Academic Program Details
Discussion Meeting on Zero Mean curvature Surfaces
@ICTS-TIFR

Tentative Schedule

	10:00-11:30	11:30-12:00	12:00-1:00	1:00-2:30	2:30-4:00	4:00-4:30	4:30-5:30 (or 6 PM)
2nd September	John Loftin	Tea Break	Franz E. Weber	Lunch	Shoichi Fujimori	Tea Break	Randall Kamien*
3rd September	John Loftin	Tea Break	Franz E. Weber	Lunch	Shoichi Fujimori	Tea Break	Randall Kamien*
4th September	John Loftin	Tea Break	Nathaniel Sagman*				
5th September	Shoichi Fujimori	Tea Break	Short Talks/ Student's Talk	Lunch	Nathaniel Sagman*	Tea Break	L Mahadevan*
6th September	Nathaniel Sagman*	Tea Break	Short Talks/ Student's Talk	Lunch		Tea Break	Rahul Kumar Singh

1. On 4th September, we will have first talk from 9:30 to 11 am, and Second talk from 11:30 am to 1 PM.
2. * these are online talks.
3. Time is Indian Standard Time (IST)
4. Last talk on the day that starts from 4:30 PM will be of 60 minutes on some days and 90 minutes on some days (we shall inform.)

Speakers list:

1. Mini Course/Research Talks
   (a) John Loftin on Equivariant Minimal Surfaces in the Symmetric Spaces via Higgs Bundles.
   (b) Shoichi Fujimori on zero mean curvature surfaces in Lorentz Minkowski spaces.
   (c) Nathaniel Sagman on Minimal surfaces in symmetric spaces and Labourie's Conjecture.
   (d) Rahul Kumar Singh on CMC surfaces of revolution in $\mathbb{E}^3_1$ and Weierstass-$\wp$ functions
2. Mini-course/Talk on the Application side
   (a) Franz E. Weber on Microarchitectures in 3D-printed bone substitutes orchestrate bone biology: From pores to TPMS (Triply Periodic Minimal Surfaces).
   (b) Randall Kamien on Minimal Surfaces in Diblock Copolymers and geometric scaffolds in materials science.
   (c) L Mahadevan on Soft surfaces bounded by soft filaments: from the Euler-Plateau problem to morphogenesis
3. Short Talks by Faculties and Students.

(a) Mohammad Saleem Lone: Title: T CMC affine translation surfaces in Minkowski 3-space.

(b) Niteesh Kumar: Title- Clifford minimal hypersurface in $S^5(c)$ of constant isotropic curvature.

(c) Simarn Bedi Title: Minimal Surfaces over harmonic shears.

(d) Rivu Bardhan Title: Higher genus maxfaces with Eneper ends.

(e) Anu Dhochak Title: Construction of higher genus maxfaces.

(f) Sai Mohanty Title: Genus Zero Complete Maximal Maps and Maxfaces with an Arbitrary Number of Ends.

(g) Subham Paul Title: Higher genus maxfaces with a given number of singular components

1. John Loftin (3 talks, each 90 minutes).

Title of the talk: Equivariant Minimal Surfaces in the Symmetric Spaces via Higgs Bundles.

I will first consider the geometry of the complex hyperbolic plane and immersed surfaces therein, in particular the cases of Lagrangian and complex surfaces. The complex surfaces are all minimal, but there are many others as well. As it is a symmetric space, the more general case of harmonic maps from a Riemann surface into the complex hyperbolic plane naturally generates holomorphic data of a Higgs bundle.

We impose a compactness condition to relate our study of minimal surfaces to Higgs bundles. Let S be a closed Riemann surface of genus at least 2. Consider then harmonic immersions of the universal cover of S into the complex hyperbolic plane which are equivariant with respect to some representation of the fundamental group of S into the group $PU(2,1)$ of holomorphic isometries of the complex hyperbolic plane. In this case, the nonlinear Hodge correspondence applies and thus there is a (poly-)stable Higgs bundle over S. In the standard case of the group $GL(n, C)$, this consists of a holomorphic vector bundle E of rank n over S together with the Higgs field: a holomorphic section Φ of $\mathrm{End}(E) \otimes K$. The cases of other groups such as our group of isometries can typically be addressed as modifications of this case.

The moduli theory of Higgs bundles over Riemann surfaces is in general rather complicated, as in particular it depends on an arbitrary vector bundle E. In important special cases we can determine the Higgs bundle in terms of simpler objects. (This approach goes back to Hitchin's original work in parametrizing Teichmuller space with quadratic differentials via Higgs bundles.) The minimal surface condition (that the harmonic map is conformal) provides enough information to classify all such equivariant minimal surfaces into the complex hyperbolic plane, as well as into a few other cases of minimal surfaces into real hyperbolic spaces of degree 3 and 4.

This is based on joint work from 2019 with Ian McIntosh, as well as work of Alessandrini-Li.

2. Shoichi Fujimori (3 talks, each 90 minutes).

First Talk (90 minutes) I would like to give a brief introduction of spacelike hypersurfaces and maximal hypersurfaces in Minkowski space. Then I would like to introduce maximal surfaces with singularities in Minkowsiki 3-space and give examples.

Second Talk (90 minutes) I would like to focus a construction of nonorientable maximal surfaces. Then I would like to give some classification theorems.

Third Talk (90 minutes) I would like to give a brief introduction of zero mean curvature surfaces as the analytic extension of maximal surfaces to timelike minimal surfacces. Then I would like to give numerous examples of zero mean curvature surfaces and discuss some properties of them such as embeddedness and symmetries.

3.Nathaniel Sagman (3 talks, each 90 minutes).

Title: Minimal surfaces in symmetric spaces and Labourie's Conjecture.
I will discuss geometric and analytic aspects of the theory of equivariant harmonic maps and (branched) minimal immersions from surfaces into Riemannian symmetric spaces of non-compact type, such as SL(n,R)/SO(n,R). We'll also highlight certain research directions and share open problems.

First, I will provide an overview of the basic existence and uniqueness theorems on harmonic maps due to Donaldson and Corlette, and I will explain how harmonic maps to symmetric spaces give rise to holomorphic objects called Higgs bundles (and moreover play a prominent role in the non-abelian Hodge correspondence). Next, I will introduce Hitchin representations and the surrounding minimal surface existence theory, and Labourie's conjecture about the uniqueness of equivariant minimal immersions for Hitchin representations. I will then detail the state of the art on high energy harmonic maps to symmetric spaces, and explain how to use this theory to construct unstable equivariant minimal surfaces for Hitchin representations, which lead to counterexamples to Labourie's conjecture.

Reference:

`https://arxiv.org/abs/1809.05747`
`https://arxiv.org/abs/math/0512070`
`https://drive.google.com/file/d/13JFgLa2Tkh3NSVYcJ_wlOpQM_X9pwEl6/view`
`https://arxiv.org/abs/1406.4637`
`https://arxiv.org/abs/1311.7101`
`https://people.maths.ox.ac.uk/~markovic/M-minimalac.pdf`
`https://arxiv.org/abs/2208.04885`

4. Rahul Kumar Singh (One 60-minute talk).

Title: CMC surfaces of revolution in E_1^3 and Weierstass-$\wp$ functions

Abstract: It is a well-known fact that in the class of regular non-zero constant mean curvature (CMC) surfaces in the Euclidean space, spheres and the right circular cylinders are the only examples of CMC surfaces which are algebraic. In this talk, first we will show for every spacelike CMC surface of revolution (except spacelike cylinders and standard hyperboloids), which is either an unduloid or a nodoid, in the Lorentz-Minkowski space $\mathbb{E}_1^3$, there is an associated Weierstrass-$\wp$ function. Next, using this association, we will show unduloid and nodoid cannot be algebraic and hence concluding only spacelike cylinders and standard hyperboloids are algebraic.

5. Kamien, Randall (2 talks each 60 to 90 minutes).

Outline of Topics:

Minimal Surfaces in Diblock Copolymers: the P,G, and D triply periodic minimal surfaces.
Smectic Liquid Crystals: the simplest crystals and they are built from surfaces.
Using minimal surfaces to knit: geometric scaffolds in materials science.

6. Franz E. Weber (2 talks, each 60 minutes).

First talk - Title: 3D-printed bone substitutes: From pores to adaptive density minimal surface microarchitecture.

Introduction: In the last decades, advances in bone tissue engineering mainly based on osteoinduction and on stem cell research. Only recently, new efforts focused on the micro- and nanoarchitecture of bone substitutes to improve and accelerate bone regeneration. By the use of additive manufacturing, diverse microarchitectures were tested to identify the ideal pore size [1], the ideal filament distance and diameter [2], or light-weight microarchitecture [3], for osteoconduction to minimize the chance for the development of non-unions. Overall, the optimal microarchitecture doubled the efficiency of scaffold-based bone regeneration without the need for growth factors or cells. Another focus is on bone augmentation, a procedure mainly used in the dental field.

Methods: For the production of scaffolds, we applied the CeraFab 7500 from Lithoz, a lithography-based additive manufacturing machine. Hydroxyapatite-based and tri-calcium-phosphate-based scaffolds were produced with Lithoz TCP 300 or HA 400 slurries.

Results: The histomorphometric analysis revealed that bone ingrowth was significantly increased with pores between 0.7-1.2 mm in diameter. Best pore-size for bone augmentation was 1.7 mm in diameter. Therefore, pore-based microarchitectures for osteoconduction and bone augmentation are different. Moreover, microporosity appeared to be a strong driver of osteoconduction and influenced osteoclastic degradation for tri-calcium phosphate-based scaffolds. For hydroxyapatite-based scaffolds, however, microporosity appears to influence osteoconductivity to a lesser extent.

Osteoclasts were able to degrade hydroxyapatite-based scaffolds irrespective of nanoarchitecture but tri-calcium phosphate-based scaffolds only at high and moderate levels of microporosity. The evaluation of the gene expression profiles at early bone healing leading to osteoconduction revealed that a reduction of the filament size from 1.25 mm to 0.5 mm yielded in differentiation of mesenchymal stem cells towards osteoclasts, enhanced angiogenesis and increased osteoconduction.

Discussion: Micro- and nanoarchitectures are key driving forces for osteoconduction and bone augmentation. A variety of microarchitectures can be realized by additive manufacturing. We identified optimized lattice, pore, filament and adjusted density minimal surface architectures for bony bridging and bone augmentation purposes. Based on these new results additive manufacturing appears as the tool of choice to produce personalized bone tissue engineering scaffolds to be used in cranio-maxillofacial surgery, dentistry, and orthopedics.

References:

(1) Ghayor et al Weber FE (2021) The optimal microarchitecture of 3D-printed β-TCP bone substitutes for vertical bone augmentation differs from that for osteoconduction. Materials and Design 204 (2021) 109650

(2) Guerrero J, et al Weber FE, (2023) Influence of Scaffold's Microarchitecture on Angiogenesis and Regulation of Cell Differentiation During the Early Phase of Bone Healing: a Transcriptomics and Histological Analysis. Int J Bioprint, 9(1): 0093.

(3) Maevskaia et al Weber FE (2022) 3D-printed hydroxyapatite bone substitutes designed by a novel periodic minimal surface algorithm are highly osteoconductive.

Second talk - Title: 3D-printed bone substitutes with triply periodic minimal surface microarchitectures.

Introduction: Additive manufacturing or 3D printing are key methodologies to produce libraries of bone substitutes to test them to identify highly osteoconductive microarchitectures for bone defects or bone augmentation. Bone is a lightweight, high strength structure and resembles in its trabecular microarchitecture a gothic style. TPMS architectures are also lightweight and high strength. Therefore, we produced triply periodic minimal surface (TPMS) lightweight-based scaffolds based on three different algorithms and tested them in a cranial defect and a bone augmentation model in rabbits.

Methodology For the production of scaffolds, we applied the CeraFab 7500 from Lithoz, a lithography-based additive manufacturing machine and studied tri-calcium phosphate- based and hydroxyapatite-based scaffolds. As in vivo test model, we used a calvarial defect and a bone augmentation model in rabbits. Histomorphometry revealed that all generatively produced structures were well osseointegrated into the surrounding bone and induced bone augmentation. The histomorphometric analysis, based solely on the middle section combined with microCT analysis showed that for triply periodic minimal surface lightweight microarchitectures, gyroid- and diamond- microarchitecture performs well in bone augmentation and cranial defect models.
Results: In essence, we have identified the optimal triply periodic lightweight microarchitecture for cranial defects (1) and for bone augmentation purposes needed for the placement of dental implants (2). We learned before that the optimal pore-based and filament-based microarchitecture for bone augmentations differ from the best for the treatment of defects. For TPMS-based microarchitectures, however, diamond and especially gyroid microarchitectures are optimal for bone augmentation and osteoconduction. Moreover, we saw that additive manufacturing appears as a promising tool for the production of personalized bone substitutes to be used in cranio-maxillofacial surgery, dentistry, and orthopedics. The addition of exosomes to further enhance osteoconduction and bone augmentation did not yield better results (3).
Discussion: Micro- and nanoarchitectures are key driving forces for osteoconduction and bone augmentation. A variety of microarchitectures can be realized by additive manufacturing. We identified optimized TPMS microarchitectures for bony bridging and bone augmentation purposes. Based on these new results additive manufacturing appears as the tool of choice to produce personalized bone tissue engineering scaffolds to be used in cranio-maxillofacial surgery, dentistry, and orthopedics.
References: (1) Maevskaia et al Weber FE (2023) TPMS-based scaffolds for bone tissue engineering: a mechanical, in vitro and in vivo study.Tissue Engineering Part A.
(2)Maevskaia E, et al Weber FE, TPMS Microarchitectures for Vertical Bone Augmentation and Osteoconduction: An In Vivo Study Materials 2024, 17, 2533.
(3) Maevskaia E, et al Weber FE, Functionalization of Ceramic Scaffolds with Exosomes from Bone Marrow Mesenchymal Stromal Cells for Bone Tissue Engineering. Int. J. Mol. Sci. 2024 25, 3826.

7. L Mahadevan (One 60-minute talk).

Title: Soft surfaces bounded by soft filaments: from the Euler-Plateau problem to morphogenesis

Short Talks (Each 15 minutes)

1. Mohamd Saleem Lone

Title: CMC affine translation surfaces in Minkowski 3-space.
Abstract: We obtain the classification results of spacelike affine translation surfaces with constant mean curvature in Minkowski space $\mathbb{E}_1^3$.

2. Niteesh Kumar

Title: Complete Hypersurfaces of Constant Isotropic Curvature in Space forms.
Abstract: In this talk, I will discuss the complete classification of complete orientable embedded hypersurfaces of constant isotropic curvature in space forms. Such a hypersurface has constant mean curvature only if it is an isoparametric hypersurface. Additionally, it is minimal if and only if it is totally geodesic or it is the Cliffordminimal hypersurface $\mathbb{S}^3(\frac{4c}{3}) \times \mathbb{S}^1(4c)$ in $\mathbb{S}^5(c)$.in $S^5(c)$.
Reference: Gururaja, H A and Kumar, Niteesh, *Complete hypersurfaces of constant isotropic curvature in space forms*, Journal of mathematical analysis and applications, (520)2023, 126876.

3. Simran Bedi

Title: Minimal Surfaces over harmonic shears.
Abstract: Harmonic mappings have long intrigued researchers due to their intrinsic connection with minimal surfaces. In this paper, we investigate shearing of two distinct classes of univalent conformal mappings that are convex in horizontal direction with appropriate dilatations. Subsequently, we unveil a family of minimal surfaces formed by lifting the harmonic mappings obtained through shear construction method given by Clunie-Sheil. Furthermore, we contribute to addressing an open problem partially, proposed by Boyd and Dorff, by identifying the resulting minimal surfaces for certain values of the parameters in one of the classes of mappings. Notably, this family of minimal surfaces transforms from the well-established Enneper's surface to a Helicoid.
Reference: https://arxiv.org/abs/2310.10258

4. Rivu Bardhan

Title: Higher genus maxfaces with Eneper ends.
Abstract: In this talk, I shall discuss the existence of higher genus complete maxfaces with one Enneper end using Weierstrass Enneper representation for the maxface. I would introduce a new geometric shape named tweezer of genus p and use it to generate p genus maxface with one Enneper end. Moreover, I shall go through all possible singularities of these new surfaces.
Reference: Rivu Bardhan, Indranil Biswas, Pradip Kumar, "Higher genus maxfaces with one Enneper end," The Journal of Geometric Analysis (2024), `https://doi.org/10.48550/arXiv.2310.00235`.

5. Anu Dhochak

Title: Construction of higher genus maxfaces.
Abstract: I will discuss the construction of one parameter family of higher genus maxfaces using the Weierstrass gluing method given by Martin Traizet.
Reference: `https://arxiv.org/abs/2402.11965`

6. Sai Mohanty

Title: Genus Zero Complete Maximal Maps and Maxfaces with an Arbitrary Number of Ends.
Abstract: We will discuss the existence of a genus-zero complete maximal map with a prescribed singularity set and an arbitrary number of simple and complete ends. We will also discuss the conditions under which this maximal map can be made into a complete maxface.

Reference: Pradip Kumar; Sai Rasmi Ranjan Mohanty. *Genus Zero Complete Maximal Maps and Maxfaces with an Arbitrary Number of Ends.* Comptes Rendus. Mathématique, Volume 361 (2023), pp. 1683-1690.

7.Subham Paul

Title: Higher genus maxfaces with a given number of singular components.
Abstract: Awaited.