Emergent Stability of Floquet Quantum Matter Under High Fields and Couplings

Arnab Das

Indian Association for the Cultivation of Science, Kolkata

Collaborators:

Asmi Haldar (MPI-PKS, Dresden), Roderich Moessner (MPI-PKS, Dresden), Frank Pollmann (TU Munich), Diptiman Sen (IISc, Bangalore), Alexander Wietek (MPI-PKS, Dresden)

Ref: PRB **97**, 245122 (2018), PRX **11** 021008 (2021)

J. Phys: Cond Matt **34** 234001 (2022) (Review)

Conservation laws and Statistical Mechanics

Gibbs' Statistical Mechanics ≡ Entropy Maximization Subject to ***Exact*** *(**local**, additive)* Conservation Laws

$$\rho_{Equi} = \frac{e^{-\sum_\kappa \lambda_\kappa \hat{\mathcal{I}}_\kappa}}{\mathcal{Z}}$$

Additive Local Conserved Quantities

Lagrange Multipliers

Many-body System

Hypothesis: ETH/Quantum Chaos

Quantum Mechanics →

$$[\mathcal{O}, H] = 0 \Rightarrow \mathcal{O} \text{ is conserved}$$
$$[\mathcal{O}, H] \neq 0 \Rightarrow \mathcal{O} \text{ is not conserved}$$

What if

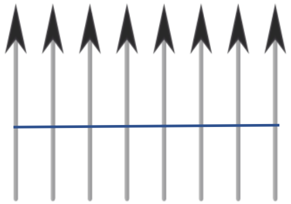
$$H = H_{Large} + H_{small}; \text{ and } [H_{Large}, H] = \epsilon \text{ (small, but } \neq 0)?$$

Is H_{Large} "Approximately Conserved" in any sense up to some ϵ?

Not known for Sure: no KAM-like "Threshold Theorem" in quantum Mechanics.

General Belief: Conservations are destroyed even for an infinitesimal ϵ !

Asking this Question by Minimally Breaking Energy Conservation: Periodically Driven Many-Body System with no conserved local operator



Interacting
Degrees
of Freedom

+

(Unitary Periodic
Drive)

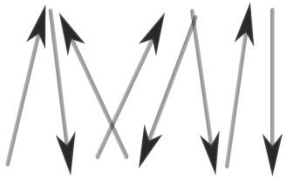
$$H(t);\ H(t+T) = H(t)$$

=

?

Intuitive Answer

Ergodicity →Unbounded "Heating" (no constraints whatsoever)
The system goes locally
to an infinite Temperature Ensemble



*A. Lazarides, **AD**, R. Moessner, PRE (2014).*

L. D'Alessio, M. Rigol, PRX (2014)

Summary

- Under a Strong Periodic Drive, an interacting, non-integrable quantum system may not thermalize to a locally infinite-T-like scenario!

- A new approximate but perpetually stable conservation law emerges due to the drive, that was not present in the undriven system! The Hilbert space approximately fragments into dynamically disjoint sectors, which are the eigen-subspaces of the emergent conserved quantity.

- The emergence occurs beyond a threshold drive strength (**reminiscent of the KAM scenario**)!

- We have found analytic signatures (Floquet-Magnus expansions) for quantum many-body systems of an enormously large class including the Ising model in a transverse field in any dimension and for any form of Ising interactions and also for any two-body Heisenberg model in any dimension, anisotropy, and bond distributions.

A Concrete Example:
STRONG Integrable Drive + Non-Integrable Static Part

(A.Haldar, R. Moessner, **AD**., PRB 2018)

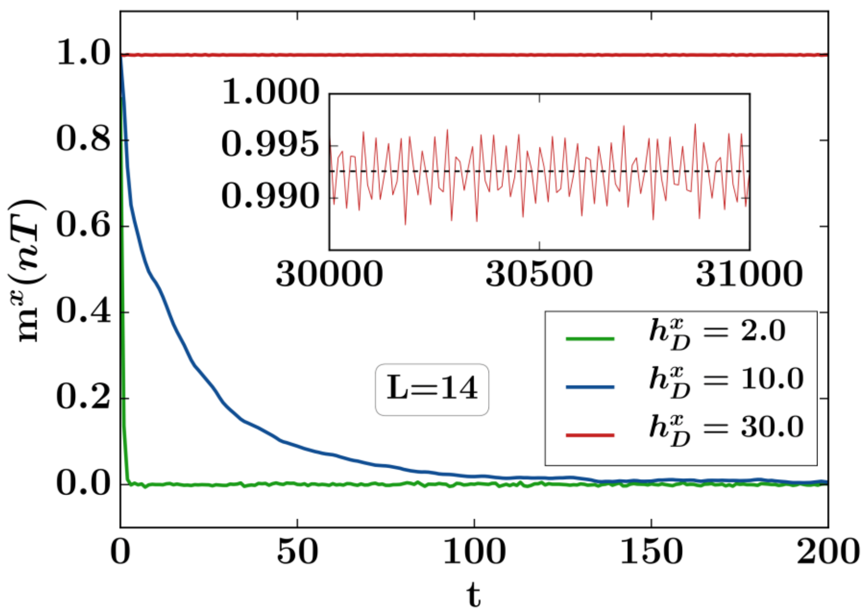
$$H(t) = H_0(t) + V, \text{ where}$$

$$H_0(t) = H_0^x + \text{Sgn}(\sin(\omega t))H_D, \text{ with}$$

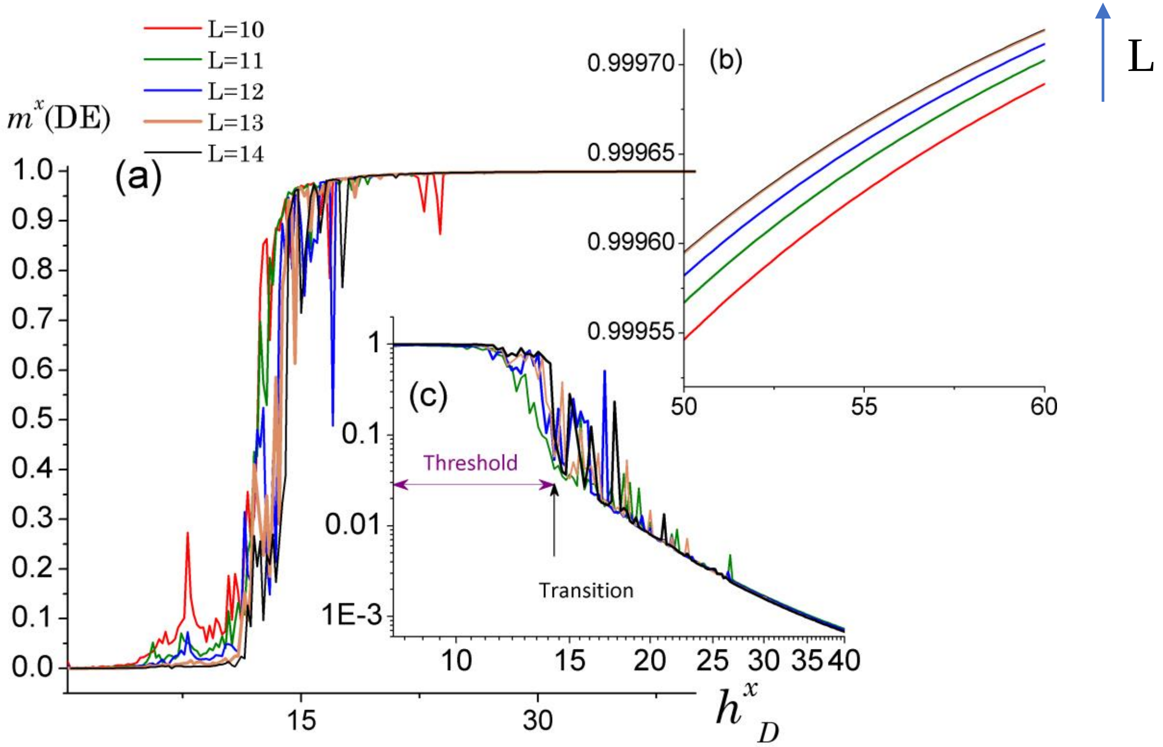
$$H_0^x = -\sum_{n=1}^{L} J\sigma_n^x\sigma_{n+1}^x + \sum_{n=1}^{L} \kappa\sigma_n^x\sigma_{n+2}^x - h_0^x\sum_{n=1}^{L}\sigma_n^x,$$

$$H_D = h_D^x \sum_{n=1}^{L}\sigma_n^x, \text{ and}$$

$$V = h^z \sum_{n=1}^{L}\sigma_n^z,$$



The Threshold: Finite L, t → ∞



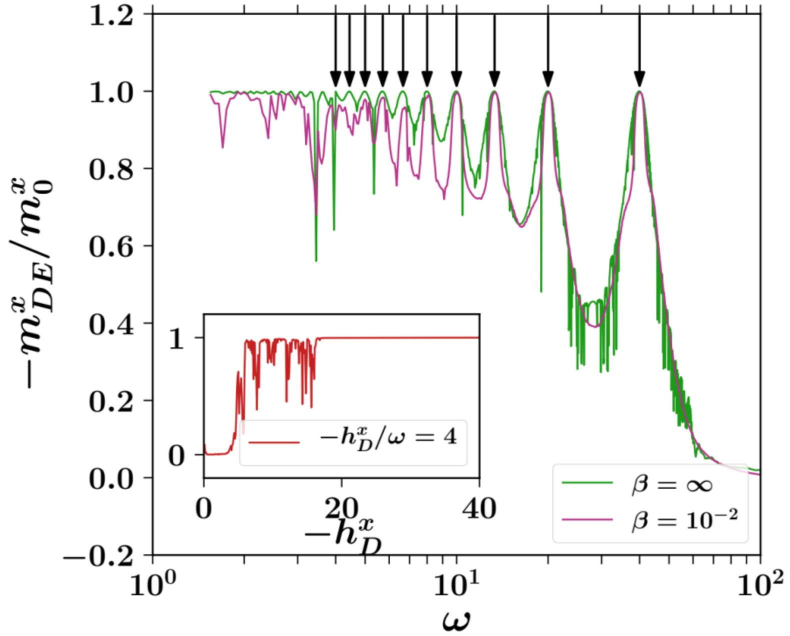
$$J = 1, \kappa = 0.7, \omega = 0.1, h_0^x = 0.01, h^z = 1.2$$

Initial State = the Ground State of *H(t=0)*

- The threshold doesn’t move with the system-size.
- Finer resolution shows, m^x is more strongly frozen for larger L above the threshold.

Freezing Points and Emergent-Conservation

$$J = 1, \kappa = 0.7\pi/3, h_0^x = e/10, h_D^x = 40, L = 14$$



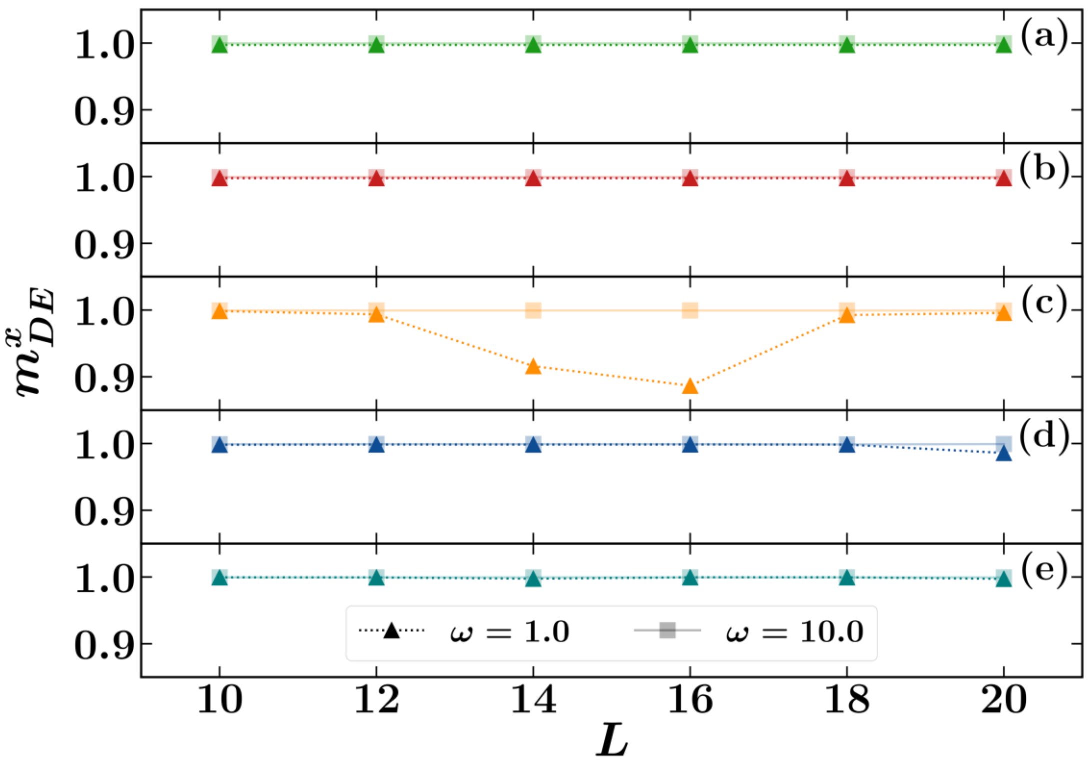
Longitudinal magnetization emerges as a quasi-conserved quantity under the Drive condition:

$$\boxed{h_D^x = k\omega}$$

This happens for a very broad range of ω

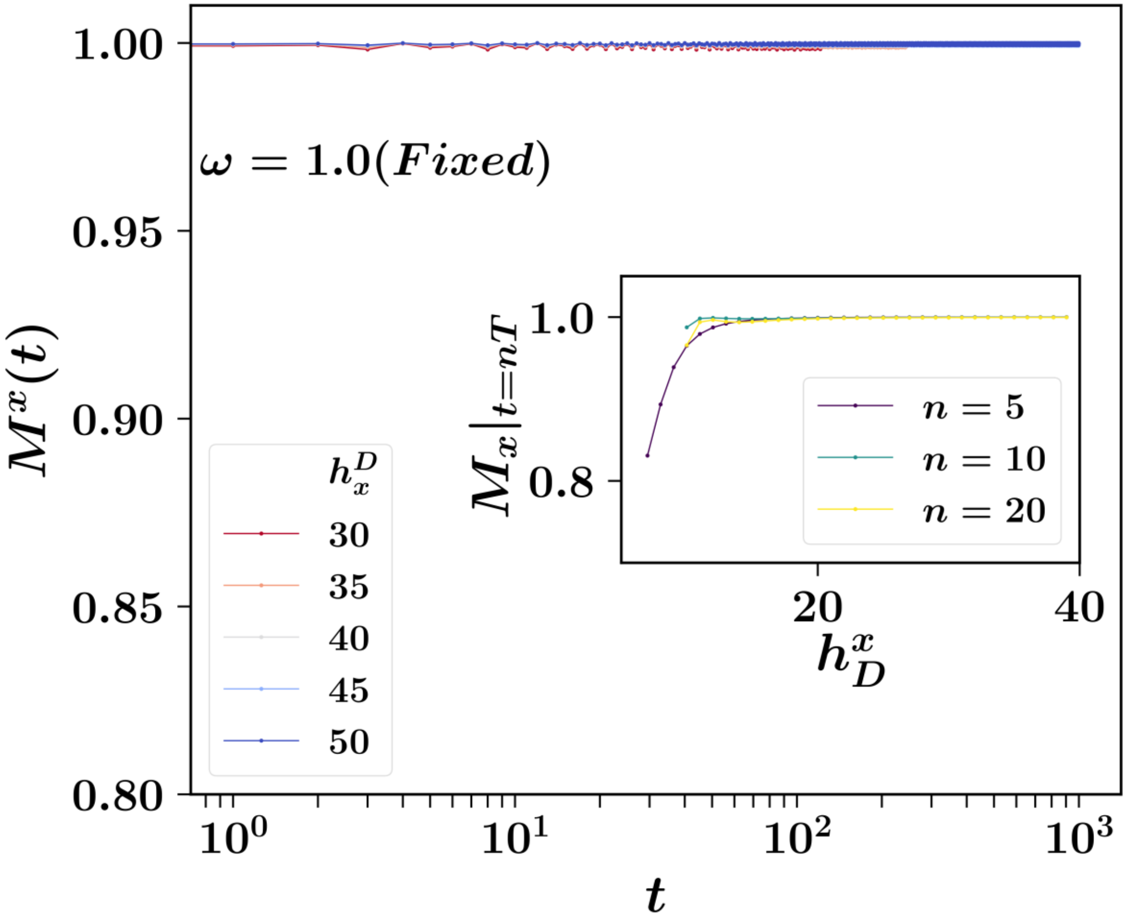
A. Haldar, D. Sen, R, Moessner and ***AD***, PRX **11** 021008 (2021)

Various Models (L-dependence at Freezing Point)



(a) Ising: NN + NNN
(b) Ising: 3-spin Interactions
(c) Ising: 1/r Interactions (long-range)
(d) Heisenberg: Homogeneous, Isotropic
(e) Heisenberg: Homogeneous, Anisotropic

Results for large t, L → ∞ (iTEBD)

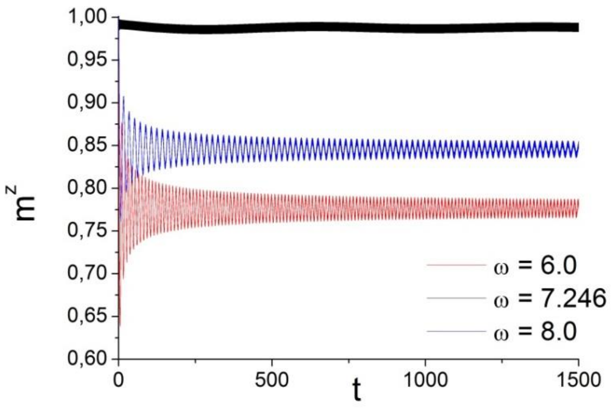


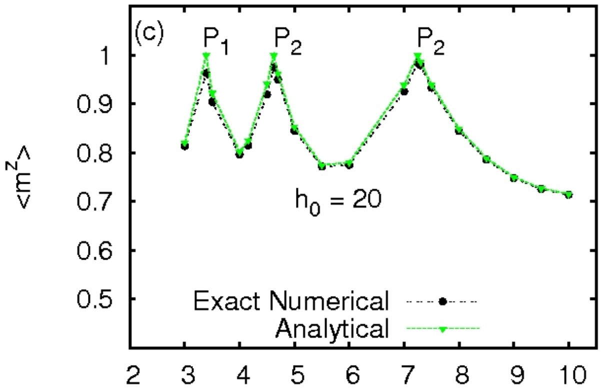
With:
Asmi Haldar
Frank Pollmann
Alexander Wietek

Periodically Driven Free fermions
Absence of *Generalized* Floquet Thermalization beyond the Threshold
In L → & t →∞

$$H = -\frac{J}{2}\left[\sum_{i=1}^{L}\sigma_i^x\sigma_{i+1}^x + h_0\cos(\omega t)\sum_i \sigma_i^z\right]$$

__AD__, PRB 82, 172402 (2010)





Stability guaranteed by Convergence of Magnus Expansion In a rotating Frame ☺

$$Q = \langle m^z\rangle = 1/[1 + J_0(2h_0/\omega)]$$

Periodic Gibbs'' Ensemble

$$\hat{H}_{eff} = \sum_{p=1}^{L}\omega_p\tilde{a}_p^\dagger\tilde{a}_p \text{ with } \mathcal{I}_p = \tilde{a}_p^\dagger\tilde{a}_p$$

Exact (periodic) Conserved Quantities

$$m^z$$

Is **NOT** among them!

$$\hat{\rho}_{PGE}(t) = \mathcal{Z}^{-1}\exp\left[-\sum_p \lambda_p\mathcal{I}_p(t)\right];\ \mathcal{Z}(t) = tr[\hat{\rho}_{PGE}(t)]$$

A. Lazarides, __AD__, R. Moessner, PRL (2014)

What We Didn't Discuss

- Our analytical Treatments – Magnus Expansion in a time-dependent Frame and A Floquet Perturbation Theory.

- Resonances: We can analytically capture all of them by the 1^{st} order perturbation Theory!

Higher Order resonances do not appear beyond the freezing threshold

→ Signature of Series strongly asymptotic to an otherwise analytical function!

- Apparent "underlying" convergence of the divergent many-body-series: Its connection with the Beyond the KAM-stability – solar stability problem etc.

- Further constraints beyond the magnetization conservation (Work in Progress).

Thanks!