Entanglement Asymmetry in non-Abelian Anyonic Systems

Nicetu Tibau Vidal, Ved Kunte, Lucia Vilchez-Estevez,
Mohit Lal Bera, and **Manabendra Nath Bera**

Reference: arXiv:2406.03546

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Quantum Trajectories - 2025, ICTS, Bengaluru

Outline

- **Charge super-selection rule:** physically allowed states and operations
- **Non-Abelian anyons:** Fibonacci anyons
- **Correlated anionic states:** marginal spectra ambiguity and all that
- **Quantum teleportation:** asymmetric entanglement sharing
- **Conclusion**

Motivation

Unpaired Majorana fermions in quantum wires

A Yu Kitaev[1]

Physics-Uspekhi, Volume 44, Number 10S

Citation A Yu Kitaev 2001 *Phys.-Usp.* **44** 131

Topological quantum computation

HTML articles powered by AMS MathViewer

by Michael H. Freedman, Alexei Kitaev, Michael J. Larsen and Zhenghan Wang

Bull. Amer. Math. Soc. **40** (2003), 31-38

Non-Abelian anyons and topological quantum computation

Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, and Sankar Das Sarma

Rev. Mod. Phys. **80**, 1083 – Published 12 September 2008

ELSEVIER

Annals of Physics

Volume 303, Issue 1, January 2003, Pages 2-30

Fault-tolerant quantum computation by anyons

A.Yu. Kitaev

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Rev. Mod. Phys. **80**, 1083 – Published 12 September 2008

nature physics

Article https://doi.org/10.1038/s41567-024-02529-6

Non-Abelian braiding of Fibonacci anyons with a superconducting processor

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Check for updates

Shibo Xu[1,7], Zheng-Zhi Sun[2,7], Ke Wang[1,7], Hekang Li[3], Zitian Zhu[1], Hang Dong[1], Jinfeng Deng[1], Xu Zhang[1], Jiachen Chen[1], Yaozu Wu[1], Chuanyu Zhang[1], Feitong Jin[1], Xuhao Zhu[1], Yu Gao[1], Aosai Zhang[1], Ning Wang[1], Yiren Zou[1], Ziqi Tan[1], Fanhao Shen[1], Jiarun Zhong[1], Zehang Bao[1], Weikang Li[2], Wenjie Jiang[2], Li-Wei Yu[4], Zixuan Song[3], Pengfei Zhang[3], Liang Xiang[3], Qiujiang Guo[3,5], Zhen Wang[1,5], Chao Song[1,5], H. Wang[1,3,5] & Dong-Ling Deng[2,5,6]

My motivation?

Charge super-selection rule: physically allowed states and operations

Fermions

Spin-Statistics Theorem → Parity super-selection rule

↓

Coherent superposition between even and odd parity (even/odd # of fermions) is not physically allowed.

Restrictions on the physically allowed states and operations/maps!

Fermionic-mode entanglement in quantum information

Nicolai Friis, Antony R. Lee, and David Edward Bruschi
Phys. Rev. A **87**, 022338 – Published 25 February 2013

Reasonable fermionic quantum information theories require relativity

Nicolai Friis[1,2]

Published 7 March 2016 • © 2016 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 18, March 2016

Citation Nicolai Friis 2016 *New J. Phys.* **18** 033014

Marginal spectra ambiguity

Charge super-selection rule: physically allowed states and operations

Fermions → Parity super-selection rule

Spin-Statistics Theorem ↗

↓

Coherent superposition between even and odd parity (even/odd # of fermions) is not physically allowed.

Restrictions on the physically allowed **fermonic** states and operations/maps!

arXiv > quant-ph > arXiv:1610.00539

Quantum Physics

[Submitted on 3 Oct 2016]

Comment on 'Reasonable fermionic quantum information theories require relativity'

Markus Johansson

Connecting it to micro-causality.

Charge super-selection rule: physically allowed states and operations

Fermions

Spin-Statistics Theorem

→ Parity super-selection rule

↓

Coherent superposition between even and odd parity (even/odd # of fermions) is not physically allowed.

Restrictions on the physically allowed **fermonic** states and operations/maps!

Quantum operations in an information theory for fermions

Nicetu Tibau Vidal, Mohit Lal Bera, Arnau Riera, Maciej Lewenstein, and Manabendra Nath Bera
Phys. Rev. A **104**, 032411 – Published 13 September 2021

Charge super-selection rule: physically allowed states and operations

Fermions → Parity super-selection rule

Spin-Statistics Theorem

↓

Coherent superposition between even and odd parity (even/odd # of fermions) is not physically allowed.

What about anyons?

Charge super-selection rule

Charge Superselection Rule
Yakir Aharonov and Leonard Susskind
Phys. Rev. **155**, 1428 – Published 25 March 1967

Superselection Rule for Charge
G. -C. Wick, A. S. Wightman, and Eugene P. Wigner
Phys. Rev. D **1**, 3267 – Published 15 June 1970

Charge super-selection rule: physically allowed states and operations

Fermions → Parity super-selection rule

Spin-Statistics Theorem

↓

Coherent superposition between even and odd parity (even/odd # of fermions) is not physically allowed.

What about anyons?

Charge super-selection rule (cSSR)

Coherent superposition between states with different topological charges is not physically allowed.

The cSSR also restricts the set of physically allowed operations/maps.

What are the ramifications of cSSR (together with the fusion rules) in quantum information theory of anyons?

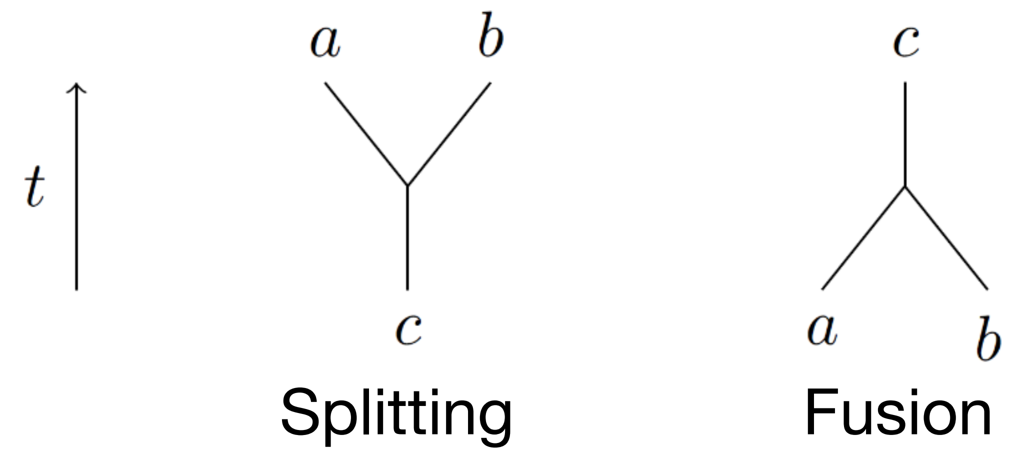
Outline

- **Charge super-selection rule:** physically allowed states and operations
- **Non-Abelian anyons:** Fibonacci anyons
- **Correlated anionic states:** marginal spectra ambiguity and all that
- **Quantum teleportation:** asymmetric entanglement sharing
- **Conclusion**

Non-Abelian anyons: Fibonacci anyons

e Vacuum

τ Quasi-excitation

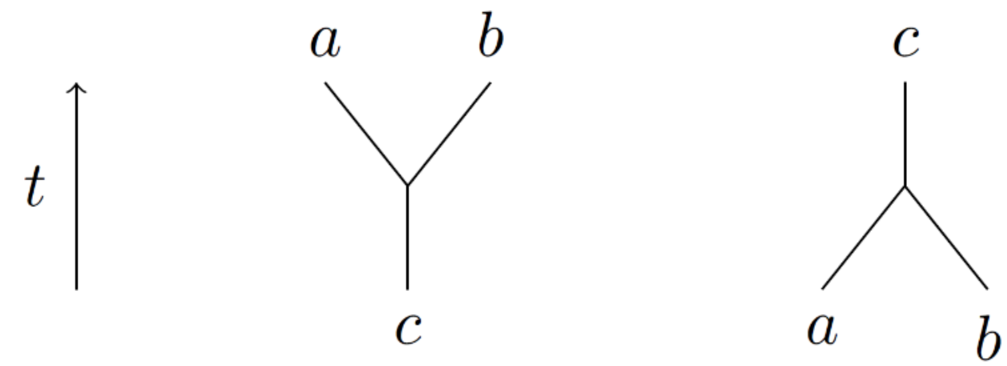


Fusion rules	Dirac notation
$e \times e = e$	$\vert e, e; e\rangle$
$\tau \times e = \tau$	$\vert \tau, e; \tau\rangle$
$e \times \tau = \tau$	$\vert e, \tau; \tau\rangle$
$\tau \times \tau = e + \tau$	$\vert \tau, \tau; e\rangle \quad \vert \tau, \tau; \tau\rangle$

Non-Abelian anyons: Fibonacci anyons

e Vacuum

τ Quasi-excitation



Splitting   Fusion

Fusion rules	Dirac notation
$e \times e = e$	$\vert e, e; e\rangle$
$\tau \times e = \tau$	$\vert \tau, e; \tau\rangle$
$e \times \tau = \tau$	$\vert e, \tau; \tau\rangle$
$\tau \times \tau = e + \tau$	$\vert \tau, \tau; e\rangle, \ \vert \tau, \tau; \tau\rangle$

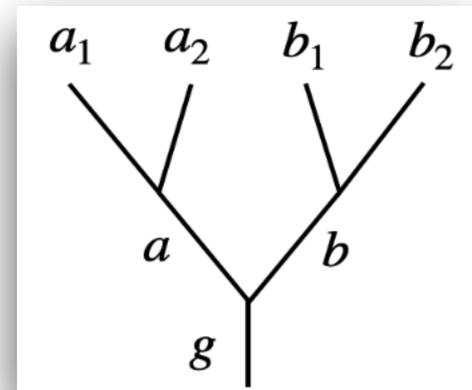
Has distinctive Hilbert/state space

$$\dim(\mathcal{H}_N) \neq d^N \longrightarrow \dim(\mathcal{H}_1) = 2, \ \dim(\mathcal{H}_2) = 5, \ \dim(\mathcal{H}_3) = 13, \ldots$$

In general $\dim(\mathcal{H}_N) = F_{2N+1}$

Non-Abelian anyons: Fibonacci anyons

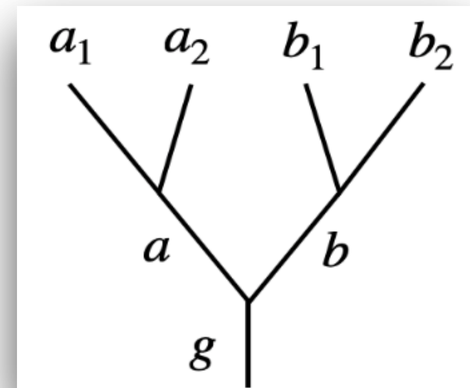
Notation and change in bases



$$\longrightarrow \quad \{|(a_1, a_2)(b_1, b_2); a, b; g\rangle\}$$

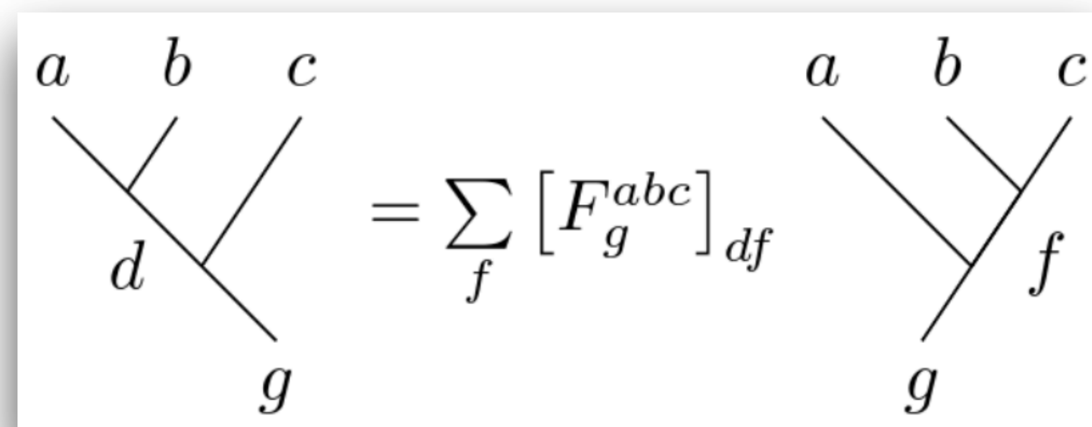
Non-Abelian anyons: Fibonacci anyons

Notation and change in bases



$\longrightarrow$ $\{|(a_1, a_2)(b_1, b_2); a, b; g\rangle\}$

Not associative and bases transformation with F-moves



$\longrightarrow$ $|(a,b),c;d,c;g\rangle = \sum_f [F_g^{abc}]_{df}\,|a,(b,c);a,f;g\rangle$

If $g = e$, i.e., trivial, then the F-moves become simpler and the components equal to 1.

Non-Abelian anyons: cSSR

Restrictions on physical states?

All states with charge e

$$|\psi_e\rangle_{AB} = \alpha_e |e, e; e\rangle + \beta_e |\tau, \tau; e\rangle$$

$$\mathcal{H}_N = \bigoplus_g \mathcal{H}_N^g$$

All states with charge τ

$$|\psi_\tau\rangle_{AB} = \alpha_\tau |\tau, e; \tau\rangle + \beta_\tau |e, \tau; \tau\rangle + \gamma_\tau |\tau, \tau; \tau\rangle$$

Non-Abelian anyons: cSSR

Restrictions on physical states?

All states with charge e

$$|\psi_e\rangle_{AB} = \alpha_e \, |e, e; e\rangle + \beta_e \, |\tau, \tau; e\rangle$$

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All states with charge τ

$$|\psi_\tau\rangle_{AB} = \alpha_\tau \, |\tau, e; \tau\rangle + \beta_\tau \, |e, \tau; \tau\rangle + \gamma_\tau \, |\tau, \tau; \tau\rangle$$

Restrictions on physical operations/maps?

All those that respect cSSR, i.e., cannot create superpositions between charges e and τ.

$$K = K_e \oplus K_\tau$$

Non-Abelian anyons: Fibonacci anyons

Like, for qudit: $|i\rangle\langle j| \rightarrow \sum_k |ik\rangle\langle jk|$

Extension: local to global operators, $A_1A_2 \rightarrow A_1A_2B_1B_2$

$$|a_1, a_2; a\rangle\langle a'_1, a'_2; a| \longrightarrow \sum_{b_1, b_2, b, g} |(a_1, a_2)(b_1, b_2); a, b; g\rangle\langle(a'_1, a'_2)(b_1, b_2); a, b; g|$$

Example $A_1 \rightarrow A_1A_2$	$U_{A_1} = \mathrm{diag}\{e^{i\phi}, e^{i\eta}\} \longrightarrow U_{A_1A_2} = \mathrm{diag}\{e^{i\phi}, e^{i\eta}, e^{i\eta}, e^{i\phi}, e^{i\eta}\}$ $\{|e\rangle, |\tau\rangle\} \longrightarrow \{|e, e; e\rangle, |\tau, \tau; e\rangle, |\tau, e; \tau\rangle, |e, \tau; \tau\rangle, |\tau, \tau; \tau\rangle\}$

Non-Abelian anyons: Fibonacci anyons

Like, for qudit: $|i\rangle\langle j| \rightarrow \sum_k |ik\rangle\langle jk|$

Extension: local to global operators, $A_1A_2 \rightarrow A_1A_2B_1B_2$

$$|a_1, a_2; a\rangle\langle a_1', a_2'; a| \longrightarrow \sum_{b_1, b_2, b, g} |(a_1, a_2)(b_1, b_2); a, b; g\rangle\langle (a_1', a_2')(b_1, b_2); a, b; g|$$

Example $A_1 \rightarrow A_1A_2$	$U_{A_1} = \mathrm{diag}\{e^{i\phi}, e^{i\eta}\} \longrightarrow U_{A_1A_2} = \mathrm{diag}\{e^{i\phi}, e^{i\eta}, e^{i\eta}, e^{i\phi}, e^{i\eta}\}$ $\{\|e\rangle, \|\tau\rangle\} \longrightarrow \{\|e, e; e\rangle, \|\tau, \tau; e\rangle, \|\tau, e; \tau\rangle, \|e, \tau; \tau\rangle, \|\tau, \tau; \tau\rangle\}$

Reduction: partial tracing, $A_1A_2B_1B_2 \rightarrow A_1A_2$

$$\mathrm{Tr}_B(|(a_1, a_2)(b_1, b_2); a, b; g\rangle\langle (a_1', a_2')(b_1', b_2'); a', b'; g|) = \delta_{b_1 b_1'}\delta_{b_2 b_2'}\delta_{bb'}\delta_{aa'} |a_1, a_2; a\rangle\langle a_1', a_2'; a| ,$$

For $A_1A_2 \rightarrow A_1$, $\quad \mathrm{Tr}_{A_2}(|a_1, a_2; g\rangle\langle a_1', a_2'; g|) = \delta_{a_2 a_2'}\delta_{a_1 a_1'} |a_1\rangle\langle a_1|$

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Anyonic states: marginal spectra ambiguity

Bipartite pure state

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|e,\tau;\tau\rangle + |\tau,\tau;\tau\rangle\right)$$

→

Marginal states

$$\rho_A = \frac{1}{2}\left(|e\rangle\langle e| + |\tau\rangle\langle\tau|\right)$$

$$\rho_B = |\tau\rangle\langle\tau|$$

Schmidt decomposition?

Questioning traditional methods to characterise entanglement!

Anyonic states: marginal spectra ambiguity

Bipartite pure state

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}} \left(|e, \tau; \tau\rangle + |\tau, \tau; \tau\rangle \right) \longrightarrow$$

Marginal states

$$\rho_A = \tfrac{1}{2} \left(|e\rangle\langle e| + |\tau\rangle\langle\tau| \right)$$

$$\rho_B = |\tau\rangle\langle\tau|$$

Schmidt decomposition?

Questioning traditional methods to characterise entanglement!

Classically correlated state

$$\rho_{AB} = \tfrac{1}{2} \left(|\tau, \tau; e\rangle\langle\tau, \tau; e| + |\tau, \tau; \tau\rangle\langle\tau, \tau; \tau| \right) \longrightarrow$$

Marginal states

$$\rho_A = |\tau\rangle\langle\tau|$$

$$\rho_B = |\tau\rangle\langle\tau|$$

Questioning traditional approach to characterise information!

Correlated anyonic states

Uncorrelated states $\mathrm{Tr}\left(\hat{O}_A\ \hat{O}_B\ \rho_{AB}\right) = \mathrm{Tr}\left(\hat{O}_A\ \rho_A\right)\mathrm{Tr}\left(\hat{O}_B\ \rho_B\right)$

All states with charge e	Uncorrelated states
$\|\psi_e\rangle_{AB} = \alpha_e \|e, e; e\rangle + \beta_e \|\tau, \tau; e\rangle$	$\alpha_e = 0 \text{ or } \beta_e = 0$

Correlated anyonic states

Uncorrelated states $\mathrm{Tr}\left(\hat{O}_A\ \hat{O}_B\ \rho_{AB}\right) = \mathrm{Tr}\left(\hat{O}_A\ \rho_A\right)\mathrm{Tr}\left(\hat{O}_B\ \rho_B\right)$

All states with charge e

$$|\psi_e\rangle_{AB} = \alpha_e\,|e,e;e\rangle + \beta_e\,|\tau,\tau;e\rangle$$

Uncorrelated states

$$\alpha_e = 0 \text{ or } \beta_e = 0$$

All states with charge τ

$$|\psi_\tau\rangle_{AB} = \alpha_\tau\,|\tau,e;\tau\rangle + \beta_\tau\,|e,\tau;\tau\rangle + \gamma_\tau\,|\tau,\tau;\tau\rangle$$

Uncorrelated states one with $\alpha_\tau = 0$ and the other with $\beta_\tau = 0$

$$\alpha_\tau\,|e,\tau;\tau\rangle + \gamma_\tau\,|\tau,\tau;\tau\rangle \text{ and } \beta_\tau\,|\tau,e;\tau\rangle + \gamma_\tau\,|\tau,\tau;\tau\rangle$$

Correlated anyonic states

Uncorrelated states $\mathrm{Tr}\left(\hat{O}_A\ \hat{O}_B\ \rho_{AB}\right) = \mathrm{Tr}\left(\hat{O}_A\ \rho_A\right)\mathrm{Tr}\left(\hat{O}_B\ \rho_B\right)$

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$$|\psi_e\rangle_{AB} = \alpha_e\,|e,e;e\rangle + \beta_e\,|\tau,\tau;e\rangle$$

Uncorrelated states

$$\alpha_e = 0 \text{ or } \beta_e = 0$$

All states with charge τ

$$|\psi_\tau\rangle_{AB} = \alpha_\tau\,|\tau,e;\tau\rangle + \beta_\tau\,|e,\tau;\tau\rangle + \gamma_\tau\,|\tau,\tau;\tau\rangle$$

Uncorrelated states $\quad$ one with $\alpha_\tau = 0$ and the other with $\beta_\tau = 0$

$$\alpha_\tau\,|e,\tau;\tau\rangle + \gamma_\tau\,|\tau,\tau;\tau\rangle \text{ and } \beta_\tau\,|\tau,e;\tau\rangle + \gamma_\tau\,|\tau,\tau;\tau\rangle$$

Maximally entangled states,
with maximally mixed marginals

with charge e $\quad \frac{1}{\sqrt{2}}\left(|e,e;e\rangle + e^{i\phi}\,|\tau,\tau;e\rangle\right)$

with charge τ $\quad \frac{1}{\sqrt{2}}\left(|e,\tau;\tau\rangle + e^{i\varphi}\,|\tau,e;\tau\rangle\right)$

Do not have a complete set maximally entangled states spanning the entire state space.

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Quantum teleportation: Asymmetric entanglement

$$|\varphi_M\rangle = \alpha\,|\tau, e; \tau\rangle + \beta\,|e, \tau; \tau\rangle$$

Quantum message (α, β)

$$|R_{AB}\rangle = \tfrac{1}{\sqrt{2}}\Big(\,|(e,e),(e,\tau); e, \tau; \tau\rangle + |(\tau,e),(\tau,e); \tau, \tau; \tau\rangle\,\Big)$$

Alice to Bob teleportation: the global state

$$\big|\psi_{M(AB)}\big\rangle = \frac{1}{\sqrt{2}}\Big(\alpha\,|(\tau,e),(e,e),(e,\tau);\tau,(e,\tau);\tau,\tau;e\rangle + \alpha\,|(\tau,e),(\tau,e),(\tau,e);\tau,(\tau,\tau);\tau,\tau;e\rangle$$
$$+\,\beta\,|(e,\tau),(e,e),(e,\tau);\tau,(e,\tau);\tau,\tau;e\rangle + \beta\,|(e,\tau),(\tau,e),(\tau,e);\tau,(\tau,\tau);\tau,\tau;e\rangle\Big)$$

Quantum teleportation: Asymmetric entanglement

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$$+\,\beta\,|(e,\tau),(e,e),(e,\tau);\tau,(e,\tau);\tau,\tau;e\rangle + \beta\,|(e,\tau),(\tau,e),(\tau,e);\tau,(\tau,\tau);\tau,\tau;e\rangle\Big)$$

Bases transformation $|(a,b),c;d,c;g\rangle = \sum_f [F_g^{abc}]_{df}\,|a,(b,c);a,f;g\rangle$

$$\left|\psi_{(MA)B}\right\rangle = \frac{1}{\sqrt{2}}\Big(\alpha\,|(\tau,e),(e,e),(e,\tau);(\tau,e),\tau;\tau,\tau;e\rangle + \alpha\,|(\tau,e),(\tau,e),(\tau,e);(\tau,\tau),\tau;\tau,\tau;e\rangle$$
$$+\,\beta\,|(e,\tau),(e,e),(e,\tau);(\tau,e),\tau;\tau,\tau;e\rangle + \beta\,|(e,\tau),(\tau,e),(\tau,e);(\tau,\tau),\tau;\tau,\tau;e\rangle\Big)$$

Quantum teleportation: Asymmetric entanglement

Global state, again

$$\left|\psi_{(MA)B}\right\rangle = \frac{1}{2}\Big(\left|\lambda_+\right\rangle \left(\alpha \left|e,\tau;\tau\right\rangle + \beta \left|\tau,e;\tau\right\rangle\right) + \left|\lambda_-\right\rangle \left(\alpha \left|e,\tau;\tau\right\rangle - \beta \left|\tau,e;\tau\right\rangle\right)$$
$$+ \left|\eta_+\right\rangle \left(\alpha \left|\tau,e;\tau\right\rangle + \beta \left|e,\tau;\tau\right\rangle\right) + \left|\eta_-\right\rangle \left(\alpha \left|\tau,e;\tau\right\rangle - \beta \left|e,\tau;\tau\right\rangle\right)\Big)$$

For MA

$$\left|\lambda_\pm\right\rangle = \frac{1}{\sqrt{2}}\Big(\left|(\tau,e),(e,e);\tau,e;\tau\right\rangle \pm \left|(e,\tau),(\tau,e);\tau,\tau;\tau\right\rangle \Big)$$

$$\left|\eta_\pm\right\rangle = \frac{1}{\sqrt{2}}\Big(\left|(\tau,e),(\tau,e);\tau,\tau;\tau\right\rangle \pm \left|(e,\tau),(e,e);\tau,e;\tau\right\rangle \Big)$$

Quantum teleportation: Asymmetric entanglement

Global state, again

$$\left|\psi_{(MA)B}\right\rangle = \frac{1}{2}\Big(\left|\lambda_+\right\rangle \left(\alpha \left|e,\tau;\tau\right\rangle + \beta \left|\tau,e;\tau\right\rangle\right) + \left|\lambda_-\right\rangle \left(\alpha \left|e,\tau;\tau\right\rangle - \beta \left|\tau,e;\tau\right\rangle\right)$$
$$+ \left|\eta_+\right\rangle \left(\alpha \left|\tau,e;\tau\right\rangle + \beta \left|e,\tau;\tau\right\rangle\right) + \left|\eta_-\right\rangle \left(\alpha \left|\tau,e;\tau\right\rangle - \beta \left|e,\tau;\tau\right\rangle\right)\Big)$$

For MA

$$|\lambda_\pm\rangle = \frac{1}{\sqrt{2}}\Big(|(\tau,e),(e,e);\tau,e;\tau\rangle \pm |(e,\tau),(\tau,e);\tau,\tau;\tau\rangle \Big)$$
$$|\eta_\pm\rangle = \frac{1}{\sqrt{2}}\Big(|(\tau,e),(\tau,e);\tau,\tau;\tau\rangle \pm |(e,\tau),(e,e);\tau,e;\tau\rangle \Big)$$

Measurements by Alice

$$\{|\lambda_+\rangle\langle\lambda_+|\,,|\lambda_-\rangle\langle\lambda_-|\,,|\eta_+\rangle\langle\eta_+|\,,|\eta_-\rangle\langle\eta_-|\}$$

Local OPs by Bob

$$\{X,Y,I,Z\}$$

$$Z = |\tau,e;\tau\rangle\langle\tau,e;\tau| - |e,\tau;\tau\rangle\langle e,\tau;\tau|$$

Bob can recover the message (α,β)

Perfect teleportation from Alice to Bob.

Quantum teleportation: Asymmetric entanglement

$$|\varphi_M\rangle = \alpha\,|\tau, e; \tau\rangle + \beta\,|e, \tau; \tau\rangle$$

Quantum message (α, β)

$$|R_{AB}\rangle = \frac{1}{\sqrt{2}}\Big(\,|(e,e),(e,\tau); e,\tau;\tau\rangle + |(\tau,e),(\tau,e);\tau,\tau;\tau\rangle\,\Big)$$

Teleportation from Bob to Alice

Quantum teleportation: Asymmetric entanglement

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Teleportation from Bob to Alice

$$\left|\xi_{(AB)M}\right\rangle = \frac{1}{\sqrt{2}}\Big(\alpha\,|(e,e),(e,\tau),(\tau,e);(e,\tau),\tau;\tau,\tau;e\rangle + \alpha\,|(\tau,e),(\tau,e),(\tau,e);(\tau,\tau),\tau;\tau,\tau;e\rangle$$
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Bases transformation by F-moves

$$\left|\xi_{A(BM)}\right\rangle = \frac{1}{\sqrt{2}}\Big(\alpha\,|(e,e),(e,\tau),(\tau,e);e,(\tau,\tau);e,e;e\rangle + \alpha\,|(\tau,e),(\tau,e),(\tau,e);\tau,(\tau,\tau);\tau,\tau;e\rangle$$
$$+\,\beta\,|(e,e),(e,\tau),(e,\tau);e,(\tau,\tau);e,e;e\rangle + \beta\,|(\tau,e),(\tau,e),(e,\tau);\tau,(\tau,\tau);\tau,\tau;e\rangle\Big)$$

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Bases transformation by F-moves

$$\left|\xi_{A(BM)}\right\rangle = \frac{1}{\sqrt{2}}\Big(\alpha\,|(e,e),(e,\tau),(\tau,e);e,(\tau,\tau);e,e;e\rangle + \alpha\,|(\tau,e),(\tau,e),(\tau,e);\tau,(\tau,\tau);\tau,\tau;e\rangle$$
$$+\,\beta\,|(e,e),(e,\tau),(e,\tau);e,(\tau,\tau);e,e;e\rangle + \beta\,|(\tau,e),(\tau,e),(e,\tau);\tau,(\tau,\tau);\tau,\tau;e\rangle\Big)$$

cSSR in action!
Cannot perform all measurements.

$$\{|(e,\tau),(\tau,e);\tau,\tau;e\rangle\,,|(e,\tau),(e,\tau);\tau,\tau;e\rangle\}$$

$$\{|(\tau,e),(\tau,e);\tau,\tau;\tau\rangle\,,|(\tau,e),(e,\tau);\tau,\tau;\tau\rangle\}$$

Regardless, the local state of Alice would be probabilistic mixture of

$$|e,e;e\rangle\langle e,e;e| \text{ and } |\tau,e;\tau\rangle\langle\tau,e;\tau|$$

Quantum teleportation: Asymmetric entanglement

$$|\varphi_M\rangle = \alpha\,|\tau, e; \tau\rangle + \beta\,|e, \tau; \tau\rangle$$

Quantum message (α, β)

$$|R_{AB}\rangle = \frac{1}{\sqrt{2}}\Big(\,|(e,e),(e,\tau); e,\tau;\tau\rangle + |(\tau,e),(\tau,e);\tau,\tau;\tau\rangle\,\Big)$$

Teleportation from Bob to Alice

$$\big|\xi_{(AB)M}\big\rangle = \frac{1}{\sqrt{2}}\Big(\alpha\,|(e,e),(e,\tau),(\tau,e);(e,\tau),\tau;\tau,\tau;e\rangle + \alpha\,|(\tau,e),(\tau,e),(\tau,e);(\tau,\tau),\tau;\tau,\tau;e\rangle$$
$$+\,\beta\,|(e,e),(e,\tau),(e,\tau);(e,\tau),\tau;\tau,\tau;e\rangle + \beta\,|(\tau,e),(\tau,e),(e,\tau);(\tau,\tau),\tau;\tau,\tau;e\rangle\Big)$$

Bases transformation by F-moves

$$\big|\xi_{A(BM)}\big\rangle = \frac{1}{\sqrt{2}}\Big(\alpha\,|(e,e),(e,\tau),(\tau,e);e,(\tau,\tau);e,e;e\rangle + \alpha\,|(\tau,e),(\tau,e),(\tau,e);\tau,(\tau,\tau);\tau,\tau;e\rangle$$
$$+\,\beta\,|(e,e),(e,\tau),(e,\tau);e,(\tau,\tau);e,e;e\rangle + \beta\,|(\tau,e),(\tau,e),(e,\tau);\tau,(\tau,\tau);\tau,\tau;e\rangle\Big)$$

No (perfect) quantum teleportation is possible from Bob to Alice!

The asymmetry in entanglement sharing is not due to unequal marginal spectra.

Conclusions

- cSSR and the fusion rules deep implications in anyonic quantum information theory.
- Marginal spectra ambiguity and ambiguity in global vs local spectra: warrants a radically new approach to understand and characterise information and correlation.
- In a bipartite pure entangled state, the parties do not have uniform access to entanglement. This asymmetric nature is manifested in teleportation.
- Possibilities of quantum tasks such as communication or cryptographic protocols where one party has the superiority in accessing correlation and manipulation of information over the other.

For more details, see arXiv:2406.03546

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About

The International Union of Scientists (IUS), Scientists Against Militarism and the Destructive Use of Science and Technology.

IUS brings together scientists from around the world who refuse to stay silent in the face of atrocities like the genocide in Palestine. We stand united against the weaponization of science and technology for war, authoritarianism, and the dehumanization of life. As scientists, we recognize that our scientific and technological research holds power and influence. We believe science is not morally neutral. It is inseparable from the social and political contexts in which it operates. Our work can either serve humanity or contribute to its downfall—and we choose to stand for humanity.

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Email your proposal today to info@iuscientists.org

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An appeal…

A photographer came to take her picture and this **little refugee toddler offered him food,** thinking he is hungry. Its such a beautiful picture yet so thought provoking!!! What's been done to these angels in the **name of War**…

Thank you