Frequency space derivation of Gravitational Wave Memory

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Hearing beyond the standard model with cosmic sources of gravitational waves

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International Centre for Theoretical Sciences
Tata Institute of Fundamental Research

Plan of the talk

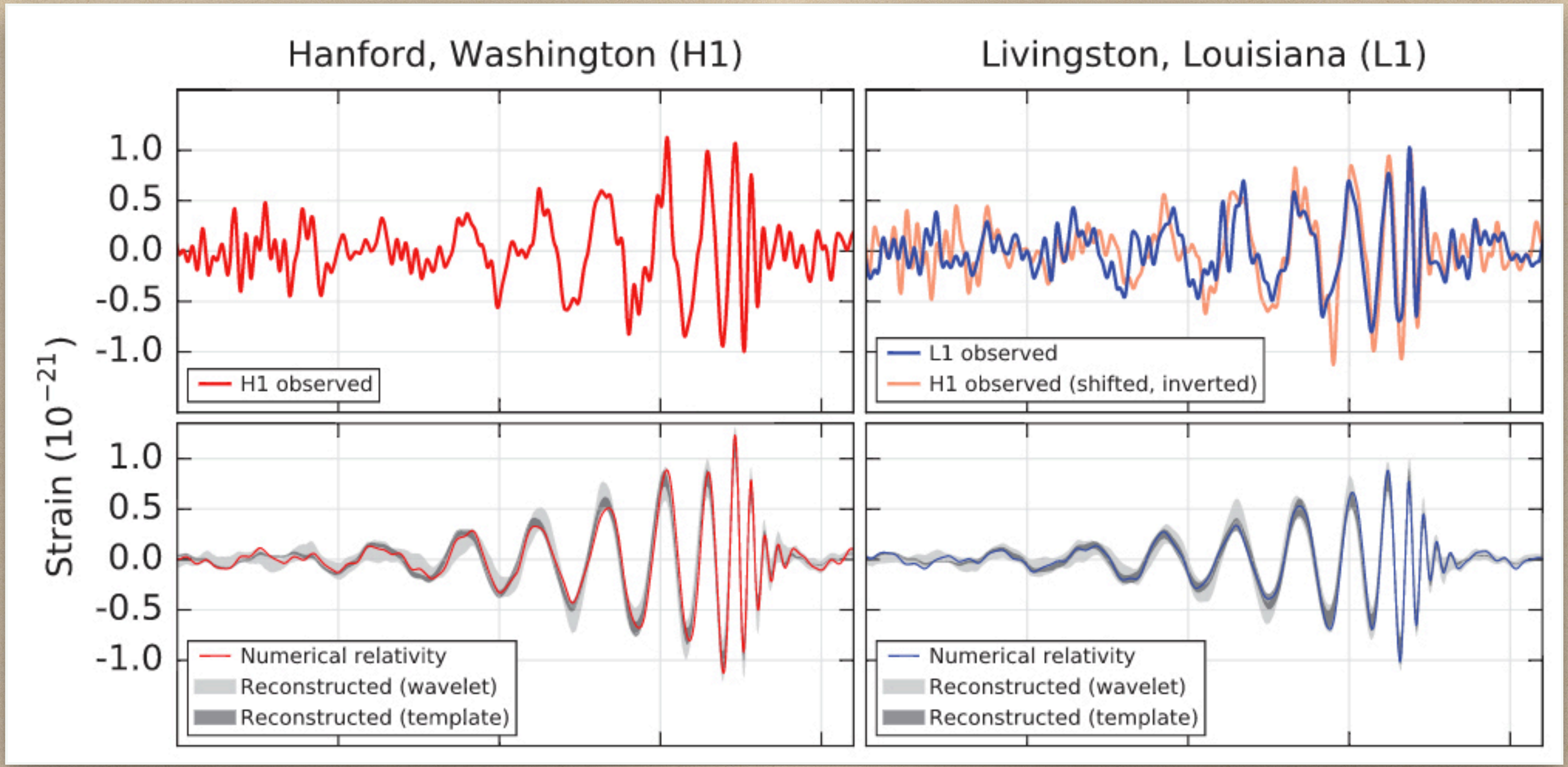
What is Gravitational wave memory?

Field-theoretic way of deriving memory signal.

Comparison with the waveform constructed using soft-graviton theorem

Advantages of the field-theoretic approach

Gravitational wave



B.P. Abbott et. al, Phys. Rev. Lett. 116, 061102

The strain rises from zero ⟶ Oscillates ⟶ Returns to zero

Gravitational wave memory

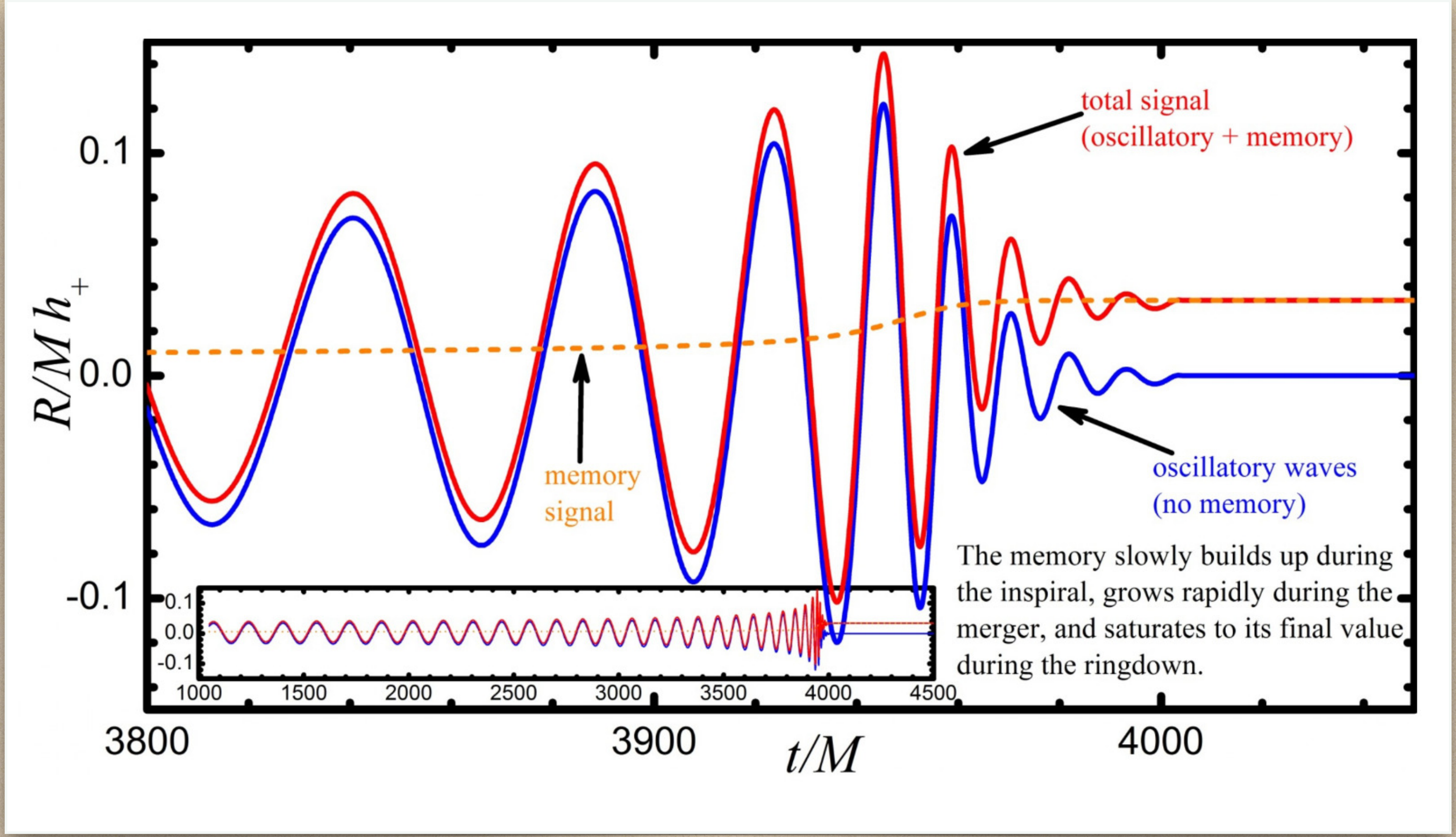


Fig. Courtesy: Marc Favata

The strain rises from zero ⟶ Oscillates ⟶ Settles down to a non-zero value

Linear and Nonlinear memory

Linear memory arises from the **non-oscillatory motion** of objects.

Example: Binaries in hyperbolic orbit, Neutrinos from Supernovas, …..

Zel'dovich & Polnarev (1974)

Braginsky & Grishchuk (1985)

Braginsky & Thorne (1987)

Nonlinear memory arises from the gravitational wave sourced by the gravitational wave.

Example: Binaries in bound orbits (Circular orbit, Elliptical orbit)

Christodoulou (1991)

Blanchet & Damour (1992)

Field theoretic way of deriving Memory

The probability amplitude of emitting a graviton of polarisation $\epsilon^{\lambda}_{\mu\nu}(n)$ from a source with stress-tensor $T^{\mu\nu}(k)$

$$\mathscr{A}_{\lambda}(k_0, \vec{n}k_0) = -\iota\frac{\kappa}{2}\epsilon^{*\lambda}_{\mu\nu}(\vec{n})T^{\mu\nu}(k_0, \vec{n}k_0)$$

Lecture by Subhendra Mohanty

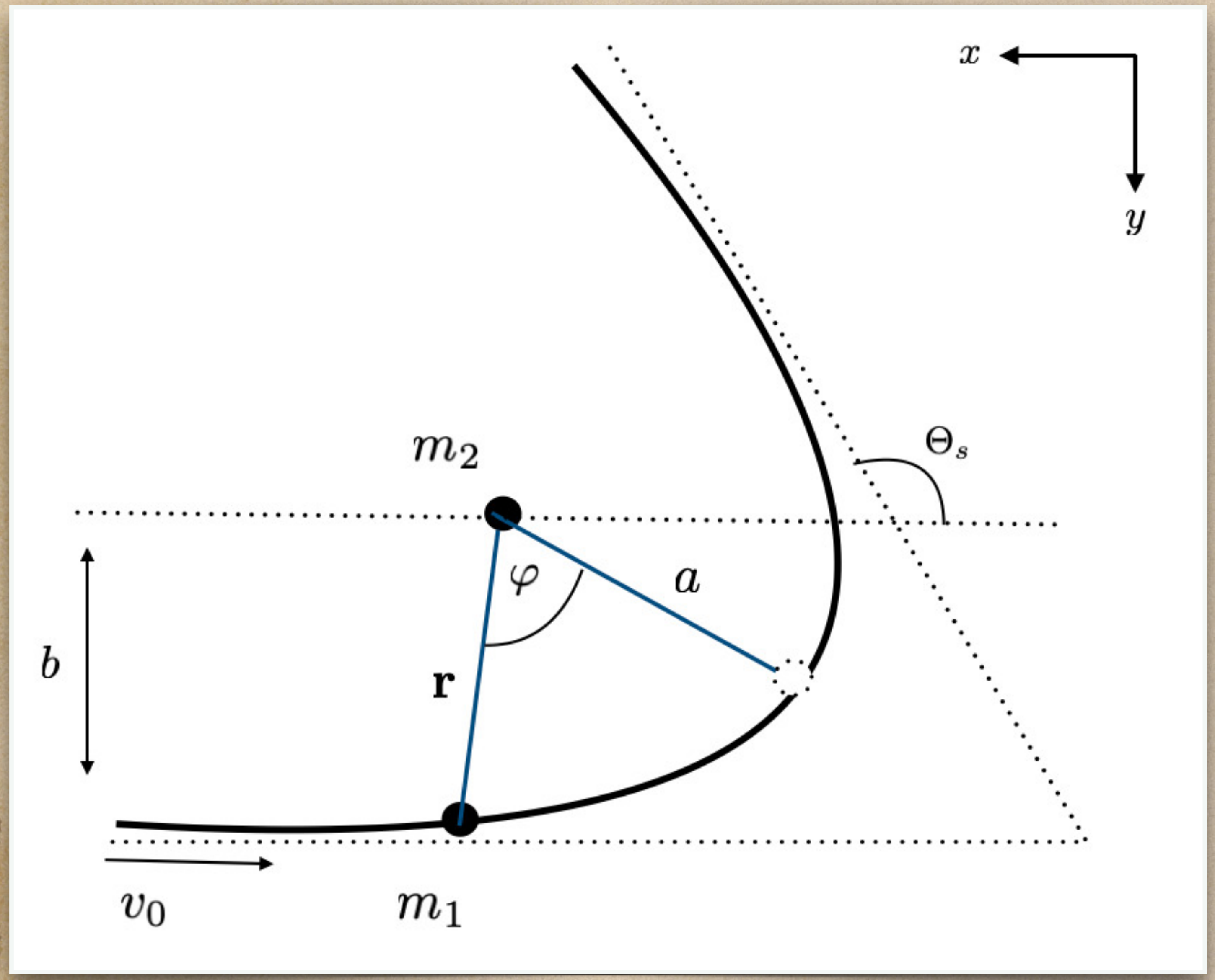
In frequency space, the stress-tensor of the source and the gravitational wave can be related as

$$[\tilde{h}_{ij}]^{\mathrm{TT}}(\vec{x}, k_0) = -\frac{4G}{r}\Lambda_{ij,kl}(\vec{n})T_{kl}(k_0, \vec{n}k_0)$$

Hait, Mohanty & Prakash (2024)

Linear memory from hyperbolic encounter

The stress-tensor component in frequency space



$$T_{xx}(\omega') = \mu\nu\omega_0 a^2 \pi \left[\frac{\imath}{\nu e^2} H_{\imath\nu}^{(1)}(ie\nu) - \left(e - \frac{1}{e}\right) H_{\imath\nu}^{(1)'}(\imath e\nu)\right]$$

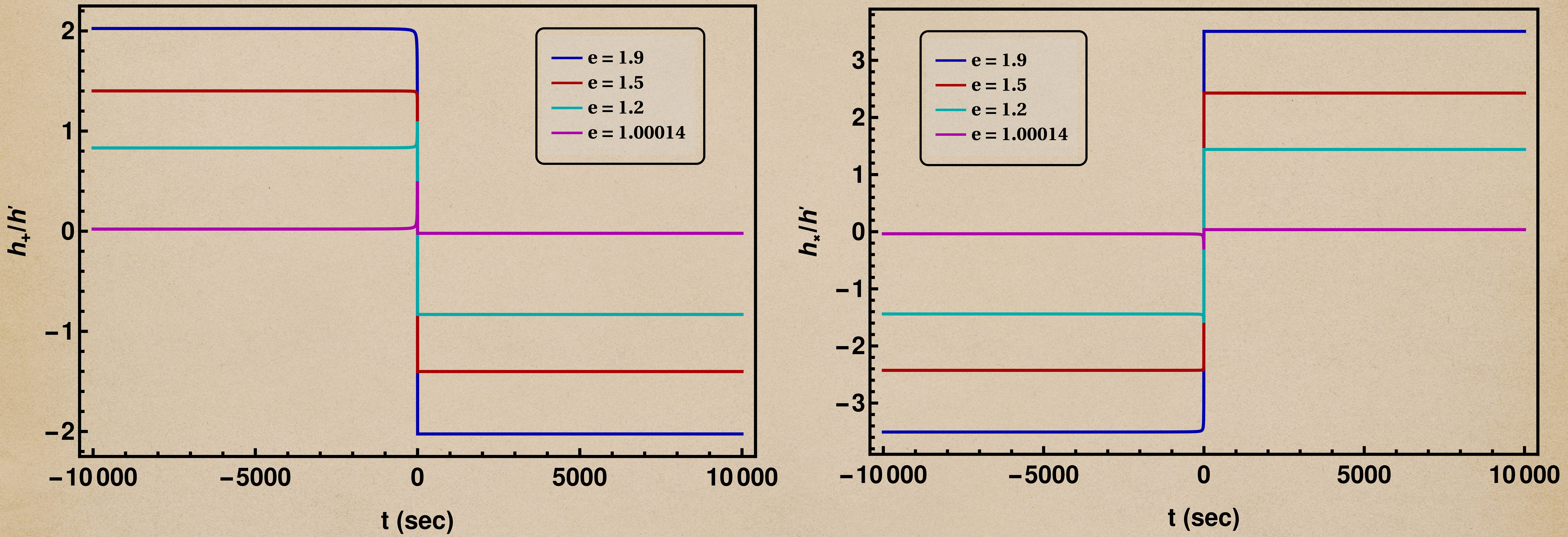
$$T_{yy}(\omega') = \mu\nu\omega_0 a^2 (e^2 - 1)\pi \left[\frac{\imath}{\nu e^2} H_{\imath\nu}^{(1)}(ie\nu) + \frac{1}{e} H_{\imath\nu}^{(1)'}(\imath e\nu)\right]$$

$$T_{xy}(\omega') = \mu\nu\omega_0 a^2 \sqrt{e^2 - 1}\,\pi \left[\left(\frac{1}{e^2} - 1\right) H_{\imath\nu}^{(1)}(ie\nu) + \frac{1}{\nu e} H_{\imath\nu}^{(1)'}(\imath e\nu)\right]$$

Hait, Mohanty & Prakash (2024)

Talk by Indranil Chakraborty on linear memory from Supernova Neutrinos (January 8, Wednesday)

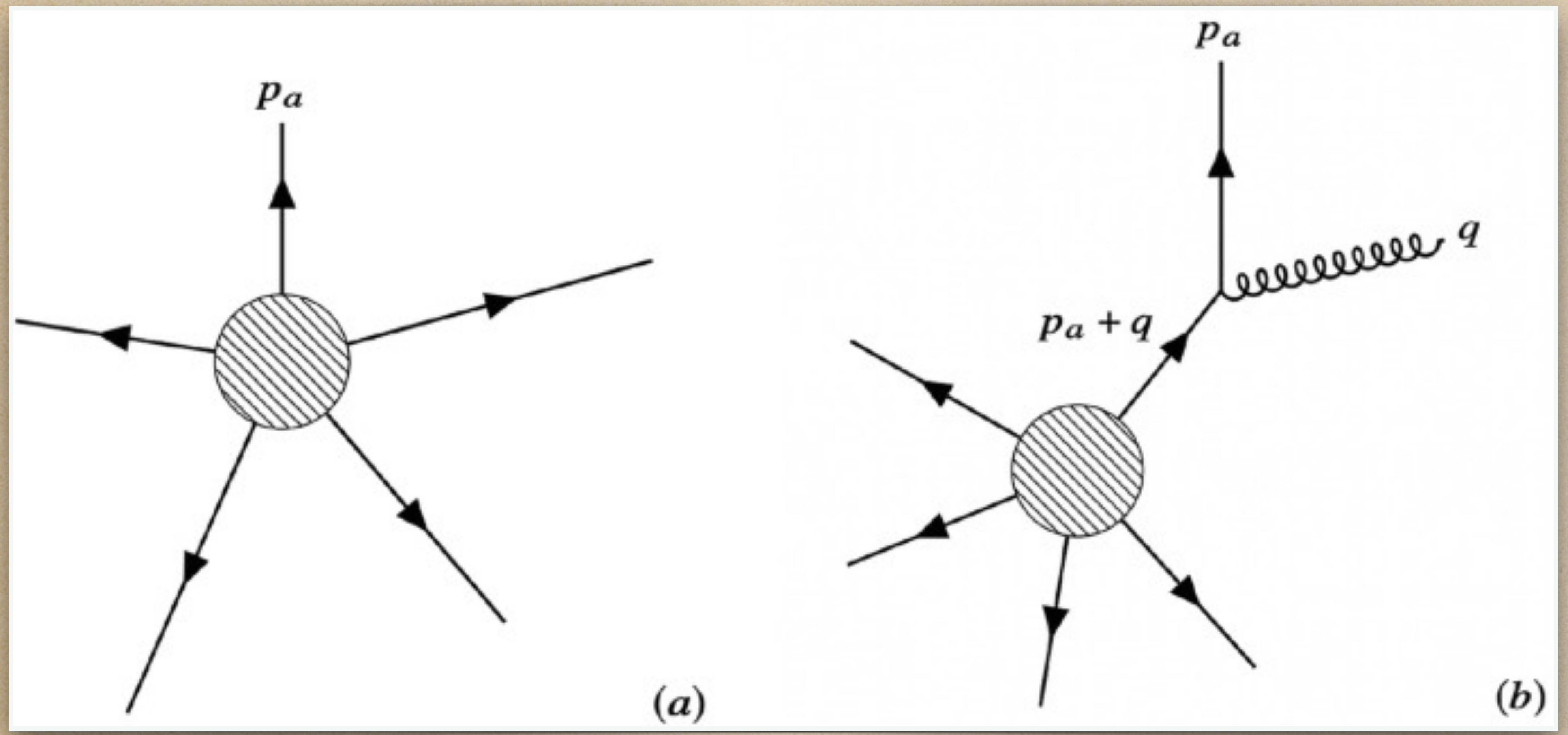
Memory waveform



Hait, Mohanty & Prakash (2024)

$$h^{\mathrm{mem}}(t) = A\Theta(t) \xleftrightarrow[\text{Transform}]{\text{Fourier}} \tilde{h}^{mem}(\omega) = \frac{A}{\omega}$$

Waveforms constructed using Soft theorem



$$\mathscr{A}_{n+1}(p_a, q) = f(p_a, q)\mathscr{A}_n(p_a)$$

The low frequency graviton signal has the form

$$h_{ij}(\omega) = \imath\omega^{-1}A_{ij} + B_{ij}\ln\omega^{-1} + \ldots\ldots$$

Laddha & Sen (2018)

Sahoo & Sen (2019)

In the zero frequency limit, the memory signal in frequency space has both the $1/\omega$ and $\ln\omega$ terms.

Nonlinear memory from circular orbits

The stress-tensor component in frequency space

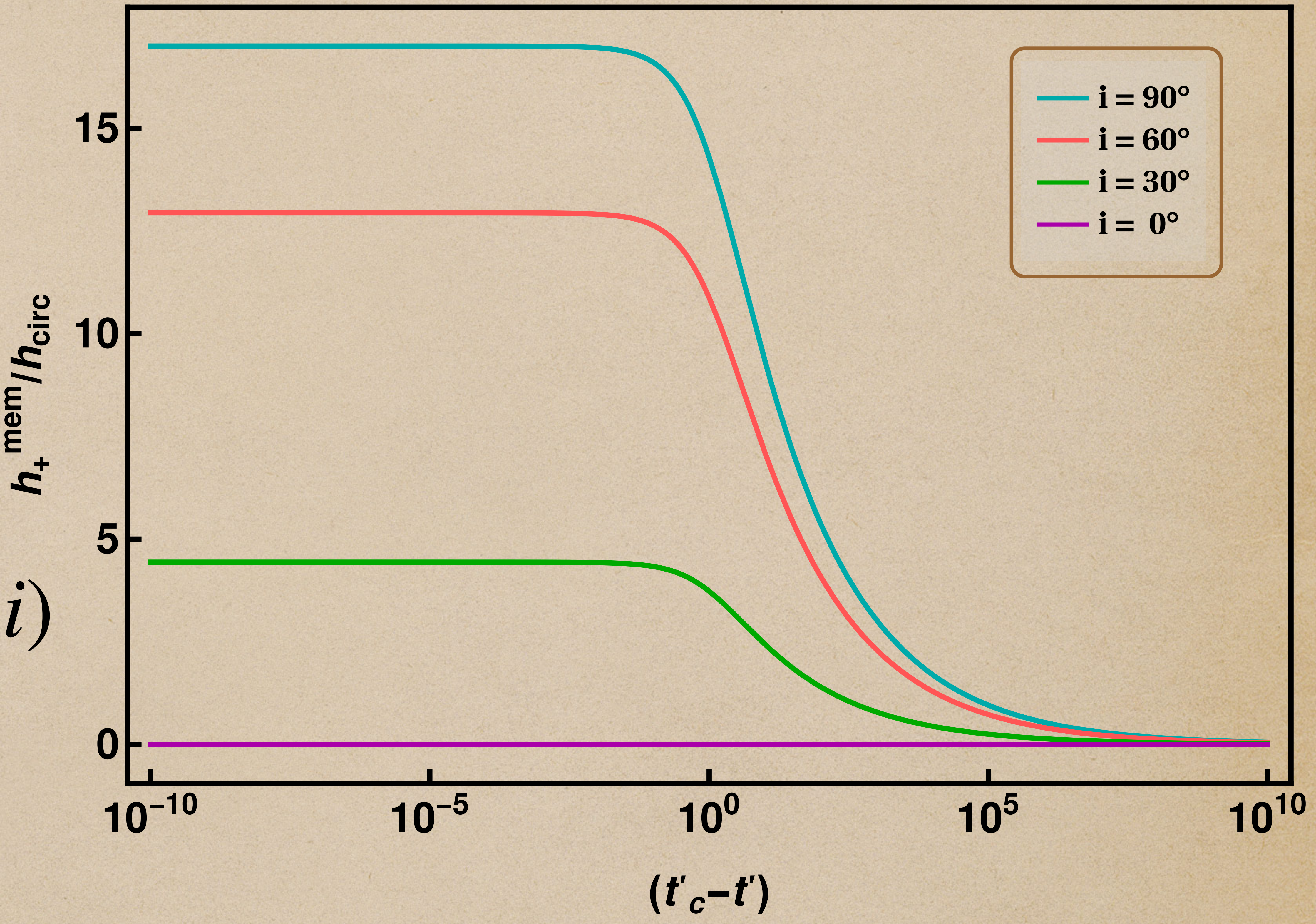
$$T_{xx} = -T_{yy} = \frac{\mu a^2 \omega_0^2}{2}, T_{xy} = \imath \frac{\mu a^2 \omega_0^2}{2}$$

$$\tilde{h}_+^{\rm mem}(t') = \frac{1}{[1 + (t'_c - t')]^{1/4}} \sin^2 i (17 + \cos^2 i)$$

Kennefick (1994)

$$h_\times^{\rm mem}(t') = 0$$

Favata (2009)



Hait, Mohanty & Prakash (2024)

Takeaways

Field-theoretic calculation reproduces the results as obtained from the classical quadrupole formula.

Field-theoretic calculation gives us the waveform directly in frequency space. These frequency domain templates are useful for extracting signal from data.

Full orbit calculation using the field- theoretic method matches with the results from soft-graviton theorem.

Thank you for your attention