Linear response theory of ecosystems to environmental perturbations

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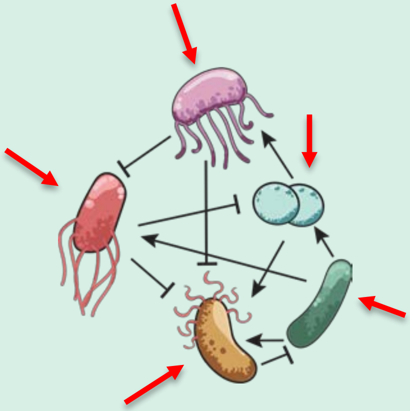
Work with Jason W. Rocks and Pankaj Mehta (BU)

How do ecosystems respond to environmental perturbations?



practical importance

central questions in ecology

e.g., Δresource supply → new state

theoretical importance

linear response is a key way to understand systems

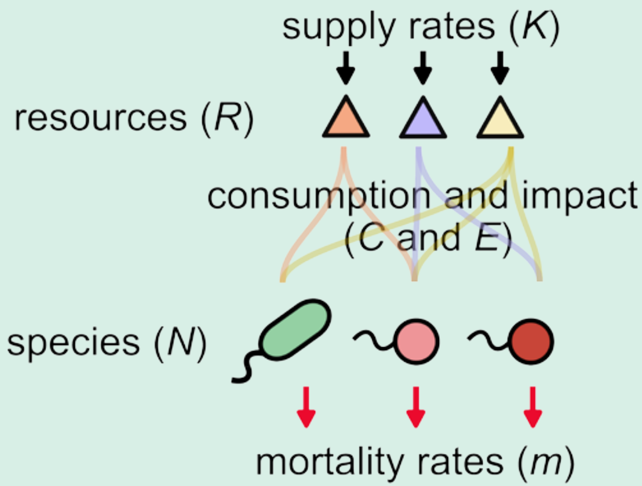
In ecology, largely studied linear response to **species perturbations**

e.g., May, Nature (1972); cartoon credit: Martina Dal Bello

Still lacking a framework for response to environmental perturbations

this talk (consumer-resource models)

MacArthur Consumer-Resource Models



$$\frac{dR_\alpha}{dt} = R_\alpha(K_\alpha - R^\alpha) - \sum_{j=1}^{S} E_{j\alpha} N_j R_\alpha$$

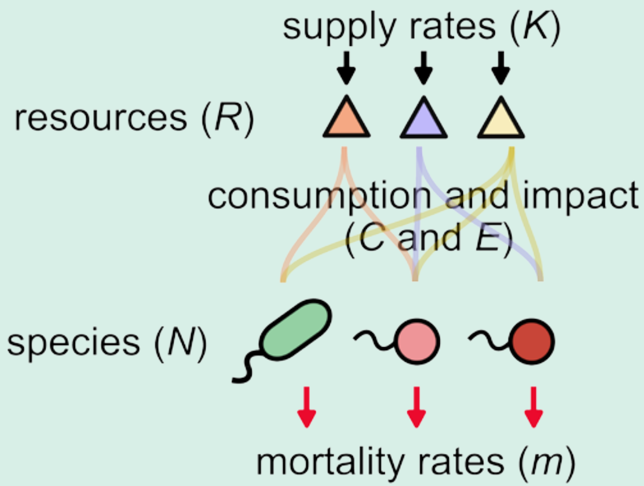
$$\frac{dN_i}{dt} = N_i\left(\sum_{\alpha=1}^{M} C_{i\alpha} R_\alpha - m_i\right)$$

env parameters

response
variables

Goyal et al, arXiv (2024)

Goal: linear response to env perturbations



env perturbations

response

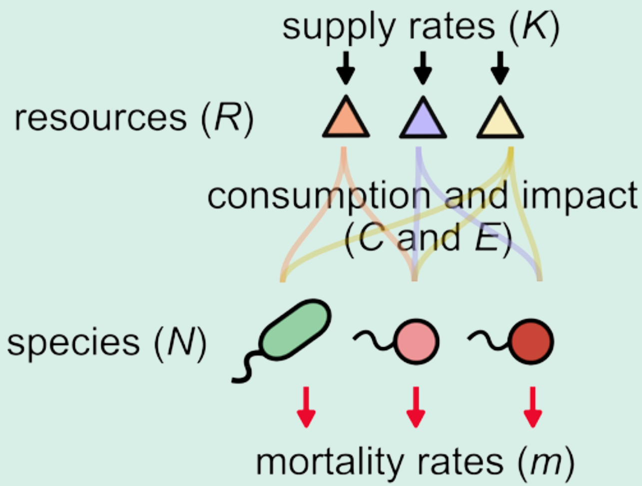
$$\Delta R = \frac{\partial R}{\partial m} \cdot \Delta m + \frac{\partial R}{\partial K} \cdot \Delta K$$

$$\Delta N = \frac{\partial N}{\partial m} \cdot \Delta m + \frac{\partial N}{\partial K} \cdot \Delta K$$

susceptibilities

Goyal et al, arXiv (2024)

Solution: linear algebra at steady states



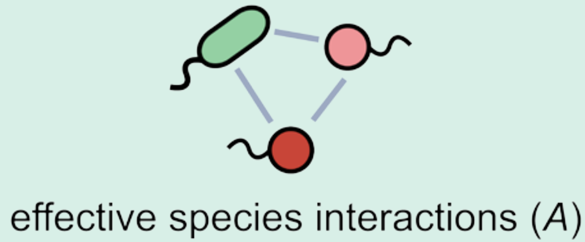
env perturbations

response

$$\Delta N^* = A^{-1}\Delta m + A^{-1} C \Delta K$$

$$\Delta R^* = E^T A^{-1}\Delta m + (1 - P)\Delta K$$

susceptibilities



Goyal et al, arXiv (2024)

Geometry: susceptibilities map b/w 4 vector spaces

env perturbations

response

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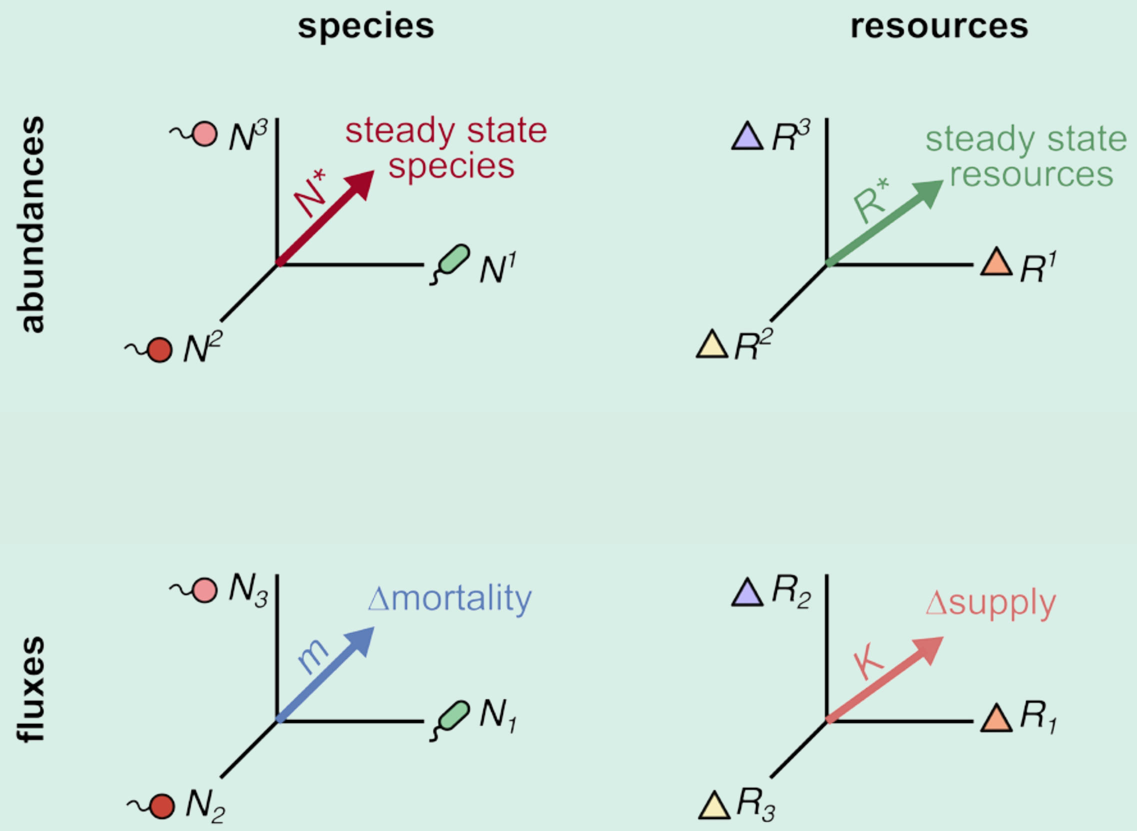
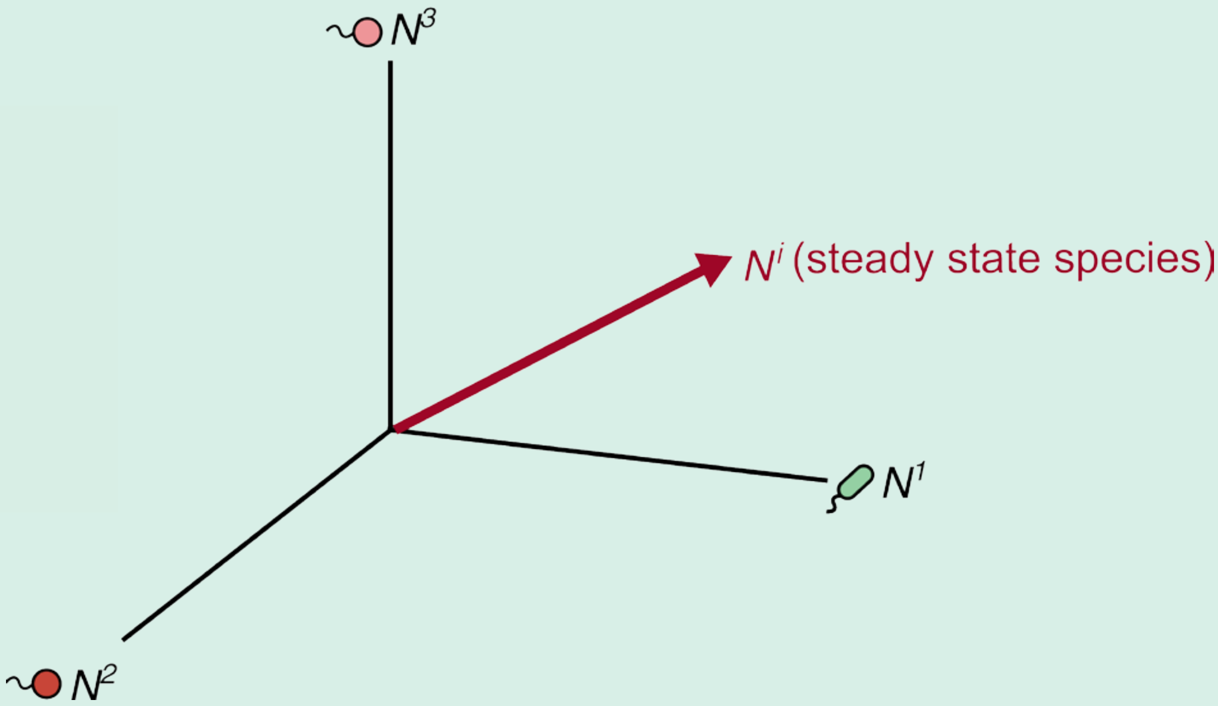


Goyal et al, arXiv (2024)

Geometry of species space



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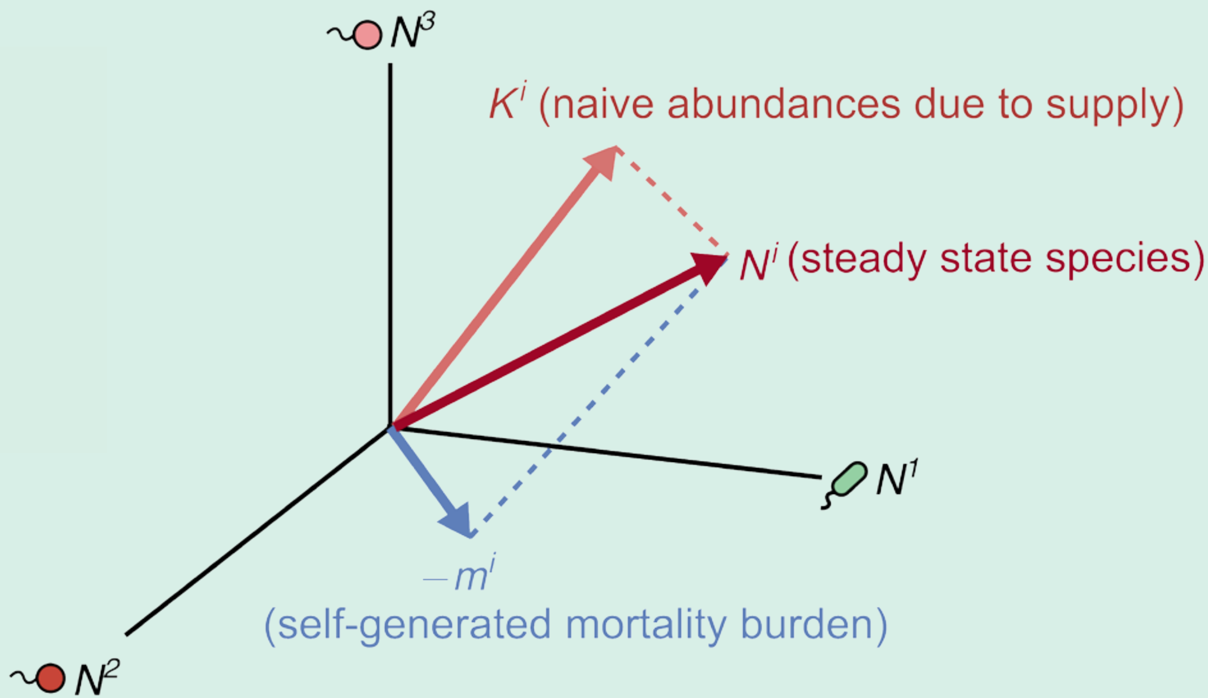
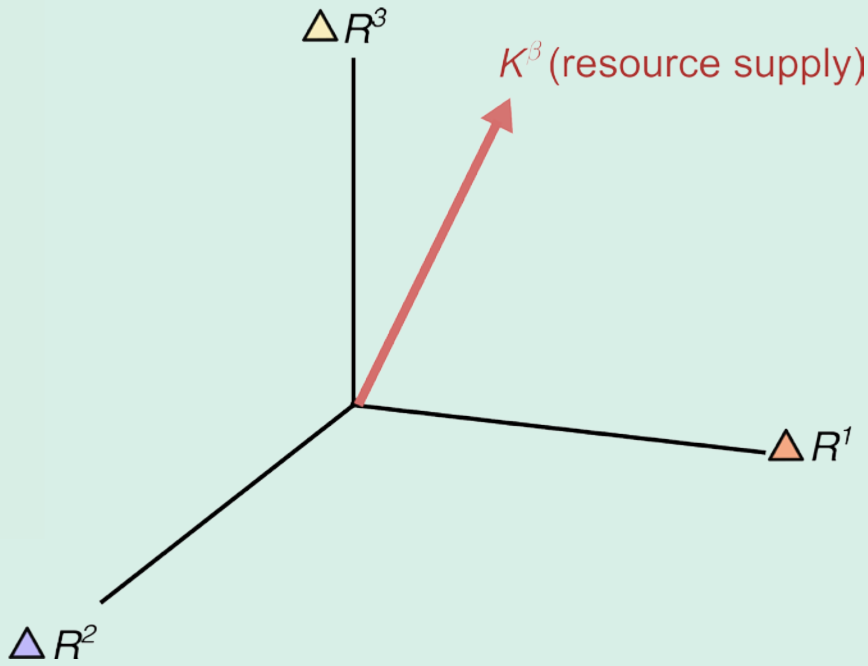


Goyal et al, arXiv (2024)

Geometry of resource space



Goyal et al, arXiv (2024)

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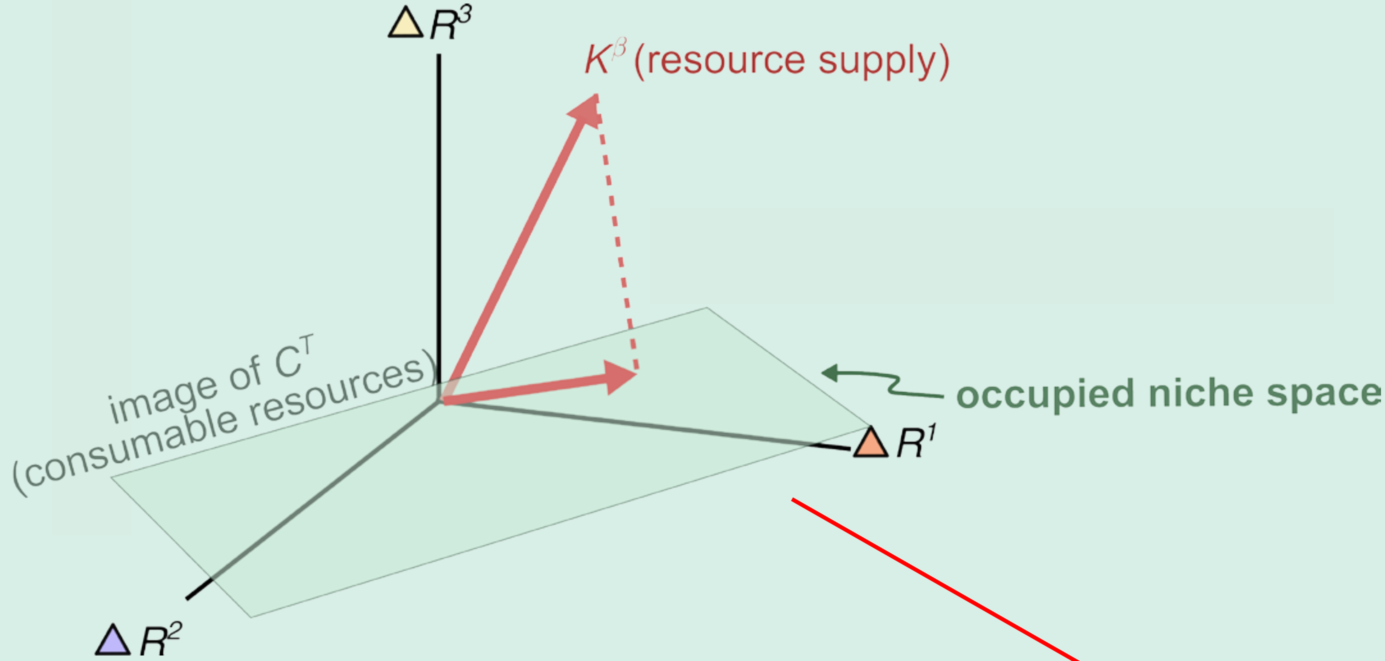
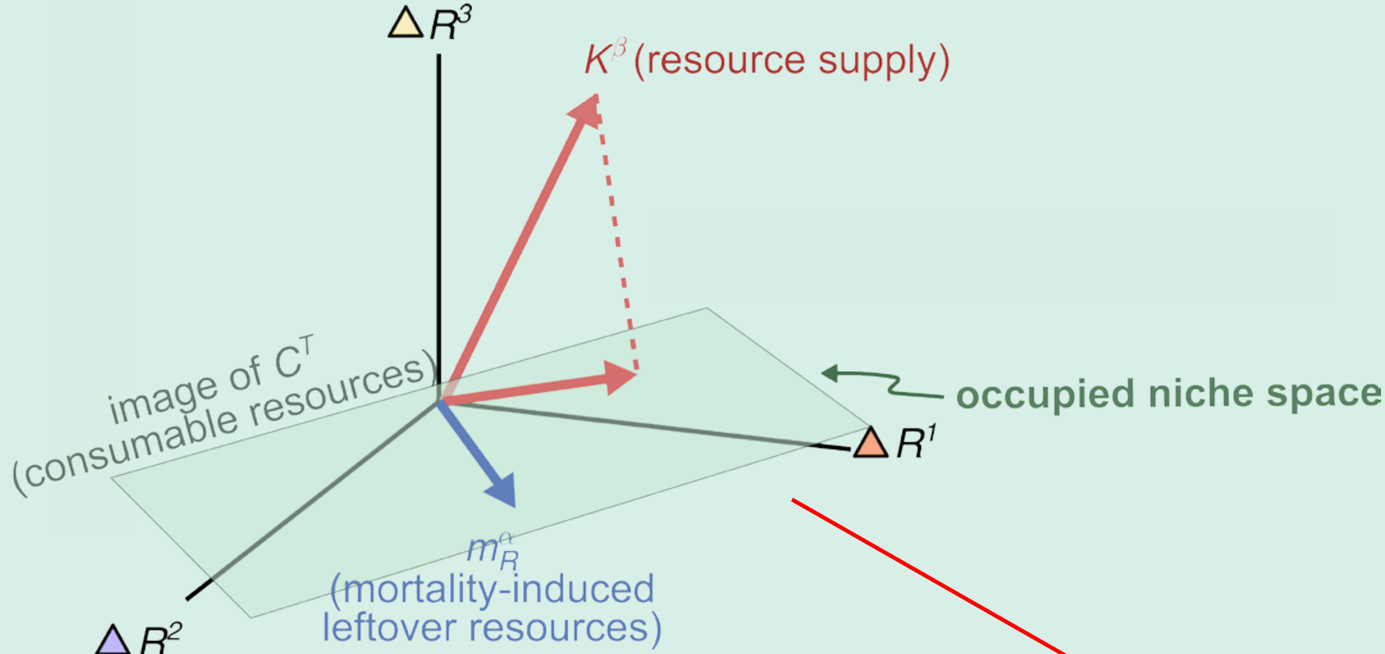


$$\Delta R^* = E^T A^{-1} \Delta m + (1 - P) \Delta K$$

Goyal et al, arXiv (2024)

Geometry of resource space



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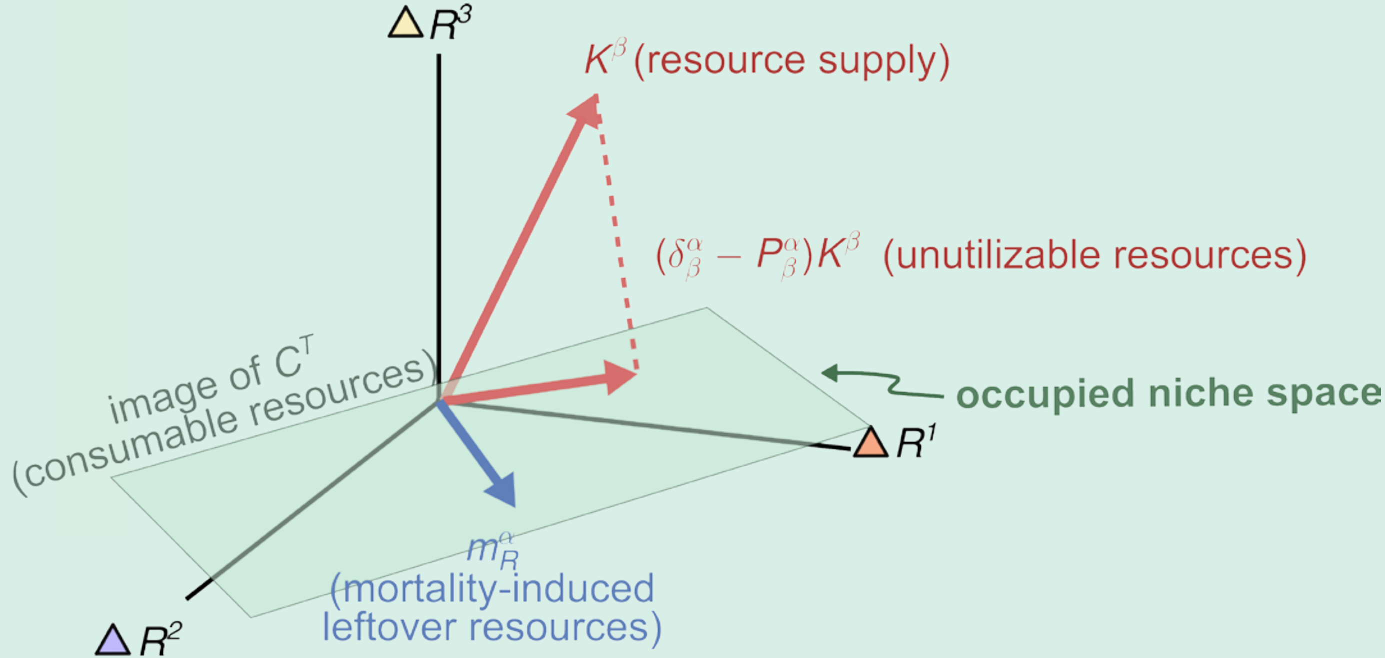
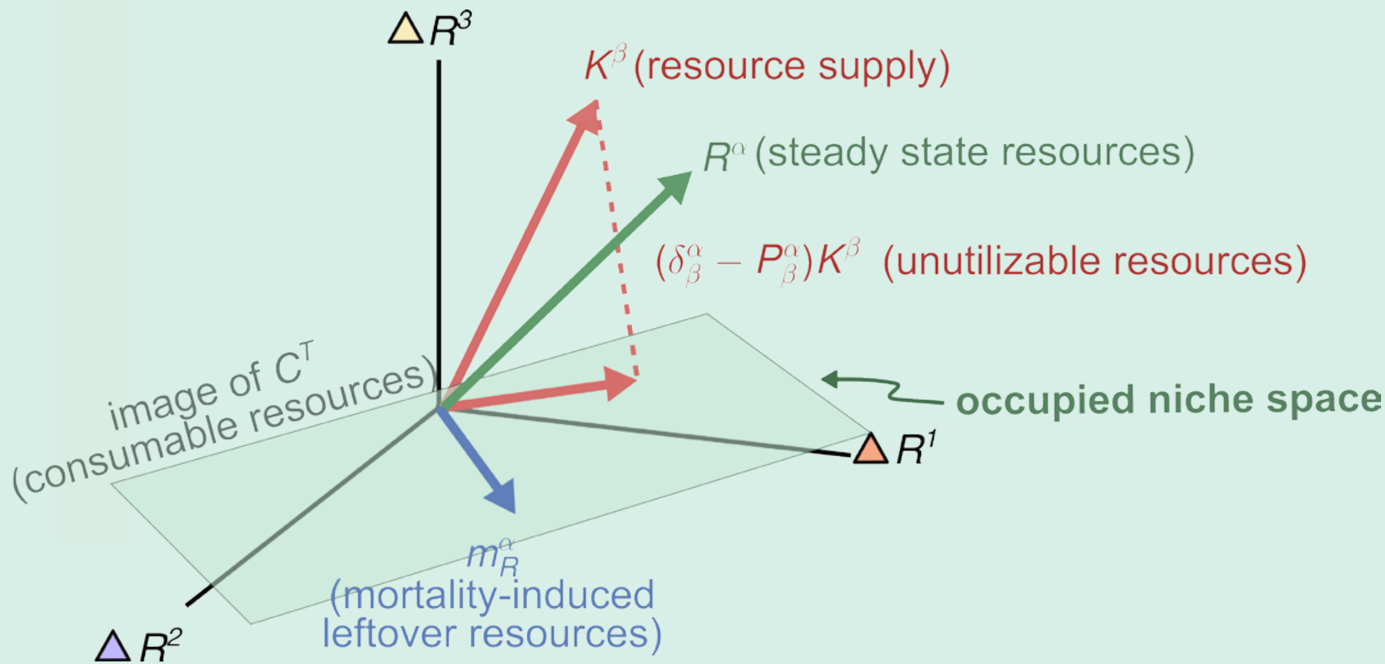


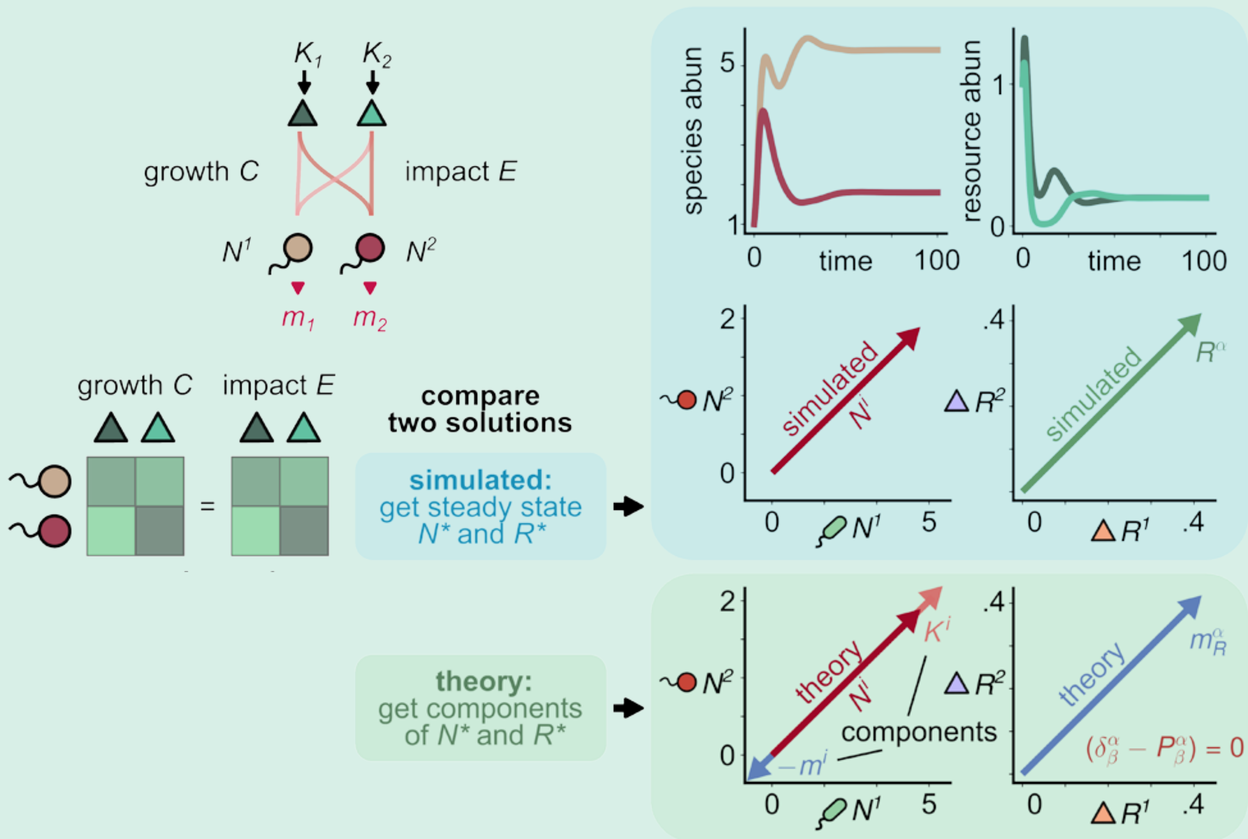
Goyal et al, arXiv (2024)

Geometry of resource space



Goyal et al, arXiv (2024)

Theory makes accurate predictions



Summary

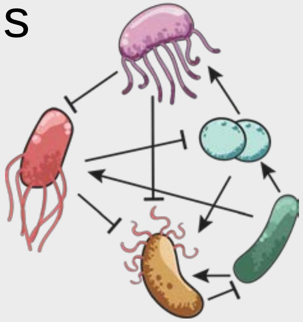
Developed a linear response theory of ecosystems to perturbations.

Susceptibilities are the key quantities connecting environmental perturbations to ecosystem response.

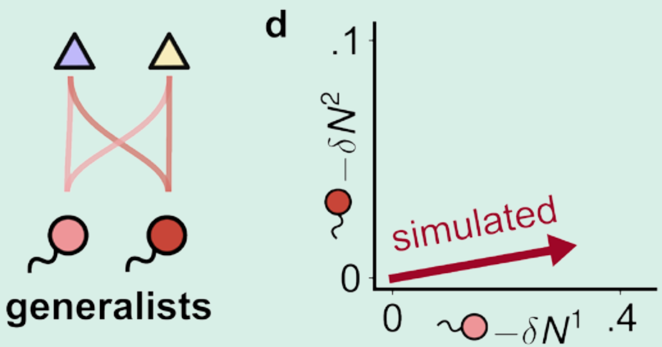
Geometrically, susceptibilities are transformations b/w 4 distinct vector spaces.

Geometry aids interpretation of complex emergent quantities like niches & pH sensitivity.

Universal: generalizes to arbitrary consumer resource models



Generalists more sensitive than specialists to pH

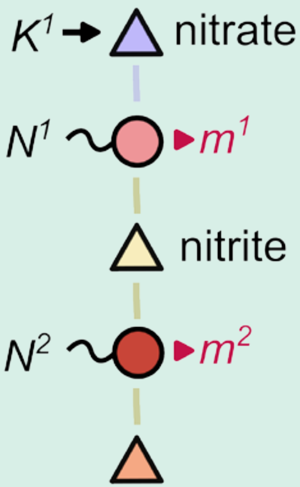


Consistent with experiments

Goyal et al, arXiv (2024)

Crocker et al, BioRxiv (2023)

Example: pH sensitivity of denitrifying communities



$$\frac{dN_i}{dt} = N_i\left[\sum_\alpha (1-\ell) r_{i\alpha} \max\left(0, \frac{pH - pH_{min}}{pH_{neutral} - pH_{min}}\right) R_\alpha - m_i\right]$$

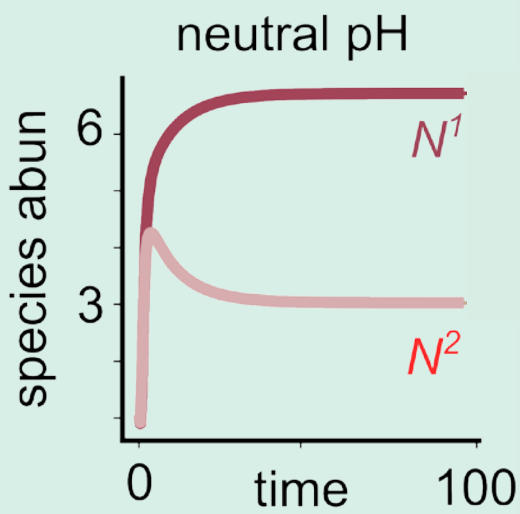
$$\frac{dR_\alpha}{dt} = K_\alpha - R_\alpha - \sum_j I_{j\alpha} N_j R_\alpha + \sum_{j,\beta} \ell D_{\alpha\beta} r_{j\beta} \max\left(0, \frac{pH - pH_{min}}{pH_{neutral} - pH_{min}}\right)$$

nitrite becomes toxic as pH decreases

Data from: Crocker et al, BioRxiv (2023)

Simulations: change pH → measure ΔN

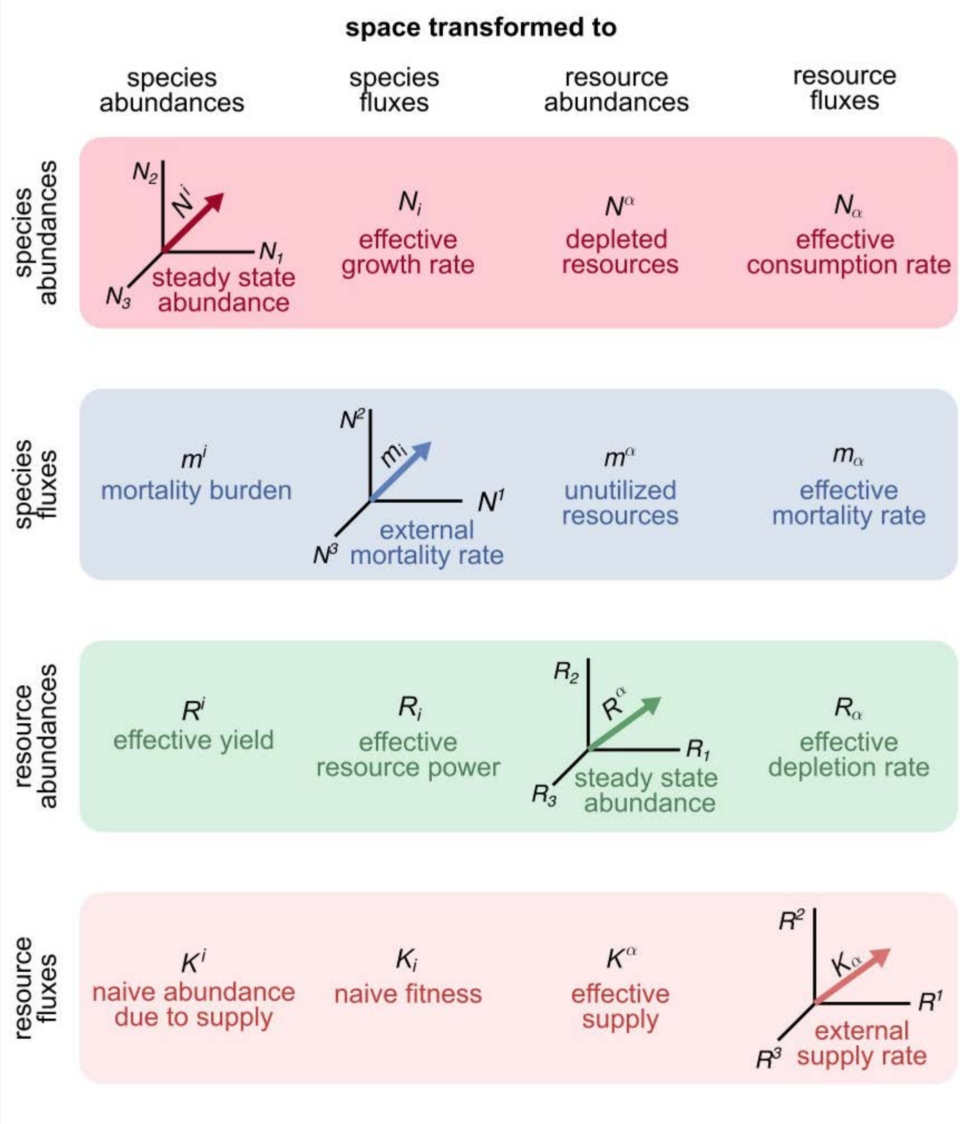
env perturbations



response

Goyal et al, arXiv (2024)

Geometry is universal,
applies to variety of community data



Matrix of transformed vectors