Gravitational Waves from Chiral Plasma Instability in the Standard Cosmology

Andrew J. Long
Rice University
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Baryogenesis "by-products"

Among the outstanding problems in modern cosmology (dark matter, dark energy, inflation, baryogenesis) … the matter / anti-matter asymmetry is uniquely challenging, because we only know one number ($n_B/s = 10^{-10}$)!

Therefore it is compelling to study models with "secondary predictions" that we can test in the lab (*e.g.*, EWBG tested by collider observables & EDMs).

However, the physics of baryogenesis may not within reach of terrestrial experiments (*e.g.*, Affleck-Dine, GUT baryogenesis, high-scale leptogenesis).

In this case, we may still probe the origin of the matter / anti-matter asymmetry through observations of baryogenesis "by-products".

Baryogenesis requires a departure from thermal equilibrium (Sakharov), and such conditions may create additional cosmological relics (*e.g.*, gravity waves and topological defects) or the OOE conditions may be provided by other relics (*e.g.*, primordial black holes and primordial magnetic fields).

If we could observe these other relics (spectra), we would gain a new handle on the origin of the matter / anti-matter asymmetry (more numbers) .

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Relic gravitational waves from the chiral plasma instability in the standard cosmological model

Axel Brandenburg,[1,2,3,4,†] Emma Clarke,[4,*] Tina Kahniashvili,[4,3,5,‡] Andrew J. Long,[6,§] and Guotong Sun[4,‖]

[1]*Nordita, KTH Royal Institute of Technology and Stockholm University, 10691 Stockholm, Sweden*
[2]*Department of Astronomy, AlbaNova University Center, Stockholm University, 10691 Stockholm, Sweden*
[3]*School of Natural Sciences and Medicine, Ilia State University, 0194 Tbilisi, Georgia*
[4]*McWilliams Center for Cosmology and Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*
[5]*Abastumani Astrophysical Observatory, Tbilisi, GE-0179, Georgia*
[6]*Department of Physics and Astronomy, Rice University, 6100 Main Street, Houston, Texas 77005, USA*

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In the primordial plasma, at temperatures above the scale of electroweak symmetry breaking, the presence of chiral asymmetries is expected to induce the development of helical hypermagnetic fields through the phenomenon of chiral plasma instability. It results in magnetohydrodynamic turbulence due to the high conductivity and low viscosity and sources gravitational waves that survive in the universe today as a stochastic polarized gravitational wave background. In this article, we show that this scenario only relies on Standard Model physics, and therefore the observable signatures, namely the relic magnetic field and gravitational background, are linked to a single parameter controlling the initial chiral asymmetry. We estimate the magnetic field and gravitational wave spectra, and validate these estimates with 3D numerical simulations.

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initial particle asymmetry

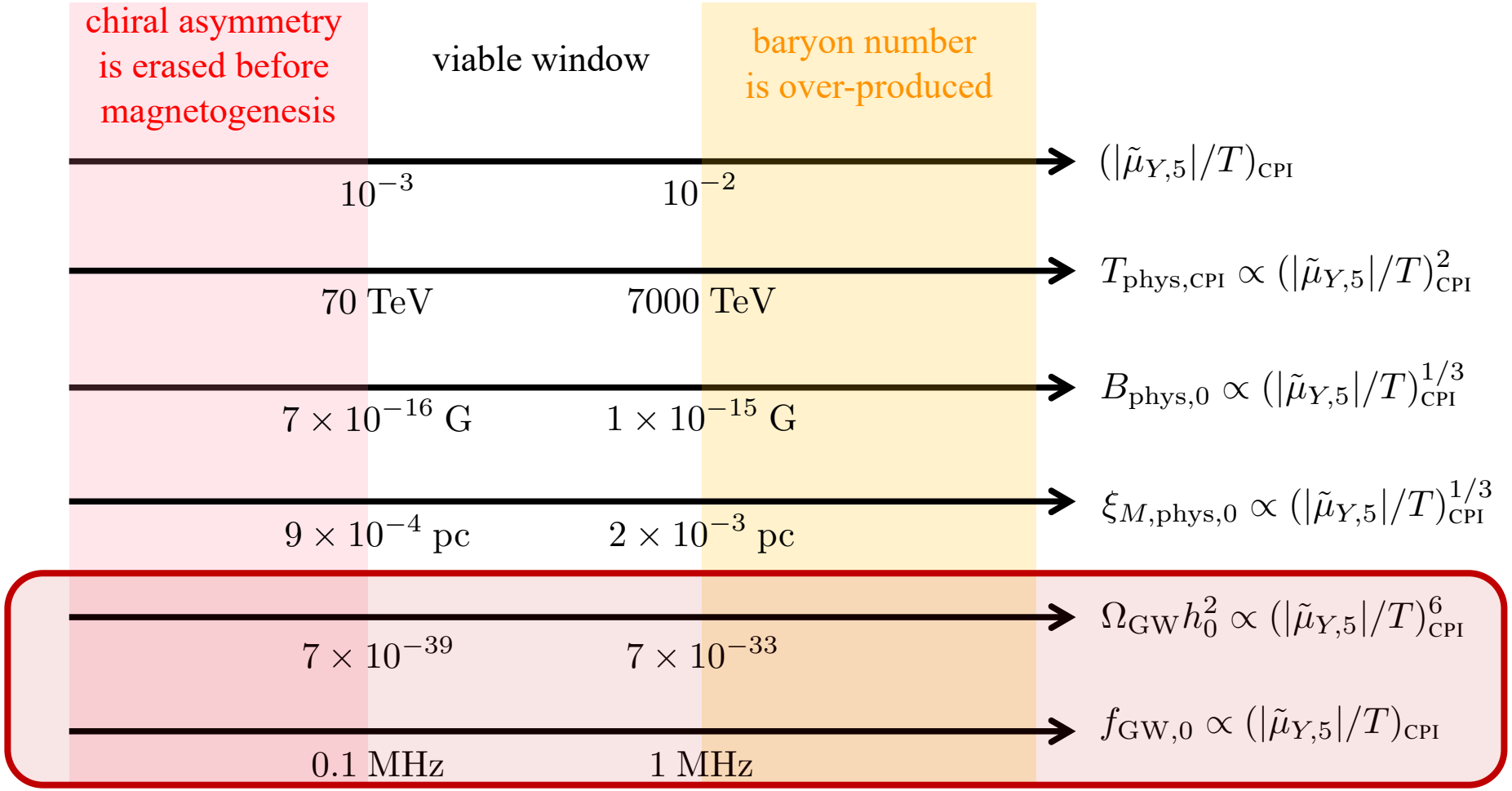
↓ chiral plasma instability

helical magnetic field

↓ Einstein's equation

chiral grav-wave

where I'm going with this:



very high-frequency and very weak – not detectable

the electroweak plasma
& particle asymmetries

The electroweak plasma

We're thinking about the primordial plasma before the electroweak epoch.

FRW spacetime (homo / iso)

$$(\mathrm{d}s)^2 = a(t)^2\left[(\mathrm{d}t)^2 - |\mathrm{d}\boldsymbol{x}|^2\right]$$

$$H(t) = \tfrac{1}{a(t)}\tfrac{\mathrm{d}}{\mathrm{d}t}a(t)$$

R-dominated

$$\mathcal{E}(t) = \tfrac{\pi^2}{30}\, g_{*E}(t)\, T(t)^4$$

comoving entropy constant

$$s(t) = \tfrac{2\pi^2}{45} g_{*S}(t) T(t)^3 = \tfrac{2\pi^2}{45} g_{*S,0} T_0^3$$

at the electroweak epoch:

$$T_{\mathrm{phys,ew}} \simeq 100\ \mathrm{GeV}$$

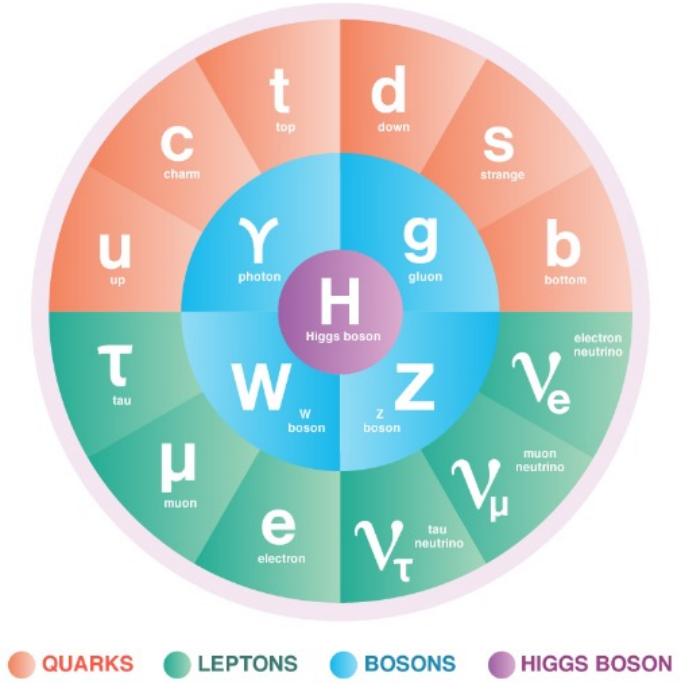
$$t_{\mathrm{phys,ew}} \simeq \left(2.3\times 10^{-11}\ \mathrm{sec}\right)\left(\tfrac{g_*}{106.75}\right)^{-1/2}$$

$$a_{\mathrm{ew}} \simeq \left(7.8\times 10^{-16}\ a_0\right)\left(\tfrac{g_*}{106.75}\right)^{-1/3}$$

$$d_{H,\mathrm{phys,ew}} \simeq \left(1.4\ \mathrm{cm}\right)\left(\tfrac{g_*}{106.75}\right)^{-1/2}$$

Particle content of the universe

The Standard Model of the Elementary Particles



electroweak unbroken phase

$$\langle \Phi \rangle = 0$$
$$\text{mass}(W^{\pm}, Z) = 0$$
$$\text{mass}(\text{quarks}, \text{leptons}) = 0$$

gauge theory

$$\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$$

field content

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (\mathbf{3}, \mathbf{2}, 1/3)$$
$$u_R \sim (\mathbf{3}, \mathbf{1}, 4/3)$$
$$d_R \sim (\mathbf{3}, \mathbf{1}, -2/3)$$
$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, -1)$$
$$e_R \sim (\mathbf{1}, \mathbf{1}, -2)$$
$$G \sim (\mathbf{8}, \mathbf{1}, 0)$$
$$W \sim (\mathbf{1}, \mathbf{3}, 0)$$
$$B \sim (\mathbf{1}, \mathbf{1}, 0)$$
$$\Phi \sim (\mathbf{1}, \mathbf{2}, 1)$$

Particle asymmetries

All of the SM particles interact (scatter, decay, inverse-decay) & maintain thermal equilibrium

momentum distribution

$$\mathrm{d}n_i(t) = g_i \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} f_i(\boldsymbol{p},t)$$

$$f_i(\boldsymbol{p},t) = \begin{cases} \left(\mathrm{e}^{[E_i(t)-\mu_i(t)]/T(t)} - 1\right)^{-1} \\ \left(\mathrm{e}^{[E_i(t)-\mu_i(t)]/T(t)} + 1\right)^{-1} \end{cases}$$

$$E_i(t) = \sqrt{|\boldsymbol{p}|^2 + a(t)^2 m_i^2}$$

number density (for small μ_i)

$$n_i(t) = g_i \begin{cases} \frac{\zeta(3)}{\pi^2} T^3 + \frac{1}{6}\mu_i T^2 + O(\mu_i^2 T) \\ \frac{3}{4}\frac{\zeta(3)}{\pi^2} T^3 + \frac{1}{12}\mu_i T^2 + O(\mu_i^2 T) \end{cases}$$

if pair annihilation & production are in thermal equilibrium:

$$\mu_i = -\mu_{\bar{i}}$$

particle asymmetries

$$\Delta n_i(t) = g_i \begin{cases} \frac{1}{3}\mu_i T^2 + O(\mu_i^2 T) \\ \frac{1}{6}\mu_i T^2 + O(\mu_i^2 T) \end{cases}$$

Standard Model Boltzmann Equations w/ Anomalous Sources

$$\frac{d\eta_{u_L^i}}{dx} = -\mathcal{S}^i_{\rm UDW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Uhu} + \mathcal{S}^{ij}_{\rm Uu} + \mathcal{S}^{ij}_{\rm Uhd}\right) - \mathcal{S}_{\rm s,sph} - \frac{N_{\rm c}}{2}\mathcal{S}_{\rm w,sph} + \left(N_{\rm c} y^2_{Q_L}\mathcal{S}^{\rm bkg}_{\rm y} + \frac{N_{\rm c}}{2}\mathcal{S}^{\rm bkg}_{\rm w} + N_{\rm c}\frac{y_{Q_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{d_L^i}}{dx} = \mathcal{S}^i_{\rm UDW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Dhd} + \mathcal{S}^{ij}_{\rm Dd} + \mathcal{S}^{ij}_{\rm Dhu}\right) - \mathcal{S}_{\rm s,sph} - \frac{N_{\rm c}}{2}\mathcal{S}_{\rm w,sph} + \left(N_{\rm c} y^2_{Q_L}\mathcal{S}^{\rm bkg}_{\rm y} + \frac{N_{\rm c}}{2}\mathcal{S}^{\rm bkg}_{\rm w} - N_{\rm c}\frac{y_{Q_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{\nu_L^i}}{dx} = -\mathcal{S}^i_{\nu\rm EW} - \sum_{j=1}^{N_{\rm g}}\mathcal{S}^{ij}_{\nu\rm he} - \frac{1}{2}\mathcal{S}_{\rm w,sph} + \left(y^2_{L_L}\mathcal{S}^{\rm bkg}_{\rm y} + \frac{1}{2}\mathcal{S}^{\rm bkg}_{\rm w} + \frac{y_{L_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{e_L^i}}{dx} = \mathcal{S}^i_{\nu\rm EW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Ehe} + \mathcal{S}^{ij}_{\rm Ee}\right) - \frac{1}{2}\mathcal{S}_{\rm w,sph} + \left(y^2_{L_L}\mathcal{S}^{\rm bkg}_{\rm y} + \frac{1}{2}\mathcal{S}^{\rm bkg}_{\rm w} - \frac{y_{L_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{u_R^i}}{dx} = \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ji}_{\rm Uhu} + \mathcal{S}^{ji}_{\rm Uu} + \mathcal{S}^{ji}_{\rm Dhu}\right) + \mathcal{S}_{\rm s,sph} - N_{\rm c} y^2_{u_R}\mathcal{S}^{\rm bkg}_{\rm y}$$

$$\frac{d\eta_{d_R^i}}{dx} = \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ji}_{\rm Dhd} + \mathcal{S}^{ji}_{\rm Dd} + \mathcal{S}^{ji}_{\rm Uhd}\right) + \mathcal{S}_{\rm s,sph} - N_{\rm c} y^2_{d_R}\mathcal{S}^{\rm bkg}_{\rm y}$$

$$\frac{d\eta_{e_R^i}}{dx} = \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ji}_{\rm Ehe} + \mathcal{S}^{ji}_{\rm Ee} + \mathcal{S}^{ji}_{\nu\rm he}\right) - y^2_{e_R}\mathcal{S}^{\rm bkg}_{\rm y}$$

$$\frac{d\eta_{\phi^+}}{dx} = -\left(\mathcal{S}_{\rm hhw} + \mathcal{S}_{\rm hw}\right) + \sum_{i,j=1}^{N_{\rm g}}\left(-\mathcal{S}^{ij}_{\rm Dhu} + \mathcal{S}^{ij}_{\rm Uhd} + \mathcal{S}^{ij}_{\nu\rm he}\right)$$

$$\frac{d\eta_{\phi^0}}{dx} = \mathcal{S}_{\rm hhw} - \mathcal{S}_{\rm h} + \sum_{i,j=1}^{N_{\rm g}}\left(-\mathcal{S}^{ij}_{\rm Uhu} + \mathcal{S}^{ij}_{\rm Dhd} + \mathcal{S}^{ij}_{\rm Ehe}\right)$$

$$\frac{d\eta_{W^+}}{dx} = \left(\mathcal{S}_{\rm hhw} + \mathcal{S}_{\rm hw}\right) + \sum_{i=1}^{N_{\rm g}}\left(\mathcal{S}^i_{\rm UDW} + \mathcal{S}^i_{\nu\rm EW}\right) .$$

$$\mathcal{S}^{ij}_{\rm Dhu} \equiv \frac{\gamma^{ij}_{\rm Dhu}}{2}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} + \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) \quad , \quad \mathcal{S}^{ij}_{\rm Uhu} \equiv \frac{\gamma^{ij}_{\rm Uhu}}{2}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Uhd} \equiv \frac{\gamma^{ij}_{\rm Uhd}}{2}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) \quad , \quad \mathcal{S}^{ij}_{\rm Dhd} \equiv \frac{\gamma^{ij}_{\rm Dhd}}{2}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\nu\rm he} \equiv \frac{\gamma^{ij}_{\nu\rm he}}{2}\left(\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} - \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) \quad , \quad \mathcal{S}^{ij}_{\rm Ehe} \equiv \frac{\gamma^{ij}_{\rm Ehe}}{2}\left(\frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) ,$$

$$\mathcal{S}^i_{\rm UDW} \equiv \gamma^i_{\rm UDW}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}^i_{\nu\rm EW} \equiv \gamma^i_{\nu\rm EW}\left(\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} - \frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm hhw} \equiv \gamma_{\rm hhw}\left(\frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm s,sph} \equiv \gamma_{\rm s,sph}\sum_{i=1}^{N_{\rm g}}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{u_R^i}}{k_{u_R^i}} - \frac{\eta_{d_R^i}}{k_{d_R^i}}\right) ,$$

$$\mathcal{S}_{\rm w,sph} \equiv \gamma_{\rm w,sph}\sum_{i=1}^{N_{\rm g}}\left(\frac{N_{\rm c}}{2}\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{N_{\rm c}}{2}\frac{\eta_{d_L^i}}{k_{d_L^i}} + \frac{1}{2}\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} + \frac{1}{2}\frac{\eta_{e_L^i}}{k_{e_L^i}}\right)$$

$$\mathcal{S}^{\rm bkg}_{\rm y} = \frac{1}{sT}\frac{\alpha_{\rm y}}{4\pi}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\langle Y_{\mu\nu}\rangle\langle Y_{\rho\sigma}\rangle}$$

$$\mathcal{S}^{\rm bkg}_{\rm w} = \frac{1}{sT}\frac{1}{2}\frac{\alpha_{\rm w}}{4\pi}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\langle W^a_{\mu\nu}\rangle\langle W^a_{\rho\sigma}\rangle}$$

$$\mathcal{S}^{\rm bkg}_{\rm yw} = \frac{1}{sT}\frac{gg'/4\pi}{4\pi}\epsilon^{\mu\nu\rho\sigma}\overline{\langle Y_{\mu\nu}\rangle\langle W^3_{\rho\sigma}\rangle} .$$

$$\mathcal{S}^{ij}_{\rm Uu} \equiv \gamma^{ij}_{\rm Uu}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Dd} \equiv \gamma^{ij}_{\rm Dd}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Ee} \equiv \gamma^{ij}_{\rm Ee}\left(\frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) ,$$

$$\mathcal{S}_{\rm hw} \equiv \gamma_{\rm hw}\left(\frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm h} \equiv \gamma_{\rm h}\frac{\eta_{\phi^0}}{k_{\phi^0}} .$$

$\eta = n/s$

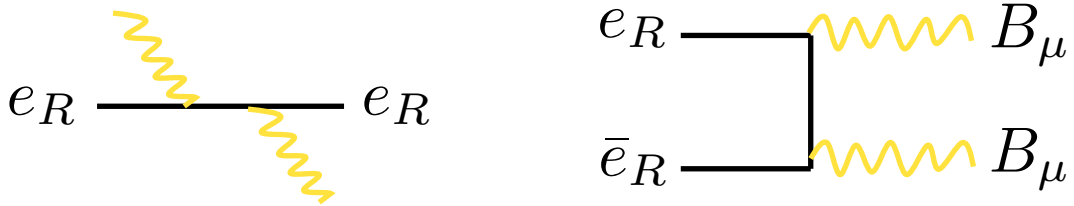
$x = T/H \sim M_{\rm pl}/T$

$k =$ # degree of freedom

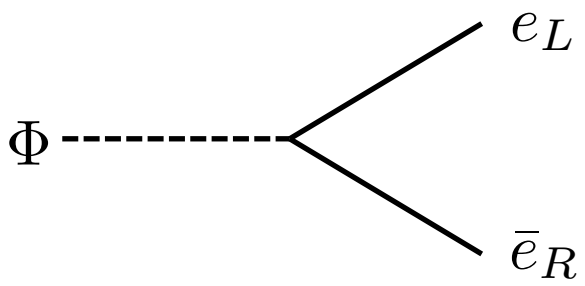
The special role of e_R

How does the asymmetry between e_R and $e\text{bar}_R$ evolve?

The hypercharge interaction does not change the e_R asymmetry



But the Yukawa interactions do create / annihilate e_R



Can we "store" a chiral asymmetry in e_R?

Chiral charge erasure

rate comes from: Bodeker & Schroder (2019)
see also: Cambbell, Davidson, Ellis, & Olive (1993)
Joyce & Shaposhnikov (1997)

The approximately conserved charges tend to be erased over time.

The limiting factor is the right-chiral electron, since it has $y_e \sim 3 * 10^{-6}$.

chiral charge evolution

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_{e_R}(t) = -\Gamma_{\mathrm{f}}(t)\,\mu_{e_R}(t)$$

interaction rate

$$\Gamma_{\mathrm{f}}(t) \sim \left(1.3 \times 10^{-2}\right) y_e^2 T(t)$$

an exponential decay:

$$\mu_{e_R}(t) = \mu_{e_R,0}\,\exp\left[-\int_{t_0}^{t} \mathrm{d}t'\,\Gamma_f(t')\right]$$

chiral-erasure epoch

$$T_{\mathrm{phys,f}} \simeq (80\ \mathrm{TeV})\left(\frac{g_*}{106.75}\right)^{-1/2}$$

$$t_{\mathrm{phys,f}} \simeq (3.7 \times 10^{-17}\ \mathrm{sec})\left(\frac{g_*}{106.75}\right)^{1/2}$$

$$a_{\mathrm{f}} \simeq (9.7 \times 10^{-19}\ a_0)\left(\frac{g_*}{106.75}\right)^{1/6}$$

$$d_{H,\mathrm{phys,f}} \simeq (2.2 \times 10^{-6}\ \mathrm{cm})\left(\frac{g_*}{106.75}\right)^{1/2}$$

key message thus far

There might be a primordial chiral asymmetry (e.g., $e_R > ebar_R$).

Yukawa interactions will erase the asymmetry once
the plasma temperature drops to ~ 80 TeV.

If we want to do something fun with the chiral asymmetry,
we need to finish before T = 80 TeV
(or continue to source the asymmetry later)

magnetic field evolution
& chiral magnetic effect

A hypermagnetic field

mag screening in QCD: Gross, Pisarski, & Yaffe (1981)
see also: Arnold, Son, & Yaffe (1996)

Where can we add magnetic fields to our primordial plasma?
(for T > 100 GeV electroweak unbroken phase)

(1) electromagnetism is in here: $A_\mu = \cos\theta_W \, B_\mu + \sin\theta_W \, W^3_\mu$

gauge interactions: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

(2) but non-Abelian magnetic fields are screened at finite temperature

$$m_{\rm mag} \sim g^2 T$$

(3) so the only place for long-range coherent magnetic fields is here

$$\boldsymbol{B}_Y(\eta, \boldsymbol{x}) \quad \text{and} \quad \boldsymbol{E}_Y(\eta, \boldsymbol{x})$$

Evolution of hypermagnetic fields

conductivity: Arnold, Moore, & Yaffe (2000)

How do hypermagnetic fields evolve in the primordial plasma?

hyper-Maxwell equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{E}_Y = \rho_Y$$
$$\boldsymbol{\nabla} \times \boldsymbol{E}_Y = -\frac{\partial}{\partial t} \boldsymbol{B}_Y$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{B}_Y = 0$$
$$\boldsymbol{\nabla} \times \boldsymbol{B}_Y = \boldsymbol{J}_Y + \frac{\partial}{\partial t} \boldsymbol{E}_Y$$

continuity equation

$$\frac{\partial}{\partial \eta} \rho_Y + \boldsymbol{\nabla} \cdot \boldsymbol{J}_Y = 0$$

constitutive relation

$$\boldsymbol{J}_Y = \underbrace{\sigma_Y \left(\boldsymbol{E}_Y + \boldsymbol{v} \times \boldsymbol{B}_Y \right)}_{\text{Ohm's Law}} + \underbrace{\frac{2}{\pi} \alpha_Y \mu_{Y,5} \boldsymbol{B}_Y}_{\text{chiral magnetic effect (CME)}}$$

fine structure constant

$$\alpha_Y = g'^2/4\pi \approx 1/100$$

conductivity

$$\sigma_Y \sim T/\alpha_Y \approx 100T$$

magnetic diffusivity

$$\eta_Y = 1/\sigma_Y \approx 0.01 T^{-1}$$

Chiral magnetic effect

Vilenkin (1980), Fukushima, Kharzeev, & Warringa (2008), Alekseev, Cheianov, & Frohlich (1998) Boyarsky, Frohlich, & Ruchaiskiy (2011)

First studied for a relativistic electron-positron plasma in QED.

CME = in the presence of a chiral asymmetry, a magnetic field induces an electric current.

From the symmetry perspective, the CME could be correct.

$$\boldsymbol{J} \propto \mu_5 \boldsymbol{B}$$

mass dimensions: ✓

parity properties: ✓

$\boldsymbol{J}$ = vector = P-odd

$\boldsymbol{B}$ = axial-vector = P-even

m_e = scalar = P-even

μ_5 = pseudo-scalar = P-odd

$\mu_5 \propto [\#\text{ R-chiral}] - [\#\text{ L-chiral}]$

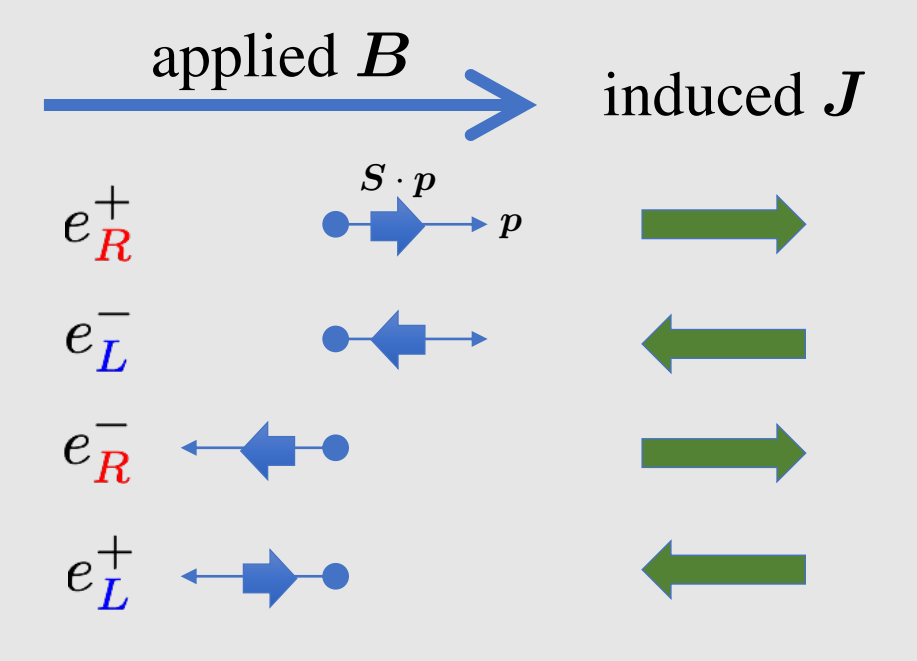
Semiclassical understanding

Tashiro, Vachaspati, & Vilenkin (2012)

Consider, applying a B-field to a collection of (relativistic) electrons & positrons.

The magnetic dipole moment μ ~ qS wants to align with B … the Hamiltonian is: $H = -\boldsymbol{\mu} \cdot \boldsymbol{B}$

without chiral asymmetry



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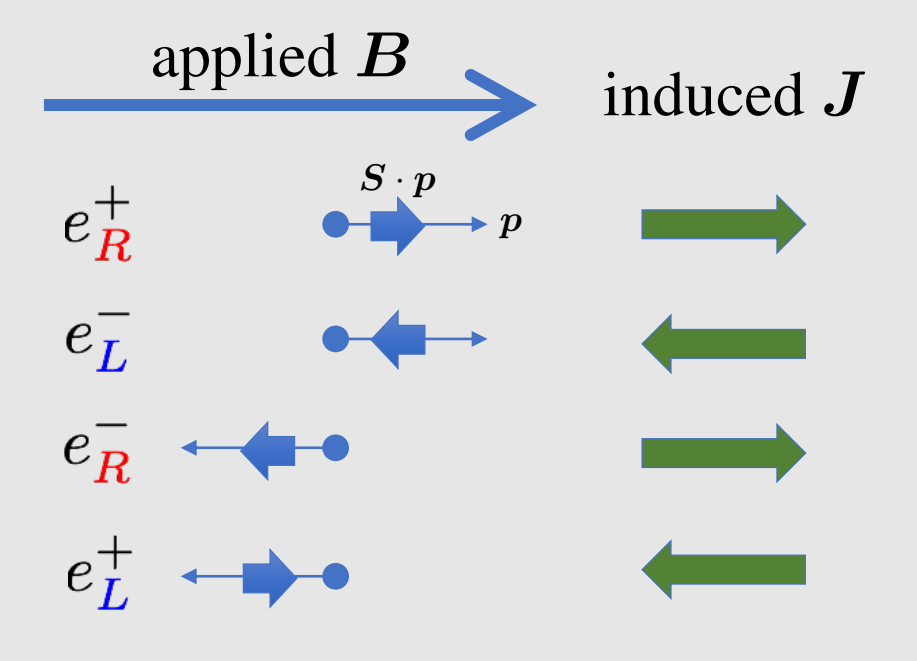
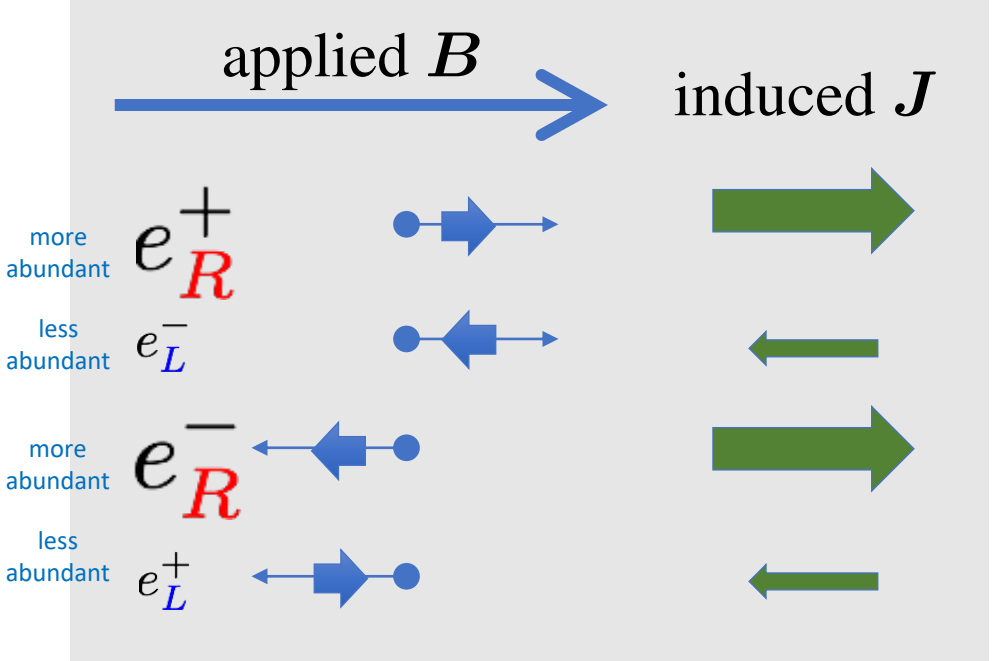


with chiral asymmetry



implies: $\boldsymbol{J} \propto q^2 \mu_5 \boldsymbol{B}$

CME recap

CME = in the presence of a chiral asymmetry, a magnetic field induces an electric current.

for electromagnetism: $\boldsymbol{J} = \frac{2}{\pi}\alpha\mu_5\boldsymbol{B}$

for hypercharge:

$$\boldsymbol{J}_Y = \frac{2}{\pi}\alpha_Y\mu_{Y,5}\boldsymbol{B}_Y$$

$$\mu_{Y,5}(t) = \sum_{\text{particles}} g_i\varepsilon_i Y_i^2\mu_i(t)$$

$\varepsilon_i = \pm 1$ for R- and L-chiral particles

$Y_i =$ hypercharge of particle species i

magnetogenesis
via chiral plasma instability

Linearized field equations

similar equations of motion to axion inflation

Combining the hyper-Maxwell equations (and neglecting velocity v)

$$0 = \dot{\boldsymbol{B}}_Y - \eta_Y \nabla^2 \boldsymbol{B}_Y - \tfrac{2}{\pi} \alpha_Y \eta_Y \mu_{Y,5} \boldsymbol{\nabla} \times \boldsymbol{B}_Y$$

Move to Fourier space (and assume homogenous μ_{Y5}).

Circular polarization modes decouple

$$0 = \dot{B}^{(R)}_{Y,k} + \eta_Y \left(k^2 - \mathrm{sgn}(\mu_{Y,5})\, k_{\mathrm{CPI}} k \right) B^{(R)}_{Y,k}$$
$$0 = \dot{B}^{(L)}_{Y,k} + \eta_Y \left(k^2 + \mathrm{sgn}(\mu_{Y,5})\, k_{\mathrm{CPI}} k \right) B^{(L)}_{Y,k}$$

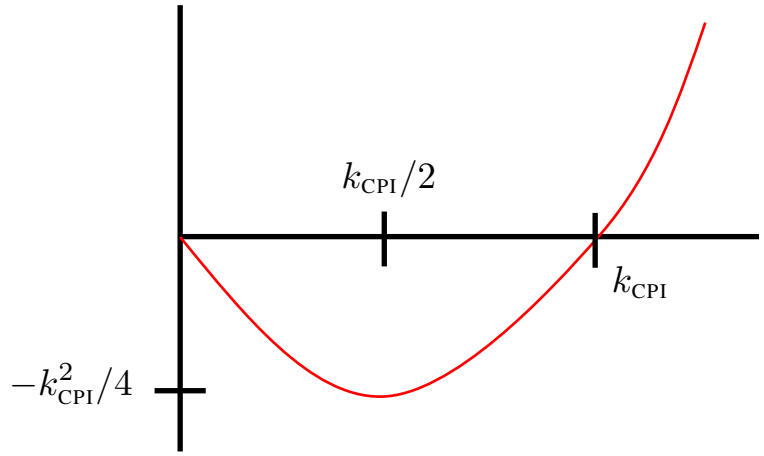
where $k_{\mathrm{CPI}} = \tfrac{2}{\pi} \alpha_Y |\mu_{Y,5}|$

Depending on the sign of μ_{Y5}, one mode or the other can be unstable.

Chiral plasma instability

Explore the dependence on wavenumber

$$0 = \dot{B}^{(R)}_{Y,k} + \eta_Y \left(k^2 - k_{\rm CPI} k \right) B^{(R)}_{Y,k}$$



- modes with $k > k_{\rm CPI}$ are decaying (Ohmic dissipation)
- modes with $k < k_{\rm CPI}$ have a tachyonic instability
- modes with $k = k_{\rm CPI}/2$ grow fastest

$$B^{(R)}_{Y,k}(t) \propto \exp\left[\tfrac{1}{4} \eta_Y k^2_{\rm CPI} t \right]$$

- typically $k_{\rm CPI}$ is deep inside the horizon

CPI = tachyonic growth of a helical magnetic field due to the CME

Helicity saturation

Since either the R- or L-polarized modes grow, the resultant magnetic field is helical.

As the helicity grows, the chiral asymmetry is depleted:

$$\partial_\mu j^\mu_{Y,5} \sim -\frac{g'^2}{16\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu} \quad \Rightarrow \quad \frac{\partial}{\partial t} n_{Y,5} \sim -\frac{1}{\pi}\alpha_Y \frac{\partial}{\partial t}\mathcal{H}_M$$

The exponential growth must saturate when the initial asymmetry is `used up'

helicity density: $\mathcal{H}_{M,\mathrm{CPI}} \sim \frac{\pi}{6}\frac{1}{\alpha_Y}|\mu_{Y,5}|T^2_{\mathrm{CPI}}$

coherence length: $\xi_{M,\mathrm{CPI}} \approx 2\pi/(k_{\mathrm{CPI}}/2)$

field strength: $B_{\mathrm{CPI}} \approx \sqrt{\mathcal{H}_{M,\mathrm{CPI}}/\xi_{M,\mathrm{CPI}}}$

(We will validate these estimates using MHD simulations.)

using
$n_{Y,5} = \frac{1}{6}\mu_{Y,5}T^2$
$Y\tilde{Y} = -4\boldsymbol{E}_Y \cdot \boldsymbol{B}_Y = 4\frac{\partial}{\partial t}(\boldsymbol{A}_Y \cdot \boldsymbol{B}_Y) + \cdots$

Magnetogenesis

Typical scales

$$a_{\rm CPI} \simeq \left(1.1\times10^{-18} a_0\right)\left(\frac{g_*}{106.75}\right)^{1/6}\left(\frac{\eta_Y}{0.01T^{-1}}\right)^{-1}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-2}$$

$$T_{\rm phys,CPI} \simeq \left(7.2\times10^{4}\ {\rm GeV}\right)\left(\frac{g_*}{106.75}\right)^{-1/2}\left(\frac{\eta_Y}{0.01T^{-1}}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{2}$$

$$d_{H,\rm CPI} \simeq \left(2.5\times10^{12}\ {\rm cm}\, a_0^{-1}\right)\left(\frac{g_*}{106.75}\right)^{1/3}\left(\frac{\eta_Y}{0.01T^{-1}}\right)^{-1}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-2}$$

$$\xi_{M,\rm CPI} \simeq \left(5.0\times10^{5}\ {\rm cm}\, a_0^{-1}\right)\left(\frac{g_*}{106.75}\right)^{1/3}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-1}$$

$$B_{\rm CPI} \simeq \left(5.0\times10^{-11}\ {\rm G}\, a_0^{2}\right)\left(\frac{g_*}{106.75}\right)^{-2/3}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)$$

$$\Omega_{B,\rm CPI} \simeq \left(3.8\times10^{-10}\right)\left(\frac{g_*}{106.75}\right)^{-1}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{2}$$

If $\mu_{Y5}/T >\sim 10^{-3}$ then the CPI develops before chiral charge erasure.

Inverse cascade

A maximally-helical field co-evolves with the plasma subject to the turbulent inverse-cascade:

$$B_0 = B_{\rm CPI}\left(\frac{t_{\rm rec}}{t_{\rm CPI}}\right)^{-1/3} \quad \text{and} \quad \xi_{B,0} = \xi_{B,\rm CPI}\left(\frac{t_{\rm rec}}{t_{\rm CPI}}\right)^{2/3} \quad \text{so} \quad \mathcal{H}_{B,0} = \mathcal{H}_{B,\rm CPI}$$

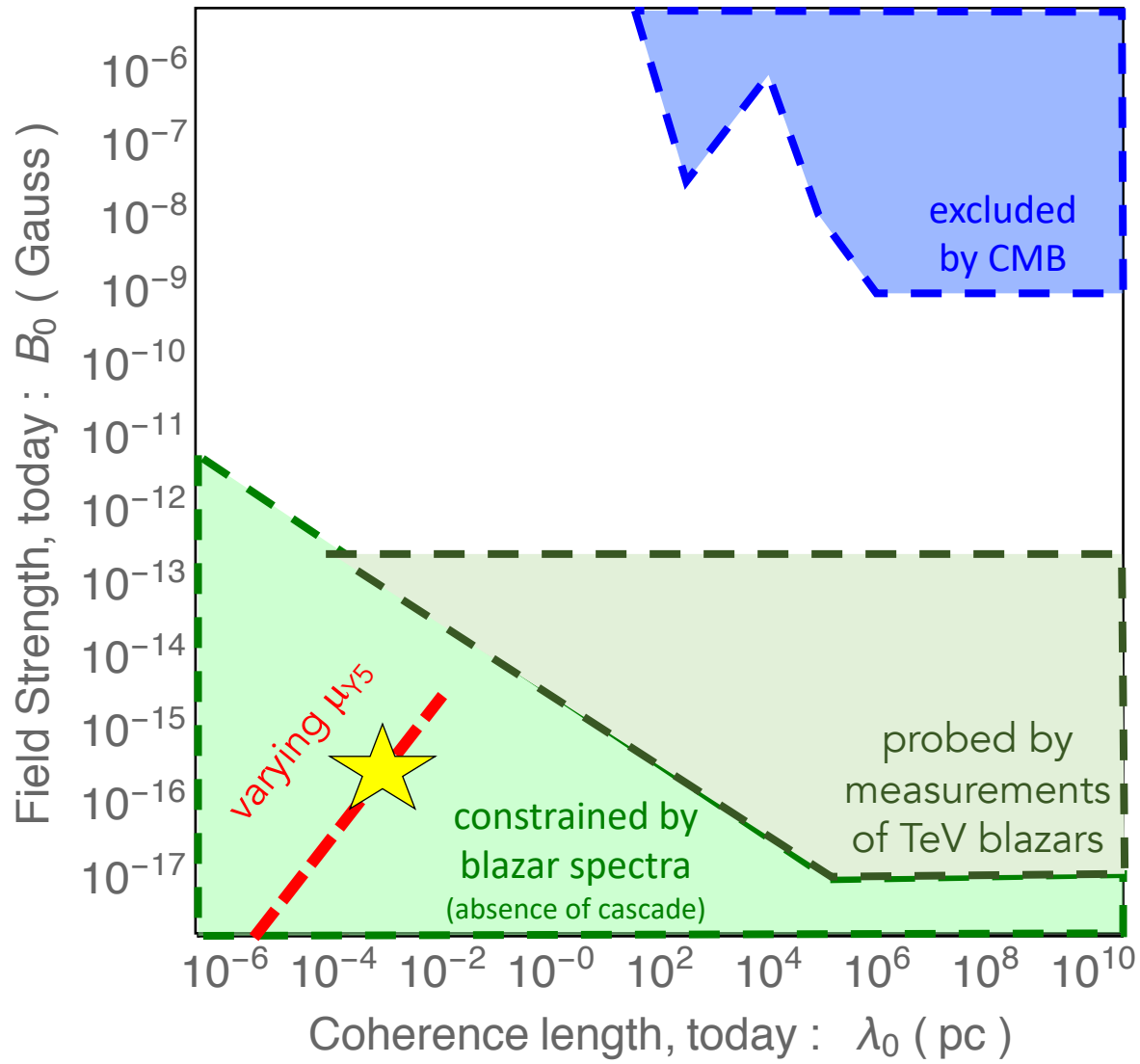
The field today is expected to be

$$\xi_{M,\rm phys,0} \simeq (9.5\times10^{-4}\ {\rm pc})\left(\tfrac{g_*}{106.75}\right)^{1/9}\left(\tfrac{\eta_Y}{0.01T^{-1}}\right)^{2/3}\left(\tfrac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

$$B_{\rm phys,0} \simeq (6.6\times10^{-16}\ {\rm G})\left(\tfrac{g_*}{106.75}\right)^{-5/9}\left(\tfrac{\eta_Y}{0.01T^{-1}}\right)^{-1/3}\left(\tfrac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

IGMF parameter space

adapted from: Durrer & Neronov (2013)



The relic of the
primordial magnetic field
is an intergalactic
magnetic field today

$$\xi_{M,\mathrm{phys},0} \simeq \left(9.5 \times 10^{-4}\ \mathrm{pc}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

$$B_{\mathrm{phys},0} \simeq \left(6.6 \times 10^{-16}\ \mathrm{G}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

- too weak to explain blazars
- too weak to seed galactic B-field

grav-wave generation
& stochastic GW background

Einstein's equation

for details: Roper Pol, Brandenburg, Kahniashvili, Kosowsky, & Mandal (2018)

The primordial magnetic field sources gravitational wave radiation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\partial_t^2 h_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}$$

$$T_{ij} \sim B_i B_j \quad \text{(transverse / traceless part)}$$

$$\mathcal{E}_{\rm gw} = \langle \partial_t h_{ij} \partial_t h_{ij} \rangle / (32\pi G)$$

parametrically:

$$h \sim (GB^2)/\xi^2$$

$$\mathcal{E}_{\rm gw} \sim \xi^2 h^2 / G \sim G\xi^2 B^4$$

GW estimates

At the time of production

$$f_{\rm gw,CPI} \approx 2/\xi_{M,\rm CPI}$$
$$\mathcal{E}_{\rm gw,CPI} \sim (G/2\pi) a_{\rm CPI}^{-2} \xi_{M,\rm CPI}^2 B_{\rm CPI}^4$$

there's a 2 because the source is quadratic in the field

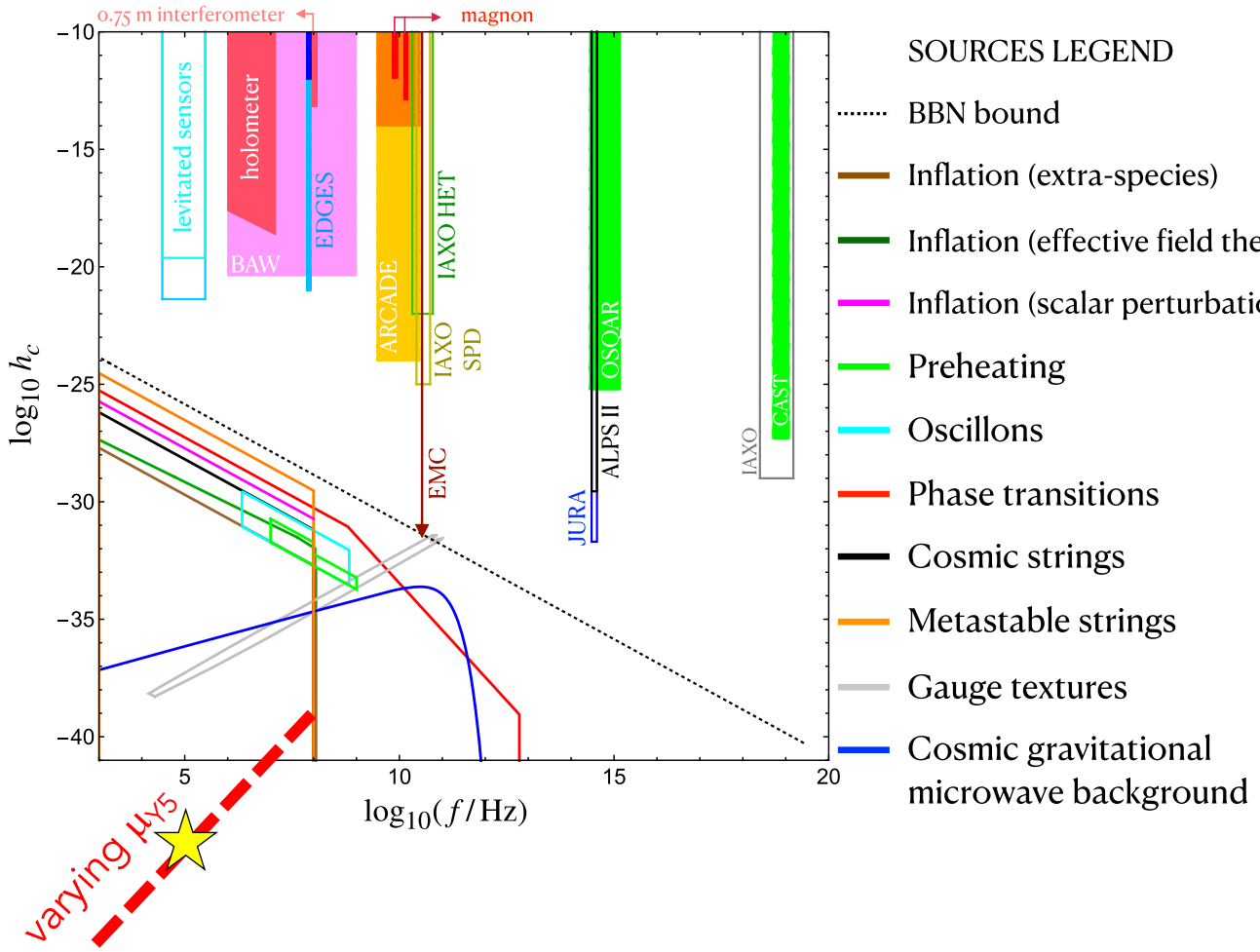
Frequency and cosmological energy fraction of stochastic GW today:

$$f_{\rm gw,phys,0} = (1\times 10^5\ {\rm Hz}) \left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)$$
$$(\Omega_{\rm gw} h^2)_0 = (7\times 10^{-39}) \left(\frac{\eta_Y}{0.01 T^{-1}}\right)^2 \left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^6$$

The GW signal is high-frequency and very low amplitude (even for $\mu_{Y5}/T = 1$)

Searches for high-freq grav waves

review on high-freq grav wave: Aggarwal et al (2021)



$$f_{\mathrm{gw},0} = \left(1 \times 10^5 \text{ Hz}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)$$

$$(\Omega_{\mathrm{gw}} h^2)_0 = \left(7 \times 10^{-39}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^6$$

$$h_c = \left(1 \times 10^{-45}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^2$$

using

$$\Omega_{\mathrm{gw}} = \frac{\pi f^2 h_c^2}{4G\mathcal{E}_{\mathrm{cr}}}$$

numerical validation with PENCIL CODE sims

MHD equations + CME

magnetic diffusivity η
kinematic viscosity ν
chiral diffusion coeff. D_5
rescaled chem. pot. μ_5
chiral depletion param. λ

Fully nonlinear system of equations

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B} + \eta(\mu_5 \boldsymbol{B} - \boldsymbol{J}), \tag{5}$$

$$\frac{\partial \mu_5}{\partial t} = -\boldsymbol{\nabla} \cdot (\mu_5 \boldsymbol{u}) - \lambda\eta(\mu_5 \boldsymbol{B} - \boldsymbol{J}) \cdot \boldsymbol{B} + D_5 \nabla^2 \mu_5, \tag{6}$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = \frac{2}{\rho}\boldsymbol{\nabla} \cdot (\rho\nu\mathsf{S}) - \frac{1}{4}\boldsymbol{\nabla} \ln \rho + \frac{\boldsymbol{u}}{3}(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right] + \frac{3}{4\rho}\boldsymbol{J} \times \boldsymbol{B}, \tag{7}$$

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho) + \frac{1}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right], \tag{8}$$

$$\mathsf{S}_{ij} = (\partial_j u_i + \partial_i u_j)/2 - \delta_{ij}\boldsymbol{\nabla} \cdot \boldsymbol{u}/3$$

$$\frac{\partial^2}{\partial t^2}\tilde{h}_{+/\times}(\boldsymbol{k}, t) + k^2 \tilde{h}_{+/\times}(\boldsymbol{k}, t) = \frac{6\, H_*}{t\, \mathcal{E}_{\rm cr}}\tilde{T}_{+/\times}(\boldsymbol{k}, t),$$

PENCIL CODE
parallelized
6th order finite differences
3rd order time-stepping
1024^3 mesh points
initially: weak seed field, vanishing plasma velocity, homogenous chem. pot.

Parameters

cf, Brandenburg, He, Kahniashivili, Rheinhardt, & Schober (2021)

Dynamic range issues prevent studying the "expected" parameters.

Run	ηH_*	$(\mathcal{E}_{\rm cr}\lambda)^{1/2}/H_*$	μ_{50}/H_*	v_μ	v_λ	$\eta\mu_{50}^2/H_*$	k_1/H_*	$\mathcal{E}_{\rm M}^{\rm max}/\mathcal{E}_{\rm cr}$	$\mathcal{E}_{\rm GW}^{\rm sat}/\mathcal{E}_{\rm cr}$	q
B1	1×10^{-6}	2×10^{4}	10^{4}	1×10^{-2}	5×10^{-1}	1×10^{2}	1×10^{2}	1.6×10^{-2}	4.7×10^{-12}	0.027
B10	1×10^{-3}	2×10^{4}	10^{4}	1×10^{1}	5×10^{-1}	1×10^{5}	1×10^{2}	6.0×10^{-2}	6.0×10^{-9}	12
A1	1×10^{-6}	5×10^{4}	10^{4}	1×10^{-2}	2×10^{-1}	1×10^{2}	1×10^{2}	4.6×10^{-3}	8.9×10^{-14}	0.032
A12	5×10^{-3}	5×10^{4}	10^{4}	5×10^{1}	2×10^{-1}	5×10^{5}	5×10^{1}	9.2×10^{-3}	3.0×10^{-10}	18
X1	5×10^{-8}	10^{10}	10^{6}	5×10^{-2}	1×10^{-4}	5×10^{4}	5×10^{3}	2.4×10^{-9}	8.8×10^{-31}	0.39
X2	5×10^{-9}	10^{10}	10^{6}	5×10^{-3}	1×10^{-4}	5×10^{3}	5×10^{3}	2.4×10^{-9}	1.6×10^{-30}	0.53
X3	5×10^{-10}	10^{10}	10^{6}	5×10^{-4}	1×10^{-4}	5×10^{2}	5×10^{3}	2.4×10^{-9}	1.1×10^{-30}	0.44
X4	5×10^{-11}	10^{10}	10^{6}	5×10^{-5}	1×10^{-4}	5×10^{1}	5×10^{3}	2.3×10^{-9}	3.1×10^{-31}	0.12
Y1	5×10^{-8}	7×10^{11}	10^{6}	5×10^{-2}	1×10^{-6}	5×10^{4}	5×10^{3}	4.9×10^{-13}	3.6×10^{-38}	0.39
Y2	5×10^{-8}	7×10^{11}	10^{6}	5×10^{-2}	1×10^{-6}	5×10^{4}	2×10^{3}	4.4×10^{-13}	3.2×10^{-37}	1.3
Y3	5×10^{-8}	7×10^{11}	10^{6}	5×10^{-2}	1×10^{-6}	5×10^{4}	1×10^{3}	3.3×10^{-13}	6.9×10^{-37}	2.5
expected	$10^{-15}\eta_2$	6×10^{12}	$5\times10^{7}\mu_3$	$6\times10^{-8}\eta_2\mu_3$	$8\times10^{-6}\mu_3$	$3\eta_2\mu_3^2$	—	$6\times10^{-15}\mu_3^2$	$7\times10^{-39}\eta_2^2\mu_3^6$	—

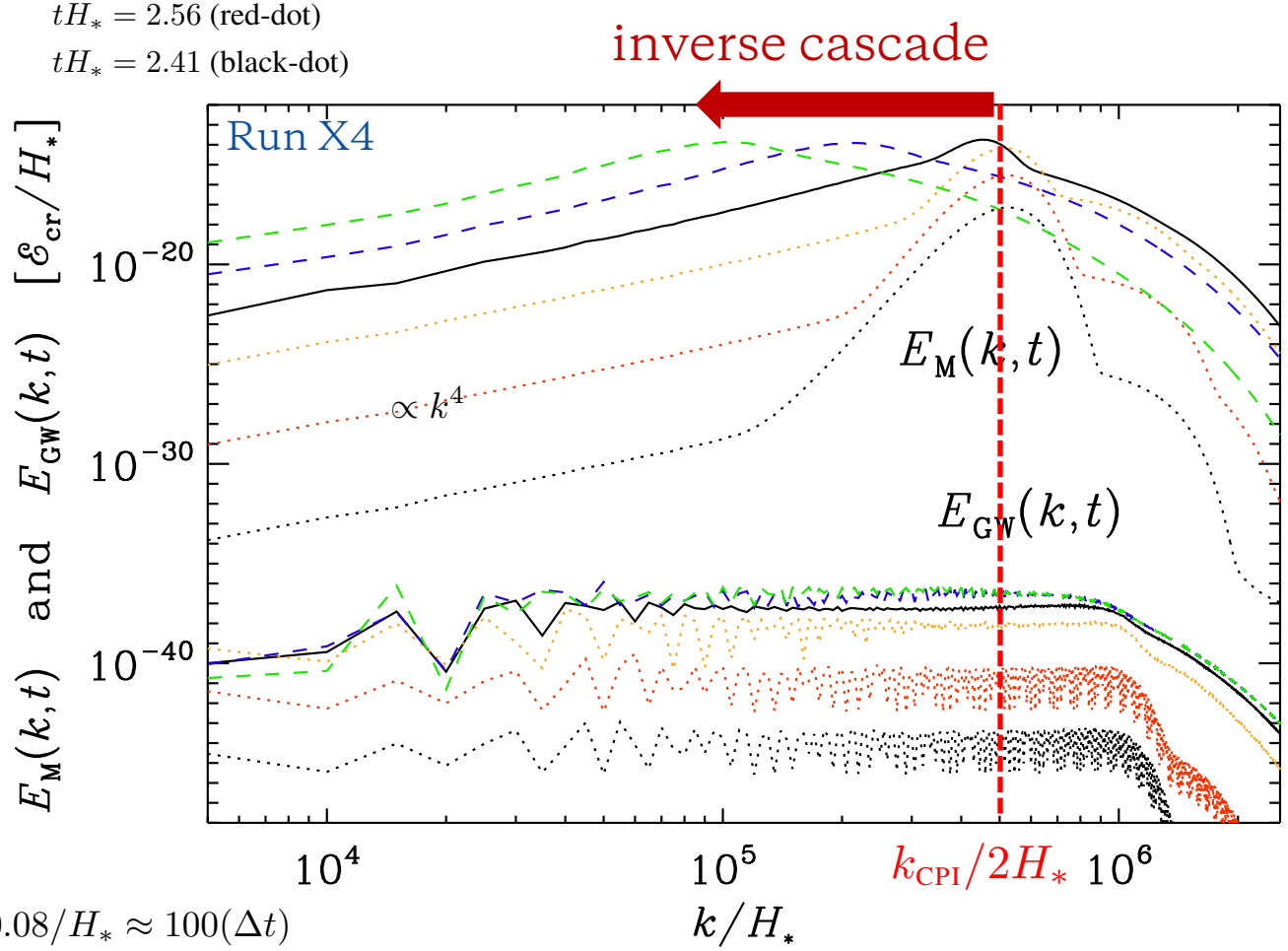
$$v_\mu = \mu_{50}\eta$$
$$v_\lambda = \mu_{50}/(\mathcal{E}_{\rm cr}\lambda)^{1/2}$$

$$\mu_{50} = 2\alpha_Y\tilde{\mu}_{Y,5}/\pi$$
$$\lambda = 192\alpha_Y^2/T^2$$
$$D_5 = \nu = \eta = 1/\sigma$$

$$\eta_2 = \eta_Y/(0.01T^{-1})$$
$$\mu_3 = \tilde{\mu}_{Y,5}/(10^{-3}T)$$

Spectra for PMF and GWs

$tH_* = 5.37$ (green-dashed)
$tH_* = 3.66$ (blue-dashed)
$tH_* = 2.98$ (solid)
$tH_* = 2.71$ (orange-dot)
$tH_* = 2.56$ (red-dot)
$tH_* = 2.41$ (black-dot)



$t_{CPI} = 0.08/H_* \approx 100(\Delta t)$

NB : $\mathcal{E}_M(\boldsymbol{x},t) = \frac{1}{2}\langle|\boldsymbol{B}(\boldsymbol{x},t)|^2\rangle = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} P_B(\boldsymbol{k}) = \int \frac{\mathrm{d}k}{k} \underbrace{\frac{k^3}{2\pi^2} P_B(k)}_{=\mathrm{d}\mathcal{E}_M/\mathrm{d}\ln k} = \int \mathrm{d}k\, E_M(k)$

- mag energy grows at $k_{CPI}/2$
- GW energy grows while mag energy grows
- mag energy reaches maximum and GW energy spectrum saturates (solid curves)
- subsequently mag energy evolves with the inverse cascade scaling (expected for helical):

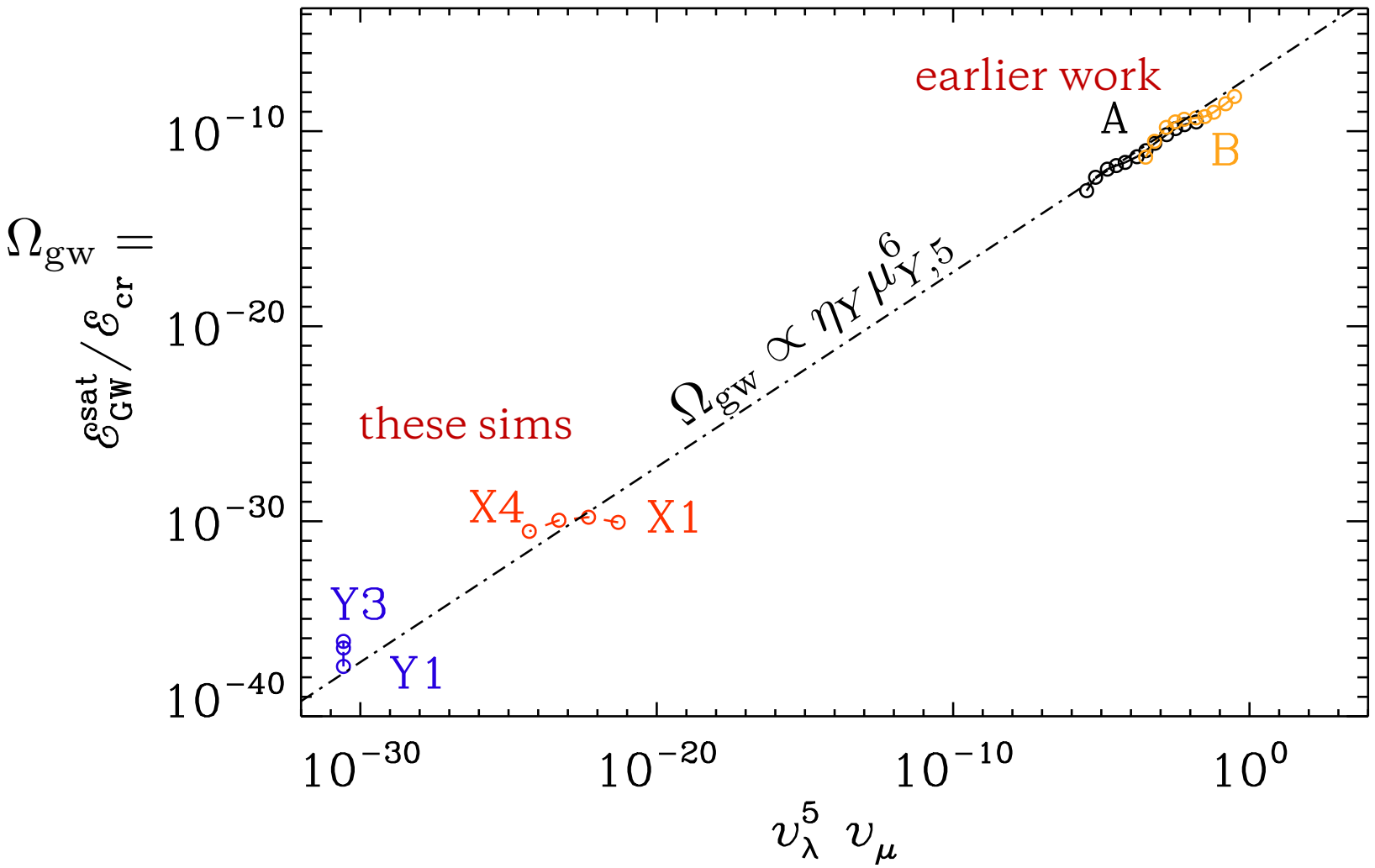
$$B \propto t^{-1/3} \quad \text{so} \quad \mathcal{E}_M \propto t^{-2/3}$$
$$\xi_M \propto t^{2/3} \quad \text{so} \quad k_M \propto t^{-2/3}$$
$$E_M \sim \mathcal{E}_M/k_M \propto t^0$$

- resultant GW energy spectrum is blue-tilted

$$\frac{\mathrm{d}\mathcal{E}_{\mathrm{gw}}}{\mathrm{d}\ln k} = kE_{\mathrm{GW}} \propto k^1 \quad (k \lesssim k_{\mathrm{CPI}})$$

  - (scaling should extend down to k_{IR}~H)
  - (most energy at k_{UV}~k_{CPI})

Comparision w/ previous studies



Earlier work studied GW from CME at the EW epoch, assuming no chiral asymmetry washout (see Run Series A & B)

We explore parameters that are closer to the expectation for early universe cosmology at T > 80 TeV (see Run Series X and Y)

Main difference is the choice of magnetic diffusivity: η_Y. (see the table on prev. slide)

For those parameters, the GW signal is found to be weaker.

Scaling with $v_\lambda{}^5 \, v_\mu$ from earlier work, confirmed over wider range.

cf., Kohei & Kyohei's talks

B-number overproduction
& an upper bound on μ_{Y5}

Sourcing baryon number

Fujita & Kamada (2016), Kamada & AL (2016a,b), Jimenez, Kamada, Schmitz,Xu (2017)
see also: Giovannini & Shaposhnikov
(also gravitational anomaly: cf, Evangelos's talk)

Varying hypermagnetic helicity sources B and L-number

$$\partial_\mu j^\mu_{\mathsf{B}} = \partial_\mu j^\mu_{\mathsf{L}} = 3\frac{g^2}{16\pi^2} W^a \tilde{W}^a - 3\frac{g'^2}{16\pi^2} Y\tilde{Y}$$

$$\dot{n}_{\mathsf{B}} = \dot{n}_{\mathsf{L}} = -3\frac{g'^2}{4\pi^2}\dot{\mathcal{H}}_Y + \cdots$$

At the EW crossover, there is a change in helicity when the B_Y field converts to B_{em}

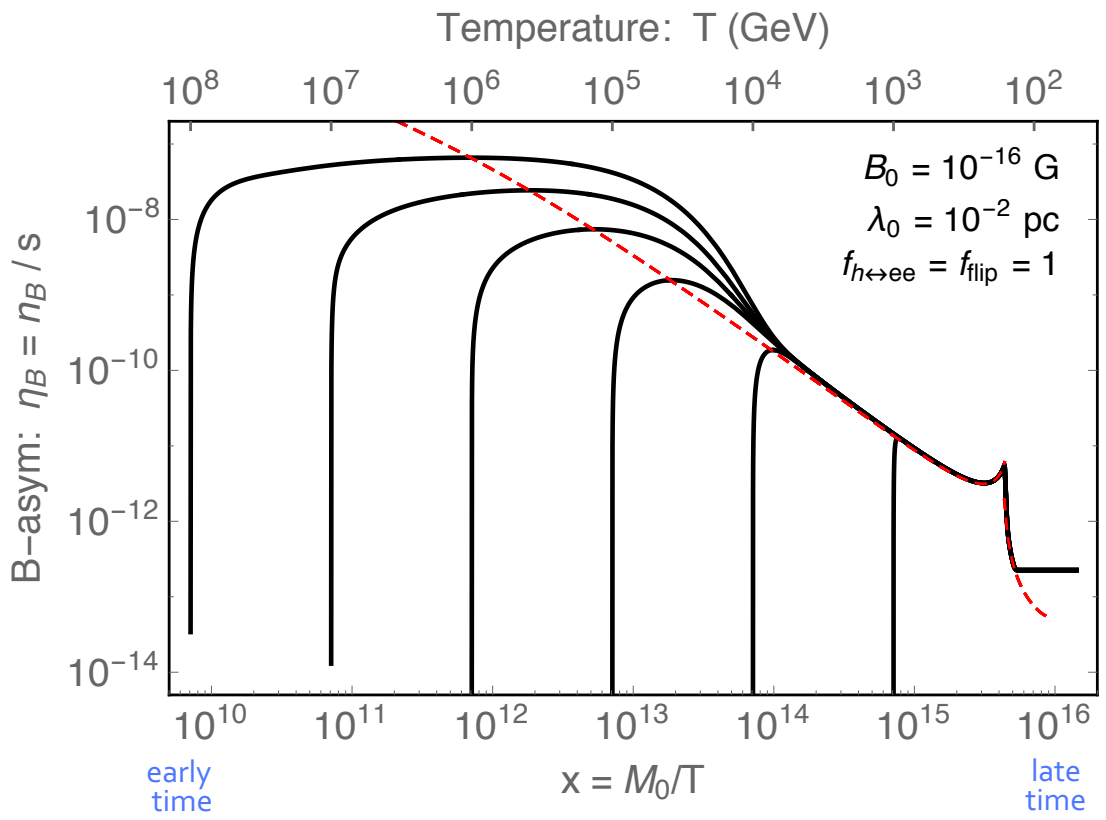
- ➔ this could explain the matter/antimatter asymmetry!
- ➔ but it would be a problem if too many baryons are created

(B+L) washout is avoided

Kamada & AL (2016)

basic idea: $\dot{n}_{\rm B} = -\Gamma_{\rm sphaleron}\, n_{\rm B} + S_{\rm helicity}$ ⇒ $n_{\rm B} \sim S_{\rm helicity} \,/\, \Gamma_{\rm sphaleron}$

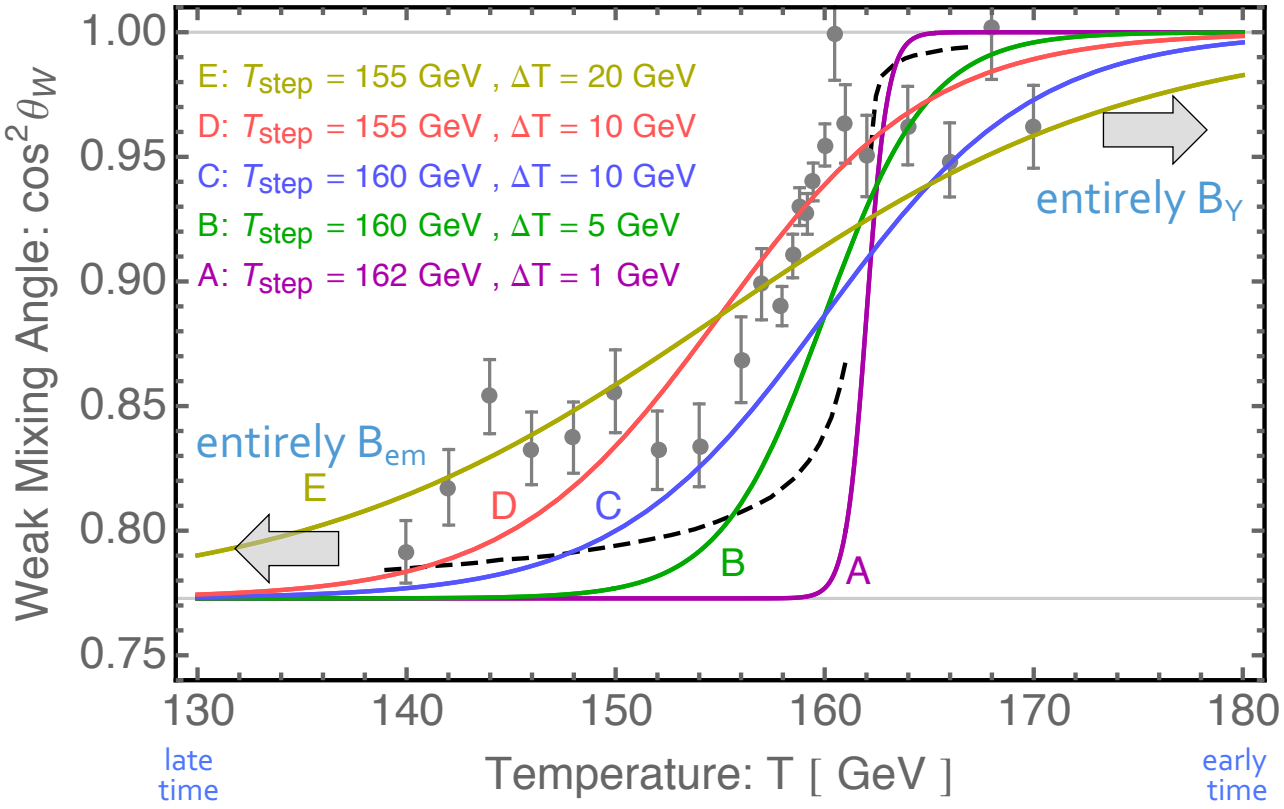
time evolution



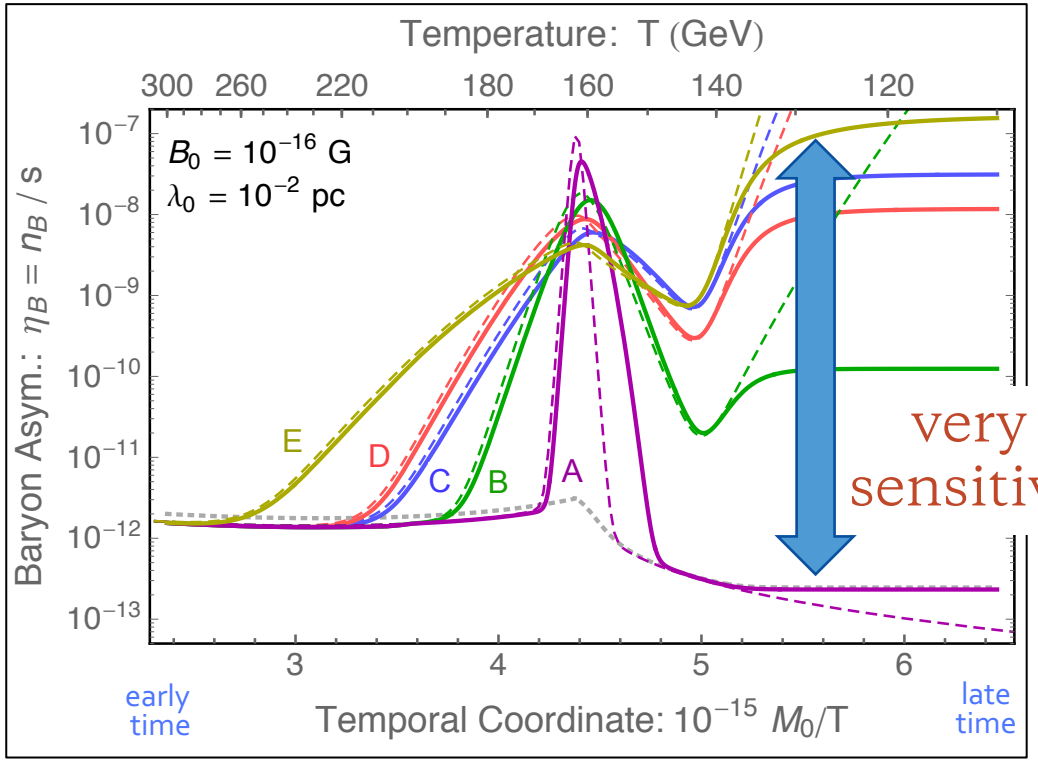
Sensitivity to the EW crossover

Kamada & AL (2016)

Z-γ mixing is proxy for B_Y to B_{em}



time evolution



We should work to develop a better understanding of how hypermagnetic fields are converted into electromagnetic fields at the EW epoch (even if it is a smooth crossover)

Avoiding baryon overproduction

[Domcke, Kamada, Mukaida, Schmitz, Yamada (2022)]

If μ_{Y5} is too large, the helical B_Y-field becomes too strong, and B-number is over-produced.

The upper limit is subject to uncertainties associated with modeling B_Y -> B_{EM} at the EW crossover.

We take $\mu_{Y,5}/T \lesssim 10^{-2}$ as a guide.

recall

$$B_{\rm phys,0} \simeq (6.6\times 10^{-16}\ {\rm G})\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

summary graphic
& remarks

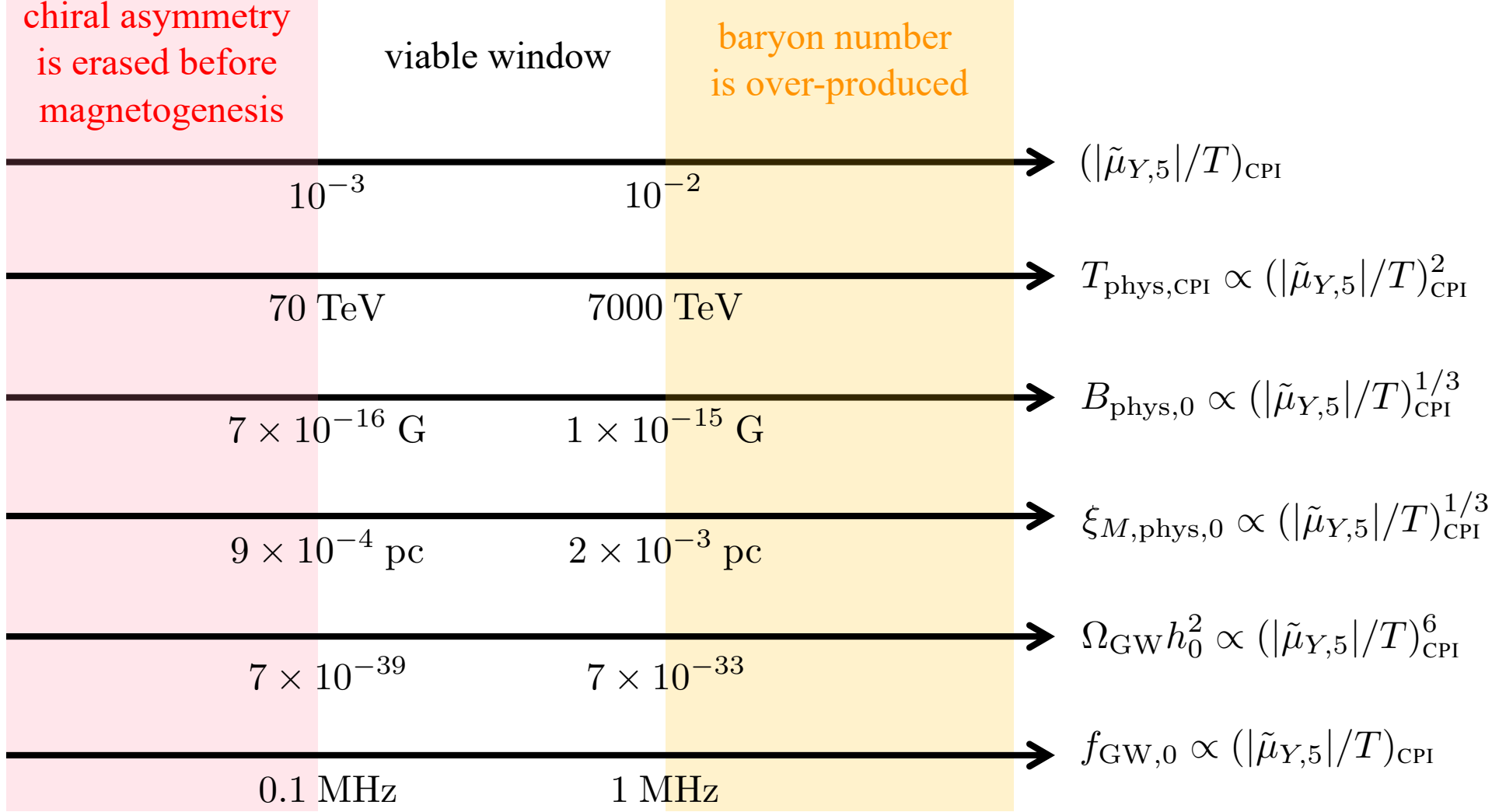
Summary

assume: chiral asymmetry (Baryo-genesis)

Chiral Plasma Instability

Magneto-genesis

Gravito-genesis



- chiral plasma instability develops at T_{CPI} > 80 TeV before washout
- development of CPI leads to helical PMFs and chiral PGWs
- the PMFs survives as an intergalactic magnetic field ~ too weak for blazars
- the PGWs are high frequency ~ too weak for laboratory detection

backup slides

A note on notation

All dimensionful variables are comoving. Some examples,

conformal time:	$\mathrm{d}t = \mathrm{d}t_{\rm phys}/a$
comoving Hubble param:	$H = aH_{\rm phys}$
comoving temperature:	$T = aT_{\rm phys}$
comoving energy density:	$\mathcal{E} = a^4\mathcal{E}_{\rm phys}$
comoving chemical potential:	$\mu = a\mu_{\rm phys}$
comoving magnetic field strength:	$\boldsymbol{B} = a^2\boldsymbol{B}_{\rm phys}$
comoving magnetic correlation length:	$\xi_M = \xi_{M,\rm phys}/a$
comoving conductivity:	$\sigma = a\sigma_{\rm phys}$
Friedmann equation:	$3a^2M_{\rm pl}^2H^2 = \mathcal{E}$

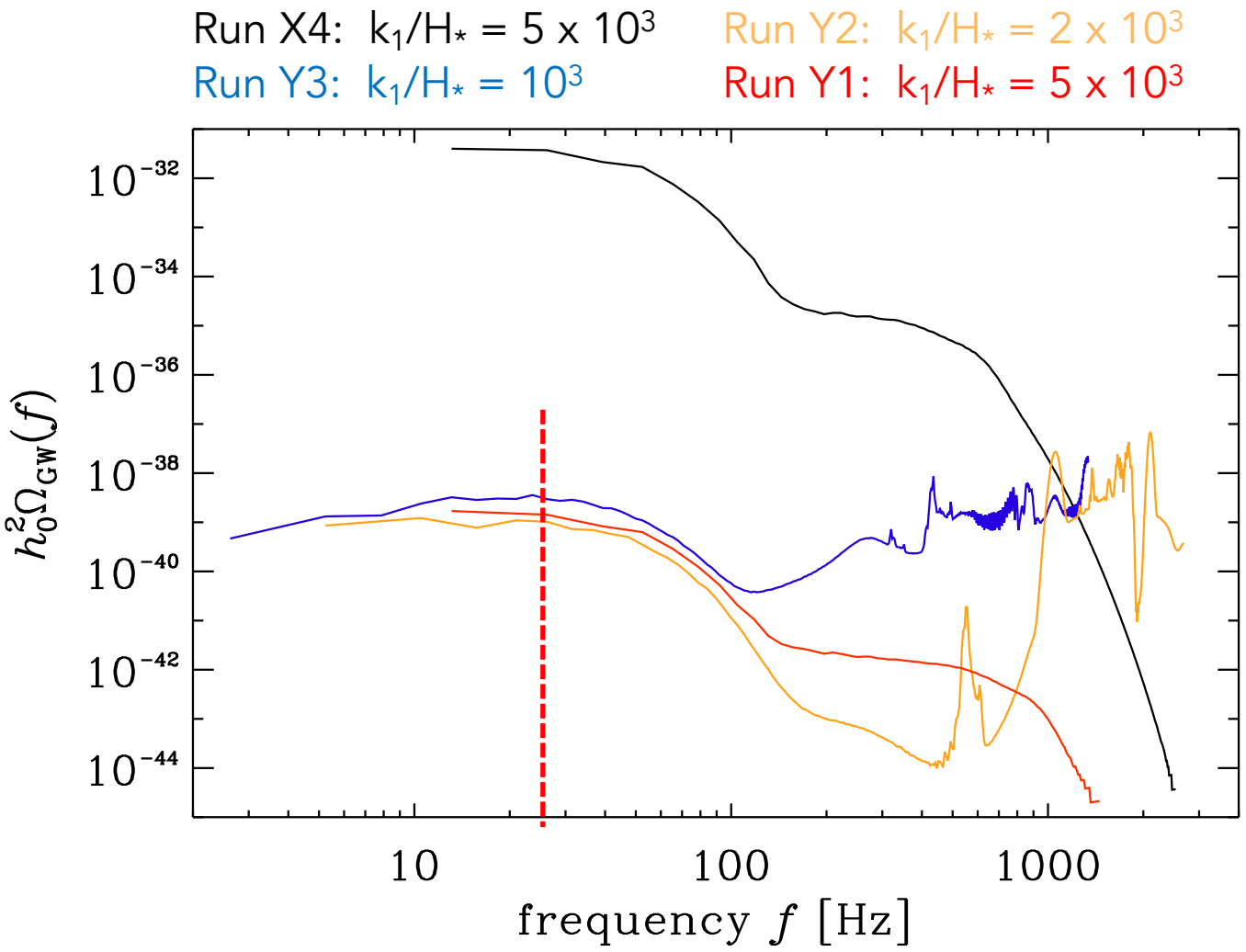
Stochastic B-field power spectra & energy

$$\langle B_i(\boldsymbol{k},t)B_j^*(\boldsymbol{k}',t)\rangle = (2\pi)^3\,\delta(\boldsymbol{k}-\boldsymbol{k}')\left[\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)P_B(k) - \mathrm{i}\epsilon_{ijm}\hat{k}_m P_{aB}(k)\right]$$

$$\begin{aligned}\langle B_i(\boldsymbol{x},t)B_j(\boldsymbol{y},t)\rangle &= \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3}\langle B_i(\boldsymbol{k},t)B_j^*(\boldsymbol{k}',t)\rangle\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}-\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{y}}\\ &= \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)P_B(k) - \mathrm{i}\epsilon_{ijm}\hat{k}_m P_{aB}(k)\right]\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}\end{aligned}$$

$$\begin{aligned}\mathcal{E}_M(\boldsymbol{x},t) &= \frac{1}{2}\sum_i\langle B_i(\boldsymbol{x},t)B_i(\boldsymbol{x},t)\rangle\\ &= \frac{1}{2}\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[(3-1)\,P_B(k)\right]\\ &= \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}P_B(k)\\ &= \int_0^\infty\frac{\mathrm{d}k}{k}\underbrace{\frac{k^3}{2\pi^2}P_B(k)}_{=\,kE_M(k)}\end{aligned}$$

Grav wave frequency spectrum today



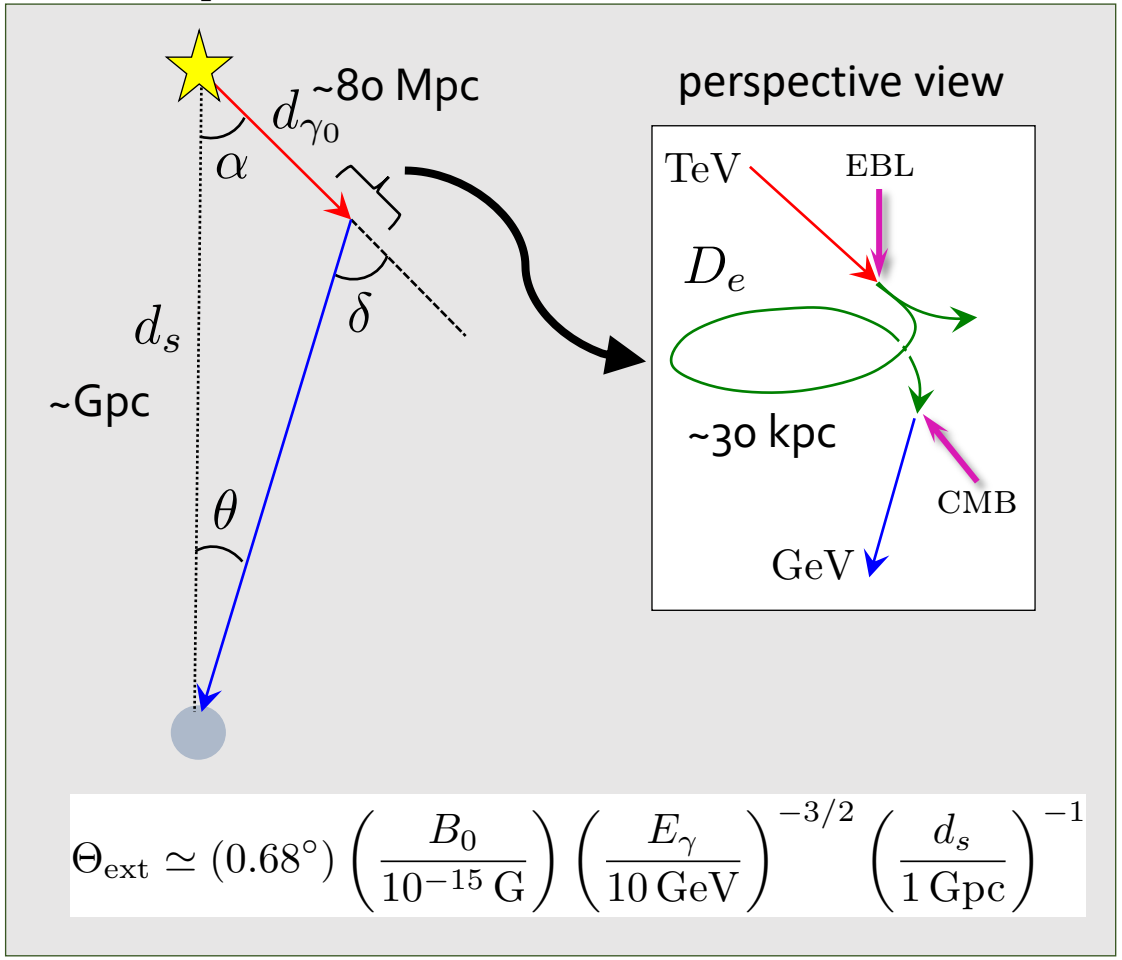
- a high-frequency & low amplitude
- UV end of the spectrum suffers from numerical instability (that depends on the k_1, highest wavenumber on the lattice)

TeV blazars
as a probe of the IGMF

[Aharonian, Coppi, & Voelk (1994); Neronov & Semikoz (2006); AL & Vachaspati (2016)]

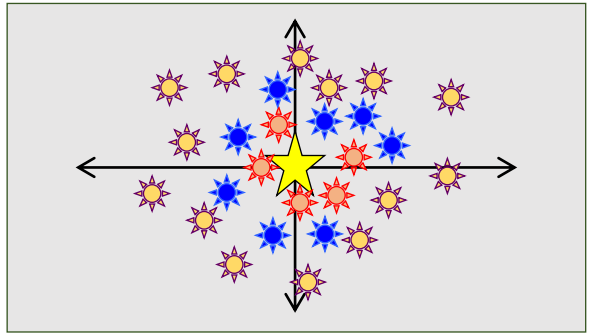
Gamma rays from TeV blazars develop an electromagnetic cascade by scattering on starlight (EBL) and cosmic microwave background (CMB) photons.

The presence of an IGMF deflects the cascade.

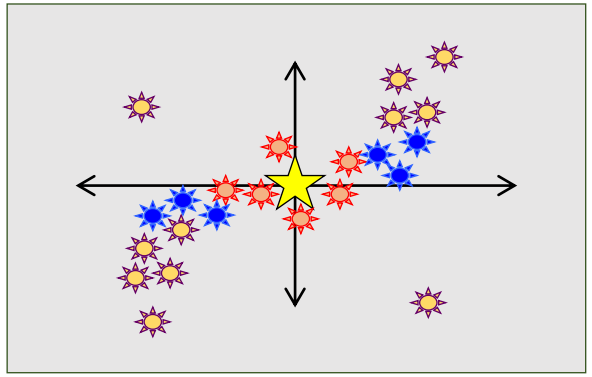


$$\Theta_{\rm ext} \simeq (0.68^\circ) \left(\frac{B_0}{10^{-15}\,{\rm G}}\right) \left(\frac{E_\gamma}{10\,{\rm GeV}}\right)^{-3/2} \left(\frac{d_s}{1\,{\rm Gpc}}\right)^{-1}$$

the blazar acquires a halo



the blazar halo becomes "twisted" by a *helical* IGMF



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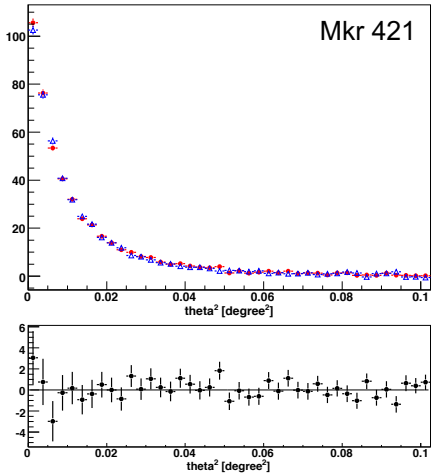
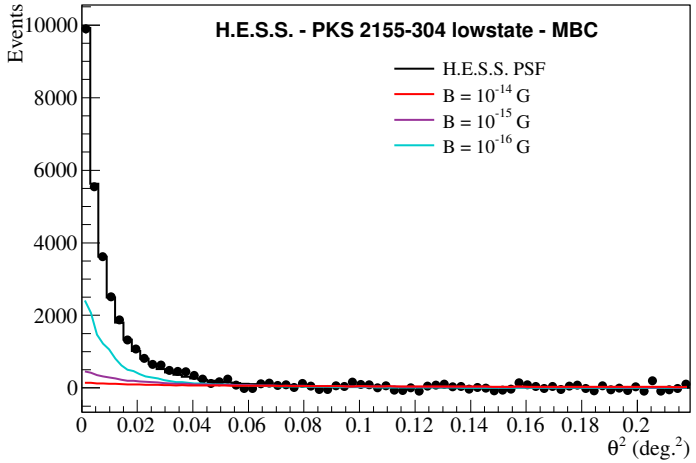
Ongoing experimental efforts …

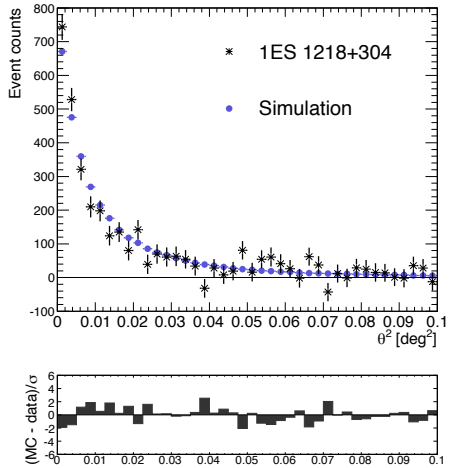
MAGIC (2010)



HESS + Fermi-LAT (2014)



VERITAS (2017)



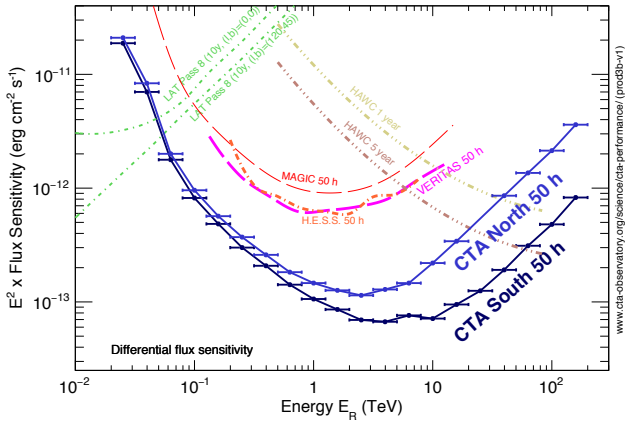
A halo is not observed ➔ some of the IGMF parameter space is excluded:

$$B_0 \sim (0.3 - 70) \times 10^{-15}\,\mathrm{G} \quad \text{for} \quad \lambda_0 = 1\,\mathrm{Mpc}$$

[Meyer, Conrad, & Dickinson (2016); CTA Science Book (2017)]

On the horizon…

The Cherenkov Telescope Array (CTA) will dramatically improve the flux sensitivity & angular resolution (PSF).



Implications for IGMF contraints:

Benchmark models (3, green). Corresponding exclusions (blue).