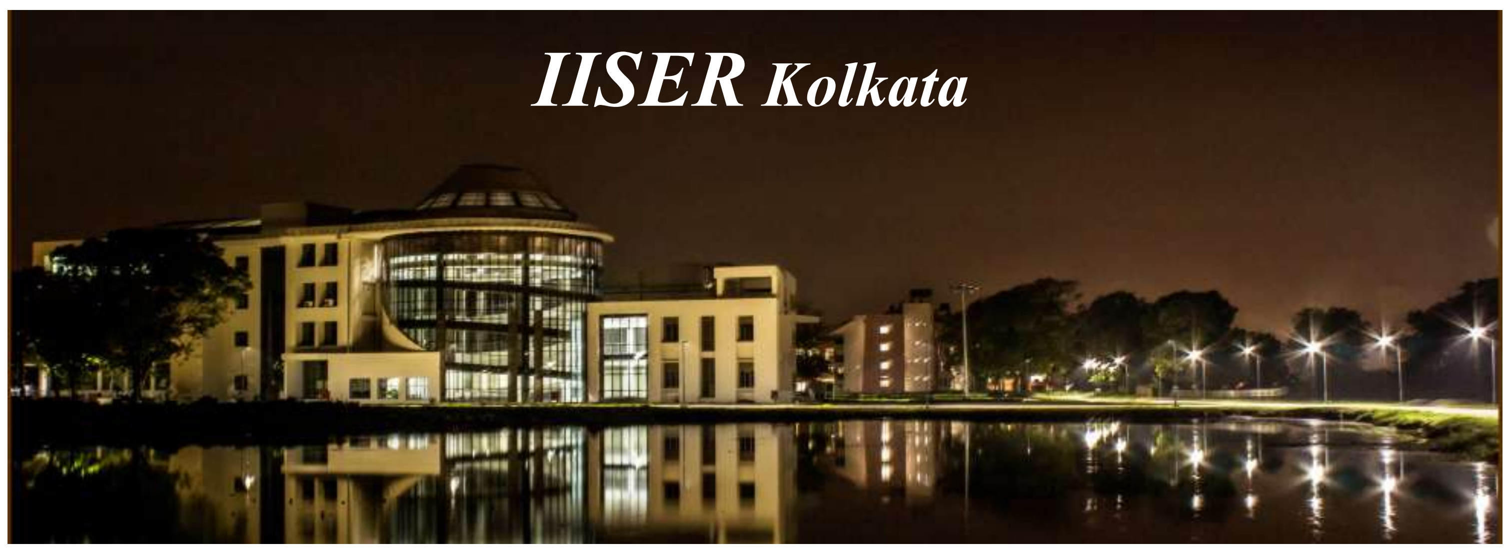
Intermediate spectral properties of the β - ensemble

arXiv:2112.11910 Phys. Rev. E 105, 054121 2022

Adway Kumar Das & Anandamohan Ghosh

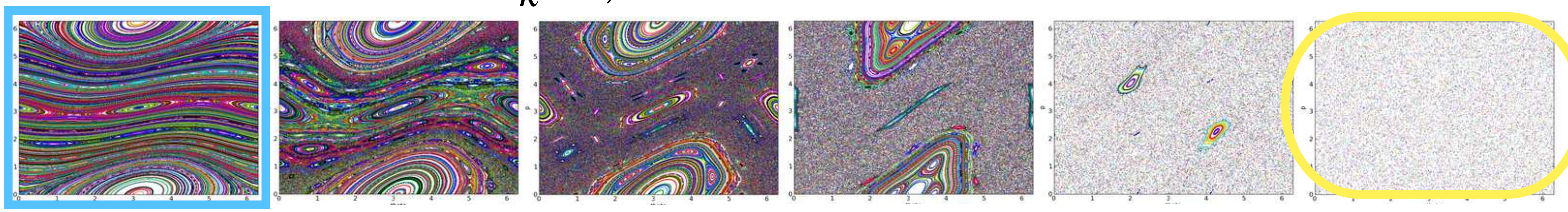


ISPCM-2023 - ICTS

Quantum chaos ⟷ Random Matrix Theory

Classical Kicked Rotor: $$H = \frac{p^2}{2} + k\cos(x) \sum_{n=-\infty}^{\infty} \delta(t-n)$$

$k \longrightarrow$



← Intermediate dynamics →

Quantum Kicked Rotor:

$i\hbar\frac{\partial\psi}{\partial t} = H\psi$

$$\mathrm{P}(\vec{E}) = \frac{1}{\mathcal{Z}_\beta} \exp\left(-\sum_{i=1}^{N} \frac{E_i^2}{2}\right) \prod_{i<j} |E_i - E_j|^\beta$$

- Classically integrable
- Poisson statistics $\beta \to 0$
- Level clustering
- Localisation - non-ergodic

- Classically chaotic
- RMT $\beta = 1$ (2, 4)
- Level repulsion
- Extended states - ergodic

Wigner 1959; Bohigas, Giannoni, Schmit 1984; Berry 1985; Haake 1991; Mehta 1991……

Intermediate Statistics $\longrightarrow$ *Generalized* Random Matrix Models

- Wigner-Dyson Ensemble ($\beta = 1,2,4$): $H_{nn} \in \mathcal{N}(0,1)$ & $H_{nm} \in \mathcal{N}(0, \frac{\beta}{2})$

- Rosenzweig-Porter Ensemble (RPE 1960): $\hat{H}_{nn} \in \mathcal{N}(0, \sigma_d^2)$ & $\hat{H}_{nm} \in \mathcal{N}(0, \sigma_o^2)$

  Berry & Shukla 2009, Das & Ghosh 2019, Khaymovich & Kravtsov 2021

- Random Banded Matrices (RBM): $G_{mn} = H_{mn}\ f(|m-n|)$

  Casati et al. 1991, Mirlin et al. 1996, Bogomolny & Giraud 2011, Pandey, Kumar, Puri 2020

- Deformed Ensemble (DGOE, DCE, DPE): $\mathscr{H} = H_0 + \lambda\ V$

  Guhr & Weidenmuller 1989, Hussein & Pato 1993, Das & Ghosh 2022

$\vdots$

Matrix model for β ensemble

Dumitriu & Edelman 2002

Wigner-Dyson ensemble $\xrightarrow{\text{Successive Householder}}$ Tridiagonal

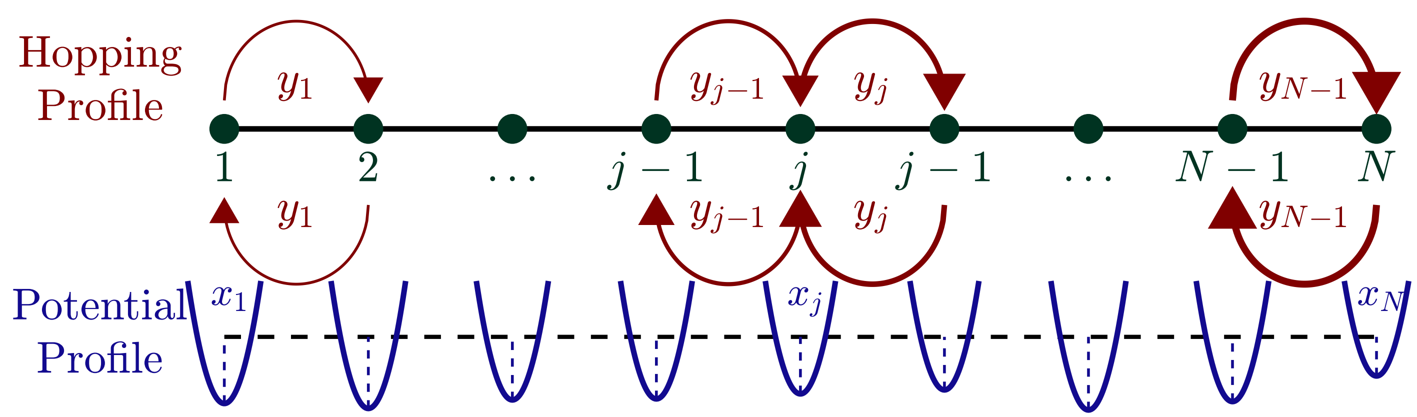
$$H_\beta = \begin{bmatrix} x_1 & y_1 & & & \\ y_1 & x_2 & y_2 & & \\ & \ddots & \ddots & \ddots & \\ & & y_{N-2} & x_{N-1} & y_{N-1} \\ & & & y_{N-1} & x_N \end{bmatrix}$$

$\beta = 1,2,4$

$x_n \sim \mathcal{N}(0,1), \ \sqrt{2} y_n \sim \chi_{n\beta} \qquad \beta \in \mathbb{R}^+$

y_n's increase along lattice

Lattice picture: $\hat{H}_\beta = \sum_{n=1}^{N} x_n \left|n\right\rangle \left\langle n\right| + \sum_{n=1}^{N-1} y_n \left(\left|n+1\right\rangle \left\langle n\right| + \text{H.c.}\right)$



- **Spectral properties ?**
- **Transitions ?**

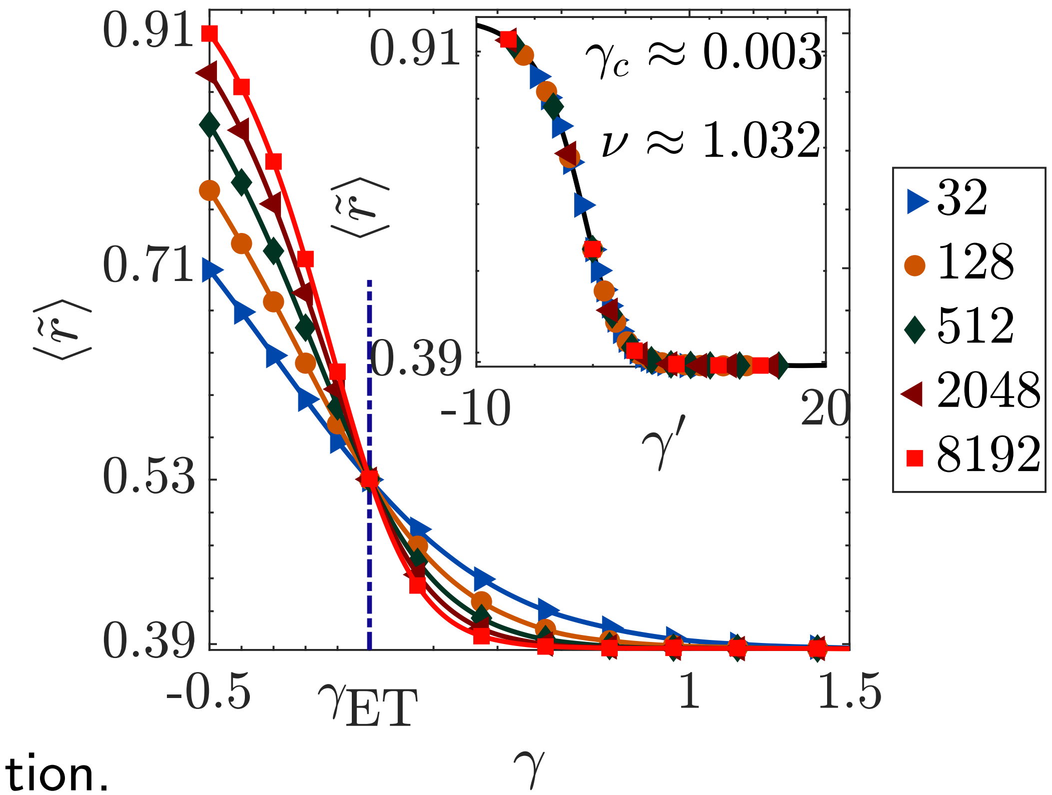
$$\beta = N^{-\gamma}$$

Energy levels: Short-range correlations in RNNS

$$r_j = \min\left(\tilde{r}_j, \frac{1}{\tilde{r}_j}\right)$$

Oganesyan & Huse 2007

$$\tilde{r}_j = \frac{E_{j+1} - E_j}{E_j - E_{j-1}}$$



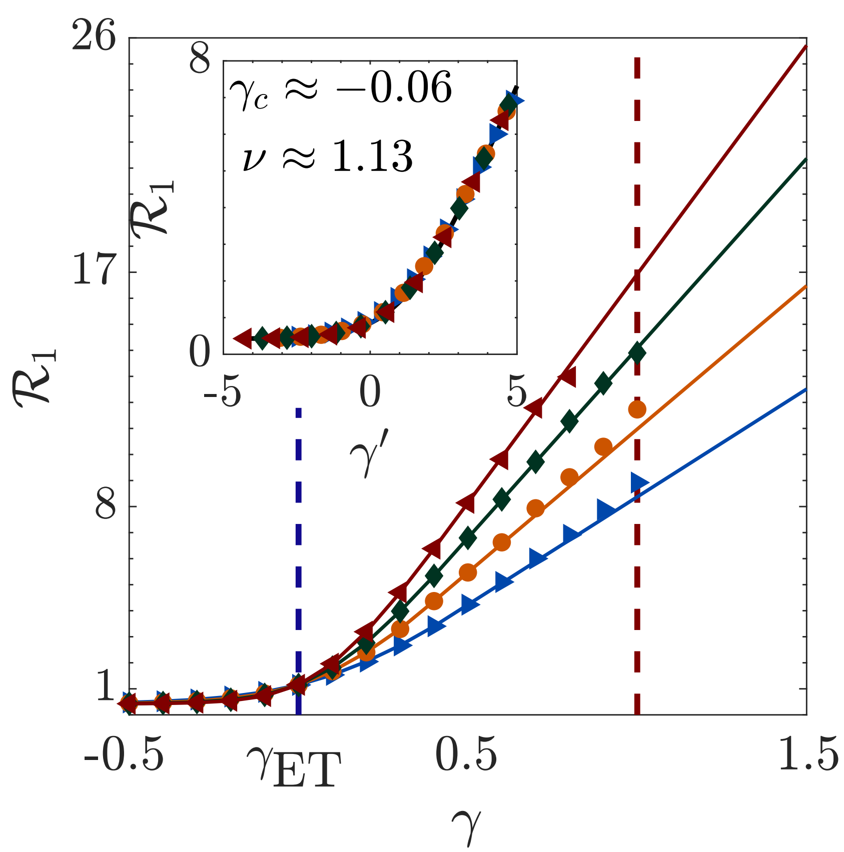
Energy states: Ergodicity

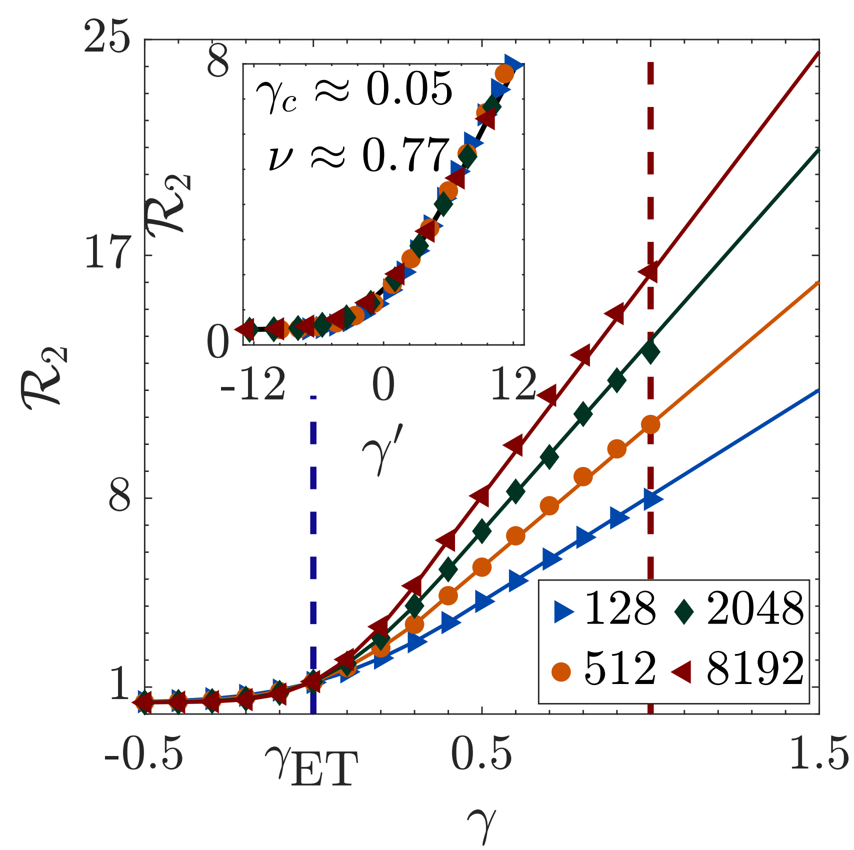
$\Psi_i^j(k) \rightarrow$ k^{th} element of i^{th} state of the j^{th} realization.

$$\mathcal{R}_1 = -2\ln\left(\sum_{k=1}^{N}\left|\Psi_i^j(k)\Psi_{i+1}^j(k)\right|\right)$$

$$\mathcal{R}_2 = -2\ln\left(\sum_{k=1}^{N}\left|\Psi_i^j(k)\Psi_{i+1}^{j'}(k)\right|\right)$$

Nagy & Romera 2015





$$A(\gamma, N) \propto f((\gamma - \gamma_c)(\ln N)^{1/\nu})$$

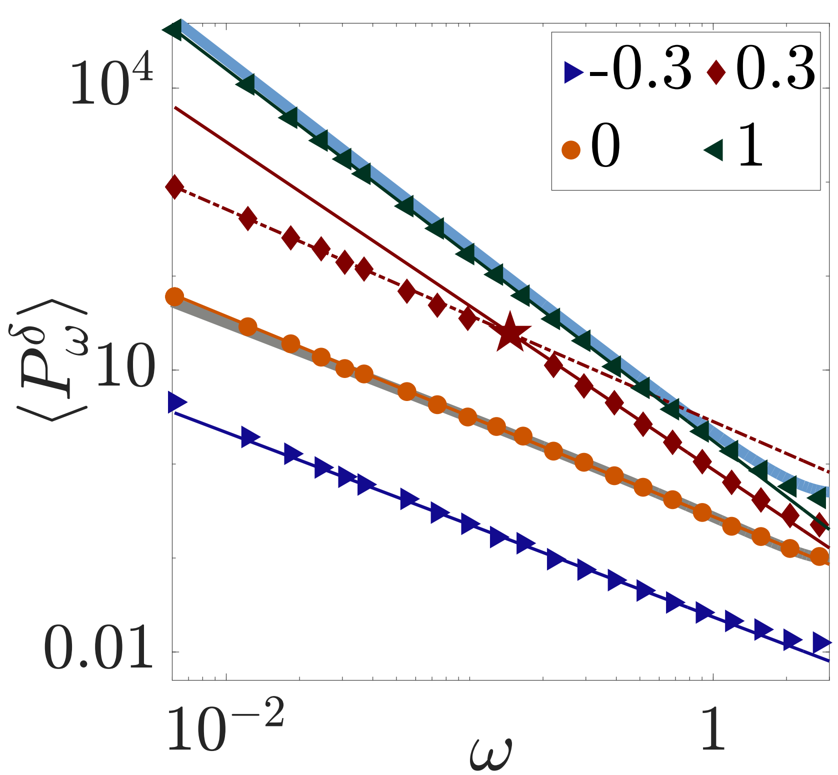
$$\nu \approx 1$$

$$\gamma = \gamma_{ET} \approx 0$$

$$\beta \approx 1$$

Energy levels: Long-range correlations in power spectrum Corps, Relano …

$$\delta_n = E_n - \bar{E}$$



- $\gamma < 0 : P_\omega^\delta \propto \dfrac{1}{\omega} \implies \{E\}$ correlated
- $0 < \gamma < 1$: heterogeneity

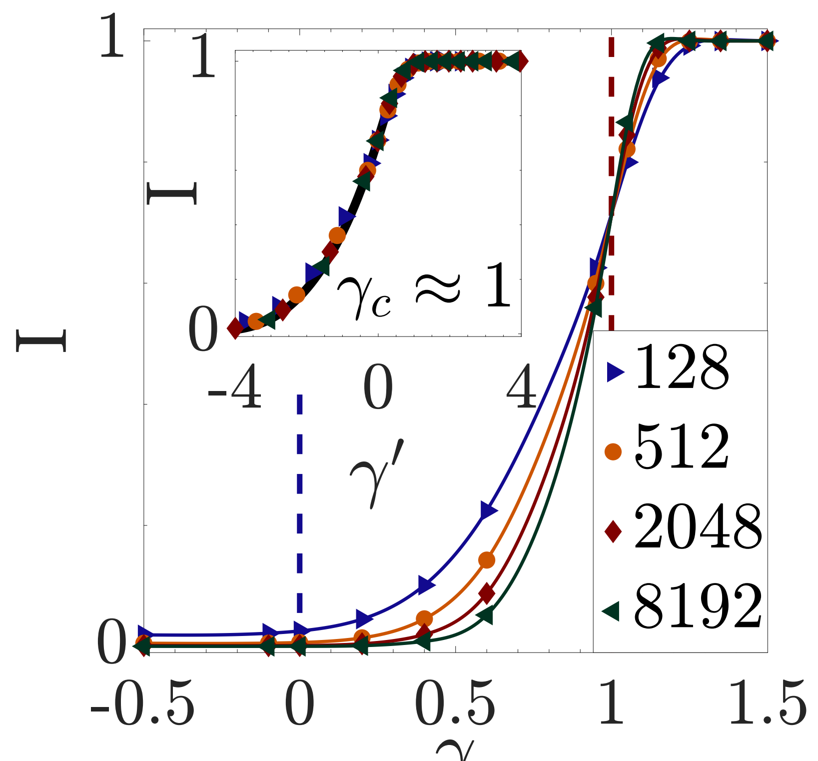
  $\omega < \omega_c : P_\omega^\delta \propto \dfrac{1}{\omega} \implies$ correlated

  $\omega > \omega_c : P_\omega^\delta \propto \dfrac{1}{\omega^2} \implies$ independent
- $\gamma > 1$: uncorrelated

Energy states: Localization

Inverse Participation Ratio (IPR):

$$I = \sum_{i=1}^{N} |\Psi_i|^4$$



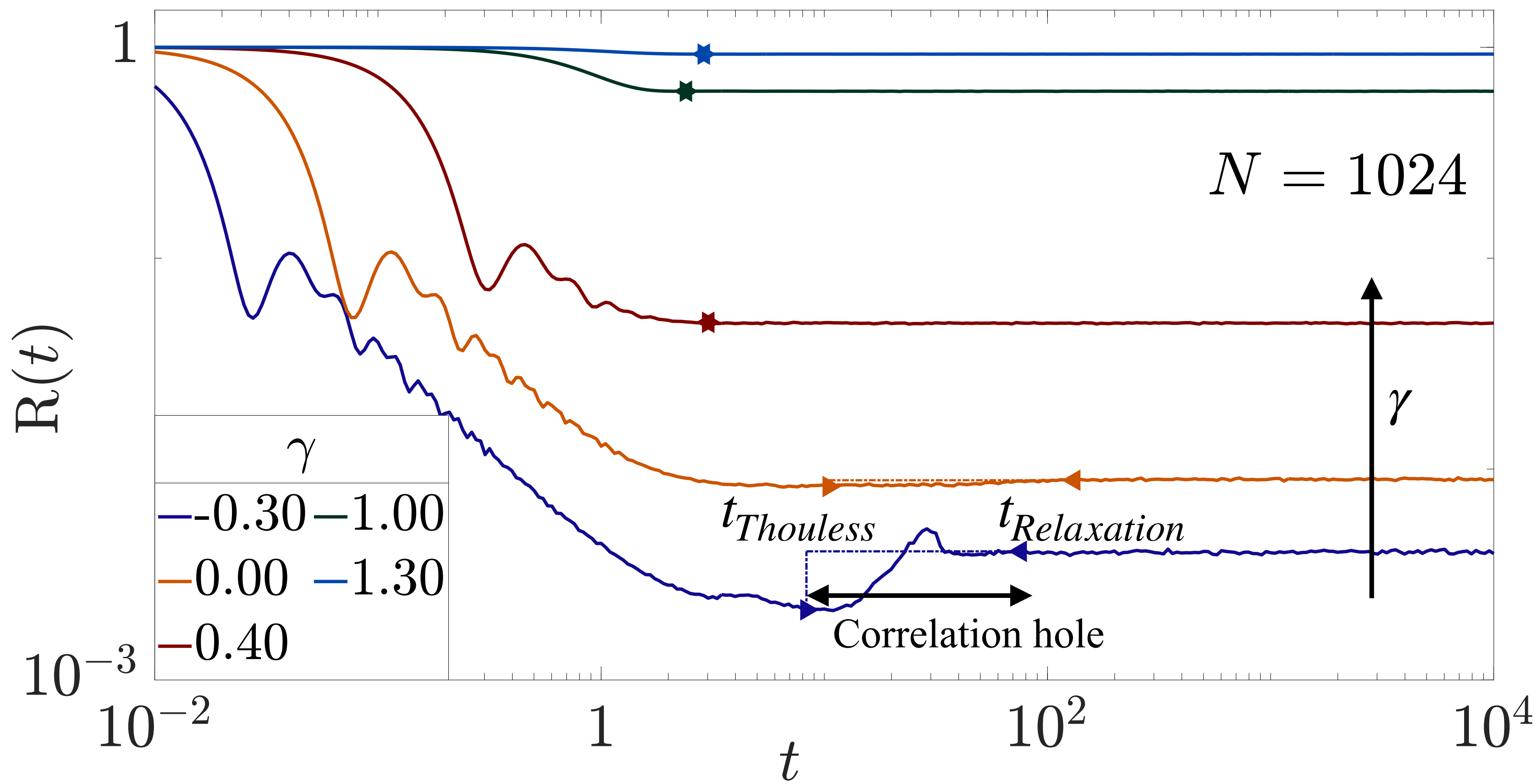
$$\gamma = \gamma_{AT} \approx 1$$

$$\beta \approx \frac{1}{N}$$

Dynamical signatures : Survival probability

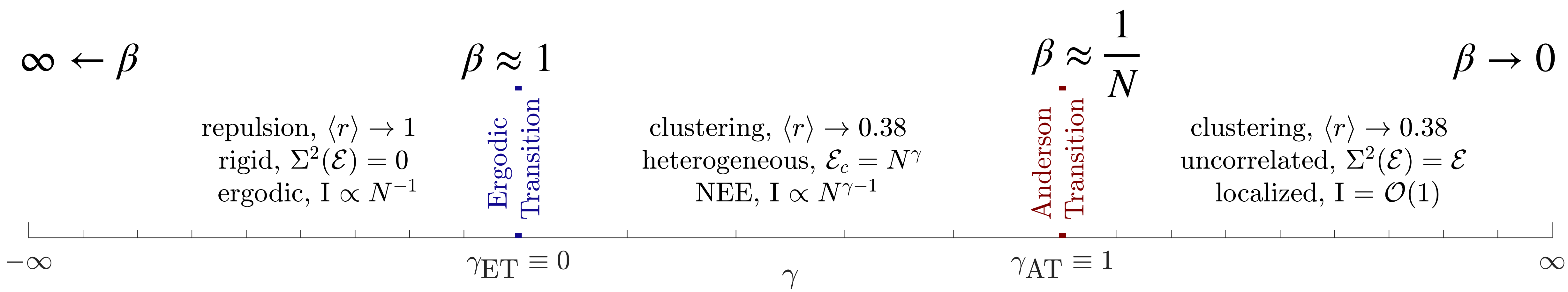
$$R(t) = |\langle j|j(t)\rangle|^2 = \left|\sum_{k=1}^{N} \left|\phi_k^{(j)}\right|^2 e^{-iE_k t}\right|^2$$

Torres-Herrara & Lea Santos 2017



R($t \to \infty$) ≡IPR

Non-Ergodic Extended (NEE) states in β ensembles

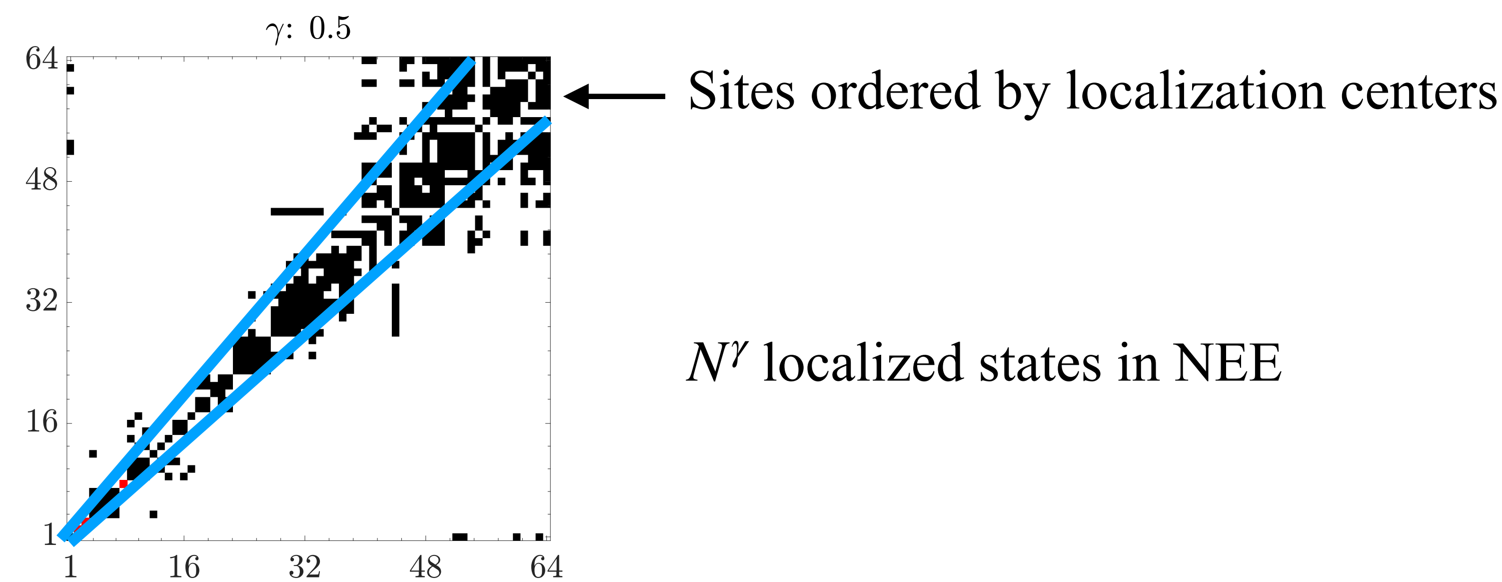


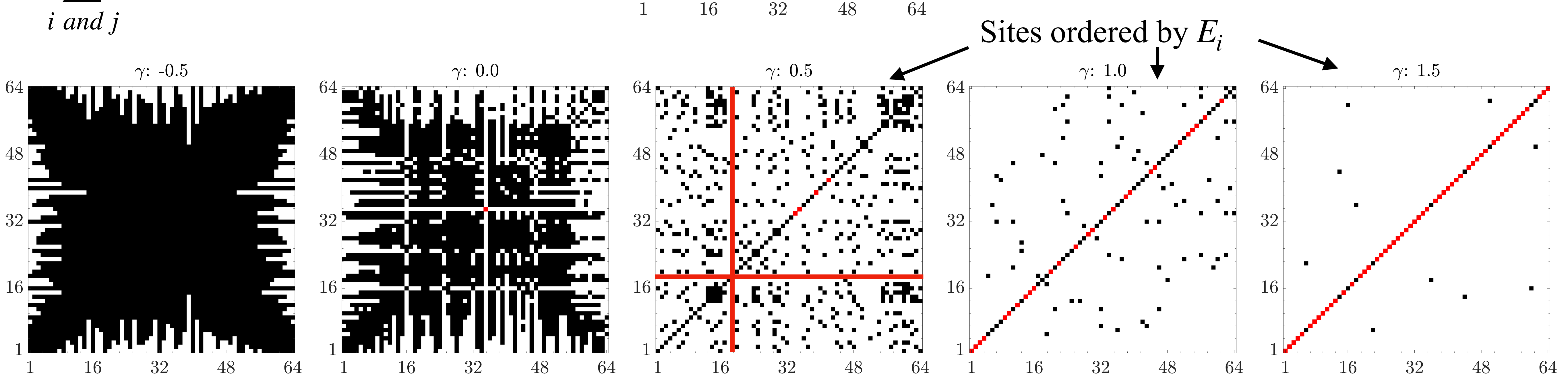
$$C_{ij} = \sum_k |\Psi_i(k)\Psi_j(k)|$$

$$= 1 \qquad i{=}j$$

$$\approx \frac{2}{\pi} \qquad \text{Ergodic (Berry)}$$

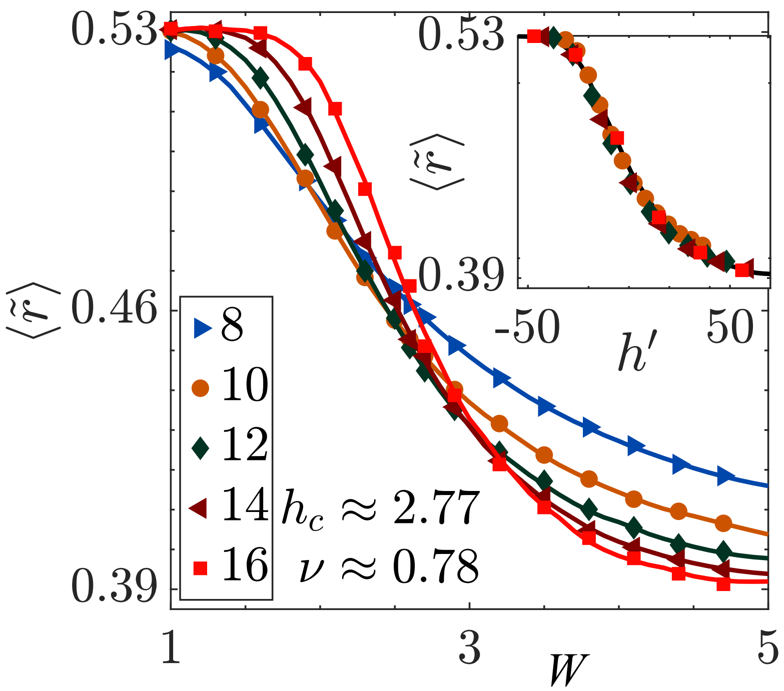
$$\sum_{i \text{ and } j} C_{ij} = 0 \qquad \text{Localized}$$

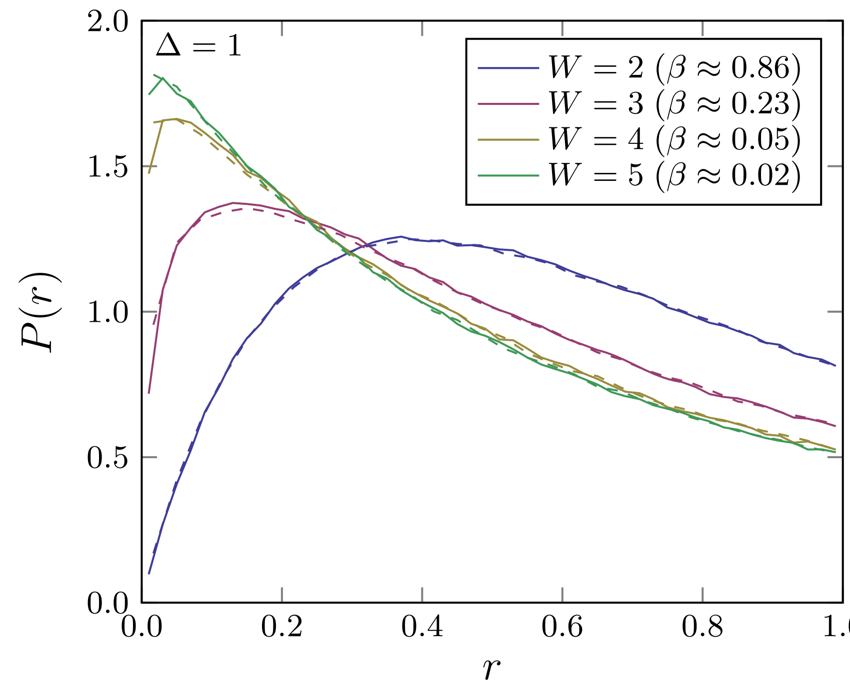




β ensembles $\longleftrightarrow$ spin chains

$$H = \sum_{i=1}^{L} (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z) + \sum_{i=1}^{L} h_i S_i^z \qquad h \in [-W : W]$$





Buijsman et al. 2019

Higher order statistics?

Long-range correlations?

$\beta - h$ ensemble

$$\mathcal{P}_h^\beta(E_1, \ldots, E_N) = Z_N^{-1} \prod_{i=0}^{N} |E_i - E_{i+1}|^\beta \cdots |E_i - E_{i+h}|^\beta$$

Sierant & Zakrzewski 2020

Summary

- Non-Ergodic Extended states
- *Neighbouring energy* states do not overlap
- Coexistence of localised - extended states

Das and Ghosh 2022

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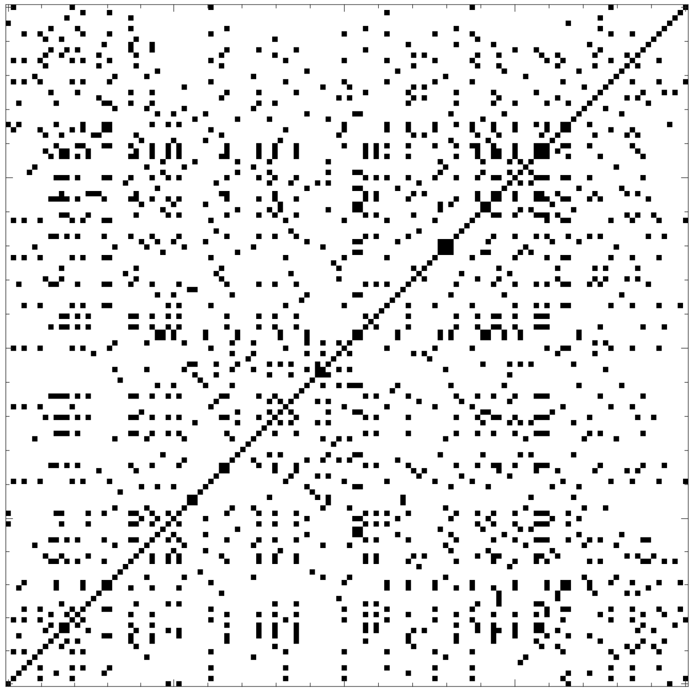

Ongoing projects….

- Anomalous Localization
- Ground states vs. Bulk states
- Absence of mobility edge
- Local equivalence to Anderson model
- $\beta - h$ model

with **Ivan Khaymovich**

Nordita

THANK YOU

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