Matter fields

matter fields : everything except Goldstone bosons.

Transform as representations of unbroken symmetry H. They do not form representations of broken group G.

Look at baryons. can easily generalize to other states.

$SU(3)_V$ octet of baryons

$$B = \begin{bmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{bmatrix}$$

Transforms as an adjoint under $SU(3)_V$

$$B \to V B V^\dagger \qquad L = R = V$$

Then for $g = (L, R) \in SU(3)_L \times SU(3)_R$

$$B \to h B h^\dagger \qquad \xi \to L \xi h^\dagger = h \xi R^\dagger$$

one can pick other choices. eg. $B_L \to L B_L L^\dagger \quad B_R \to R B_R R^\dagger$

Then $$\xi^\dagger B_L \xi \to h \left(\xi^\dagger B_L \xi \right) h^\dagger$$
$$\xi B_R \xi^\dagger \to h \left(\xi B_R \xi^\dagger \right) h^\dagger$$

which is a field redefinition.

To write Lagrangian, useful to introduce

$$V_\mu = \frac{1}{2} \left(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right) = \frac{1}{2f^2} [\pi, \partial_\mu \pi] + \cdots$$
$$A_\mu = \frac{i}{2} \left(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right) = \frac{\partial_\mu \pi}{f} + \cdots$$

A_μ hermitian $\qquad V_\mu$ antihermitian

$$\partial\xi \to L\,\partial\xi\, h^\dagger - L\,\xi\, h^\dagger \partial h\, h^\dagger$$

$$\xi^\dagger \partial \xi \to h\, \xi^\dagger \partial \xi\, h^\dagger - \partial h\, h^\dagger$$

$$\xi\, \partial \xi^\dagger \to h\, \xi\, \partial \xi^\dagger\, h - \partial h\, h^\dagger$$

$$A_\mu \to h\, A_\mu\, h^\dagger$$

$$V_\mu \to h\, V_\mu\, h^\dagger - \partial h\, h^\dagger$$

$D_\mu = \partial_\mu + V_\mu$ = chiral covariant derivative

$\psi \to h\psi$ then $D_\mu \psi = \partial_\mu \psi + V_\mu \psi$

$$\to h\,(D_\mu \psi)$$

$$\Rightarrow \quad D_\mu B = \partial_\mu B + [V_\mu, B]$$

$$D_\mu B \to h\, D_\mu B\, h^\dagger$$

$$\mathcal{L} = \operatorname{tr} \bar{B}(i\not{D} - m_B) B + D \operatorname{tr} \bar{B}\gamma^\mu \gamma_5 \{A_\mu, B\}$$
$$+ F \operatorname{tr} \bar{B}\gamma^\mu\gamma_5 [A_\mu, B] + \mathcal{L}_{meson}$$

$\mathcal{L}$ baryon is order p.

$m_B \sim 1$ GeV and leads to a breakdown of the chiral power counting. The solution is to treat the baryon as in HQET, with an expansion in $1/m_B$.

$$B_v(x) = e^{im_B(v\cdot x)}\, \frac{1+\not{v}}{2}\, B(x)$$

We write the most general $\mathcal{L}$ using $B_v(x)$. We do not have to first use $B(x)$ and then convert. Can write B_v Lagrangian directly.

$$\mathcal{L}_v = \mathrm{tr}\, \bar{B}_v (iv\cdot D) B_v + D\, \mathrm{tr}\, \bar{B}\, \gamma^\mu \gamma_5 \{A_\mu, B_v\}$$
$$+ F\, \mathrm{tr}\, \bar{B}_v \gamma^\mu \gamma_5 [A_\mu, B_v] + O\left(\frac{1}{m_B}\right) + \mathcal{L}_{meson}$$

The baryon mass is not present in the leading order Lagrangian, and corrections are $1/m_B$ $\Rightarrow$ power counting valid.

Baryon number is conserved, so baryon lines go through diagrams, and we are shifting their energies by m_B.

When SU(3) symmetry breaking is included, m_B is an average baryon mass, and all baryon masses get shifted $m \longrightarrow m - m_B$.

$\mathcal{L}_v$ is order p. The baryon propagator is $\frac{i}{k\cdot v}$

The power counting rule in the one-baryon sector is

$$D - 1 = 2L + \sum V_k (k-2) + \sum W_k (k-1)$$

V_k = p^k vertices in purely mesonic Lagrangian

W_k = p^k vertices in one-baryon sector L_v.

Note $V_k \neq 0$ for $k \geqslant 2$ $W_k \neq 0$ for $k \geqslant 1$ so all terms on r.h.s. are non-negative.

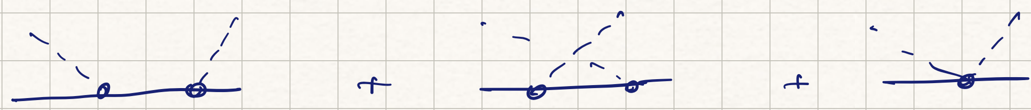
The naive dimensionless estimate of coefficients is

$$\mathcal{L} \sim \underbrace{\Lambda_\chi^2 f^2}_{\frac{\Lambda_\chi^2}{16\pi^2}} \left(\frac{\pi}{f}\right)^a \left(\frac{\partial}{\Lambda_\chi}\right)^b \underbrace{\left(\frac{B}{f\sqrt{\Lambda_\chi}}\right)^c}_{\left(\frac{4\pi B}{\Lambda_\chi^{3/2}}\right)^c}$$

since B is a fermion

πN scattering lengths

πN scattering at threshold. $SU(2)$ $B_v = \begin{pmatrix} p_v \\ n_v \end{pmatrix}$



$A_\mu \sim \partial_\mu \pi$

so $\bar{B}_v \gamma^\mu \gamma_5 B_v \frac{\partial_\mu \pi}{f}$ coupling

But B_v is a "heavy field" with $\not{v} B = 0$

$\Rightarrow \bar{B}_v \gamma^\mu \gamma_5 B_v = 0$ if $\mu = 0$

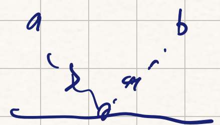
$\gamma^i \gamma_5 \longrightarrow \sigma^i$

$\partial_\mu \pi \longrightarrow p_\pi^\mu = (E_\pi, \vec{0})$ so first two graphs vanish.

The 2π interaction is $\frac{i}{2f^2} \bar{B}_v [\pi, \partial_\mu \pi] v^\mu B_v$ from $(iv \cdot D)$

$[\pi, \partial_\mu \pi] = i f_{cgh} \pi^g \partial_\mu \pi^h$

π are in the adjoint rep so $(T_\pi^c)_{hg} = i f_{agh}$



interaction $\frac{i}{2f^2} (T_\pi^c)_{hg} \bar{B}_v T^c B_v (\pi^g v \cdot \partial \pi^h)$

$$A = i \cdot \frac{i}{2f^2} (T_B^c)(T_\pi^c)(i M_\pi) \cdot 2$$

$$= -\frac{i}{f^2} M_\pi (T_\pi \cdot T_B)(2 m_N)$$ switching to relativistic normalization of spinors.

Weinberg - Tomozawa formula.

$$a \equiv \frac{-i A}{8\pi (m_\pi + m_N)} = \text{scattering length}$$

$$a = -\frac{1}{6\pi f^2}(T_\pi \cdot T_B)\frac{2m_N m_\pi}{m_\pi + m_N} \qquad L = \frac{m_\pi}{8\pi f^2}$$

$$= -L\,(2T_\pi \cdot T_B)\frac{1}{(1+m_\pi/m_N)}$$

$$2T_\pi \cdot T_B = (T_\pi + T_B)^2 - T_\pi^2 - T_B^2$$

$$= (T_\pi + T_B)^2 - 2 - \frac{3}{4} = \begin{cases} I = 1/2 & -2 \\ I = 3/2 & 1 \end{cases}$$

$$a_{1/2} = 2L\left(1+\frac{m_\pi}{m_N}\right)^{-1}$$

$$a_{3/2} = -L\left(1+\frac{m_\pi}{m_N}\right)^{-1}$$

$$a_{1/2} + 2a_{3/2} = 0$$

<u>Baryon masses</u> E. Jenkins NPB 368 (1992) 190

$$M \to LMR^\dagger \qquad \xi \to L\xi h^\dagger = h\xi R^\dagger$$

$$\xi^\dagger M \xi^\dagger \to hMh^\dagger \qquad \xi M^\dagger \xi \to hM^\dagger h^\dagger$$

$$\begin{aligned} L_m = {} & b_D \langle \bar{B}_v \{\xi^\dagger M \xi^\dagger + \xi M^\dagger \xi, B_v\}\rangle \\ & + b_F \langle \bar{B}_v [\xi^\dagger M \xi^\dagger + \xi M^\dagger \xi, B_v]\rangle \\ & + \sigma\, Tr(M\Sigma^\dagger + M^\dagger \Sigma)\,\bar{B}_v B_v \end{aligned}$$ = σ-term

In limit $m_u = m_d = 0$ $m_s \neq 0$.

$$\begin{aligned} m_N &= m_B - 2(b_D - b_F)m_s - 2\sigma m_s \\ m_\Lambda &= m_B - \frac{8}{3}b_D m_s - 2\sigma m_s \\ m_\Sigma &= m_B - 2\sigma m_s \\ m_\Xi &= m_B - 2(b_D + b_F)m_s - 2\sigma m_s \end{aligned}$$

σ : can be absorbed into m_B for masses. But σ leads to πB terms, but m_B does not.

Gell-Mann Okubo : $\frac{1}{2}(m_N + m_\Xi) = \frac{3}{4} m_\Lambda + \frac{1}{4} m_\Sigma$

1128 MeV $\simeq$ 1135 MeV.

Λ, Σ : uds but they do not have same mass
$3\Lambda + \Sigma$ not avg since $1\ \Lambda$ and $3\ \Sigma$ states

B, B^*, D, D^* χPT

$$H = \frac{1+\slashed{v}}{2}\left(B^*_\mu \gamma^\mu - B\gamma_5\right)$$

$b\bar{u}$
$b\bar{d}$
$b\bar{s}$

$H \longrightarrow S_Q H$ spin transformation

$H \longrightarrow H h^\dagger$ under $SU(3) \times SU(3)$

H = 4×4 spinor matrix $\sim Q\bar{q}$ light quark index on right, heavy quark on left.

$$\mathcal{L} = \operatorname{tr} \bar{H}(iv\cdot D)H + g \operatorname{tr} \bar{H} H \gamma^\mu \gamma_5 A_\mu$$

$$D_\mu H = \partial_\mu H - H V_\mu$$

$B^*\ g\ B^*$ $\qquad$ $B^*\ g\ B$ $\qquad$ related by heavy quark spin symmetry.

$\langle \bar{H} \gamma^\mu \gamma_5 H A_\mu \rangle \frac{1}{m_Q}$ since violates spin symmetry

calculate loops, etc.

$\frac{f_{B_s}}{f_B}$ etc.

Sources

$$u = e^{i\pi/f} \qquad U = u^2 = e^{2i\pi/f}$$

$$u \to R u h^\dagger = h u L^\dagger \qquad U \to R U L^\dagger$$

and $\quad u \to \xi^\dagger \qquad U \to \Sigma^\dagger \qquad \pi \to -\pi$

To understand pion dynamics, including weak and EM interactions, treat masses and EW gauge fields as weakly coupled background fields.

$$\mathcal{L} = \bar{q}_L (i\not{D} + \not{l}) q_L + \bar{q}_R (i\not{D} + \not{r}) q_R$$
$$- \bar{q}_R \, m \, q_L - \bar{q}_L \, m^\dagger q_R$$

$$\left.\begin{aligned} l_\mu &= l_\mu^a T^a \\ r_\mu &= r_\mu^a T^a \end{aligned}\right\} \; 3 \times 3 \text{ hermitian gauge field}$$

can include a $U(1)_V$ gauge field coupled to baryon number, but <u>not</u> one coupled to $U(1)_A$.

W, Z, γ are included in l, r.

$\mathcal{L}$ still has $SU(3)_L \times SU(3)_R$ chiral symmetry provided

$$l_\mu \to L \, l_\mu L^\dagger - i \, \partial_\mu L L^\dagger$$
$$r_\mu \to R \, r_\mu R^\dagger - i \, \partial_\mu R R^\dagger$$
$$m \to R \, m \, L^\dagger$$

With background gauge fields, it is a <u>local</u> symmetry.

$$F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu] \qquad \sim O(p^2)$$

$$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]$$

$$\begin{aligned}
\mathcal{L} &= \bar{q}(i\not{D} + \not{l})P_L q + \bar{q}(i\not{D} + \not{r})P_R q \\
&\quad - \bar{q}\, m P_L q - \bar{q}\, m^\dagger P_R q \\
&= \bar{q}\, i\not{D} q + \bar{q}\,\tfrac{1}{2}(\not{l} + \not{r}) q + \bar{q}\,\tfrac{1}{2}(\not{r} - \not{l})\gamma_5 q \\
&\quad - \bar{q}\,\tfrac{1}{2}(m + m^\dagger) q - \bar{q}\,\tfrac{1}{2}(m^\dagger - m)\gamma_5 q \\
&= \bar{q}\, i\not{D} q + \bar{q}(\not{v} + \not{a}\gamma_5) q - \bar{q}(s - ip\gamma_5) q
\end{aligned}$$

$$\left.\begin{aligned} v_\mu &= \tfrac{1}{2}(r_\mu + l_\mu) & a_\mu &= \tfrac{1}{2}(r_\mu - l_\mu) \\ s &= \tfrac{1}{2}(m + m^\dagger) & p &= \tfrac{i}{2}(m^\dagger - m) \end{aligned}\right\} \text{hermitian}$$

$$s + ip = m \qquad \chi = 2B(s + ip) = 2Bm$$

$$u_\mu \equiv i\{u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i l_\mu)u^\dagger\} \sim O(p)$$

$$\sim -\frac{2\partial\pi}{f} + \dots = 2A_\mu$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \qquad \sim O(p^2)$$

Transform as $h\, u_\mu h^\dagger$ $\quad h\, \chi_\pm h^\dagger$.

Choose building blocks that transform under h. Can always convert L or R to h using u to $u^\dagger$.

$$\chi_+^\dagger = \chi_+ \qquad \chi_-^\dagger = -\chi_-$$

$$u_\mu^\dagger = -i\left\{ \partial_\mu u^\dagger u + i u^\dagger r_\mu u - \partial_\mu u u^\dagger - i u l_\mu u^\dagger \right\}$$

$$= -i\left\{ -u^\dagger \partial_\mu u + i u^\dagger r_\mu u + u \partial_\mu u^\dagger - i u l_\mu u^\dagger \right\}$$

$$= u_\mu$$

$$\mathcal{L}_2 = \frac{f^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle$$

$$\langle \chi_+ \rangle = \langle u^\dagger \chi u^\dagger + u \chi^\dagger u \rangle$$

$$= \langle \chi (u^\dagger)^2 + \chi^\dagger u^2 \rangle = \langle \chi U^\dagger + \chi^\dagger U \rangle$$

had $\frac{Bf^2}{2}$ previously now $\frac{f^2}{4}$ so $2B$ is absorbed into definition of χ.

$$U = u^2$$

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$$

$$= \partial_\mu u u + u \partial_\mu u - i r_\mu u^2 + i u^2 l_\mu$$

$$u^\dagger D_\mu U u^\dagger = u^\dagger \partial_\mu u + \partial_\mu u u^\dagger - i u^\dagger r_\mu u + i u l_\mu u^\dagger$$

$$\partial_\mu u = - u \partial_\mu u^\dagger u$$

$$u^\dagger D_\mu U u^\dagger = u^\dagger \partial_\mu u - u \partial_\mu u^\dagger - i u^\dagger r_\mu u + i u l_\mu u^\dagger$$

$$= u^\dagger (\partial_\mu - i r_\mu u) - u (\partial_\mu u^\dagger - i l_\mu u^\dagger)$$

$$= -i u_\mu .$$

$$u D_\mu U^\dagger u = i u_\mu$$

$$\langle u_\mu u^\mu \rangle = \langle u D_\mu u^\dagger u \, u^\dagger D^\mu U u^\dagger \rangle = \langle D_\mu U^\dagger D^\mu U \rangle$$

so this is the usual L_2 Lagrangian.

$$f_\pm^{\mu\nu} \equiv u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u \qquad O(p^2)$$

$$\Gamma_\mu = \frac{1}{2} \left\{ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger \right\}$$

$$= V_\mu \qquad \Gamma_\mu^\dagger = -\Gamma_\mu$$

$$\Gamma_\mu \to h \Gamma_\mu h^\dagger - \partial_\mu h \, h^\dagger$$

$$\nabla_\mu = \partial_\mu + V_\mu \qquad \text{covariant derivative for } h$$

$$D_\mu = \partial_\mu - i r_\mu \qquad \text{for } R$$

$$D_\mu = \partial_\mu - i l_\mu \qquad \text{for } L$$

$$h_{\mu\nu} \equiv \nabla_\mu u_\nu + \nabla_\nu u_\mu \qquad O(p^2)$$

$$\chi_{\pm\mu} \equiv u^\dagger D_\mu \chi u^\dagger \pm u D_\mu \chi^\dagger u \qquad O(p^3)$$

verify

$$\nabla^\nu u^\mu - \nabla^\mu u^\nu = f_-^{\mu\nu} \quad \text{is not independent}$$

$$[\nabla_\mu, \nabla_\nu] = \Gamma_{\mu\nu} = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2} f_{+\mu\nu}$$

$$\nabla_\mu \Gamma_{\nu\rho} + \text{cyclic} = 0 \quad \text{(Bianchi identity)}$$

$$L_4 = \hat{L}_0 \langle u_\mu u_\nu u_\mu u_\nu \rangle + \cdots$$

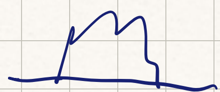
$$\hat{L}_5 \langle u \cdot u \, \chi_+ \rangle + \cdots$$

$$\hat{L}_6 \langle \chi_+^2 \rangle \cdots$$

$$i \hat{L}_9 \langle f_{+\mu\nu} u_\mu u_\nu \rangle$$

$$+ \frac{L_{10}}{4} \langle f_{+\mu\nu}^2 - f_{-\mu\nu}^2 \rangle$$

$$\vdots$$



$$\left(\frac{e}{4\pi}\right)^2 f^2 \Lambda_\chi^2 \langle (Q_R U - U Q_L)(U^\dagger Q_R - Q_L U^\dagger) \rangle$$

$$Q = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix} \in SU(3)$$

$$\Delta m^2 \sim \frac{\alpha}{4\pi} \Lambda_\chi^2$$

$$1000 \text{ MeV} \sim 800 \text{ MeV}$$

exchange $SU(3)_R$: $\langle T_a U U^\dagger T^a \rangle \rightarrow$ constant