Fingerprints of Composite Fermion Lambda Levels in Scanning Tunneling Microscopy

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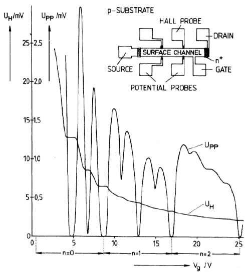
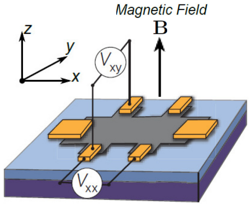
Songyang Pu, Ajit C. Balram, Yuwen Hu, Yen-Chen Tsui, Minhao He, Nicolas Regnault, Michael P. Zaletel, Ali Yazdani, Zlatko Papić, arXiv:2312.06779

Plan of the talk

- Integer quantum Hall effect (IQHE)
- Fractional quantum Hall effect (FQHE) in the Lowest Landau level (LLL): composite fermion (CF) theory
- Scanning tunneling microscopy of FQHE states in the LLL (see also Gattu *et al.*, Phys. Rev. B **109**, L201123 (2024) and Xinlei Yue and Ady Stern, arXiv:2406.09382)
- Conclusion and outlook

Experimental discovery of the integer QHE

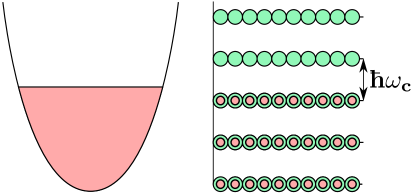




K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980)

- Plateau in Hall resistance $R_{xy} = h/(ne^2)$ where n is an integer
- forms the standard of resistance: Klitzing constant $R_{\mathrm{K}} = h/e^2 = 25812.8074593045\cdots\ \Omega$

IQHE arises from the formation of Landau levels (LLs)



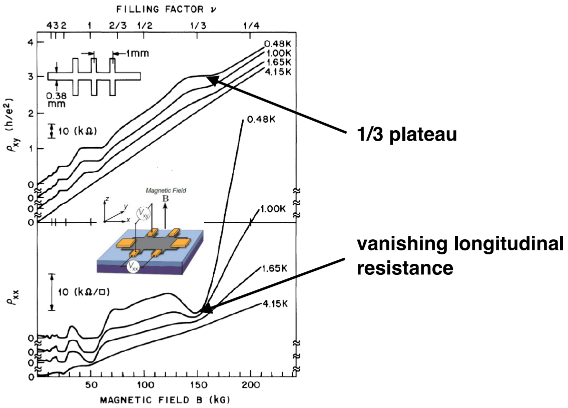
- Excitation gap is set by the cyclotron energy $\rightarrow \hbar\omega_{\rm c} = \hbar\frac{eB}{m_{\rm eff}}$

$$\Phi_1 = \prod_{i<j}(z_i - z_j) \times \exp\left[-\frac{1}{4\ell^2}\sum_i |z_i|^2\right]$$

$$z = x - iy, \text{ magnetic length } \ell = \sqrt{\frac{\hbar c}{eB}}, \text{ filling } \nu = \frac{\rho\phi_0}{B}, \; \phi_0 = \frac{hc}{e}.$$

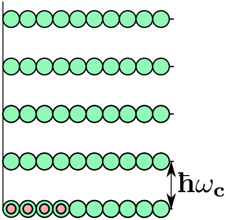
No closed form expression for the wave function of higher LLs.

Plateau at $h/(\frac{1}{3}e^2)$



D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982)

FQHE arises from electron-electron interactions



- Electrons interacting via Coulomb forces:

$$\mathcal{H} = \frac{e^2}{\epsilon} \sum_{i<j} \frac{1}{|r_i - r_j|}$$

- Quantum mechanics $\rightarrow$ *lowest Landau level* constraint for $B \rightarrow \infty$
- Interactions $\rightarrow$ a unique state from the degenerate manifold

Laughlin's ansatz for $\nu = 1/m$

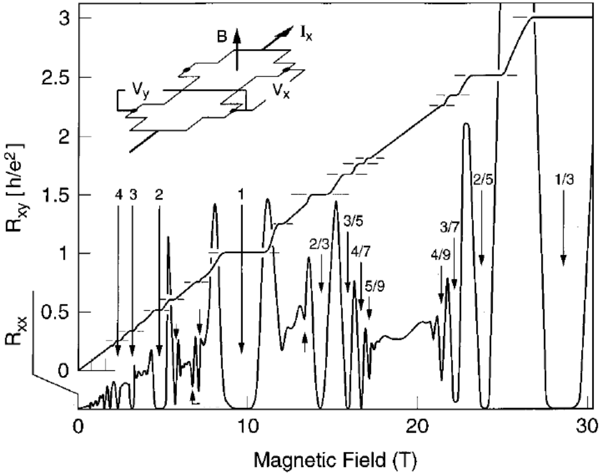
- assumed a Jastrow (pairwise) correlated state.

$$\Psi_{1/m}^{\text{Laughlin}} = \prod_{i<j}(z_i - z_j)^m$$

- fermionic wave functions must be antisymmetric, hence m is odd integer
- fluid with fractionally charged particles obeying fractional braid statistics

R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983)

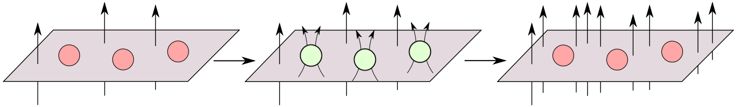
Zoo of fractions at $\nu = n/(2pn \pm 1)$



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

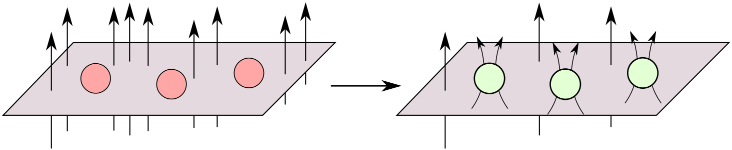
FQHE as IQHE of composite fermions

A composite fermion (CF) is a bound state of an electron and even number of vortices/flux quanta.



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE as IQHE of composite fermions

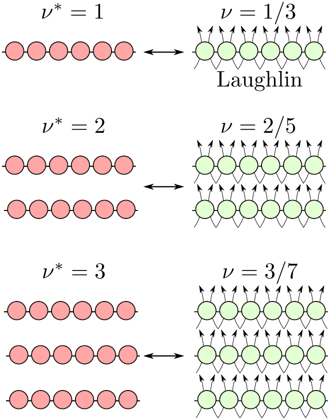


$$B^* = B - 2p\rho\phi_0, \quad \phi_0 = hc/e$$

$$\nu = \frac{\rho\phi_0}{B}, \quad \nu^* = \frac{\rho\phi_0}{|B^*|}, \quad \nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

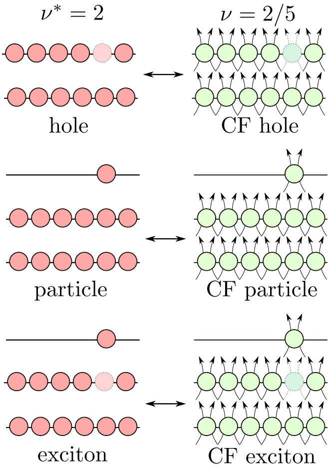
J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE ground states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE excited states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE wave functions can be built from IQHE ones

- Jain wave functions at $\nu = n/(2pn \pm 1)$:

$$\Psi^{\mathrm{CF}}_{\nu=\frac{n}{2pn\pm1}} = \mathcal{P}_{\mathrm{LLL}}\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p}\right).$$

(dropped Gaussian factor for ease of notation)

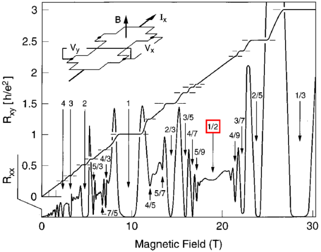
Φ_n wave function of n filled LLs.

$\mathcal{P}_{\mathrm{LLL}}$ implements lowest Landau level projection.

- no adjustable parameters in these wave functions
- wave functions can be evaluated for large system sizes

J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989)

Mystery of the $\nu = 1/2$ state

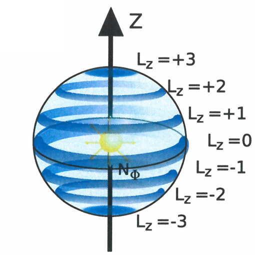


- composite fermions absorb all of the magnetic flux: $B^* = 0$

  Halperin, Lee and Read, Phys. Rev. B **47**, 7312 (1993)

- In zero effective magnetic field CFs form a Fermi sea

Spherical geometry



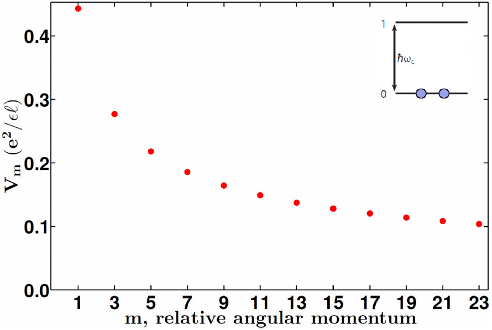
$$l = |Q|, |Q|+1, |Q|+2, \cdots \quad l_n = |Q|+n \quad m = -l, -l+1, \cdots, l-1, l$$

L and its z-component L_z are good quantum numbers

$$N_\phi = 2Q = \nu^{-1}N - \mathcal{S}, \quad \mathcal{S} \rightarrow \text{shift, characterizes the state}$$

F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

Haldane pseudopotentials parametrize the interaction



V_m: energy of two electrons in a state of relative angular momentum m

fully spin-polarized electrons $\rightarrow$ only odd pseudopotentials relevant

F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

Overlaps of CF states with LLL Coulomb ground states

overlaps obtained from direct projected states

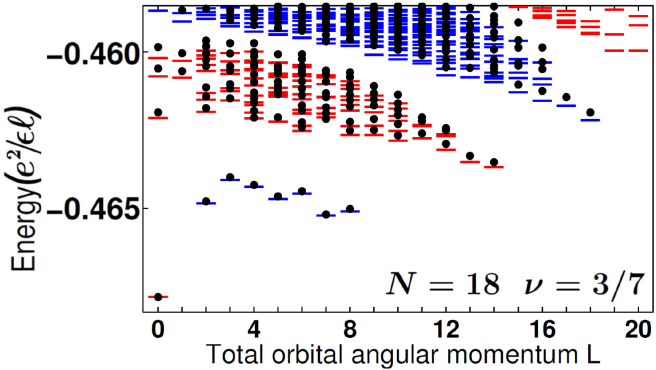
ν	N	Hilbert space dimension	$\lvert\langle\Psi^{\rm 0LL}\lvert\Psi^{\rm CF}\rangle\rvert$
1/3	15	2×10^9	0.9876 (Laughlin)
1/5	11	4×10^8	0.9413 (Laughlin)
2/5	12	3×10^5	0.9971
3/7	12	6×10^4	0.9988
2/9	10	1×10^7	0.9744

$\lvert\Psi^{\rm 0LL}\rangle$ is obtained by brute-force exact diagonalization

Ajit C. Balram and A. Wójs, Phys. Rev. Research **2**, 032035(R) (2020)
B. Yang and Ajit C. Balram, New J. Phys. **23**, 013001 (2021)

Ajit C. Balram, SciPost Phys. **10**, 083 (2021)

CF theory is extremely accurate in the lowest Landau level



dashes are obtained by brute-force exact diagonalization

$\sim 10^6$ states at each total orbital angular momentum L

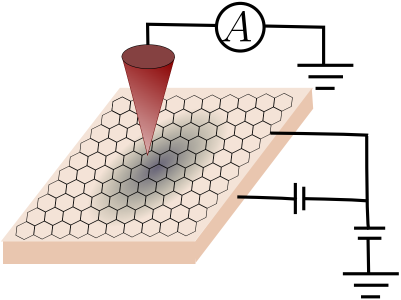
Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

Questions?

Onward to STM

Why do scanning tunneling microscopy (STM) now?

- 2DEG resides deep inside the GaAs-AlGaAs heterostructures



- extensive FQHE observed in exposed graphene samples
- improvements in the tip quality: non-invasive measurements

G. Farahi *et al.*, Nat. Phys. **19**, 1482 (2023) and Y. Hu *et al.*, arXiv:2308.05789

What does the STM measure?

- Upto certain caveats, the STM measures the energy-resolved local density of states (LDOS)
- LDOS can be probed for removing (hole) or injecting an electron at the position under the tip $\vec{r}$

$$\begin{aligned}\mathrm{LDOS}(E,\vec{r}) &= \sum_n \delta(E-E_n^-)|\langle n|c_{\vec{r}}|\Omega\rangle|^2 \\ &+ \sum_n \delta(E-E_n^+)|\langle n|c_{\vec{r}}^\dagger|\Omega\rangle|^2.\end{aligned}$$

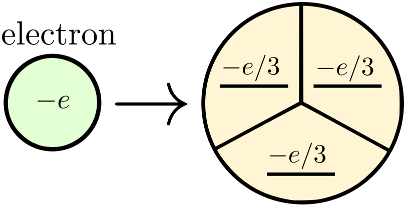
$|\Omega\rangle \rightarrow$ ground state of N particles
$|n\rangle \rightarrow$ eigenstate of $N \pm 1$ particles

- CFs are the fundamental quasiparticles of FQHE: how does the high-energy electron or hole excitation fractionalize into CFs?

Z. Papić *et al.*, Phys. Rev. X **8**, 011037 (2018) and S. Pu *et al.*, Phys. Rev. B **106**, 045140 (2022)

tunneled electron/hole is a tightly bound state of CFs

- at $\nu = n/(2n+1)$ a single hole and electron excitations have finite overlap on a tightly-bound state of $(2n+1)$ CFs

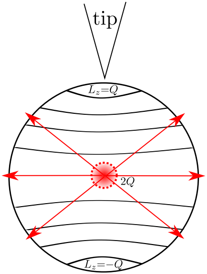


J. K. Jain and M. R. Peterson, Phys. Rev. Lett. **94**, 186808 (2005)

LDOS on the sphere from exact diagonalization

- LDOS for removing or injecting an electron at the north pole

$$\begin{aligned}\mathrm{LDOS}(E, L_z{=}Q) &= \sum_n \delta(E - E_n^-)|\langle n|c_{-Q}|\Omega\rangle|^2 \\ &+ \sum_n \delta(E - E_n^+)|\langle n|c_Q^\dagger|\Omega\rangle|^2.\end{aligned}$$



S. Pu *et al.*, Phys. Rev. B **106**, 045140 (2022)

LDOS from CF diagonalization (CFD)

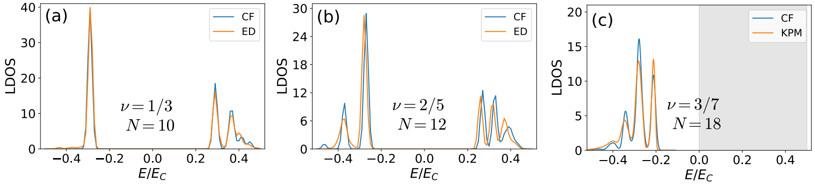
- CFD wth $(2n+1)$ particles/holes at reduced flux

$$\mathrm{LDOS}(E)=\sum_n \left(\delta(E-E_n^-)|\eta_n^-|^2+\delta(E-E_n^+)|\eta_n^+|^2\right).$$

- overlaps with the electron and hole excitations are $\eta_n^+=\langle\alpha_n^+|c_Q^\dagger|\Omega\rangle$ and $\eta_n^-=\langle\alpha_n^-|c_{-Q}|\Omega\rangle$
- orthonormal states obtained from CFD are $|\alpha_n^\pm\rangle$ with energy eigenvalues $E_n^\pm$

S. Pu, Ajit C. Balram *et al.*, arXiv:2312.06779

excellent agreement between exact and CF LDOS



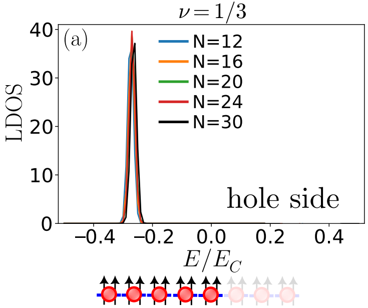
- hole sides has much better agreement than particle side
- particle side has to necessarily be truncated since there are CF-LLs with arbitrarily high ΛL index
- agreement can be improved by keeping more CF states

S. Pu, Ajit C. Balram *et al.*, arXiv:2312.06779

unique peak in the LDOS on the hole side of Laughlin

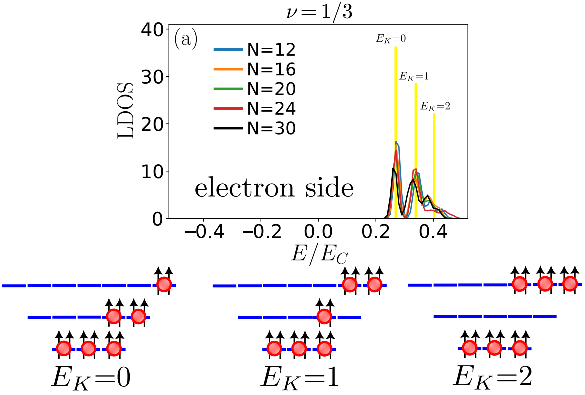
- single hole at the origin = 3 Laughlin quasiholes at the origin

$$(\prod_i z_i)^3 \prod_{i<j}(z_i - z_j)^3$$



E. H. Rezayi, Phys. Rev. B **35**, 3032 (1987)

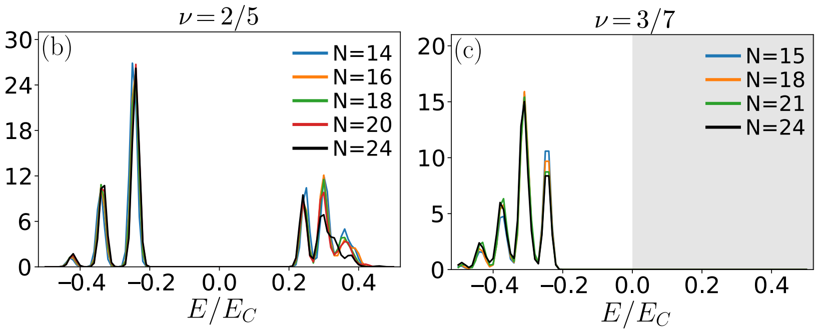
multiple LDOS peaks on the electron side of Laughlin



- $E_K = \sum_{l=1}^{2n+1}(n_l - n) - E_0$, where n_l is ΛL index of the $l^{\rm th}$ particle and $E_0 = \min \sum_{l=1}^{2n+1}(n_l - n)$ in the $L_z = Q$ sector

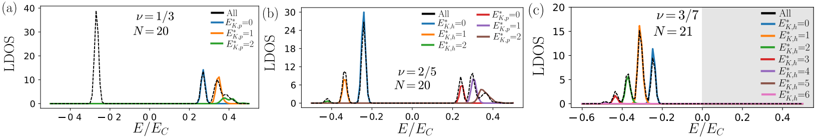
S. Pu, Ajit C. Balram *et al.*, arXiv:2312.06779

multiple LDOS peaks in 2/5 and 3/7 Jain states



S. Pu, Ajit C. Balram *et al.*, arXiv:2312.06779

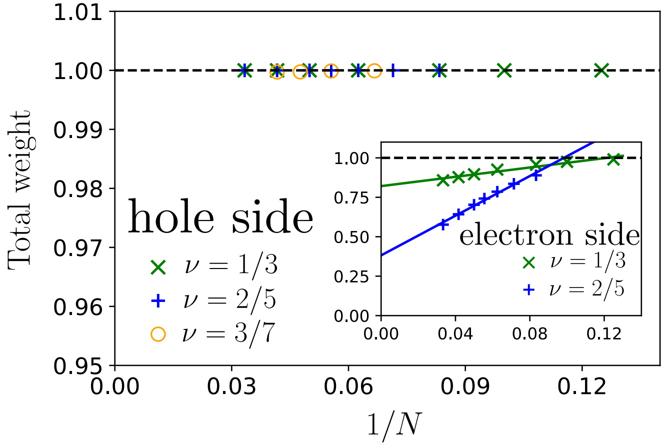
each peak corresponds to a particular CF-kinetic energy



- LDOS reconstructed by populating successive ΛLs with CFs
- each LDOS peak associated to a particular CF-kinetic energy

S. Pu, Ajit C. Balram *et al.*, arXiv:2312.06779

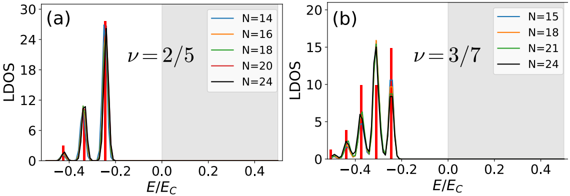
hole is fully captured by the CF-basis but electron is not



S. Pu, Ajit C. Balram *et al.*, arXiv:2312.06779

simulating the STM signal with an oversimplified model

- higher E_K states are higher-order processes and thus weaker
- assume intensity $I_K = n_K t^K$, where $t < 1$ is strength of order-K process and n_K is the number of excitations with energy E_K
- For very high-K, I_K goes down quickly and n_K goes down and t^K decays: this would be invisible to the STM spectra

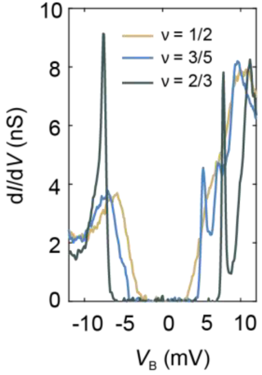


- CF basis are not orthogonal and states at a given E_K contain states with lower E_K in them which we got rid of using Gram-Schmidt orthogonalization

S. Pu, Ajit C. Balram *et al.*, arXiv:2312.06779

better modeling required to match with experiments

- multiple peaks are observed in the experiment but their numbers and energies do not match the theoretical predictions



- tip creates a potential, can screen the interaction, etc.

Y. Hu *et al.*, arXiv:2308.05789

Outlook

- Can STM be used to identify the underlying topological phase of matter at ν=5/2? Resolve the long-standing question of Pfaffian vs. anti-Pfaffian vs. PH-Pfaffian.
- Can we use STM to see signatures of partons? Potentially at ν=2/7 or 2/9 (might require tunneling of two electrons) or 1/4 Pfaffian.

STM signal can serve as a unique fingerprint of strongly interacting topological phases of matter, thereby opening a new direction for studying fractionalized excitations and identifying the nature of many-body states in the FQHE

Thanks