Detection Principles of Gravitational Wave Observatories: II

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Acknowledgment

- *I gratefully acknowledge the discussions and lecture notes of Profs. Gerhard Schäfer, Claus Lämmerzahl, Norbert Straumann, Bernad Schutz, Bala Iyer ++*

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  However, any lack of clarity that I may impose on you is entirely my own.

Outline

- Interferometric Milli-Hz GW Observatories
- nHz GW Observatories

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- nHz GW Observatories
- Spacecraft Tracking

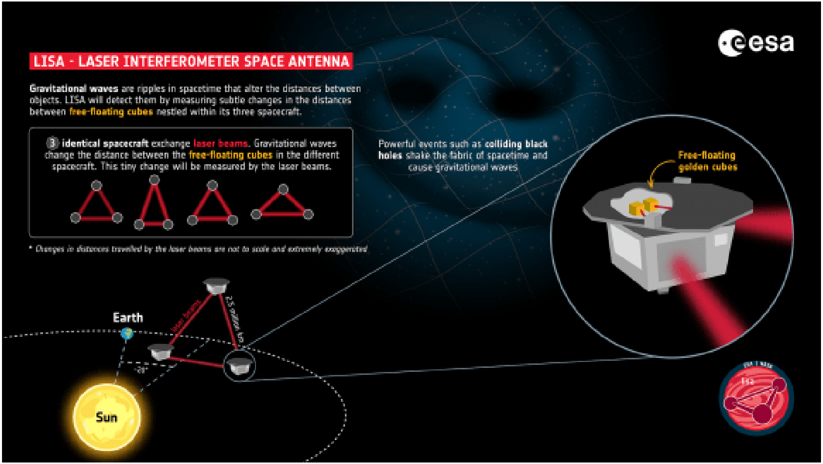
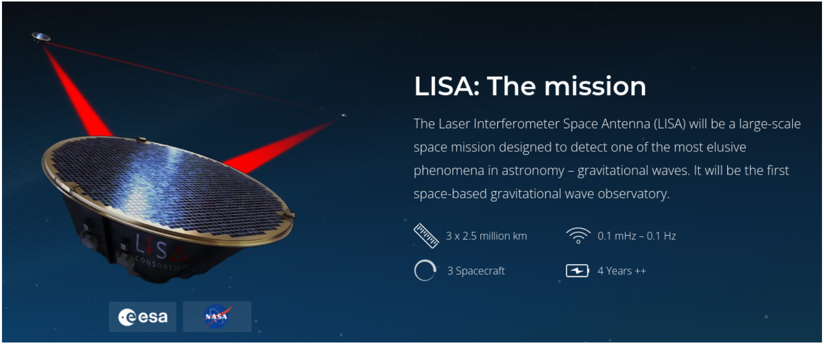


Figure: Caption

It's an amazing GW observatory(ies) between $1 \rightarrow 10$ milli-Hz


LISA: The mission
The Laser Interferometer Space Antenna (LISA) will be a large-scale space mission designed to detect one of the most elusive phenomena in astronomy – gravitational waves. It will be the first space-based gravitational wave observatory.
3 x 2.5 million km
0.1 mHz – 0.1 Hz
3 Spacecraft
4 Years ++
LISA
CONSORTIUM
esa
NASA

LISA

- LISA consists of 3 independent spacecrafts arrayed as an equilateral triangle, with laser beams along each of the arms.
  Spacecrafts are essentially point particles following geodesics,
- Given its 3 arms, LISA can be regarded as having three independent L-shaped interferometers, made from the arms that join at any two of the three vertices.
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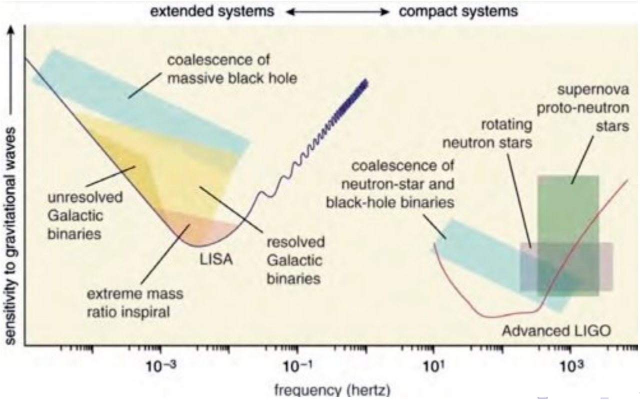
- These interferometers sense two different GW polarization states (without requiring another space interferometer)
- Any two detectors share a common arm, their instrumental noise in the two detectors is not independent.
  $\Rightarrow$ It is NOT advisable to do a cross-correlation experiment between its two detectors in order to improve its sensitivity to a stochastic GW background!!.

LISA-LIGO Sources

Astrophysical sources are mainly self-gravitating compact objects

$$f_0 = \frac{1}{4\,\pi}\left(\frac{3\,G\,M}{R^3}\right)^{1/2} \Rightarrow f_p \sim 1\ kHz\left(\frac{10\,M_\odot}{M}\right)$$



Orbits of BH Binaries

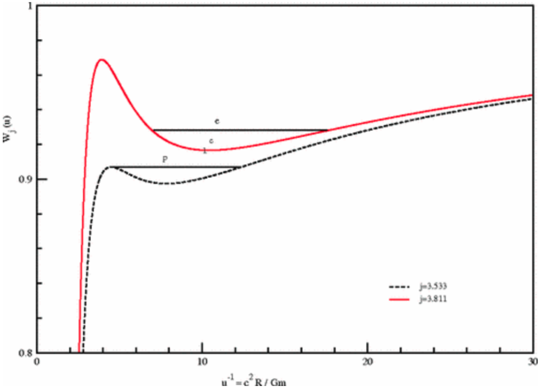


Figure: Certain effective radial potential Vs a dimensionless radial variable

The left end of the line p is tangent to the effective potential and it leads to an unstable circular orbit

LISA Sources: I

- LISA is an amazing GW observatory between $1 \to 10$ milli-Hz

$$f_p \sim 1 \; kHz \left(\frac{10 \, M_\odot}{M} \right)$$

- LISA should be able to hear GWs from merging SMBH Binaries (with total mass $10^5 - 10^7 \, M_\odot$ if it occurs during the mission lifetime & essentially anywhere in the Universe

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- $\Rightarrow$ Coordinated EM observing campaigns!!

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- $\Rightarrow$ Coordinated EM observing campaigns!!
- We should be able to probe the growth, evolution, and the initial mass spectrum of such SMBHs
- Their inspiral should occur along 'c'/'e' type orbits

LISA Sources: II

- EMRI: Extreme Mass-Ration Inspiral sources
  Stellar mass BHs can sink toward the Galactic Nuclei where SMBHs reside
  $\Rightarrow$ Overdesnsity of stellar mass BHs near the vicinity of SMBHs
- Occasionally a random close encounter should put one of these Stellar BHs on a 'plunge' orbit, a highly eccentric orbit aimed nearly directly at the central SMBH
- If the impact parameter is sufficiently small, the first pass near the SMBH should radiate enough energy in GWs
  $\Rightarrow$ A hyperbolic to an eccentric orbit

LISA Sources: III

- Stellar mass BH should perform a large number of orbits during which its periholon distance remains roughly constant but its apholon distance shrinks
- Such events should be 'hearable' upto few tenths of z

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- Such events should be 'hearable' upto few tenths of z
- There may be up to 10^5 orbits before stellar mass BH plunges into the SMBH's horizon!!
- Various details of its orbital behavior should be contained in the phase evolution of its GWs
- LISA should be able to examine in detail the geometry of the black hole exterior, and test the uniqueness theorem of general relativity (essentially all black holes will be members of the Kerr family)

Are such events possible?

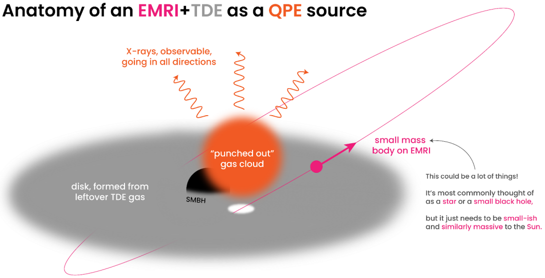
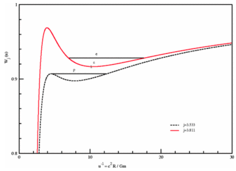


Figure: `https://astrobites.org/2024/01/24/quasi-periodic-eruptions-galactic-nuclei/`



LISA Sources: IV

- IMRI: Intermediate Mass-Ratio Inspiral sources
- It is possible that Population III stars may have produced an abundance of heavy black holes, in the range from a few hundred to a few thousand solar masses.
- These intermediate-mass black holes (IMBH) could be captured by Galactic Center SMBHs

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  Compact binaries containing White Dwarves, Neutron Stars & Stellar mass BHs

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  Compact binaries containing White Dwarves, Neutron Stars & Stellar mass BHs
- SKA-LISA should observe Pulsar binaries with Neutron Stars/Black Holes

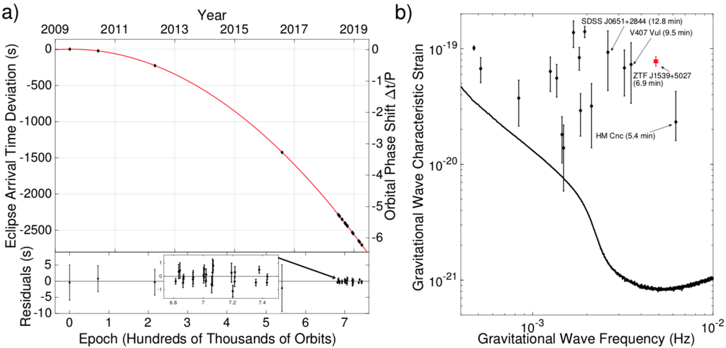

Figure: ZTF J1539+5027

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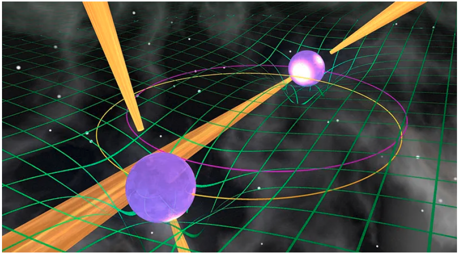

Figure: Double Pulsar

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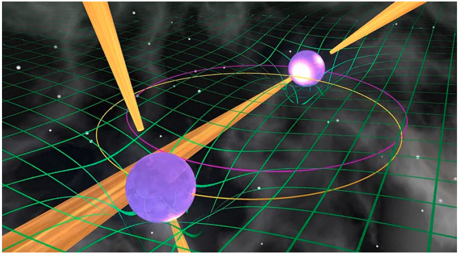

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- LISA can only detect the stochastic GW background if its noise is higher than instrumental or confusion noise from foreground objects
- If the LISA output is higher than the Sagnac noise, then LISA may have detected a GW background

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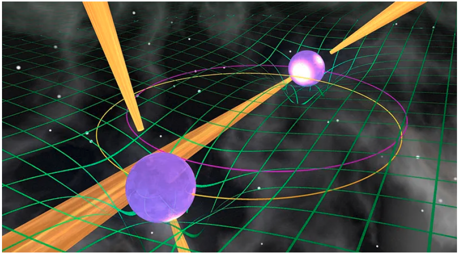

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- Unexpected GW sources

$$g_{ij}(x^0,0,0,0)(x^i-0)(x^j-0)$$
$$=\delta_{ij}x^ix^j+|A_+|\left((x^1)^2-(x^2)^2\right)\cos(\varphi-\omega t)+2|A_\times|x^1x^2\cos(\psi-\omega t).$$

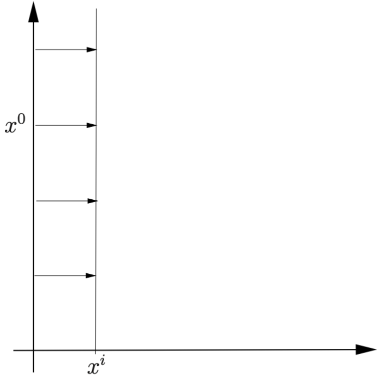

Figure: The x^0-lines are the worldlines of freely falling particles. For any such particle the (x^1, x^2, x^3)-coordinates remain constant. This does not mean that GWs do not affect freely falling particles. The distance, as measured with the metric, between neighboring x^0-lines is not constant!!

How Does LISA Work?: I

- Let one arm of an interferometer is along the x-direction & our plane GW is moving in the z-direction
  Restrict to $+$ polarization & $h_+(t)$ doesn't depend on x

- In this case, the null geodesic moving in the x-direction satisfies

$$ds^2 = -dt^2 + (1 + h_+)dx^2 + (1 - h_+)dy^2 + dz^2 = 0,$$

- We extract an effective speed

$$\left(\frac{dx}{dt}\right)^2 = \frac{1}{1 + h_+}.$$

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$$\left(\frac{dx}{dt}\right)^2 = \frac{1}{1 + h_+}.$$

- This is a *coordinate speed*, with no contradiction to special relativity.

How Does LISA Work?: II

- A photon emitted at time t from the origin reaches the other end, at a fixed coordinate position $x = L$, at the coordinate time

$$t_{\rm far} = t + \int_0^L \left[1 + h_+(t(x))\right]^{1/2} dx,$$

where $t(x)$ denotes the fact that one must know the time to reach position x in order to calculate the wave field.

- In our linearized EFEqs, $t(x) = t + x$ and we should expand the square root

$$t_{\rm far} = t + L + \frac{1}{2}\int_0^L h_+(t+x)dx.$$

- In a Space interferometer, the light is reflected back $\Rightarrow$

$$t_{\rm return} = t + 2L + \frac{1}{2}\left[\int_0^L h_+(t+x)dx + \int_0^L h_+(t+L+x)dx\right].$$

How Does LISA Work?: III

- For a space-based GW observatory, one essentially monitors changes in the time for the return trip as a function of time at the origin!

$$\frac{dt_{\rm return}}{dt} = 1 + \frac{1}{2}\left[h_+(t+2L) - h_+(t)\right].$$

Interestingly, it does not involve the wave amplitude at the other end.

- The wave amplitude at the other end does get involved if the wave travels at an angle θ to the z-axis in the $x - z$ plane.

$$\frac{dt_{\rm return}}{dt} = 1 + \frac{1}{2}\Big\{(1-\sin\theta)h_+(t+2L) - (1+\sin\theta)h_+(t)$$
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We need to allow the phase of the wave to depend on x in the appropriate way to incorporate the fact that h_{xx} is comparatively reduced

It's a bit more complicated!!

C. LISA's response function

With its three arms, LISA functions as a pair of two-arm detectors, outputting two orthogonal signals. Let l_1^i, l_2^i, l_3^i be unit vectors, each along one of LISA's three arms, and let L be LISA's average arm length. Let also $L_i(t)$ be the length of the i'th arm when LISA measures an incident GW, and denote $\delta L_1(t) \equiv L_i(t) - L$. We refer to the two-arm detector formed by arms 1 and 2 as "detector I." The strain amplitude in this detector is given by

$$h_I(t) \equiv [\delta L_1(t) - \delta L_2(t)]/L = \frac{1}{2} h_{ij}(t)(l_1^i l_1^j - l_2^i l_2^j). \tag{11}$$

The second, orthogonal signal is then given by [10]

$$\begin{aligned} h_{II}(t) &= 3^{-1/2}[\delta L_1(t) + \delta L_2(t) - 2\,\delta L_3(t)]/L \\ &= \frac{1}{2\sqrt{3}} h_{ij}(t)(l_1^i l_1^j + l_2^i l_2^j - 2 l_3^i l_3^j). \end{aligned} \tag{12}$$

For GW wavelengths much larger than the LISA arm length, $h_I(t)$ and $h_{II}(t)$ coincide with the two "Michelson variables" [26], describing the responses of a pair of two-arm/90° detectors. We can then write $h_I(t)$ and $h_{II}(t)$ as a sum over n-harmonic contributions,

$$h_\alpha(t) = \sum_n h_{\alpha,n}(t) \quad (\alpha = I, II), \tag{13}$$

$$h_{\alpha,n}(t) = \frac{1}{D}\frac{\sqrt{3}}{2}[F_\alpha^+(t)A_n^+(t) + F_\alpha^\times(t)A_n^\times(t)]. \qquad (14)$$

Here $A_n^{+,\times}(t)$ are the two polarization coefficients [given, in our model, by Eq. (10) above], the factor $\sqrt{3}/2$ accounts for the fact that the actual angle between LISA arms is 60° rather than 90°, and $F_\alpha^{+,\times}$ are the "antenna pattern" functions, reading [23,27]

$$F_I^+ = \frac{1}{2}(1+\cos^2\theta)\cos(2\phi)\cos(2\psi) - \cos\theta\sin(2\phi)\sin(2\psi),$$

$$F_I^\times = \frac{1}{2}(1+\cos^2\theta)\cos(2\phi)\sin(2\psi) + \cos\theta\sin(2\phi)\cos(2\psi), \qquad (15a)$$

$$F_{II}^+ = \frac{1}{2}(1+\cos^2\theta)\sin(2\phi)\cos(2\psi) + \cos\theta\cos(2\phi)\sin(2\psi),$$

$$F_{II}^\times = \frac{1}{2}(1+\cos^2\theta)\sin(2\phi)\sin(2\psi) - \cos\theta\cos(2\phi)\cos(2\psi). \qquad (15b)$$

Figure: Caption

In these expressions, (θ, ϕ) is the source's sky location in a detector-based coordinate system and ψ is the "polarization angle" describing the orientation of the "apparent ellipse" drawn by the projection of the orbit on the sky—see Fig. 1 in Ref. [23] and the explicit relation (17) given below.

It is more convenient to express the above response function in terms of angles defined not in the rotating, detector-based system, but rather in a fixed, ecliptic-based coordinate system. The angles θ, ϕ are related to θ_S, ϕ_S—the source location in an ecliptic-based system—through

$$\cos\theta(t) = \frac{1}{2}\cos\theta_S - \frac{\sqrt{3}}{2}\sin\theta_S \cos[\bar{\phi}_0 + 2\pi(t/T) - \phi_S],$$

$$\phi(t) = \bar{\alpha}_0 + 2\pi(t/T) + \tan^{-1}\left[\frac{\sqrt{3}\cos\theta_S + \sin\theta_S \cos[\bar{\phi}_0 + 2\pi(t/T) - \phi_S]}{2\sin\theta_S \sin[\bar{\phi}_0 + 2\pi(t/T) - \phi_S]}\right], \tag{16}$$

where $T = 1$ year and $\bar{\phi}_0, \bar{\alpha}_0$ are constant angles specifying, respectively, the orbital and rotational phase of the detector at $t = 0$. (See Cutler [10] for a complete definition of these angles; note, though, that the angle $\bar{\alpha}_0 = 0$ in this paper is referred to as $\alpha_0 = 0$ in Cutler [10].)

Figure: Papers by Curt Cutler

LISA & TDI

- In the $1 - 10$ mHz regime, the fact that the signals from the three interferometers are not independent leads to a noise veto:
- There exists a linear combination of the three TDI output signals that cancels out all GW signals (the Sagnac mode)

LISA & TDI

- In the $1 - 10$ mHz regime, the fact that the signals from the three interferometers are not independent leads to a noise veto:
- There exists a linear combination of the three TDI output signals that cancels out all GW signals (the Sagnac mode)

An amazing way to measure the instrumental noise!!

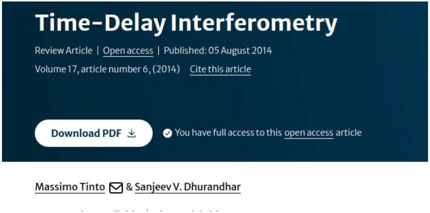

Figure: It should be possible to adapt it for PTAs!!

PTA: Introduction

- A millisecond pulsar (MSP) emits radio pulses at a rate that is highly stable
  Stability of their rotational frequency is comparable to the stablity of Atomic Clocks!!
- However, pulses from MSPs does NOT arrive a constant frequency here on our Radio Telescopes like uGMRT!!
- Pulse Times-of-Arrival (TOAs) are affected by many many effects like the relative motion of the pulsar and the Earth, by the influence of the gravitational field of the Sun and of other masses the pulses might pass, and by the interstellar medium ++++
- We have developed deterministic Timing Formulae to ensure that many such delays are taken into account by regularly monitoring MSPs for years

- In other words, we have approaches to predict 'TOAs' from close to 100 MSPs
- The time-series that arise from the difference between the predicted & observed TOAs ⇒ Pulsar Timing Residuals
- A passing GW should lead to such Timing Residuals
- The idea of searching for GWs in the Pulsar timing residuals of pulsars are due to
  M. Sazhin ["Opportunities for detecting ultralong gravitational waves" Astron. Zh. 55, 65 (1978)]
  S. Detweiler ["Pulsar timing measurements and the search for gravitational waves" Astrophys. J. 234, 1100 (1979)].

The Nanohertz Gravitational Wave Astronomer

Stephen R. Taylor

Gravitational waves are a radically new way to peer into the darkest depths of the cosmos. Pulsars can be used to make direct detections of gravitational waves through precision timing. When a gravitational wave passes between a pulsar and the Earth, it stretches and squeezes the intermediate space-time, leading to deviations of the measured pulse arrival times away from model expectations. Combining the data from many Galactic pulsars can corroborate such a signal, and enhance its detection significance. This technique is known as a Pulsar Timing Array (PTA). Here I provide an overview of PTAs as a precision gravitational-wave detection instrument, then review the types of signal and noise processes that we encounter in typical pulsar data analysis. I take a pragmatic approach, illustrating how searches are performed in real life, and where possible directing the reader to codes or techniques that they can explore for themselves. The goal is to provide theoretical background and practical recipes for data exploration that allow the reader to join in the exciting hunt for very low frequency gravitational waves.

Comments: Draft of a short technical book to be published later this year by Taylor & Francis. 156 pages. Comments and errata are welcome
Subjects: **High Energy Astrophysical Phenomena (astro-ph.HE)**; General Relativity and Quantum Cosmology (gr-qc); Applications (stat.AP)
Cite as: arXiv:2105.13270 **[astro-ph.HE]**
(or arXiv:2105.13270v1 **[astro-ph.HE]** for this version)
https://doi.org/10.48550/arXiv.2105.13270

Figure: arXiv:2105.13270

How Does a PTA Work: I

- Let the pulsar and the Earth be at rest in a Minkowski background such that

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x),$$

  and we employ the TT gauge such that
  A) $h_{0\mu} = 0$ & B) the curves $(x^\mu(t)) = (ct, \vec{r}_0)$ with a constant $\vec{r}_0$, are geodesics.
  (the worldlines of constant spatial coordinates are geodesics)

- Our $h_{\mu\nu}$ is an arbitrary superposition of GWs & Our metric $\rightarrow$

$$g_{\mu\nu}(x) dx^\mu dx^\nu = -c^2 dt^2 + \delta_{ij} dx^i dx^j + h_{ij}(x) x^i dx^j$$

How Does a PTA Work: II

- $$g_{\mu\nu}(x)dx^\mu dx^\nu = -c^2 dt^2 + \delta_{ij} dx^i dx^j + h_{ij}(x) x^i dx^j$$
- Along the wordlines of Pulsar & Earth, the time coordinate t coincides with proper time.
- Therefore, we can identify frequencies with respect to the time coordinate t

  Frequencies with respect to proper time intervals of the pulsar or of the Earth.

We want to figure out $\nu_E(t_E)$ when linearized GWs are around

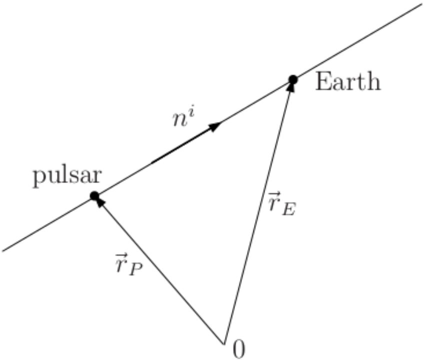

Figure: Pulsar emits signals at a fixed frequency ν_P. These pulses arrive at the Earth with a frequency $\nu_E(t_E)$

How Does a PTA Work: III

- Along a light ray from the pulsar to the Earth, we must have

$$0 = -c^2 dt^2 + \delta_{ij} dx^i dx^j + h_{ij}(x) dx^i dx^j$$

$\Rightarrow$

$$c^2 \left(\frac{dt}{d\ell} \right)^2 = 1 + h_{ij}(x) \frac{dx^i}{d\ell} \frac{dx^j}{d\ell},$$

- We have the arclength with respect to the flat background metric ℓ

$$d\ell^2 = \delta_{ij} dx^i dx^j.$$

- When a GW is around

$$\frac{dx^i}{d\ell} = n^i + O(h),$$

$$\Rightarrow c \frac{dt}{d\ell} = \sqrt{1 + h_{ij}(x) n^i n^j + O(h^2)},$$

How Does a PTA Work: IV

- Integrating over the path of the light ray, from its emission time t_P to the arrival time t_E $\rightarrow$

$$\int_{t_P}^{t_E} \left(1 - \frac{1}{2} h_{ij}(x) n^i n^j\right) dt = \frac{L}{c},$$

where L is the distance from the pulsar to the Earth measured in the flat background.
We have a way to obtain t_E as a function of t_P.

- Differentiate with respect to t_P yields

$$\frac{dt_E}{dt_P}\left(1 - \frac{1}{2} h_{ij}(ct_E, \vec{r}_E) n^i n^j\right) - \left(1 - \frac{1}{2} h_{kl}(ct_P, \vec{r}_P) n^k n^\ell\right) = 0,$$

- $$\frac{dt_P}{dt_E} = \frac{\left(1 - \frac{1}{2} h_{kl}(ct_E, \vec{r}_E) n^k n^\ell\right)}{\left(1 - \frac{1}{2} h_{ij}(ct_P, \vec{r}_P) n^i n^j\right)}$$

How Does a PTA Work: V

$$\frac{dt_P}{dt_E} = 1 + \frac{n^i n^j}{2} \left(h_{ij}(ct_P, \vec{r}_P) - h_{ij}(ct_E, \vec{r}_E) \right).$$

- Pulses, which are emitted with a constant frequency ν_P, arrive on Earth with a frequency $\nu_E(t_E)$

$$\frac{\nu_E(t_E) - \nu_P}{\nu_P} = \frac{dt_P}{dt_E} - 1 = \frac{n^i n^j}{2} \left(h_{ij}(ct_P, \vec{r}_P) - h_{ij}(ct_E, \vec{r}_E) \right).$$

How Does a PTA Work: V

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- Clearly, the frequency shift depends on the projection of the wave amplitude h_{ij} onto n^i (both at the pulsar and at the Earth) .
- If our GW is propagating in the x^3 direction, n^i should have a non-vanishing components in the $x^1 - x^2$ plane

How GWs affect TOAs?

- If a GW propagates along the positive z direction

$$z(t) = \frac{\nu_P - \nu(t)}{\nu_P} = \frac{\alpha^2 - \beta^2}{2(1+\gamma)} \Delta h_+(t) + \frac{\alpha\,\beta}{1+\gamma} \Delta h_\times(t)\,,$$

  α, β, and γ provide a pulsar's direction cosines w.r.t the x, y, and z axes (Detweiler 1979)
  Where $\Delta h_{+,\times} = h^P_{+,\times} - h^E_{+,\times}$

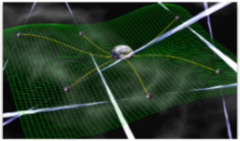
- The observed/measured TOAs are compared with their predictions, based on an appropriate pulsar timing model
  The resulting differences are usually referred to as the 'Pulsar Timing Residuals'

- Observable timing residuals, induced by the GWs, are given by the integral over time of the above Equation

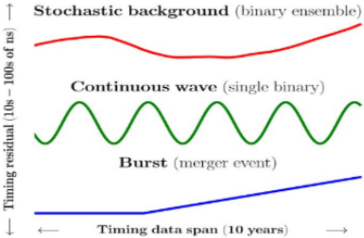
$$R(t) = \int_0^t \frac{\nu_0 - \nu(t')}{\nu_0} \delta t'$$

Pulsar Timing Array's GW Sources

- Observe a bunch of MSPs at different locations in the sky using the ***best radio telescopes*** of our times.
- Search for the correlated timing residual for detecting nHz GWs.



Courtesy: Web



Stochastic GWB from massive BH binaries ?: I

Cosmological population of MBH binaries is expected to provide a diffusive GW background for PTAs !!
We need to compute # of sources in a frequency interval $\Delta f = 1/T_{\rm obs}$

- $$\Delta N = \left(\frac{dN}{df}\right)\Delta f = \frac{dN}{dt}\left(\frac{df}{dt}\right)^{-1}\Delta f \qquad (1)$$
- $$\Delta N \propto \frac{dN}{dt}\left({\mathcal{M}_c}^{-5/3}\, f_{\rm GW}^{-11/3}\right)\Delta f \qquad (2)$$
- There are some 10^{11} galaxies in our Universe and each galaxy is likely to experience one merger with another galaxy in Hubble time (10^{10} year)
- $$\rightarrow \frac{dN}{dt} \sim 10 \text{ mergers/year}$$

Stochastic GWB from massive BH binaries ?: I

- $\rightarrow$ a rough estimate for the number of binary BH sources in a frequency interval $\Delta f = 1/T_{\rm obs}$
-

$$\Delta N \sim 3 \times 10^{12} \left(\frac{\mathcal{M}}{10^9\, M_\odot}\right)^{-5/3} \left(\frac{f_{\rm GW}}{10^{-8}\, Hz}\right)^{-11/3} \left(\frac{T_{\rm obs}}{10\, yr}\right)^{-1} \times \left(\frac{dN/dt}{10 \rm merger/yr}\right)$$

- This is clearly $>> 1$

  This ensures a diffuse GW background in the PTA GW frequency window from merging massive BHs in the universe

- It is NOT very difficult to show that its characteristic strain spectrum is given by $h_c(f) = A_{1yr} \times (f/{\rm yr}^{-1})^{-2/3}$

Next Decades...

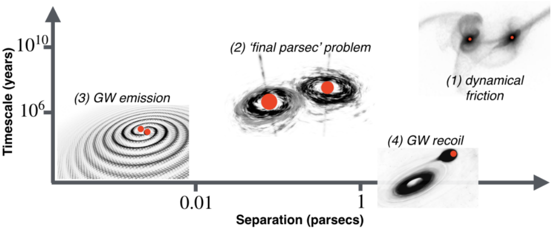


Figure: SKA-PTA, LISA, DESI,LSST,++

Ranging to Spacecrafts

- The paths of interplanetary spacecrafts like Cassini are routinely monitored with the help of Doppler tracking by both NASA & EAS We essentially monitor the return time of communication signals with interplanetary spacecrafts
- For missions to Jupiter and Saturn, for example, the return times are of order $2 - 4x10^3$s.
- The idea is to search in the Doppler tracking data for GW signatures
- The idea of using Doppler tracking data for detecting gravitational waves came up in the early 1970s. The mathematical formalism was worked out by F. Estabrook and H. Wahlquist [Gen. Rel. Grav. 6, 439 (1975)].

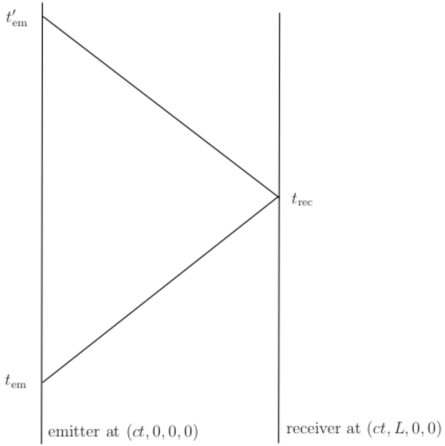


Figure: Any GW event shorter than this will appear 3 times in the time-delay:

J. Armstrong; Low-frequency gravitational wave searches using spacecraft Doppler tracking, Living Rev. Relativity 9, (2006)

Some Formulae

- $$z = \frac{\nu_{em}}{\nu'_{em}} - 1 = \frac{dt'_{em}}{dt_{em}} - 1$$

- $$\frac{dt'_{em}}{dt_{rec}} \times \frac{dt_{rec}}{dt_{em}} - 1$$
  $$= A_+ \sin(\omega T) \sin(\omega t_{em} + \omega T)$$

  The resulting red shift z oscillates, as a function of t_{em}, with the frequency ω
  $T = t'_{em} - t_{rec}$

- It should be possible to achieve sensitivities $\sim 10^{-13} - -10^{15}$

Very Promising Paths Ahead

- Multi-messenger GW Astronomy with Terrestrial, Solar System & Galaxy-based GW observatories are going to revolutionise Astronomy/Astrophysics/Theoretical Physics in the coming decades

Very Promising Paths Ahead

- Multi-messenger GW Astronomy with Terrestrial, Solar System & Galaxy-based GW observatories are going to revolutionise Astronomy/Astrophysics/Theoretical Physics in the coming decades

- There are efforts to create a GW observatory in the Moon https://www.vanderbilt.edu/lunarlabs/

Gravitational-Wave Lunar Observatory for Cosmology

Karan Jani, Abraham Loeb

Several large-scale experimental facilities and space-missions are being suggested to probe the universe across the gravitational-wave (GW) spectrum. Here we propose Gravitational-wave Lunar Observatory for Cosmology (GLOC) - the first concept design in the NASA Artemis era for a GW observatory on the Moon. Using feasible interferometer technologies, we find that a lunar-based observatory is ideal for probing GW frequencies in the range between deci-Hz to 5 Hz, an astrophysically rich regime that is very challenging for both Earth- and space-based detectors. GLOC can survey binaries with neutron stars, stellar and intermediate-mass black holes to $\gtrsim 70\%$ of the observable volume of our universe without significant background contamination. The sensitivity at □$(1\ \mathrm{Hz})$ allows a unique window into calibrating Type Ia supernovae. At its ultimate sensitivity limits, GLOC would trace the Hubble expansion rate up to redshift $z \sim 3$ and test General Relativity and ΛCDM cosmology up to $z \sim 350$.

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Thank you!