SYNOPSIS OF PHD THESIS

Effective theory of Fluctuating Hydrodynamics from Holography

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ABSTRACT: Fluid/Gravity correspondence suggests that the long wavelength deformations of AdS blackbranes are dual to configurations of a dissipative fluid. We revisit this duality, now with the aim of incorporating the effect of Hawking radiation from the blackbrane. Using holographic duality we argue that this problem is equivalent to constructing a theory of fluctuating hydrodynamics. We explicitly derive the effective description of a simmering holographic neutral plasma by studying and Hawking excitations of the Schwarzschild-AdS$_{d+1}$ blackbrane. This article is a synopsis of the salient results of [1] and [2].

1 Introduction

AdS/CFT correspondence or the holographic duality predicts that certain strongly coupled CFTs admit an alternate description as weakly coupled gravitational theories. For example, the duality identifies thermal states of d dimensional CFTs with blackholes in AdS_{d+1}. This relation opens up an avenue to explore non-trivial physics on either side of the duality. For instance, it has been extensively used to study novel phases of matter arising at strong coupling, leading to a whole new industry of holographic condensed matter theory [3]. On the other hand, it provides a more tractable definition of quantum gravity in AdS spacetimes.

In particular, the duality suggests that hydrodynamic excitations of a planar CFT are equivalent to long wavelength deformations of dual AdS blackbranes. The hydrodynamics of the CFT is dominated by gapless excitations such as Goldstone bosons (from broken symmetries), conserved charge densities (global charges and energy-momentum) etc. Such excitations are dual to gauge field and metric perturbations of the blackbrane, which indeed display gapless quasi-normal spectrum. This identification can be made more rigorous to derive a corollary of the duality, the so called fluid/gravity correspondence – the large scale deformations of blackbranes in AdS spacetimes are dual to the configurations of dissipative conformal fluids [4, 5].

It is well understood that dissipative physical systems should admit completions as stochastically driven systems. The reason is that dissipation is the manifestation of energy transfer to underlying microscopic degrees of freedom, which in turn inevitably source fluctuations in the system. The stochastic completion of the dissipative dynamics is not arbitrary either – the noise distribution is constrained by the transport characteristics, which is the essence of fluctuation-dissipation theorems. In the context of dissipative fluids this implies that a more realistic description should include fluctuations of the hydrodynamic or long-lived modes themselves. Conventional fluid dynamics is not equipped to include such effects, for long-lived fluctuations are in tension with one of its central tenets, that fluctuations die out much before the hydrodynamic time scale [6].

There are multiple challenges we need to address in order to build a framework for *fluctuating hydrodynamics* (FH). How can we relax the assumption of short-lived fluctuations and allow the fluid degrees of freedom to simmer? How should we distill out hydrodynamic fluctuations from short-lived ones which decay within the characteristic time scale of the fluid? While it is conceptually easier to answer these questions starting from the underlying microscopic theory, in practice such an exercise is not feasible – deriving the hydrodynamic limit involves a complicated rewriting of the collective degrees of freedom, which is beyond the scope of standard analytical techniques.

However, we argue that within AdS/CFT, this problem can be dealt with in an analytic fashion. The preceding discussion on the stochastic completion of dissipative systems has a natural counterpart in the gravitational description. While blackbranes admit dissipative perturbations as quasi-normal modes, they also source fluctuations, both stimulated and spontaneous, as Hawking radiation. This is required for consistency of the semi-classical field theory on the blackbrane. Situated within the context of fluid/gravity correspondence, this suggests that a theory of FH can be constructed via holography – one only needs to account for Hawking effect on the blackbrane, specifically with respect to the gapless sector of its deformations, which from the perspective of the CFT, causes its hydrodynamic degrees of freedom to fluctuate. Secondly, as we will show, the underlying gauge theoretical structure suggests a natural prescription to separate out the short-lived and long-lived excitations of the system, henceforth referred to as *Markovian* and *non-Markovian* respectively .

The central proposition of our work is that FH should be envisaged as an open effective field theory which ought to be dealt with in a Wilsonian fashion. This perspective is motivated from the following question – what is the effective theory that describes the evolution of a probe coupled to a fluctuating fluid? The answer to this question depends on the details of the probe-fluid coupling. If the probe couples to Markovian excitations, it is expected that over the hydrodynamical times scale its evolution should be local, allowing us to completely integrate out Markovian degrees of freedom from the effective theory. But how should we treat probes that couple to non-Markovian excitations which show long-term memory? Clearly, integrating out non-Markovian excitations is undesirable as it would transfer the long-term memory to the dynamics of the probe, or in other words make its evolution non-local. The intuitive remedy is to not integrate out the non-Markovian degrees of freedom, instead, to retain them in the effective description. The resulting description is a Wilsonian open effective theory which describes the local evolution of the probe-FH system, but includes the *local influence* of the forgotten Markovian degrees of freedom.

Using the Schwinger-Keldysh (SK) formalism for open field theories, we show that the Wilsonian effective field theory of the probe-FH system is characterized by a quantity we term the Wilsonian influence functional (WIF). The WIF should be understood as an object of mixed character – it serves as the correlation generating functional for the Markovian fields and effective action for the non-Markovian fields. The open effective action of a general probe-FH system can be obtained by identifying the Markovian sources in the WIF with appropriate probe operators and turning on desired interactions with the non-Markovian fields.

We are specifically interested in the WIF dual to graviton and gauge field perturbations of a Schwarzschild-AdS$_{d+1}$ blackbrane which describes the evolution of a probe coupled to a neutral plasma in the dual CFT. As mentioned before, guided by the gauge invariance of the gravitational theory, we establish a formalism to derive the WIF of this holographic FH system by cleanly separating out the Markovian and non-Markovian dynamics. As described in [1], we show that the Markovian and diffusive subset of the non-Markovian dynamics can be treated in a unified manner in terms of certain auxiliary probes of the blackbrane. The treatment of the non-Markovian mode describing energy-momentum transfer is technically more involved due to its propagating nature and is described in [2].

We conclude this section by briefly commenting on other approaches to FH developed in the recent literature. Several authors have attempted to systematically derive an FH description of varied low-dimensional, weakly coupled systems [7, 8] – these techniques typically coarse grain the dynamics within a chosen scale and thereby implement a *projection* on to the hydrodynamical variables. The details of such hydrodynamic projections are often model specific, and in particular, the treatment of fluctuations is often carried out in a bottom-up fashion using insights from fluctuation-dissipation relations. While these approaches provide significant insight into the dynamics of specific models, it is not immediately clear how to generalize them to other models including higher dimensional systems. There have also been parallel efforts to characterize the universal features of FH systems from the perspective of SK effective descriptions (see [9] and references there in). The SK formalism is well suited for investigations of this nature as it packages the constraints from microscopic unitarity and thermality of the system in a unified manner. However, the constraints on such effective actions are mostly motivated from intuitions gained from working with simple, weakly coupled models where explicit analysis is possible. It behooves us to check the validity of these predictions in the less understood regime of strongly coupled systems.

Holography provides a variety of toy models where these checks can be performed. Their success in capturing the universal features of strongly coupled systems suggests that open effective theories generated from holography should also yield qualitatively relevant predictions for strongly coupled fluctuating systems. As an example, we refer to the result of [10, 11] which derived a non-linear generalization of the Langevin equation. Their results demonstrated that non-linear corrections in the Langevin equation and non-Gaussianities in the fluctuations are tightly constrained. These predictions were validated from an independent holographic approach in [12] (see also [13] for a generalization to scalar fields). Our approach to FH is a generalization of these holographic models to the case of fluid/gravity correspondence. By doing so, we expect to perform an unbiased and independent check of various ideas developed in the context of relativistic FH.

We organize the rest of this synopsis as follows. §2 gives a brief overview of the SK formalism. In §3 and §4 we summarize the work of [1] and [2] respectively. In §5 we conclude with some future directions.

2 Open systems as Schwinger–Keldysh evolutions

From a technical viewpoint, the basic difference between an open and a closed quantum system is that the initial states of the former are not pure, but mixed states, as they are produced by tracing out a selected subset from the degrees of freedom of a parent system. We will now briefly outline the path integral formalism used to study the evolution of such mixed states (and also general density matrix elements) in field theory, known as the Schwinger-Keldysh (SK) formalism (see [14]). We motivate how such path integrals appear naturally in the study of open field theories.

2.1 Schwinger–Keldysh formalism in field theory

The generating functional for correlations within an initial state ρ_{init} is given by

$$\mathcal{Z}_{\text{SK}}[J_{\text{R}}, J_{\text{L}}] = \text{Tr}\left\{\mathcal{U}[J_{\text{R}}]\rho_{\text{init}}\mathcal{U}^\dagger[J_{\text{L}}]\right\} \;, \tag{2.1}$$

the SK generating functional for the state ρ_{init}. An essential feature of SK path integrals is that they are defined over a doubled version of the field space of the original theory. The field doubling is due the fact that we need to sum over all possible paths, independently[1] followed by the ket and bra co-ordinates of the density matrix. For a generic field φ, we will denote its ket and bra copies in the SK path integral as φ_{R} and φ_{L} respectively.

Once we adopt the SK formalism, it is straightforward, at least formally, to selectively integrate out any degrees of freedom we deem as part of the *bath*. This yields an open effective action for the *system*, which includes additional interactions originally mediated by the forgotten bath variables. Such interactions induced into the system's effective action are collectively termed the *influence functional*. As described before, we are interested in a Wilsonian analogue of this object, which we term the Wilsonian influence functional and denote as $\mathcal{S}_{\text{WIF}}$.

2.2 Schwinger–Keldysh formalism in holography

When situated within AdS/CFT, the SK formalism suggests the existence of a semi-classical geometry such that the gravitational path integral on it is dual to the SK path integral of the CFT. While many authors have attempted to construct such a saddle of the gravitational path integral, the recent proposal of [15] has proven to be particularly useful to understand such geometries. We now briefly review this proposal by Crossley, Glorioso and Liu. We refer to the saddle conjectured by them as the gravitational SK (grSK) saddle.

Recall that we are interested in the hydrodynamics of a d dimensional neutral plasma. Therefore the dual gravitational dynamics should describe the deformations and Hawking fluctuations around a Schwarzschild-AdS$_{d+1}$ blackbrane. Not surprisingly, the prescription of [15] envisages the grSK saddle as a particular analytical continuation of the Schwarzschild-AdS$_{d+1}$ blackbrane, which we now summarize. The prescription starts by considering the blackbrane in the ingoing Eddington-Finkelstein co-ordinates. To wit,

$$ds^2_{(0)} = -r^2 f(br) dv^2 + 2dvdr + r^2 d\mathbf{x}^2 \ , \qquad f(\xi) = 1 - \xi^{-d} \ . \tag{2.2}$$

Here $\mathbf{x}$ represents co-ordinates on $\mathbb{R}^{d-1}$ and $b = \frac{1}{r_h} = \frac{d}{4\pi}\beta$ is the reduced inverse temperature[2]. In order to implement the aforementioned analytical continuation, it is useful to introduce a *mock tortoise co-ordinate* ζ defined as

$$\frac{dr}{d\zeta} = \frac{i\beta}{2} r^2 f(br) \ , \qquad \zeta(r_c + i\varepsilon) = 0 \ , \qquad \zeta(r_c - i\varepsilon) = 1 \ , \tag{2.3}$$

where $r = r_c$ is the cut-off surface. Notice that ζ as a function of r has a logarithmic singularity at the horizon. The final step is to lift the radial co-ordinate of the blackbrane solution to a contour on the complex radial plane as given in Fig. 1. The resultant complexified blackbrane solution, parameterized by ζ, is the grSK saddle.

[1] φ_{R} and φ_{R} are subjected to additional boundary conditions at a future boundary, which we may ignore for our purposes.

[2] We will continue to use the variable $\xi = br$ as a dimensionless radial co-ordinate.

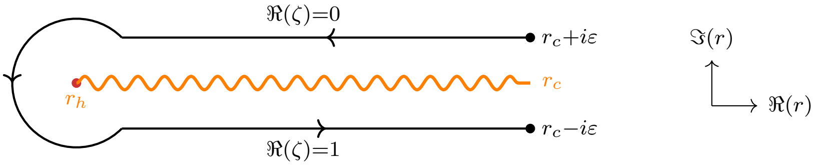

Figure 1: The complex r plane with the locations of the two boundaries and the horizon marked. The grSK contour is a co-dimension-1 surface in this plane (drawn at fixed v). As indicated the direction of the contour is counter-clockwise and it encircles the branch point at the horizon.

As evident from Fig. 1, this geometry has two asymptotic boundaries at a given time v, which represent two copies of the CFT inherent to SK formalism. The boundaries at at $\zeta = 0$ and $\zeta = 1$ stand for the bra (L) and ket (R) copies of the CFT respectively. The standard AdS/CFT dictionary given by GKPW rules when applied to this geometry is conjectured to yield the SK generating functional of the boundary plasma.

2.2.1 Time reversal isometry and Hawking fluctuations

Though not readily apparent, the metric (2.2) has a $\mathbb{Z}_2$ time-reversal isometry; it is invariant under the transformation $v \mapsto i\beta\zeta - v$. We will use this isometry to construct outgoing or Hawking modes of fields propagating on the grSK saddle from their ingoing or quasi-normal modes. More explicitly, given an ingoing scalar mode $\varphi^{\text{in}}(r,\omega,k)$ (in the Fourier domain), the time reversal isometry maps it to

$$\varphi^{\text{out}}(r,\omega,k) = e^{-\beta\omega\zeta}\varphi^{\text{in}}(r,-\omega,k) \ , \quad \mathbb{D}_+\varphi^{\text{out}}(r,\omega,k) = e^{-\beta\omega\zeta}\mathbb{D}_-\varphi^{\text{in}}(r,-\omega,k) \ , \tag{2.4}$$

where in the second equation we introduced the time-reversal covariant derivative $\mathbb{D}_+$, with $\mathbb{D}_\pm \equiv r^2 f\partial_r \pm \partial_v \sim r^2 f\partial_r \mp i\omega$. We will use it to express our results in a manifestly time reversal covariant manner. We also deduce from the above that unlike ingoing modes which are analytic functions of the radial co-ordinate, Hawking modes have non-trivial monodromy along the grSK contour as they explicitly depend on ζ.

3 Effective theory of stochastic diffusion from holography

In this section we summarize the results of [1] which characterize the dynamics of probes that couple to the conserved currents of a neutral plasma, due to global charges or energy-momentum, which exhibit long-lived diffusive behaviour.

3.1 Designer scalar and gauge probes

We introduce two families of bulk probes, the designer scalar and gauge probes, which we will argue are dual to diffusive dynamics in the CFT. The first consists of massless scalar fields $\varphi_{_\mathcal{M}}$ coupled to gravity via a dilaton χ_s. Their action is given by

$$S_{\text{ds}} = -\frac{1}{2}\int d^{d+1}x\sqrt{-g}\; e^{\chi_s}\, \nabla^A\varphi_{_\mathcal{M}}\nabla_A\varphi_{_\mathcal{M}} + S_{\text{ds, bdy}} \ , \qquad e^{\chi_s} \equiv r^{\mathcal{M}+1-d} \ , \tag{3.1}$$

where upper Latin letters A, $B\ldots$ denote bulk co-ordinates. Clearly, in the case $e^{\chi_s}=1$ the above action reduces to the Klein-Gordon action. $S_{\text{ds, bdy}}$ denotes possible boundary terms (including counterterms). The second is a family of Abelian 1-form gauge fields, but again dilaton coupled, with the action

$$S_{\text{dv}} = -\frac{1}{4}\int d^{d+1}x\,\sqrt{-g}\;e^{\chi_v}\,\mathcal{C}^{AB}\mathcal{C}_{AB} + S_{\text{dv, bdy}}\,, \qquad e^{\chi_v}\equiv r^2\,e^{\chi_s} = r^{\mathcal{M}+3-d}\,, \tag{3.2}$$

which when $e^{\chi_v}=r^2$ reduces to the usual Maxwell action. We argue that such gauge probes are dual to general diffusive currents in the CFT. In particular, they include the dynamics of conserved charges and transverse momentum currents in the dual plasma. We find it convenient to parse the description of these probes by separating out its gauge invariant sub-sectors, which we accomplish via the transverse $\mathrm{SO}(d-2)$ harmonic decomposition[3] following [16]. An interesting conclusion of our analysis is that each gauge invariant sector within the gauge probes map to scalar probes with different values of the parameter $\mathcal{M}$. Therefore, we will first give a detailed analysis of the latter, and use the results to describe the dynamics of the former.

Let us first introduce our notations for various $\mathrm{SO}(d-2)$ harmonics. $\mathbb{S}=e^{-i\omega v+i\mathbf{k}\cdot\mathbf{x}}$ are scalar plane waves. $\mathbb{V}_i^\alpha$ with $\alpha=1,\ldots,N_V=d-2$, stand for transverse vector plane waves that satisfy $k_i\mathbb{V}_i^\alpha=0$. Similarly $\mathbb{T}_{ij}^\sigma$ with $\sigma=1,\ldots,N_T=\frac{d(d-3)}{2}$, are transverse, traceless tensor plane waves and satisfy $k_i\mathbb{T}_{ij}^\sigma=0$. All of the above plane waves are orthonormalized. We also use the shorthand $\int_k\equiv\int\frac{d\omega}{2\pi}\frac{dk^{d-1}}{(2\pi)^{d-1}}$.

3.2 $\mathcal{M}$ as a Markovianity index

Recall that in AdS/CFT, the CFT observables are read-off from the asymptotic fall-offs of the spacetime field configurations – non-normalizable fall-offs correspond to source values in the CFT and normalizable fall-offs specify the corresponding operator's one point functions. With the aim of obtaining a local WIF in the boundary, we ask how should one parameterize the bulk field configurations within derivative expansion. Owing to the non-trivial near horizon structure of the grSK saddle, we find a simple dictum that answers this question. To summarize, there is another useful classification of the bulk field modes, namely, as those which are analytic and non-analytic near the horizon. We find that, *in order to get solutions on the grSK saddle that accommodate derivative expansion, they need to be parameterized based on the asymptotic fall-off exhibited by the analytic mode.*

Markovian probes : For $\mathcal{M}+1>0$, the analytic mode is non-normalizable. These modes are hence to be parameterized by the SK source fields $J_{\text{L/R}}$ in the CFT. Such probes are dual to Markovian dynamics as they can be integrated out by specifying arbitrary SK sources (or non-normalizable modes) to derive local (equivalent to bulk derivative expansion) contributions to the influence functional.

Non-Markovian probes : For $\mathcal{M}+1<0$, the analytic mode is normalizable. This

[3] This labels perturbations of the plasma based on the representations of the rotations which keep its spatial momentum $\mathbf{k}$ fixed.

implies that in order to derive a local influence functional, such probes should be parameterized in terms of the 1-pt functions of dual SK fields in the CFT, which we denote by $\breve{\Phi}_{\text{L/R}}$. In the terminology of [1], these fields are the *hydrodynamic moduli* which parameterize the non-Markovian contributions to the influence functional. They represent the long-wavelength or collective modes describing the effective theory.

3.3 Ingoing Green's functions and CFT observables

We now quickly summarize the results of [1] pertaining to the solutions to the equations of motions derived from (3.1). We will first discuss the case of Markovian scalars and motivate how these solutions can be used to bootstrap the solutions for non-Markovian scalars.

3.3.1 Markovian probes

We solve for the ingoing solutions in derivative expansion. The solutions denoted by $G^{\text{in}}_{\mathcal{M}}(\omega, r, \mathbf{k})$ satisfy the boundary condition

$$\lim_{\xi\to\infty} G^{\text{in}}_{\mathcal{M}} = 1 \,. \tag{3.3}$$

Using standard rules of AdS/CFT, the dual CFT operator corresponding to the Markovian probe is its momentum conjugate to the radial evolution evaluated at the boundary. To wit,

$$\pi_{\mathcal{M}}\big|_{\text{ren}} = -r^{\mathcal{M}}\, \mathbb{D}_+ G^{\text{in}}_{\mathcal{M}} + \text{counterterms}[G^{\text{in}}_{\mathcal{M}}]\,, \tag{3.4}$$

where we have indicated that, as usual various counterterms have to be supplemented to remove divergences. Evaluating the above explicitly we derive the corresponding boundary retarded two-point function $K^{\text{in}}_{\mathcal{M}}(\omega, k) \equiv -\lim\limits_{r\to\infty} \pi_{\mathcal{M}}\big|_{\text{ren}}$ to be

$$\begin{aligned} K^{\text{in}}_{\mathcal{M}}(\omega, k) = \frac{1}{b^{\mathcal{M}+1}} \Big\{ & -i\,\mathfrak{w} - \frac{\mathfrak{q}^2}{\mathcal{M}-1} - \mathfrak{w}^2 \Delta(\mathcal{M}, 1) \\ & + 2i\,\mathfrak{w}\left[\mathfrak{q}^2 H_k(\mathcal{M}, 1) + \mathfrak{w}^2\, H_\omega(\mathcal{M}, 1)\right] + \cdots \Big\}\,, \end{aligned} \tag{3.5}$$

where $\mathfrak{w} = b\omega$, $\mathfrak{q} = bk$ and $\Delta(\mathcal{M}, 1)\,, H_k(\mathcal{M}, 1)$ etc. are defined via series expansions.

3.3.2 Non-Markovian probes

An efficient way to derive the solutions of the non-Markovian scalar is via analytically continuing the Markovian scalar solutions in the parameter $\mathcal{M}$ via $\mathcal{M} \to -\mathcal{M}$. We denote such non-Markovian solutions by $G^{\text{in}}_{-\mathcal{M}}(\omega, r, \mathbf{k})$. As explained before, we wish to parameterize them using their normalizable modes, which are harder to extract due to the presence of dominant non-normalizable pieces. On the other hand, we find a simple recipe to extract the non-normalizable pieces. They can be read off from the radial conjugate momentum at the boundary as

$$G^{\text{in}}_{-\mathcal{M}} \sim \frac{r^{\mathcal{M}-1}}{\mathcal{M}-1} K^{\text{in}}_{-\mathcal{M}}(\omega, \mathbf{k})\,, \qquad \pi_{-\mathcal{M}} = -r^{-\mathcal{M}}\, \mathbb{D}_+ G^{\text{in}}_{-\mathcal{M}}(\omega, r, \mathbf{k}) = -K^{\text{in}}_{-\mathcal{M}}(\omega, \mathbf{k})\,. \tag{3.6}$$

The latter relation is consequence of the fact that the analytical continuation from Markovian to non-Markovian solutions exchanges their normalizable and non-normalizable components. The normalizable part of $G^{\text{in}}_{-\mathcal{M}}$ can indeed be extracted out by supplementing the correct counterterms as

$$\lim_{r\to\infty}\left(G^{\text{in}}_{-\mathcal{M}} + \text{counterterms}[\pi_{-\mathcal{M}}]\right) = 1\ . \tag{3.7}$$

Together with the equation (3.6), (3.7) implies that $K^{\text{in}}_{-\mathcal{M}}$ is proportional to the *inverse retarded two-point function* of the non-Markovian excitations of the plasma. As a curious coincidence, we find that the same function $K^{\text{in}}_{\pm\mathcal{M}}$ defines the retarded two point functions of the probes belonging to both the Markovian and non-Markovian classes, albeit admitting different interpretations as the Green's function and inverse Green's function respectively.

Notice that (3.7) suggests that the hydrodynamic moduli can be defined only via a *renormalized Dirichlet boundary condition*, where the counterterms are functionals of $\pi_{-\mathcal{M}}$. This would at first glance appear inconsistent with the action (3.1) which accommodates a variational principle with Dirichlet boundary condition (or fixed $\varphi_{-\mathcal{M}}$). While this issue seems to warrant additional boundary terms that implement a variational principle with Neumann boundary conditions (or fixed $\pi_{-\mathcal{M}}$), we argue that this peculiarity is rather desirable. In fact, in examples where non-Markovian designer scalars arise in holography, such extra boundary terms do exist, implying that the conventional gravitational actions (with GKPW boundary conditions) compute the generating functional of correlations in the CFT. An interesting take away from our analysis is that, it is by canceling precisely these boundary terms that one implements the Legendre transformation from correlation generating functionals to the effective action of the hydrodynamic moduli. To summarize, the action (3.1) for the non-Markovian scalars (sans additional boundary terms) with the boundary condition (3.7) directly computes the contribution of the hydrodynamic moduli $\breve{\Phi}_{\text{L,R}}$ to the WIF.

3.4 Solutions on the SK contour and on-shell action

The ingoing solutions summarized in the previous sections can be straightforwardly lifted to solutions on the full grSK contour. The key step here is to use the time–reversal isometry described in § 2.2.1 to generate the outgoing or Hawking solution, which we express using $G^{\text{rev}}_{\mathcal{M}}(\omega, r, \mathbf{k}) \equiv G^{\text{in}}_{\mathcal{M}}(-\omega, r, \mathbf{k})$.

3.4.1 Markovian probes

The solution on the grSK saddle satisfies the boundary condition $\lim\limits_{r\to\infty\pm i0}\varphi^{\text{SK}}_{\mathcal{M}} = J_{\text{L/R}}$, and is given by

$$\varphi^{\text{SK}}_{\mathcal{M}}(\omega,\zeta,\mathbf{k}) = G^{\text{in}}_{\mathcal{M}}\left[(n_B+1)\, J_{\text{R}} - n_B\, J_{\text{L}}\right] - G^{\text{rev}}_{\mathcal{M}}\, n_B\,(J_{\text{R}} - J_{\text{L}})\, e^{\beta\omega(1-\zeta)}\ , \tag{3.8}$$

where $n_B \equiv \frac{1}{e^{\beta\omega}-1}$ is the Bose-Einstein factor. The one point functions of the dual operators in the CFT (sourced by $J_{\text{L/R}}$) are obtained by substituting (3.8) into (3.4) and are given by

$$
\begin{aligned}
\langle \mathcal{O}_{\mathrm{R}}(\omega,\mathbf{k})\rangle &= -K^{\mathrm{in}}_{\mathcal{M}}(\omega,\mathbf{k})\left[(n_{_B}+1)\,J_{\mathrm{R}} - n_{_B}\,J_{\mathrm{L}}\right] + n_{_B}\,K^{\mathrm{rev}}_{\mathcal{M}}(\omega,\mathbf{k})\left[J_{\mathrm{R}}-J_{\mathrm{L}}\right]\ ,\\
\langle \mathcal{O}_{\mathrm{L}}(\omega,\mathbf{k})\rangle &= -K^{\mathrm{in}}_{\mathcal{M}}(\omega,\mathbf{k})\left[(n_{_B}+1)\,J_{\mathrm{R}} - n_{_B}\,J_{\mathrm{L}}\right] + (n_{_B}+1)K^{\mathrm{rev}}_{\mathcal{M}}(\omega,\mathbf{k})\left[J_{\mathrm{R}}-J_{\mathrm{L}}\right]\ ,
\end{aligned}
\tag{3.9}
$$

with $K^{\mathrm{rev}}_{\mathcal{M}}(\omega,\mathbf{k}) \equiv K^{\mathrm{in}}_{\mathcal{M}}(-\omega,\mathbf{k})$. Evaluating the action (3.1) on the solution (3.8) we derive the generating functional for the correlations of $\mathcal{O}_{\mathrm{L/R}}$ or equivalently its contribution to the WIF to be

$$
\mathcal{S}^{(\mathcal{M})}_{\mathrm{WIF}}[J_{\mathrm{L}},J_{\mathrm{R}}] \equiv S[\varphi_{_\mathcal{M}}]\Big|_{\text{on-shell}} = -\int_k (J_{\mathrm{R}}-J_{\mathrm{L}})^{\dagger}\,K^{\mathrm{in}}_{\mathcal{M}}\Big((n_{_B}+1)\,J_{\mathrm{R}} - n_{_B}\,J_{\mathrm{L}}\Big)\,. \tag{3.10}
$$

It is easily checked that differentiating the above with sources $J_{\mathrm{R/L}}$ reproduces (3.9).

3.4.2 Non-Markovian probes

The analogue of the solution (3.8) for non-Markovian scalars is obtained by analytically continuing $\mathcal{M} \to -\mathcal{M}$ and then replacing $J_{\mathrm{R/L}} \to \breve{\Phi}_{\mathrm{R/L}}$. As explained before, the one point functions of the dual operators can be directly read off from the hydrodynamic moduli as $\langle\breve{\mathcal{O}}\rangle_{\mathrm{R/L}} \equiv \breve{\Phi}_{\mathrm{R/L}}$. The analogue of (3.9) is given by

$$
\begin{aligned}
\breve{J}_{\mathrm{R}} &= K^{\mathrm{in}}_{-\mathcal{M}}\left[(n_{_B}+1)\,\breve{\Phi}_{\mathrm{R}} - n_{_B}\,\breve{\Phi}_{\mathrm{L}}\right] - n_{_B}K^{\mathrm{rev}}_{-\mathcal{M}}\left[\breve{\Phi}_{R}-\breve{\Phi}_{\mathrm{L}}\right]\ ,\\
\breve{J}_{\mathrm{L}} &= K^{\mathrm{in}}_{-\mathcal{M}}\left[(n_{_B}+1)\,\breve{\Phi}_{\mathrm{R}} - n_{_B}\,\breve{\Phi}_{\mathrm{L}}\right] - (n_{_B}+1)K^{\mathrm{rev}}_{-\mathcal{M}}\left[\breve{\Phi}_{R}-\breve{\Phi}_{\mathrm{L}}\right]\ ,
\end{aligned}
\tag{3.11}
$$

where $\breve{J}_{\mathrm{R/L}}$ represent the source configuration required to support a given configuration of $\breve{\Phi}_{\mathrm{R/L}}$. Evaluating the action of non-Markovian probe on the SK solution, we obtain its contribution to the WIF to be

$$
\mathcal{S}^{(-\mathcal{M})}_{\mathrm{WIF}}[\breve{\Phi}_{\mathrm{L}},\breve{\Phi}_{\mathrm{R}}] \equiv S[\varphi_{_{-\mathcal{M}}}]\Big|_{\text{on-shell}} = -\int_k \left[\breve{\Phi}_{R}-\breve{\Phi}_{\mathrm{L}}\right]^{\dagger} K^{\mathrm{in}}_{-\mathcal{M}}\left[(n_{_B}+1)\,\breve{\Phi}_{\mathrm{R}} - n_{_B}\,\breve{\Phi}_{\mathrm{L}}\right]\ , \tag{3.12}
$$

which we interpret as the effective action for the $\breve{\Phi}_{\mathrm{R/L}}$. It is easily checked that turning on additional source of the form $\int_k \left[\breve{J}_{\mathrm{R}}\breve{\Phi}_{\mathrm{R}} - \breve{J}_{\mathrm{L}}\breve{\Phi}_{\mathrm{L}}\right]$ in this action and varying it with $\breve{\Phi}_{\mathrm{R/L}}$ reproduces (3.11).

3.5 Dynamics of designer gauge probes

We will now discuss the designer gauge probes. From the action (3.2) we read off the equations of motion to be $\partial_A\left(\sqrt{-g}\; r^{\mathcal{M}+3-d}\;\mathcal{C}^{AB}\right) = 0$. In order to solve them, we start by writing the gauge potential $\mathcal{C}_A$ decomposed along SO$(d-2)$ harmonics as

$$
\begin{aligned}
&\mathcal{C}_r(v,r,\mathbf{x}) = \int_k \bar{\Psi}_r(r,\omega,\mathbf{k})\,\mathbb{S}\,,\quad \mathcal{C}_v(v,r,\mathbf{x}) = \int_k \bar{\Psi}_v(r,\omega,k)\,\mathbb{S}\ ,\\
&\mathcal{C}_i(v,r,\mathbf{x}) = \int_k \left[\sum_{\alpha=1}^{N_V} \bar{\Phi}_\alpha(r,\omega,\mathbf{k})\,\mathbb{V}^\alpha_i - \frac{k_i}{k}\bar{\Psi}_x(r,\omega,\mathbf{k})\,\mathbb{S}\right].
\end{aligned}
\tag{3.13}
$$

It is straightforward to check that the vector sector perturbations $\bar{\Phi}_\alpha$ are gauge invariant and decouple from the scalar sector perturbations $\bar{\Psi}$. Their equations of motion (and also contribution to the action) can be mapped to that of N_V decoupled Markovian probes with $\bar{\Phi}_\alpha = \varphi_{\mathcal{M}}$. Hence their contribution to the WIF can be extracted using the results outlined in the previous subsections. To summarize, the vector sector displays Markovian dynamics, and do not contribute to charge diffusion.

We can isolate the diffusive component of the gauge potentials as $\mathcal{C}^{\text{D}} \equiv \mathcal{C}|_{\bar{\Phi}_\alpha=0}$, which is comprised of scalar perturbations. We find that this sector of perturbations is more subtle as it inherits various intricacies from the underlying gauge symmetry, such as Gauss constraint, Bianchi identity etc. It exclusively captures the diffusion of charges in the plasma defined by the current

$$J_v^{\text{CFT}} = -\lim_{r\to r_c} r^{\mathcal{M}+2}\,\mathcal{C}^{\text{D}}_{rv}\,, \qquad J_i^{\text{CFT}} = -\lim_{r\to r_c} r^{\mathcal{M}}\left(r^2 f \mathcal{C}^{\text{D}}_{ri} + \mathcal{C}^{\text{D}}_{vi}\right)\,, \tag{3.14}$$

which is easily checked to be conserved on the solutions of equations of motion.

As our primary focus is the dynamics of Hawking radiation, we parameterize the solutions in a gauge invariant manner as follows. We first solve the equations of motion by parameterizing the field strengths as

$$r^{\mathcal{M}}\mathcal{C}^{\text{D}}_{ri} = ik_i\frac{d}{dr}\bar{\Phi}_{\text{D}}\,, \quad r^{\mathcal{M}}\left(r^2 f\mathcal{C}^{\text{D}}_{ri} + \mathcal{C}^{\text{D}}_{vi}\right) = -\omega k_i\,\bar{\Phi}_{\text{D}}\,, \quad r^{\mathcal{M}+2}\mathcal{C}^{\text{D}}_{rv} = k^2\,\bar{\Phi}_{\text{D}}\,. \tag{3.15}$$

where $\bar{\Phi}_{\text{D}}$ is an effective diffusion scalar. In order to impose the self-consistency of this parameterization, we impose the Bianchi identity on it, and show that it sets $\bar{\Phi}_{\text{D}} = \varphi_{-\mathcal{M}}$. We further provide a map from the diffusive gauge probes to the non-Markovian scalar at the level of their actions[4], hence proving the result advertised before, that the diffusive currents in the CFT are dual to the non-Markovian sector of the designer scalar probes.

3.6 Dynamics of transverse vector and tensor gravitons

Now we will turn to the dynamics of tensor and vector perturbations in the metric. The deformed metric is given by

$$\begin{aligned}
ds^2 &= ds^2_{(0)} + \Big[(h_{AB})^{\text{Tens}} + (h_{AB})^{\text{Vec}}\Big]dx^A dx^B \\
(h_{AB})^{\text{Tens}}\,dx^A dx^B &= r^2\int_k \sum_{\sigma=1}^{N_T} \Phi_\sigma(r,\omega,\mathbf{k})\,\mathbb{T}^\sigma_{ij}\;dx^i dx^j\;, \\
(h_{AB})^{\text{Vec}}\,dx^A dx^B &= r^2\int_k \sum_{\alpha=1}^{N_V}\Big[2\,\left(\Psi^\alpha_r(r,\omega,\mathbf{k})\,dr + \Psi^\alpha_v(r,\omega,\mathbf{k})dv\right)\mathbb{V}^\alpha_i dx^i \\
&\qquad -\;\Psi^\alpha_x(r,\omega,\mathbf{k})\;\frac{1}{k}\left(k_i\mathbb{V}^\alpha_j + k_j\mathbb{V}^\alpha_i\right)\;dx^i dx^j\Big],
\end{aligned} \tag{3.16}$$

We find it illuminating to repackage these perturbations into an alternate form involving scalars and Abelian 1-forms as the following.

$$\begin{aligned}
\Phi_\sigma(v,r,\mathbf{x}) &\equiv \int_k \Phi_\sigma(r,\omega,\mathbf{k})\,\mathbb{S}\,, \\
\mathcal{A}^\alpha_B(v,r,\mathbf{x})\,dx^B &\equiv \int_k\left[\Psi^\alpha_r(r,\omega,\mathbf{k})\,dr + \Psi^\alpha_v(r,\omega,\mathbf{k})dv + \frac{k_i}{k}\Psi^\alpha_x(r,\omega,\mathbf{k})\,dx^i\right]\,\mathbb{S}\;.
\end{aligned} \tag{3.17}$$

[4]As explained before, here we account for the boundary Legendre transformation needed to transition from the correlation generating functional to the WIF.

The field strength corresponding to $\mathcal{A}^\alpha$ is defined as $\mathcal{F}_{BC} \equiv \partial_B \mathcal{A}^\alpha_C - \partial_C \mathcal{A}^\alpha_B$. In this parameterization, linearised Einstein equations simplify to

$$R_{AB} + d\, g_{AB} = 0 \quad \Longrightarrow \quad \nabla_A \nabla^A \Phi^\sigma = 0 \quad \text{and} \quad \nabla_A \left(r^2\, \mathcal{F}_\alpha^{AB} \right) = 0\,, \tag{3.18}$$

which correspond to N_T massless Klein-Gordon fields and N_V diffusive gauge fields with $\mathcal{M} = d-1$. This mapping can be in fact shown at the level of actions. Starting with the Einstein-Hilbert action with Gibbons-Hawking boundary term, which we denote by S_{grav}, we show that

$$S_{\text{grav}} = \sum_{\sigma=1}^{N_T} S[\Phi_\sigma] + \sum_{\alpha=1}^{N_V} S[\Psi_\alpha] + S_{\text{ideal}}[\mathbf{b}^\mu]\ , \tag{3.19}$$

where $S[\Phi_\sigma]$ and $S[\Psi_\alpha]$ stand for the $\mathcal{M} = d-1$ designer scalar and the $\mathcal{M} = d-1$ designer gauge actions respectively. In the above we have separately denoted the contribution from the near-equilibrium ideal fluid as S_{ideal}, which is a functional of the local reduced thermal vector $\mathbf{b}^\mu = b(\partial_v)^\mu$.

To summarize, our analysis shows that the gravitational perturbations introduced in (3.16) fall into the class of designer probes, scalars and gauge 1-forms, by establishing a mapping between these alternate descriptions at the level of the action. This enables us to use the results described in the previous subsections to derive the WIF corresponding to these perturbations.

4 Effective description of energy transport from holography

In this section, we summarize the analysis given in [2] of the scalar graviton perturbations of the Schwarzchild-AdS$_{d+1}$ blackbrane. The dual interpretation of such perturbations is as the energy density perturbations of a neutral plasma of the CFT, and hence they describe the physics of sound propagation in the plasma.

The general ansatz for the metric perturbation is

$$\begin{aligned} ds^2_{(1)} &= \left(h_{AB}\, dx^A dx^B \right)^{\text{Scal}} \\ &= \int_k \Bigg\{ \left(2\,\Psi_{\text{S}}\, ds^2_{(0)} + \Psi_{vv}\, dv^2 + 2\,\Psi_{vr}\, dv dr + \Psi_{rr}\, dr^2 \right) \mathbb{S} \\ &\qquad - 2r \left[\frac{ik_i}{k} (\Psi_{vx}\, dv + \Psi_{rx}\, dr)\, dx^i + r \left(\frac{k_i k_j}{k^2} - \frac{\delta_{ij}}{d-1} \right) \Psi_{\text{T}}\, dx^i dx^j \right] \mathbb{S} \Bigg\}. \end{aligned} \tag{4.1}$$

In order to distill out the physical degree of freedom described by these perturbations, we reduce the dynamics to that of a single scalar degree of freedom. The take away from our analysis is that Ψ_{vx}, Ψ_{rx} and Ψ_{T} can be set to vanish via a gauge choice and Ψ_{rr}, for non-zero spatial momentum $\mathbf{k}$, can be eliminated using an algebraic constraint within the equations of motion[5]. We then introduce a parameterization for the remaining

[5]This implies that our analysis has an implicit IR cut-off setting $k \geq k_{\text{IR}}$. We briefly comment on the $k = 0$ sector in Appendix E of [2].

perturbations in terms of a single scalar field $\mathcal{Z}$ as

$$\begin{aligned}\Psi_{\rm s} \equiv&\frac{1}{2r^{d-2}}\,\Phi_{\rm w}\,, \quad \Psi_{vv}+r^2 f\,\Psi_{vr} \equiv -\frac{i\omega}{r^{d-3}}\,\Theta\,, \quad \Psi_{vv} \equiv \frac{1}{r^{d-3}}\mathbb{D}_+\Theta\,,\\ \Theta \equiv&\frac{r}{\Lambda_k}\left(\mathbb{D}_+ - \frac{1}{2}\,r^2\,f'\right)\mathcal{Z}\,, \qquad \Phi_{\rm w} \equiv \frac{r}{\Lambda_k}\left(\mathbb{D}_+ + \frac{k^2}{d-1}\frac{1}{r}\right)\mathcal{Z}\,,\end{aligned} \tag{4.2}$$

where $\Lambda_k(r) \equiv k^2 + \frac{d-1}{2}\,r^3\,f'$. Surprisingly, this parameterization first motivated in [16] reduces all Einstein equations to an equation for the variable $\mathcal{Z}$ given by

$$r^{d-3}\,\Lambda_k(r)^2\,\mathbb{D}_+\left(\frac{1}{r^{d-3}\,\Lambda_k(r)^2}\,\mathbb{D}_+\mathcal{Z}\right) + \left(\omega^2 - k^2 f\left[1-\frac{d\,(d-2)}{b^d\,r^{d-2}\,\Lambda_k(r)}\right]\right)\mathcal{Z} = 0\,. \tag{4.3}$$

We further show that the identification of $\mathcal{Z}$ as the true degree of freedom is possible at the level of the action by manipulating the Einstein-Hilbert action to the form

$$\begin{aligned}\frac{1}{c_{\rm eff}}\,S[\mathcal{Z}] =& -\frac{d}{8}\,\nu_s\int_k k^4\int dr\,\sqrt{-g}\;e^{\chi_s}\left[\frac{1}{r^2 f}\,\mathbb{D}_+\mathcal{Z}^\dagger\,\mathbb{D}_+\mathcal{Z} + V_{\mathcal{Z}}(r)\mathcal{Z}^\dagger\,\mathcal{Z}\right] + S_{\rm bdy}[\mathcal{Z}]\,,\\ V_{\mathcal{Z}}(r) =& -\frac{\omega^2}{r^2 f} + \frac{k^2}{r^2}\left(1-\frac{(d-2)\,r^3\,f'}{\Lambda_k}\right),\end{aligned} \tag{4.4}$$

where $S_{\rm bdy}$ denotes boundary terms (including counterterms) and

$$c_{\rm eff} = \frac{\ell_{\rm AdS}^{d-1}}{16\pi G_N}\,, \qquad \nu_s \equiv \frac{2\,(d-2)}{d\,(d-1)}\,, \qquad e^{\chi_s} \equiv \frac{1}{r^{2(d-2)}\,\Lambda_k^2}\,. \tag{4.5}$$

We find that $\mathcal{Z}$ has a more complicated dynamics compared to the designer probes studied before, with non-trivial potential and dilaton, both spatially modulated through Λ_k. Nevertheless, via explicit analysis of the variational principle, counterterms, Legendre transformation etc., we show that the main conclusions from the analysis of non-Markovian designer probes hold in this case as well. Namely,

- The standard AdS/CFT treatment (or the GKPW dictionary) yielding correlation generating functionals (for energy-momentum density correlations) corresponds to imposing Neumann boundary condition on $\mathcal{Z}$.
- Imposing renormalized Dirichlet boundary condition on $\mathcal{Z}$, one can derive an effective action (WIF) for the hydrodynamic moduli $\breve{\mathcal{Z}}_{\rm L/R}$ defined as its boundary value.

We find that $\breve{\mathcal{Z}}_{\rm L/R}$ can be gauge-invariantly expressed using

$$\langle \widehat{T}^i_v\rangle_{\rm L/R} = -\int_k \frac{k_i\,\omega}{d-1}\,\breve{\mathcal{Z}}_{\rm L/R}\,\mathbb{S}\,, \tag{4.6}$$

where $\widehat{T}_\mu^\nu$ denote the stress-energy tensor of the plasma sans the contribution from the background fluid. We further compute the WIF of the scalar gravitons, and find that it cleanly separates into contributions from the ideal fluid and fluctuations similar to the case of vector and tensor gravitons. For completeness, below we quote the the analogue of the function $K^{\text{in}}_{-\mathcal{M}}(\omega, \mathbf{k})$ for scalar gravitons, the sound dispersion function $K_s(\omega, \mathbf{k})$, which defines the poles of the energy-momentum 2-pt functions.

$$
\begin{aligned}
K_s(\omega, \mathbf{k}) &\equiv -\mathfrak{w}^2 + \frac{\mathfrak{q}^2}{d-1} + \nu_s\, \mathfrak{q}^2\, \Gamma_s(\omega, \mathbf{k})\,, \\
\Gamma_s(\omega, \mathbf{k}) &= -i\, \mathfrak{w} - \mathfrak{w}^2 \left[(d-2)\, H_k(d-1,1) - \frac{1}{d-2}\right] + \frac{d-3}{(d-1)\,(d-2)}\, \mathfrak{q}^2 + \cdots .
\end{aligned}
\tag{4.7}
$$

5 Summary and directions

As summarized in the previous sections, the results of [1] and [2] show that the FH of holographic neutral plasmas can be consistently defined using a WIF. Our analysis is guided by the mutual consistency of multiple ingredients in the dual gravitational theory. The gauge invariance of the gravitational description lead us to consider the transverse $\mathrm{SO}(d-2)$ decomposition of the fluid excitations for distilling out Markovian and non-Markovian dynamics. Quite satisfyingly, the analytic structure of grSK saddle suggested a parameterization (boundary conditions) for these excitations which fitted naturally with the philosophy of the WIF motivated in §1.

We remind that, while it is promising that we get consistent results for the quadratic theory, a convincing evaluation of our proposal is possible only after constructing an interacting theory of FH. In order to do this, we need to derive the corrections to the WIF due to gravitational interactions by computing appropriate Feynman-Witten diagrams. It will be interesting to see if the boundary conditions we propose continue to yield a local WIF in this case.

A crucial ingredient in our analysis is the grSK saddle, which we remind, is only a conjectured prescription which has survived many consistency checks so far. Its validity beyond the probe approximation (i.e., after including gravitational back reaction) is a question of high interest. Further, a clear derivation of the grSK saddle could inform us how to define it away from equilibrium, thereby facilitating the derivation of a more complete description of FH.

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