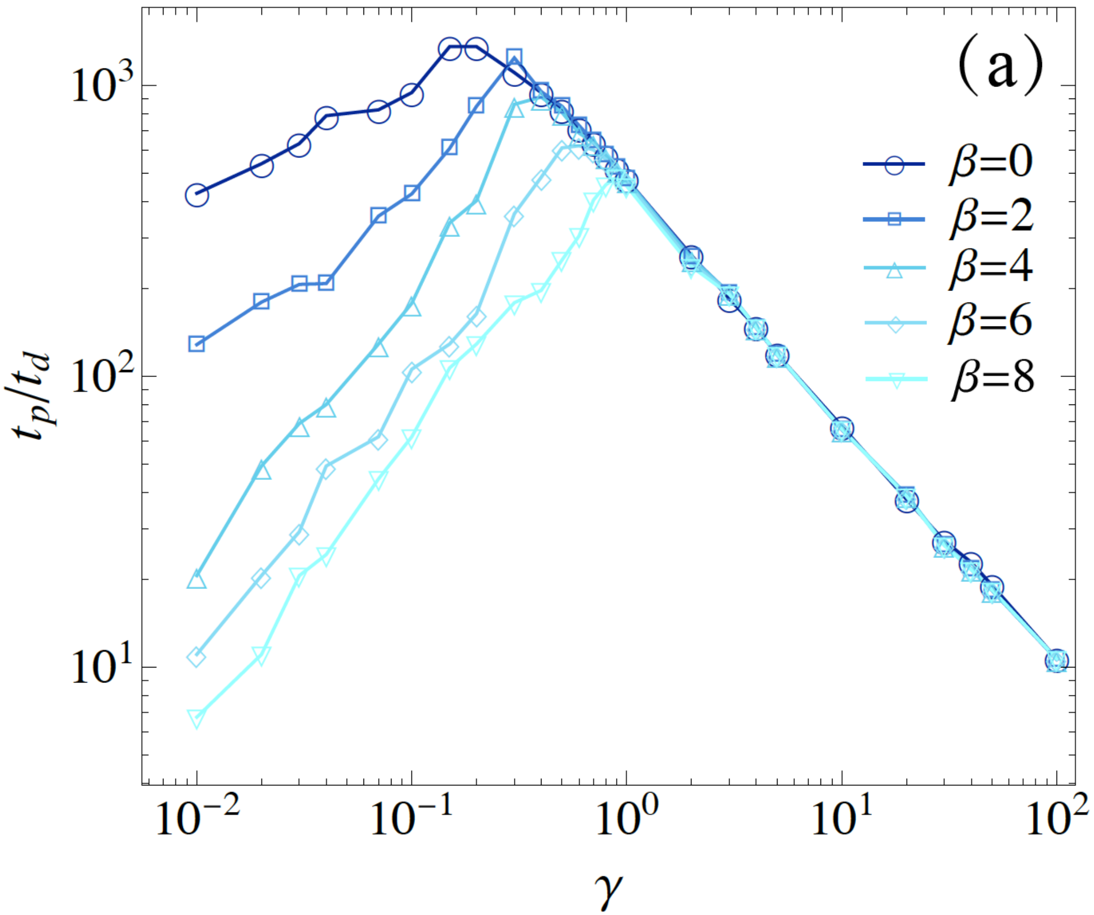
Spectral filtering enhances chaos



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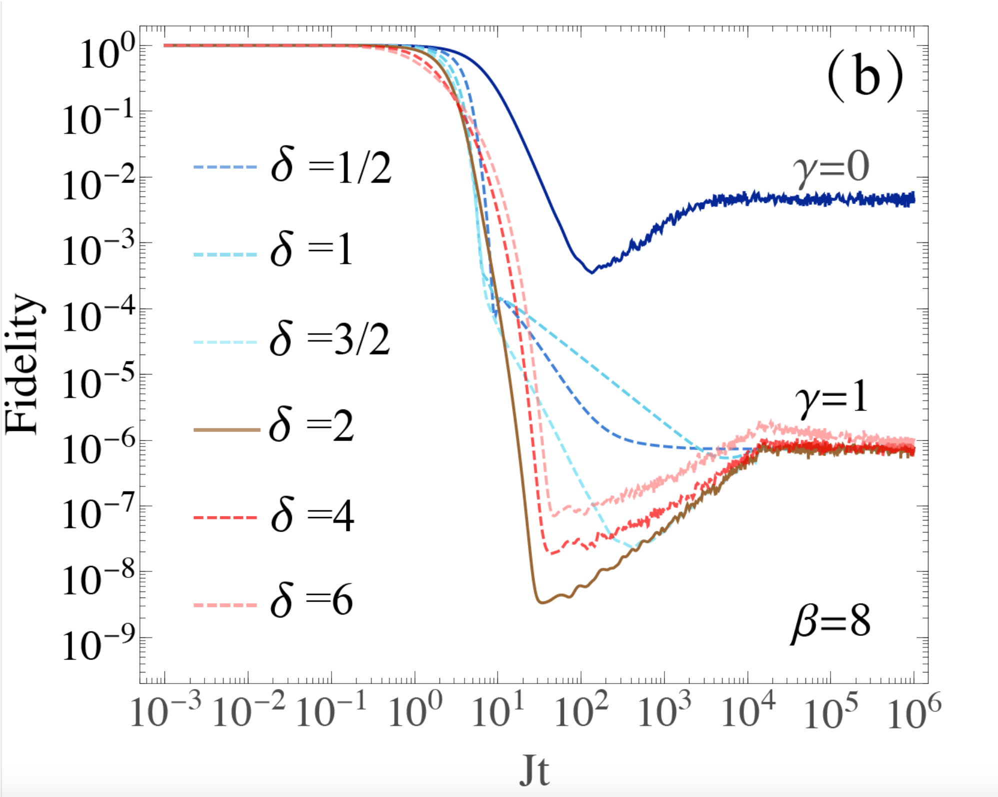
Optimal filter: Energy dephasing + BGL

$$F_t = \frac{\left|\sum_n e^{-(\beta+it)E_n} g(E_n)\right|^2}{Z_0(\beta)\sum_j e^{-\beta E_j} g(E_j)^2} \qquad g(E) = \exp(-\gamma t|E|^\delta)$$



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Spectral filtering as unifying framework

Energy dephasing = Frequency filtering = Time averaging = Liouvillian deformation

$$\mathrm{SFF}_w(t) = \frac{1}{2\pi}\widetilde{w}(t) * \mathrm{SFF}(t)$$

$$\mathbb{L}(\cdot) = -i[H,\cdot] \qquad W(\mathbb{L}) = \log w(i\mathbb{L}) \qquad \frac{d}{dt}|\rho_t) = \left[\mathbb{L} + \dot{\chi}(t)W(\mathbb{L})\right]|\rho_t)$$

$$\frac{d}{dt}\rho_t = -i[H,\rho_t] + \dot{\chi}(t)\sum_{n=0}^{\infty}\frac{W^{(2n)}(0)}{(2n)!}\mathrm{ad}_H^{2n}\rho_t$$

Energy dephasing with no jumps: Eigenvalue filtering = Hamiltonian deformation

$$Z_w(\beta) = \mathrm{Tr}\left[e^{-\beta\left(H - \frac{1}{\beta}\log w(H)\right)}\right] \qquad F_\beta = H - \frac{1}{\beta}\log w(H)$$

$$\mathrm{SFF}_w(t) = \left|\frac{\mathrm{Tr}\left(e^{-\beta F_\beta - itH}\right)}{\mathrm{Tr}\left(e^{-\beta F_\beta}\right)}\right|^2$$

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Matsoukas-Roubeas et al PRA 108, 062201 (2023)

Summary

Spontaneous Symmetry Breaking

- Extreme decoherence rate in RMT, exponential in system size
- Noise-induced anti-Kibble-Zurek scaling
- Symmetry breaking governed by monitoring

Quantum chaos

- Fidelity-based Spectral Form Factor for Open Systems
- Energy dephasing *suppresses* quantum chaos (jumps)
- Balanced gain and loss *enhance* quantum chaos (no jumps)

(last two points:) spectral filtering