Breather modes in a periodically driven anharmonic chain

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[Work in progress.]

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Nonequilibrium phase transition in a driven circuit QED lattice

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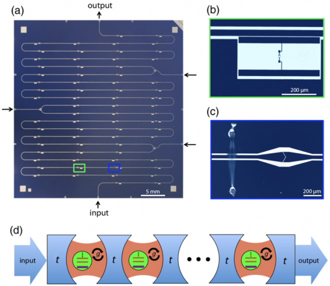
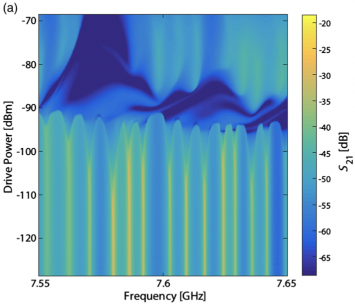
Observation of a Dissipative Phase Transition in a One-Dimensional Circuit QED Lattice

Mattias Fitzpatrick,[1] Neereja M. Sundaresan,[1] Andy C. Y. Li,[2] Jens Koch,[2] and Andrew A. Houck[1]





- Sharp reduction in phonon transimission beyond a threshold drive power.

Theoretical models - I

Photonic transport and phase transition in an array of optical cavities —
Debnath, Mascarenhas, Savona (NJP, 2017)

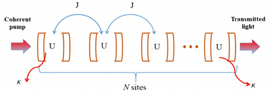


Figure 1. The schematic representation of the boundary driven and dissipative non linear arrays of optical cavities.

Model: 1D Bose-Hubbard model with boundary drive and dissipation

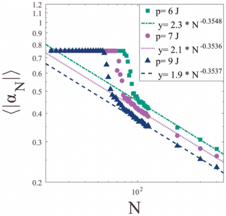
$$H = \sum_i -\delta \hat{a}_i^\dagger \hat{a}_i + U \hat{a}_i^\dagger \hat{a}_i \hat{a}_i^\dagger \hat{a}_i + J(\hat{a}_{i+1}^\dagger \hat{a}_i + \text{h.c.}) + p(\hat{a}_1 + \hat{a}_1^\dagger),$$

Semi-classical mean-field analysis:

$$\dot{\alpha}_1 = -\left(\frac{\kappa}{2} + \mathrm{i}\delta\right)\alpha_1 - \mathrm{i}J\alpha_2 - 2\mathrm{i}U|\alpha_1|^2\alpha_1 - \mathrm{i}p,$$

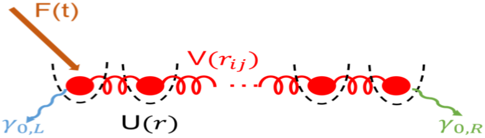
$$\dot{\alpha}_i = -\mathrm{i}\delta\alpha_1 - \mathrm{i}J\alpha_{i+1} - \mathrm{i}J\alpha_{i-1} - 2\mathrm{i}U|\alpha_i|^2\alpha_i,$$

$$\dot{\alpha}_N = -\left(\frac{\kappa}{2} + \mathrm{i}\delta\right)\alpha_N - \mathrm{i}J\alpha_{N-1} - 2\mathrm{i}U|\alpha_N|^2\alpha_N,$$



Theoretical models - II

A driven dissipative Klein-Gordon chain —
Prem, Bulchandani, Sondhi (arXiv:2209.03977)

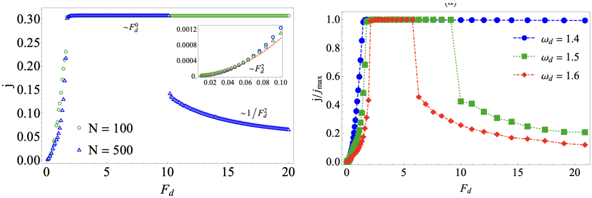


$$\begin{aligned}
\ddot{q}_1 &= -q_1 - q_1^3 + \epsilon(q_2 - q_1) - \gamma\dot{q}_1 + F\cos(\omega_d t) \\
\ddot{q}_j &= -q_j - q_j^3 + \epsilon(q_{j+1} + q_{j-1} - 2q_j) \quad j = 2, 3, \ldots, (N-1) \\
\ddot{q}_N &= -q_N - q_N^3 + \epsilon(q_{N-1} - q_N) - \gamma\dot{q}_N.
\end{aligned}$$

- Dimensionless control parameters: F, ω_d (set $\epsilon = \gamma = 1$). Harmonic spectrum: $\omega_q^2 = 1 + 2(1 - \cos q)$.
- The nonequilibrium drive leads to a energy current (J) carrying steady state in the system. [Dynamical system: goes to some attractor at long times]
- Interesting transitions as we change F.

Some of the interesting results

Prem, Bulchandani, Sondhi (arXiv:2209.03977)



- For any ω_d within harmonic band-width, there is a range of $F = (F_1, F_2)$ for which the current stays constant.
- Sharp transitions at F_1 and F_2.
- Non-chaotic periodic solution in this range — breather mode.
- $F_1,\ F_2$ depend on ω_d.
- F_2 depends on N — no transitions in the $N \to \infty$ limit.
- Anomalous - Ballistic - Diffusive scaling of current.

Breather mode

The breather mode is a periodic solution of the form $q_n = a_n e^{i\omega_d t}$. Ignoring higher frequencies, the amplitudes satisfy the nonlinear equations:

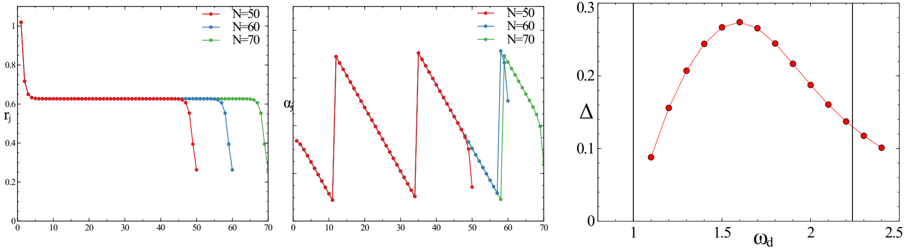
$$(\omega^2 - 1)a_1 - 3|a_1|^2 a_1 + (a_2 - a_1) - i\omega a_1 = \frac{F}{2}$$
$$(\omega^2 - 1)a_j - 3|a_j|^2 a_j + (a_{j+1} + a_{j-1} - 2a_j) = 0 \qquad j = 2, \ldots, (N-1)$$
$$(\omega^2 - 1)a_N - 3|a_N|^2 a_N + (a_{N-1} - a_N) - i\omega a_N = 0.$$

- Bulk solution: $a_n = re^{i\omega_d t}$
- $r = \sqrt{\frac{\omega_d^2 - 1}{3}}$.
- Breather mode is linearly stable.

Some open questions:
(i) a more detailed understanding of the breather mode, in particular it's stability,
(ii) effect of finite temperature noise (at the boundaries).

New results (for $T = 0$)

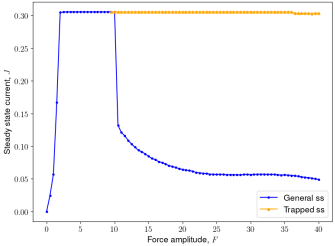
- Bulk solution: $a_n = re^{i(\Delta n - \omega_d t)}$ — constant phase factor Δ, that is independent of F and only depends on ω_d.
- $r = \sqrt{\frac{\omega_d^2 - 1 - 2(1 - \cos\Delta)}{3}}$.
- Steady state heat current: $J = -2r^2\omega_d \sin\Delta$

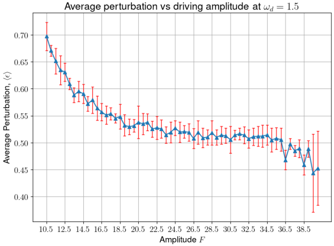


New results — Stability of breather mode

Earlier results were obtained by starting from random initial conditions — implying that the domain of attraction is very large (all of phase space ?) and becomes finite as one approaches the transition point.

- By starting with initial conditions close to the breather mode, we find that the region of stability can be extended (indefinitely ??)

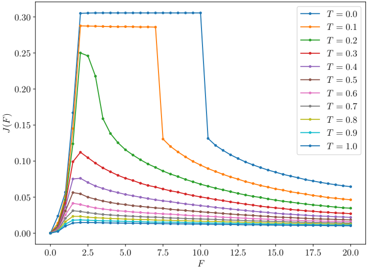


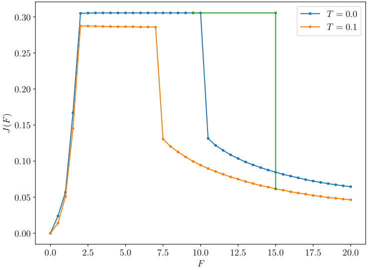


Effect of finite temperature

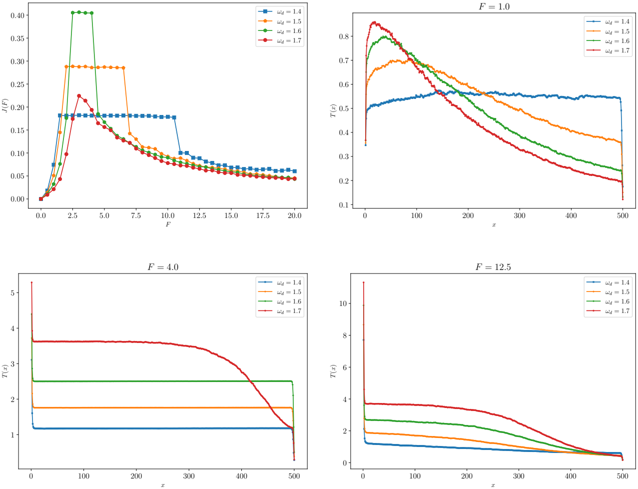
Dissipation always comes with noise satisfying fluctuation dissipation theorem.
Question: Does this kill the breather mode?

$$\ddot{q}_1 = -q_1 - q_1^3 + \epsilon(q_2 - q_1) - \gamma\dot{q}_1 + \sqrt{2T\gamma}\,\eta_L(t) + F\cos(\omega_d t)$$
$$\ddot{q}_j = -q_j - q_j^3 + \epsilon(q_{j+1} + q_{j-1} - 2q_j) \quad j = 2, \ldots (N-1)$$
$$\ddot{q}_N = -q_N - q_N^3 - \epsilon(q_N - q_{N-1}) - \gamma\dot{q}_N + \sqrt{2T\gamma}\,\eta_R(t),$$





Temperature profiles



Summary

- Nonequilibrium phase transition in a driven dissipative chain
- Presence of multiple attractors — a breather mode and a chaotic attractor.
- Strong signatures of the breather mode at finite temperatures — non-unique steady states.
- Some analytic understanding of the breather mode

Open questions:
(i) Complete solution of the breather, including form at boundary and form of Δ.

(ii) Signatures of the breather mode in a fully quantum calculation.