Complex Langevin Study of Spontaneous SO(10) Symmetry Breaking in Euclidean IKKT Matrix Model

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Ongoing work with
Anosh Joseph and Piyush Kumar at IISER Mohali

PLAN OF TALK

Complex Langevin Study of Spontaneous SO(10) Symmetry Breaking in Euclidean IKKT Matrix Model

1. IKKT Matrix Model
2. SSB and Sign-Problem
3. Complex Langevin to Euclidean IKKT Model
4. Mass Deformations
5. Preliminary Results
6. Summary and Future Work

IKKT MATRIX MODEL

Ishibashi, Kawai, Kitazawa, Tsuchiya (1996)

- Promising candidate for non-perturbative formulation of superstring theory

IKKT: *0d matrix model in large-N limit*	IIB: *10d superstring theory*

- Originally from Green-Schwarz action of type IIB superstring with Schild gauge
- Can also be derived from Kaluza-Klein compactification of *10d* $\mathcal{N} = 1$ SYM theory

Euclidean IKKT Model

Wick rotated: $X_0 \to iX_{10},\ \Gamma^0 \to -i\Gamma_{10}$

$$Z = \int \mathcal{D}X\, \mathcal{D}\psi\, e^{-(S_b + S_f)}$$

$$S_b = -\frac{1}{4} N \,\mathrm{tr}\left([X_\mu, X_\nu]^2\right)$$

$$S_f = -\frac{1}{2} N \,\mathrm{tr}\left(\psi_\alpha\, \Gamma^\mu_{\alpha\beta}\, [X_\mu, \psi_\beta]\right)$$

Vectors X_μ

Majorana-Weyl spinors ψ_α

X_μ, ψ_α : $N \times N$ *Hermitian traceless matrices*

Γ^μ : $2^4 \times 2^4$ *gamma matrices*

$\mu, \nu = 1,2\ldots,10$

$\alpha, \beta = 1,2,\ldots,16$

Z finite Krauth, Nicolai and Staudacher (1998)

Austing and Wheater (2001)

EUCLIDEAN IKKT MODEL: SYMMETRIES

$$Z = \int \mathscr{D}X\, \mathscr{D}\psi\; e^{-(S_b + S_f)}$$

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- Euclidean action is invariant under following symmetries

  A. SU(N) gauge symmetry
  B. $\mathcal{N} = 2$ supersymmetry
  C. SO(10) rotational symmetry

A. Gauge Symmetry: SU(N)

- Inherited from *10d* $\mathcal{N} = 1$ SYM, for $U \in \mathrm{SU}(N)$, model invariant under

$$X_\mu \to X'_\mu = U^\dagger X_\mu U \qquad \psi_\alpha \to \psi'_\alpha = U^\dagger \psi_\alpha U$$

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B. Supersymmetry: $\mathcal{N} = 2$

- Two set of supersymmetries: $\delta^{(1)} S = \delta^{(2)} S = 0$

$$\delta^{(1)} X_\mu = i \bar{\epsilon}_1 \Gamma_\mu \psi \qquad \delta^{(2)} X_\mu = 0$$

$$\delta^{(1)} \psi = -\frac{i}{2} \left[X_\mu, X_\nu \right] \Gamma^{\mu\nu} \epsilon_1 \qquad \delta^{(2)} \psi = \epsilon_2$$

EUCLIDEAN IKKT MODEL: SYMMETRIES

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C. Rotational Symmetry: SO(10)

$$X_\mu \to X'_\mu = \Lambda^\rho_\mu X_\rho$$

- Spontaneously broken:

$$SO(10) \to SO(d)$$

matrix d.o.f.	gravitational d.o.f.

- Realized as $SO(d)$ symmetric vacuum
  Nishimura, Okubo and Sugino (2011)

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integrating out fermions

$$Z = \int \mathscr{D}X \; \mathrm{Pf}\mathscr{M} \; e^{-S_b} = \int \mathscr{D}X \; e^{-S_{\mathrm{eff}}}$$

$$S_{\mathrm{eff}} = S_b - \ln(\mathrm{Pf}\mathscr{M})$$

expected cause of SSB

C. Rotational Symmetry: SO(10)

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IKKT MODEL: SSB AND SIGN PROBLEM

$$Z = \int \mathscr{D}X \; \mathrm{Pf}\mathscr{M} \; e^{-S_b} = \int \mathscr{D}X \; e^{-S_{\mathrm{eff}}}$$

$$S_{\mathrm{eff}} = S_b - \ln(\mathrm{Pf}\mathscr{M})$$

$$S_b = -\frac{1}{4} N \,\mathrm{tr}\left([X_\mu, X_\nu]^2\right)$$

- After integrating out fermions, we obtain Fermion matrix $\mathscr{M}$:

  anti-symmetric matrix: $16(N^2-1) \times 16(N^2-1)$

  complex nature: $\boxed{\mathrm{Pf}\mathscr{M} = |\mathrm{Pf}\mathscr{M}|\, e^{i\theta}}$

  θ fluctuates wildly ~~$\theta \approx 0$~~

- MC and $1/D$ expansion to bosonic IKKT model - No SSB
  Hotta, Nishimura and Tsuchiya (1998)

- MC to phase-quenched IKKT model with $|\mathrm{Pf}\mathscr{M}|$ - No SSB
  Ambjorn, Anagnostopoulos, Bietenholz, Hotta and Nishimura (2000)
  Anagnostopoulos, Azuma and Nishimura (2013)

  why phase quenched?

  $e^{-S_{\mathrm{eff}}} \rightarrow$ ~~probability weight~~

  sign-problem in Monte Carlo!

- Phase $e^{i\theta}$ responsible for SSB!

  how to incorporate complex phase?

  complex Langevin

COMPLEX LANGEVIN DYNAMICS IN A NUTSHELL

- Complex partition functions: $Z = \int \mathscr{D}\Phi \; e^{-(S_{re}+iS_{im})}$

$e^{-(S_{re}+iS_{im})} \rightarrow$ ~~probability weight~~

sign-problem in Monte Carlo!

Stochastic Quantization
expectation values ↔ equilibrium values

Klauder (1983), Parisi (1983)
Damgaard and Huffel (1987)

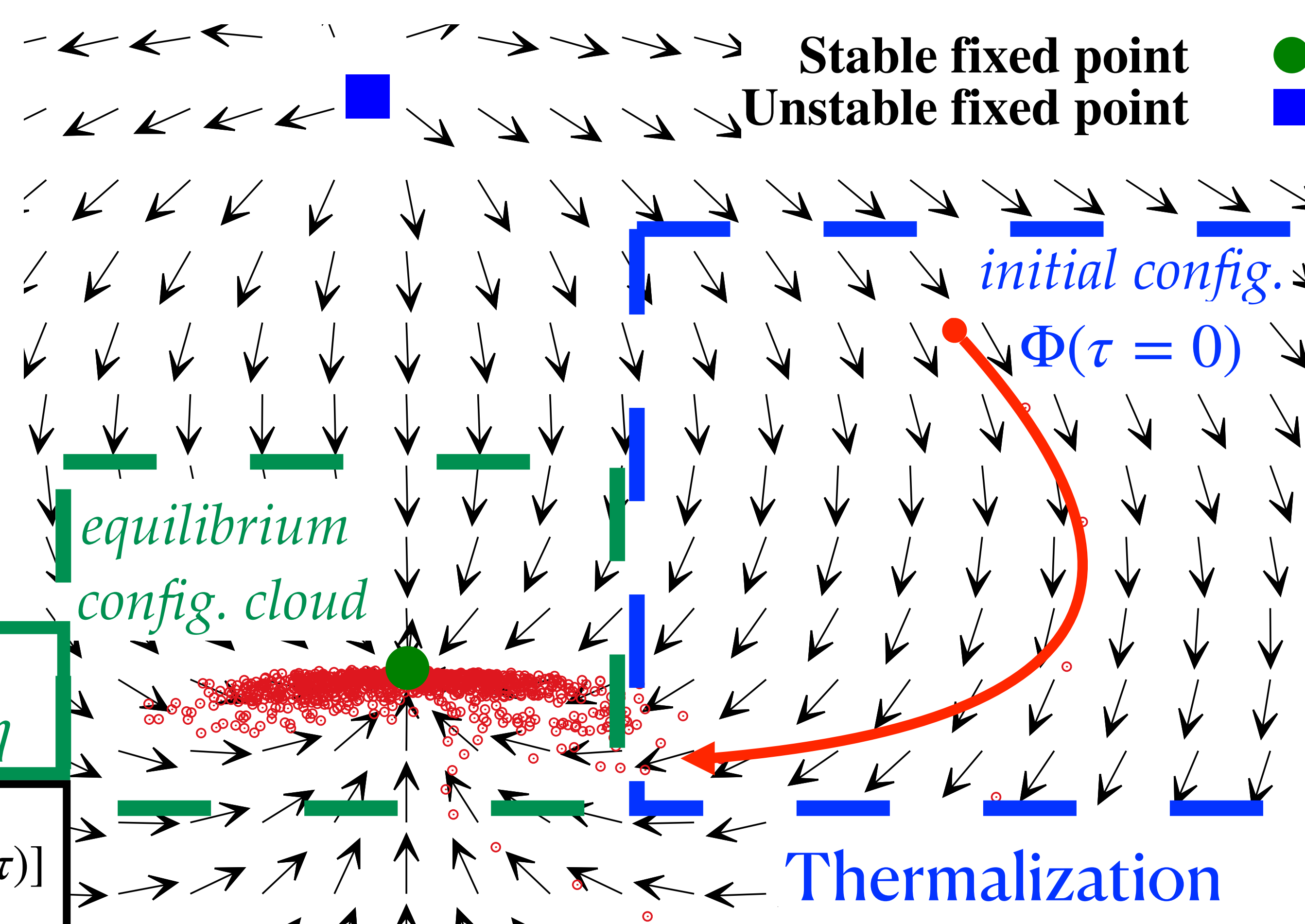
$$\Phi = \phi_x + i\phi_y$$

- Consider 0-d theory: action $S[\Phi]$

$$\partial_\tau \Phi(\tau) = v(\Phi,\tau) + \eta(\tau)$$

$$-\frac{\delta S[\Phi]}{\delta \Phi(\tau)} = v(\Phi,\tau)$$

$$\langle \eta(\tau) \rangle = 0$$
$$\langle \eta(\tau)\eta(\tau') \rangle = 2\delta(\tau - \tau')$$



$$\langle \mathscr{O}[\Phi(\tau)] \rangle_\eta$$

$$\langle \mathscr{O}[\Phi(\tau)] \rangle_\eta = \int d\phi_x d\phi_y \; P[\phi_x, \phi_y; \tau] \; \mathscr{O}[\Phi(\tau)]$$

COMPLEX LANGEVIN CORRECT CONVERGENCE

- At large Langevin time τ

$$\tau \to \infty : P[\phi_x, \phi_y; \tau] \sim e^{-S[\Phi]} ?$$

- When action is complex, no such proof of convergence exists!
  need correctness criteria

Aarts, James, Seiler, and Stamatescu (2010)

$$\langle L^T \mathcal{O} \rangle = 0$$

L: Langevin operator in Fokker-Planck equation

$$\frac{\partial P[\phi_x, \phi_y; \tau]}{\partial \tau} = L^T P[\phi_x, \phi_y; \tau]$$

Nagata, Nishimura, and Shimasaki (2016)

magnitude of drift : $u = \left| \frac{\partial S[\Phi]}{\partial \Phi} \right|$

Probability distribution $P(u)$ falls off exponentially or faster

APPLYING COMPLEX LANGEVIN METHOD

Bosonic IKKT model

- Langevin evolution of X_μ at Langevin time τ

$$\frac{d(X_\mu)_{ji}}{d\tau} = -\frac{\partial S_b}{\partial (X_\mu)_{ji}} + (\eta_\mu)_{ij}(\tau)$$

$$\frac{\partial S_b}{\partial (X_\mu)_{ji}} = -N\left(\left[X_\nu, [X_\mu, X_\nu]\right]\right)_{ij}$$

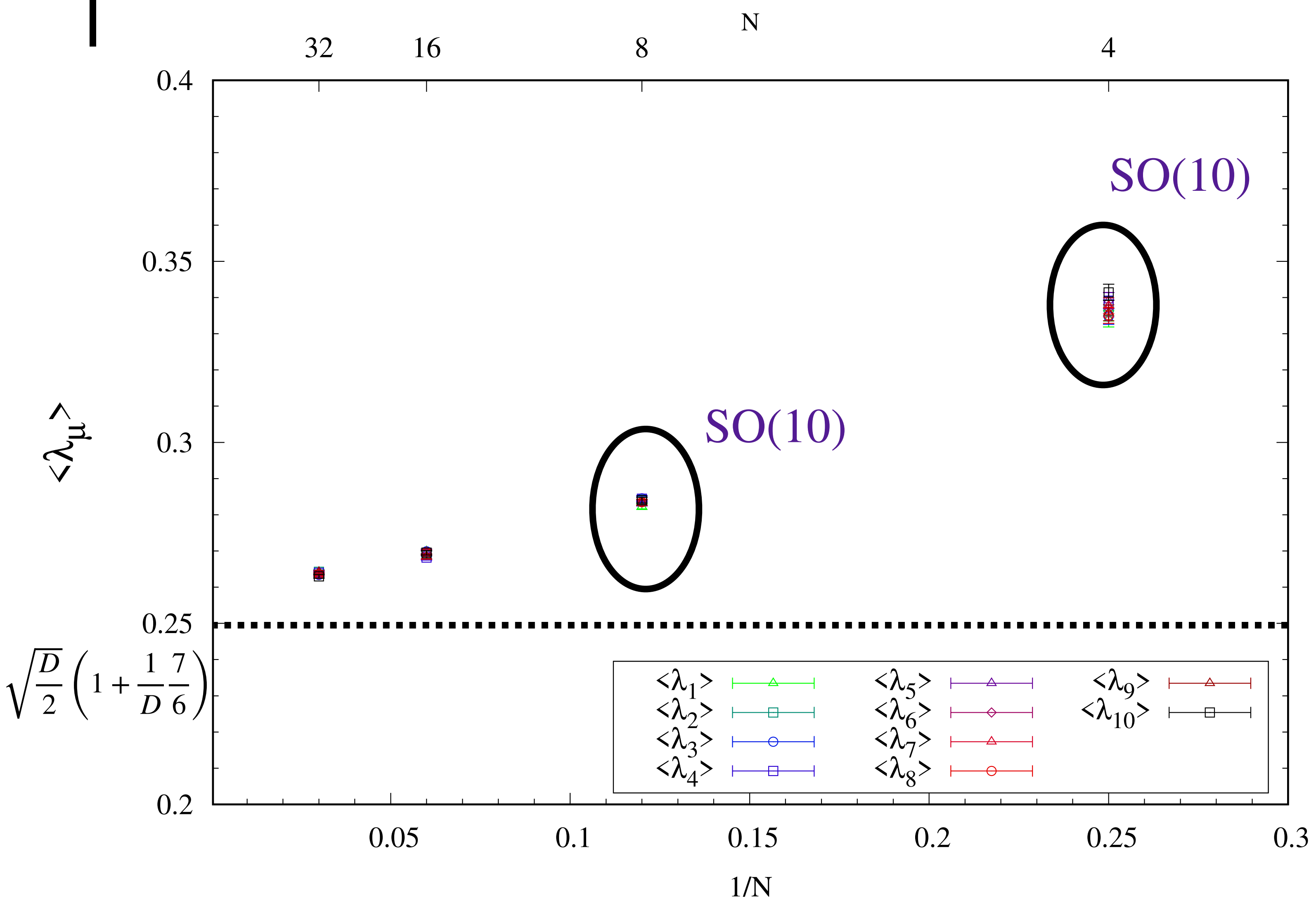
$\eta_{\mu,ij}(\tau)$ Hermitian noise obeying

$$\exp\left(-\frac{1}{4}\int \mathrm{Tr}\left(\eta_\mu^2(\tau)\right) d\tau\right)$$

Extent of space-time as an order parameter

$$\langle \lambda_\mu \rangle = \left\langle \frac{1}{N}\mathrm{tr}\left(X_\mu\right)^2 \right\rangle$$

- Results: No SSB
  SO(10) *at finite N*



APPLYING COMPLEX LANGEVIN METHOD

IKKT model

- Langevin evolution of X_μ at Langevin time τ

$$\frac{d(X_\mu)_{ji}}{d\tau} = -\frac{\partial S_{\text{eff}}}{\partial (X_\mu)_{ji}} + (\eta_\mu)_{ij}(\tau)$$

$$\frac{\partial S_{\text{eff}}}{\partial (X_\mu)_{ji}} = \frac{\partial S_b}{\partial (X_\mu)_{ji}} - \frac{1}{2}\mathrm{Tr}\left(\frac{\partial \mathcal{M}}{\partial (X_\mu)_{ji}}\mathcal{M}^{-1}\right)$$

$\eta_{\mu,ij}(\tau)$ Hermitian noise obeying

$$\exp\left(-\frac{1}{4}\int \mathrm{Tr}\left(\eta_\mu^2(\tau)\right) d\tau\right)$$

- Violation of correctness criteria when

A. Excursion problem
B. Singular drift problem

PROBLEMS WHEN APPLYING CLM FOR IKKT MODEL

Excursion problem:

- Langevin evolution $X_\mu : \mathrm{SU}(N) \to \mathrm{SL}(N, \mathbb{C})$
- X_μ too far away from Hermitian: results are unreliable

GAUGE COOLING

Seiler, Sexty and Stamatescu (2012)

- Hermiticity Norm $\mathcal{N}_H = -\frac{1}{10N} \sum_{\mu=1}^{10} \mathrm{Tr}\left[\left(X_\mu - X_\mu^\dagger\right)^2\right]$ Nagata, Nishimura and Shimasaki (2016)

- X_μ is invariant under $X_\mu \to X'_\mu = g X_\mu g^{-1}$ where $g \in \mathrm{SL}(N, \mathbb{C})$

$\mathcal{N}_H$ not invariant under the extra symmetry

$$g = \mathrm{e}^{\alpha\, \delta\mathcal{N}_\mathrm{H}}, \quad \delta\mathcal{N}_\mathrm{H} = -\frac{1}{N} \sum_{\mu=1}^{10} \left[X_\mu, X_\mu^\dagger\right].$$

α : step size (constant/adaptive)

- Each Langevin step: repeat above extra symmetry → minimized $\mathcal{N}_H$

PROBLEMS WHEN APPLYING CLM FOR IKKT MODEL

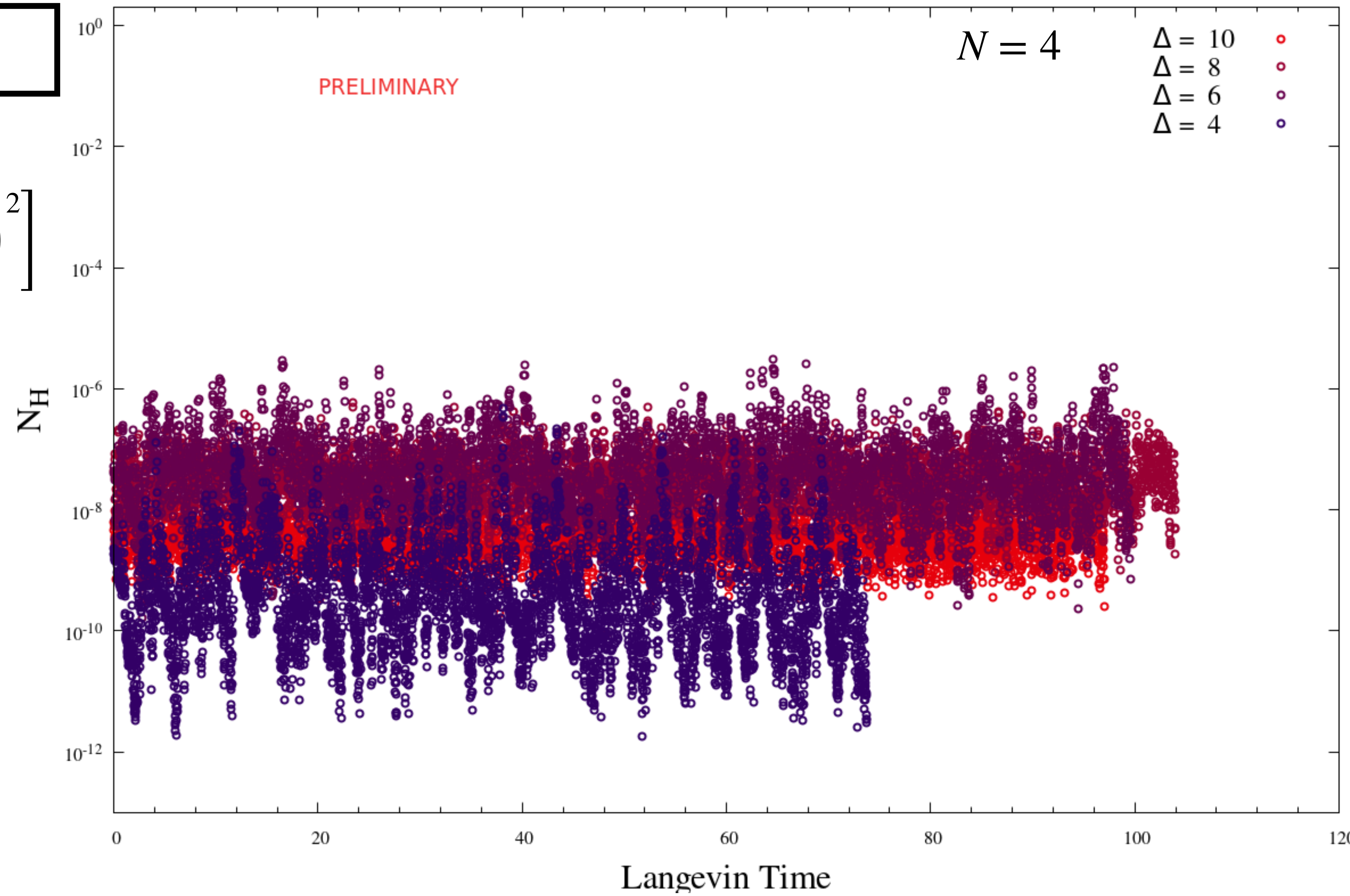
Excursion problem:

- Langevin evolution $X_\mu : \mathrm{SU}(N) \to \mathrm{SL}(N, \mathbb{C})$
- X_μ too far away from Hermitian: results are unreliable

GAUGE COOLING

$$\mathcal{N}_H = -\frac{1}{10N} \sum_{\mu=1}^{10} \mathrm{Tr}\left[\left(X_\mu - X_\mu^\dagger\right)^2\right]$$

Gauge cooling resolves excursion problem



PROBLEMS WHEN APPLYING CLM FOR IKKT MODEL

Singular drift problem:

- Fermion operator $\mathscr{M}$ has near-zero eigenvalues
- The drift term diverges: results unreliable

$$\mathscr{M}_{a\alpha,b\beta} = \frac{N}{2}\Gamma^{\mu}_{\alpha\beta}\mathrm{Tr}\left(X_\mu\left[t^a,t^b\right]\right)$$

$$\frac{\partial S_\mathrm{f}}{\partial (X_\mu)_{ji}} = -\frac{1}{2}\mathrm{Tr}\left(\frac{\partial \mathscr{M}}{\partial (X_\mu)_{ji}}\mathscr{M}^{-1}\right)$$

MASS DEFORMATIONS: $S \to S = S_{\mathrm{IKKT}} + \Delta S$

Ito and Nishimura (2016)

- In general, $\Delta S = \Delta S_b + \Delta S_\mathrm{f}$

$$\Delta S_b \propto \mathrm{Tr}\left(M_{\mu\nu}X_\mu X_\nu\right)$$

$$\Delta S_\mathrm{f} \propto \mathrm{Tr}\left(\psi_\alpha \gamma_{\alpha\beta}\psi_\beta\right)$$

shifts eigenvalue distribution of $\mathscr{M}$

$$\mathscr{M}_{a\alpha,b\beta} \to \tilde{\mathscr{M}}_{a\alpha,b\beta} = \frac{N}{2}\Gamma^{\mu}_{\alpha\beta}\mathrm{tr}\left(X_\mu\left[t^a,t^b\right]\right) + \gamma_{\alpha\beta}\delta_{ab}$$

- Explicitly break the SO(10) rotational symmetry *supersymmetry?*
- Recover original model: $\lim \Delta S \to 0$

INTRODUCING MASS DEFORMATIONS

Mass deformations: $S \to S = S_{\text{IKKT}} + \Delta S$

- Recently, Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis (2020)

breaks SO(10) symmetry: $\Delta S_{\text{b}} = \epsilon \frac{N}{2} \sum_{\mu} m_{\mu} \text{Tr} \left(X_{\mu} \right)^2$

shifts eigenvalue distribution of $\mathscr{M}$: $\Delta S_{\text{f}} = - m_{\text{f}} \frac{N}{2} \text{Tr} \left(\psi_{\alpha} \gamma_{\alpha\beta} \psi_{\beta} \right); \ \gamma = i \Gamma^8 \Gamma^{9\dagger} \Gamma^{10}$

supersymmetry explicitly broken

- Probe SSB in original IKKT model by extrapolations:

N=128
A. $N \to \infty$
B. $\epsilon \to 0$
C. $m_{\text{f}} \to 0$

$\Longrightarrow$ $\text{SO}(10) \to \text{SO}(d) \times \text{SO}(10-d)$

$\text{SO}(d)$ symmetric vacua

- Results:

$m_{\text{f}} = 3.0 \implies \text{SO}(7)$
$m_{\text{f}} = 1.4 \implies \text{SO}(4)$
$m_{\text{f}} = 1.0,\ 0.9,\ 0.7 \implies \text{SO}(3)$

consistent with GEM study
Nishimura, Okubo, Sugino (2011)

SUSY PRESERVING MASS DEFORMATIONS

MASS DEFORMATIONS: $S \to S = S_{\rm IKKT} + \Delta S$

- We introduce SUSY preserving mass deformations: Bonelli and Natuurkunde (2002)

N_3: fermion mass matrix

$$\Delta S = N \,\mathrm{tr} \left(-M^{\mu\nu} X_\mu X_\nu + \frac{i}{8} \overline{\psi} N_3 \psi + i N^{\mu\nu\rho} X_\mu [X_\nu, X_\rho] \right)$$

$M_{\mu\nu}$: bosonic mass matrix *Myers term*

- SUSY preserving mass deformation (similar to BMN)

$$\delta X^\mu = -\frac{1}{2} \bar{\epsilon} \Gamma^\mu \psi \qquad \delta \psi = \frac{1}{4} \left[X^\mu, X^\nu \right] \Gamma_{\mu\nu} \epsilon - \frac{i}{16} X^\mu \left(\Gamma_\mu N_3 + 2 N_3 \Gamma_\mu \right) \epsilon$$

provided flux constraint: $\left[N_3 (\Gamma^\mu N_3 + 2 N_3 \Gamma^\mu) + 4^3 M^{\mu\nu} \Gamma_\nu \right] \epsilon = 0$

- Simplest solution:

$$N_3 = -\Delta \Gamma^8 \Gamma^{9^\dagger} \Gamma^{10}, \; N^{\mu\nu\rho} = \frac{\Delta}{3!} \sum_{\mu\nu\rho=8}^{10} \epsilon^{\mu\nu\rho} \quad \text{and} \quad M = -\frac{\Delta^2}{4^3} \left(\mathbb{I}_7 \oplus 3 \mathbb{I}_3 \right)$$

SUSY PRESERVING MASS DEFORMATIONS

MASS DEFORMATIONS: $S \to S = S_{\text{IKKT}} + \Delta S$

- We introduce SUSY preserving mass deformations:

fermion mass term

$$\Delta S = N\ \mathrm{Tr}\left(\frac{\Delta^2}{4^3} \sum_{i=1}^{7} X_i^2 + \frac{3\Delta^2}{4^3} \sum_{a=8}^{10} X_a^2 + \frac{i\Delta}{3!} \epsilon^{abc} X_a [X_b, X_c] - \frac{N\Delta}{8} \psi_\alpha \gamma_{\alpha\beta} \psi_\beta \right)$$

bosonic mass terms *Myers term*

$$\gamma_{\alpha\beta} = i(\Gamma^8 \Gamma^{9\dagger} \Gamma^{10})_{\alpha\beta}$$

- Avoids singular drift problem?

shifts eigenvalue distribution of $\mathscr{M}$

$$\mathscr{M}_{a\alpha,b\beta} \to \tilde{\mathscr{M}}_{a\alpha,b\beta} = \frac{N}{2} \Gamma^\mu_{\alpha\beta} \mathrm{tr}\left(X_\mu \left[t^a, t^b \right] \right) - \frac{N\Delta}{8} \gamma_{\alpha\beta} \delta_{ab}$$

SUSY PRESERVING MASS DEFORMATIONS

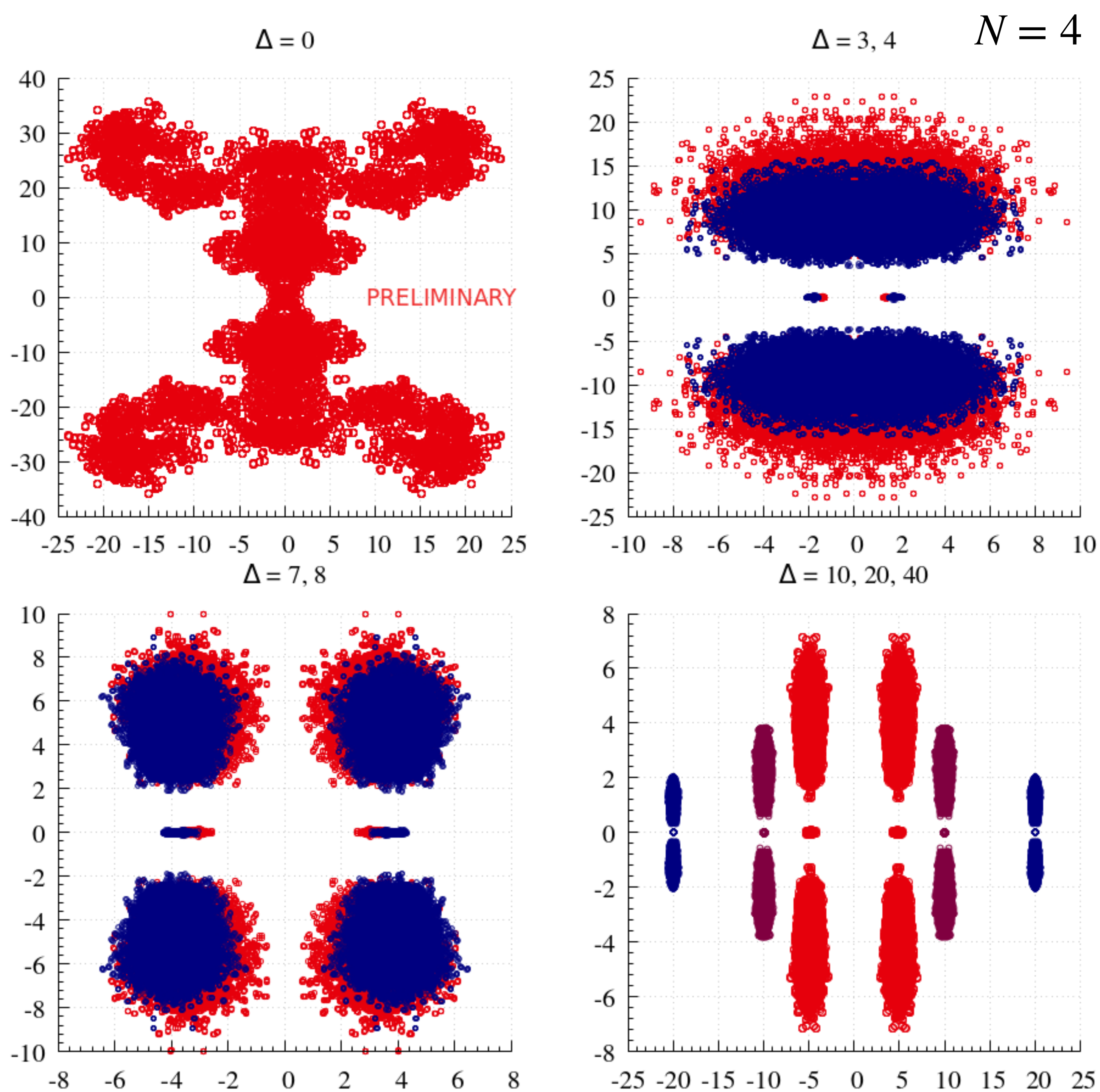
MASS DEFORMATIONS: $S \to S = S_{\text{IKKT}} + \Delta S$

Eigenvalues distribution $\mathscr{M}(\Delta)$

$$\mathscr{M}_{a\alpha,b\beta} \to \mathscr{M}_{a\alpha,b\beta}(\Delta) = \frac{N}{2}\Gamma^{\mu}_{\alpha\beta}\text{tr}\left(X_{\mu}\left[t^a, t^b\right]\right) - \frac{N\Delta}{8}\gamma_{\alpha\beta}\delta_{ab}$$

SUSY preserving mass deformations evade singular drift problem!

A. Excursion problem
B. Singular drift problem



PROBING SSB: SUSY PRESERVING MASS DEFORMATIONS

Mass deformations: $S \to S = S_{\mathrm{IKKT}} + \Delta S$ → $\lim \Delta S \to 0$

- SUSY preserving mass deformations:

fermion mass term

$$\Delta S = N \ \mathrm{Tr} \left(\frac{\Delta^2}{4^3} \sum_{i=1}^{7} X_i^2 + \frac{3\Delta^2}{4^3} \sum_{a=8}^{10} X_a^2 + \frac{i\Delta}{3!} \epsilon^{abc} X_a [X_b, X_c] - \frac{N\Delta}{8} \psi_\alpha \gamma_{\alpha\beta} \psi_\beta \right)$$

bosonic mass terms *Myers term*

$$\gamma_{\alpha\beta} = i(\Gamma^8 \Gamma^{9\dagger} \Gamma^{10})_{\alpha\beta}$$

- Probe SSB in IKKT model

A. $N \to \infty$
B. $\Delta \to 0$

Bosonic IKKT model with Myers

$$S_{\mathrm{b}} = S_{\mathrm{bIKKT}} + N\ \mathrm{Tr}\left(\frac{\Delta^2}{4^3}\sum_{i=1}^{7} X_i^2 + \frac{3\Delta^2}{4^3}\sum_{a=8}^{10} X_a^2 + \frac{i\Delta}{3!}\epsilon^{abc} X_a [X_b, X_c] \right)$$

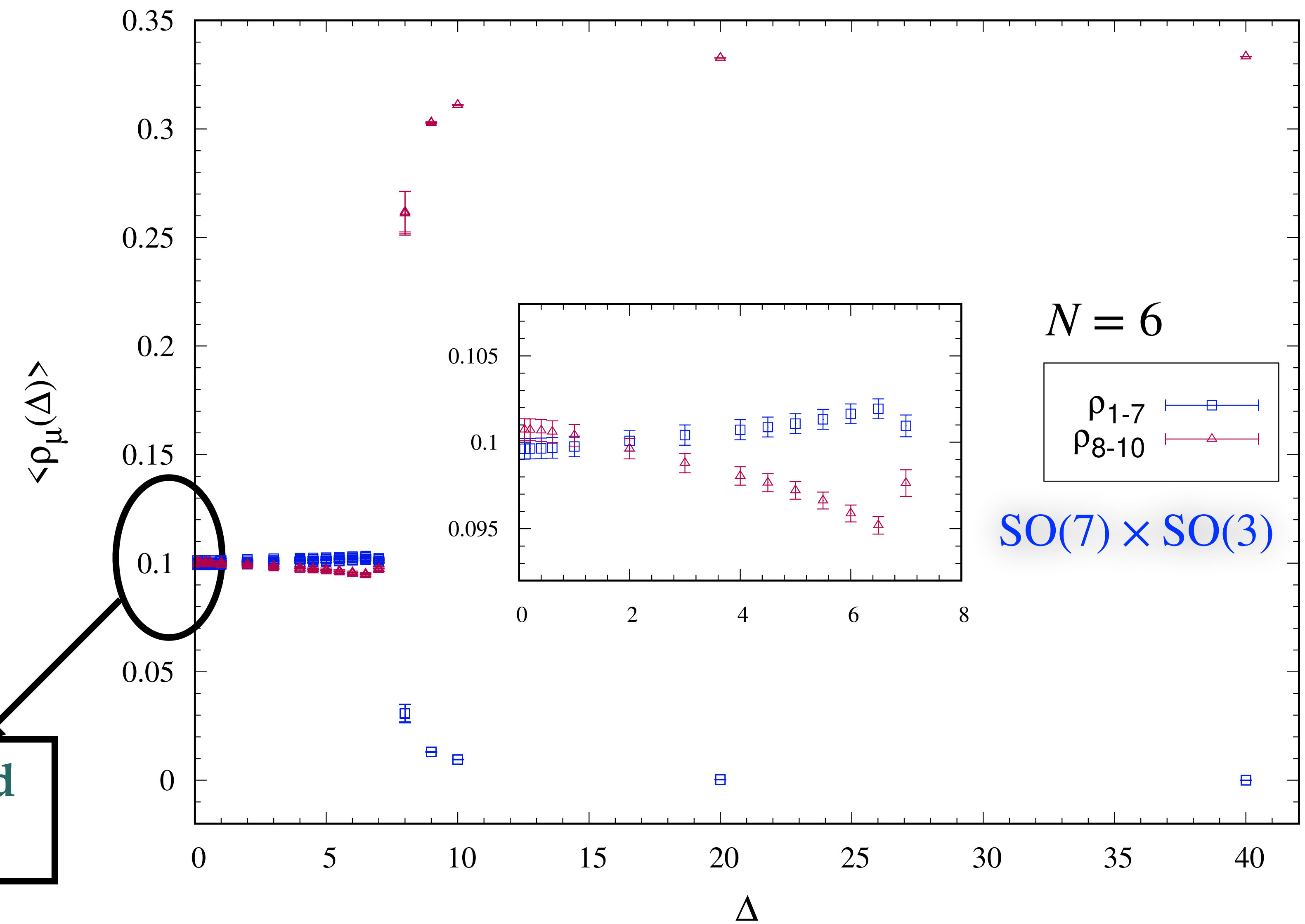
bosonic mass terms *Myers term*

Extent of space-time as an order parameter

$$\lambda_\mu(\Delta) = \frac{1}{N}\mathrm{tr}\left(X_\mu\right)^2$$

$$\rho_\mu(\Delta) = \frac{\lambda_\mu(\Delta)}{\sum_\mu \lambda_\mu(\Delta)}$$

SO(10) restored
$\Longrightarrow$ No SSB



IKKT model

fermion mass term $\gamma_{\alpha\beta} = i(\Gamma^8 \Gamma^{9^\dagger} \Gamma^{10})_{\alpha\beta}$

$$S = S_{\mathrm{IKKT}} + N\ \mathrm{Tr}\left(\frac{\Delta^2}{4^3} \sum_{i=1}^{7} X_i^2 + \frac{3\Delta^2}{4^3} \sum_{a=8}^{10} X_a^2 + \frac{i\Delta}{3!} \epsilon^{abc} X_a [X_b, X_c] - \frac{N\Delta}{8} \psi_\alpha \gamma_{\alpha\beta} \psi_\beta \right)$$

bosonic mass terms *Myers term*

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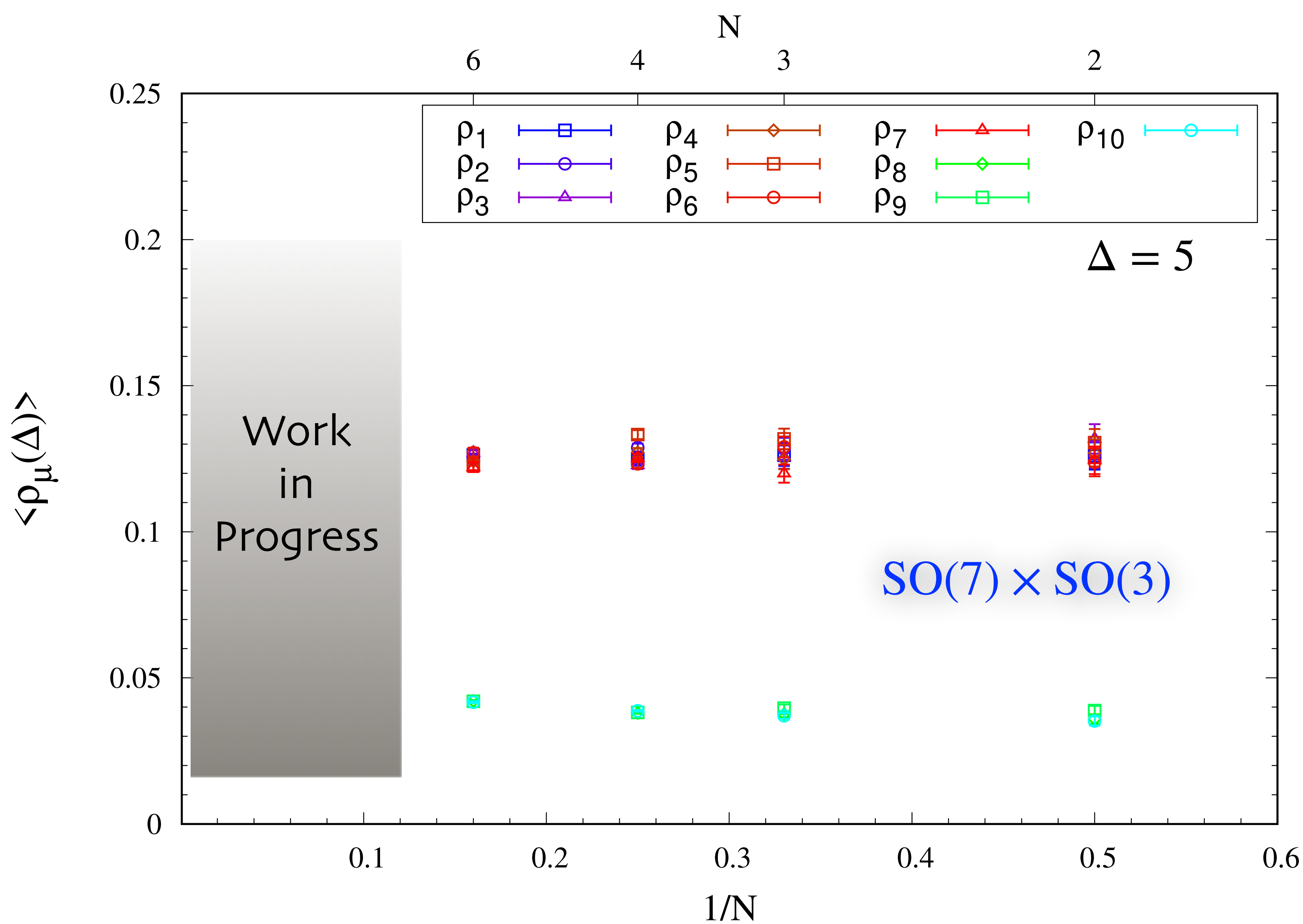


PROBING SSB: SUSY PRESERVING MASS DEFORMATIONS

MASS DEFORMATIONS: $S \to S = S_{\text{IKKT}} + \Delta S$ → $\lim \Delta S \to 0$

- Spontaneously broken: $SO(10) \to SO(d) \times SO(10-d)$

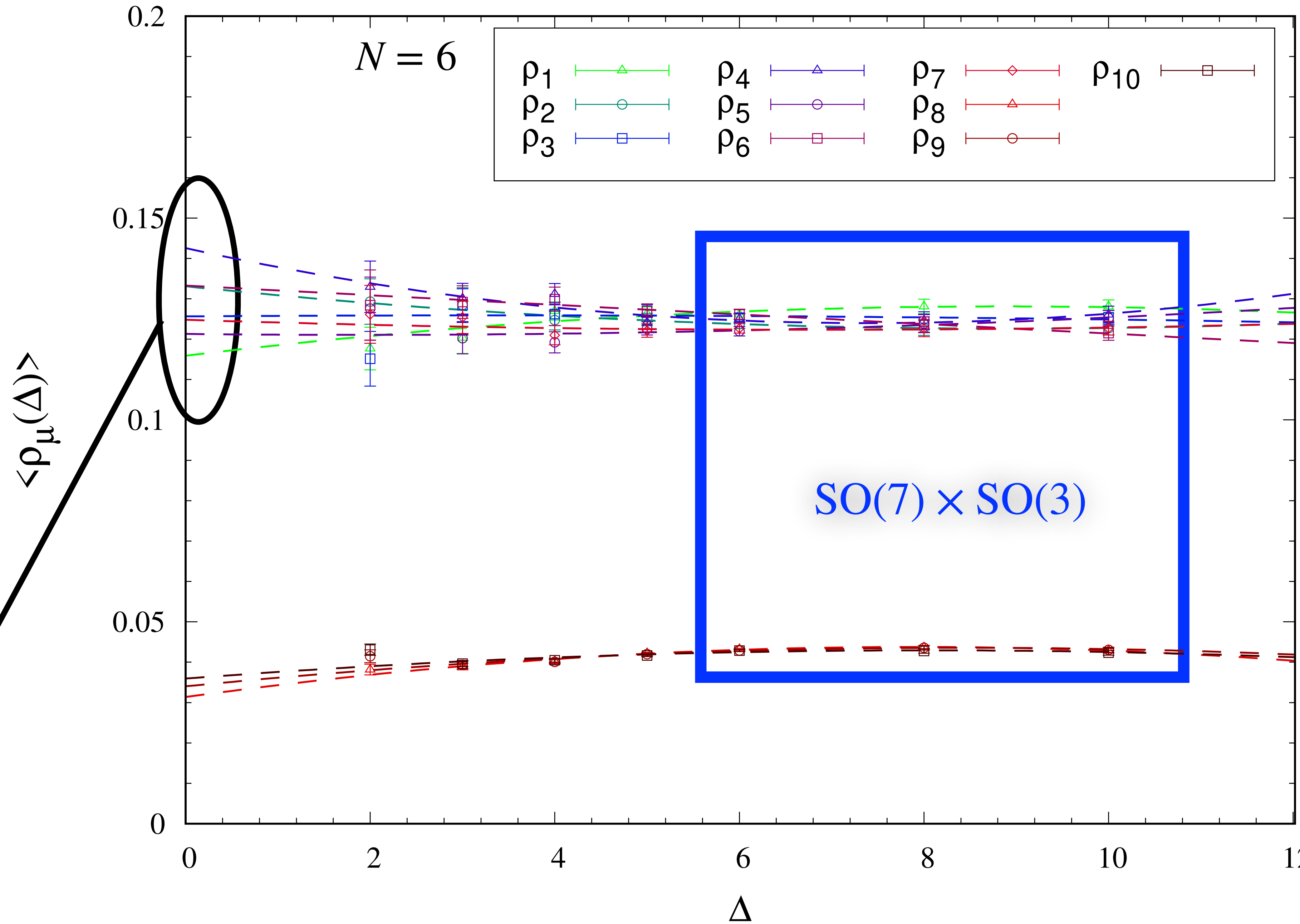
Extent of space-time as an order parameter

$$\lambda_\mu(\Delta) = \frac{1}{N}\text{tr}\left(X_\mu\right)^2$$

$$\rho_\mu(\Delta) = \frac{\lambda_\mu(\Delta)}{\sum_\mu \lambda_\mu(\Delta)}$$

- Even for $N = 6$ extents are not same

$\Longrightarrow$ hints SO(d) : $d < 7$



SUMMARY AND FUTURE WORK

- Complex Langevin is a powerful tool to study theories with complex actions.
- Complex Langevin method can be used to detect spontaneous symmetry breaking
- SUSY-preserving mass deformations successfully evade singular drift problem
- Large-N extrapolations for SUSY-preserving mass deformations and investigate SO(d) symmetric vacuum *(Noisy estimators)*
- Probe SO(D) spontaneous symmetry breaking using SUSY-preserving deformations in dimensionally reduced $D = 4,\ 6$ super Yang-Mills models

  Ito and Nishimura (2016)
  Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis (2018)

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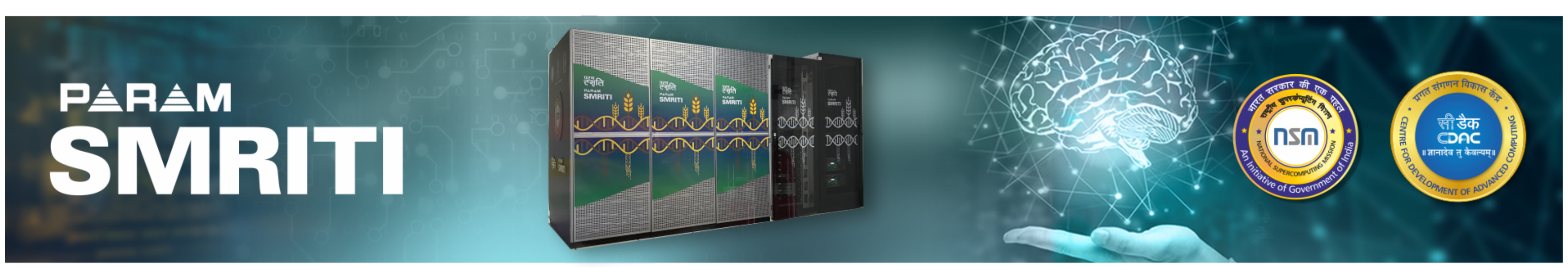

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