Active Force Patterning in a Mixture of Contractile Stresslets

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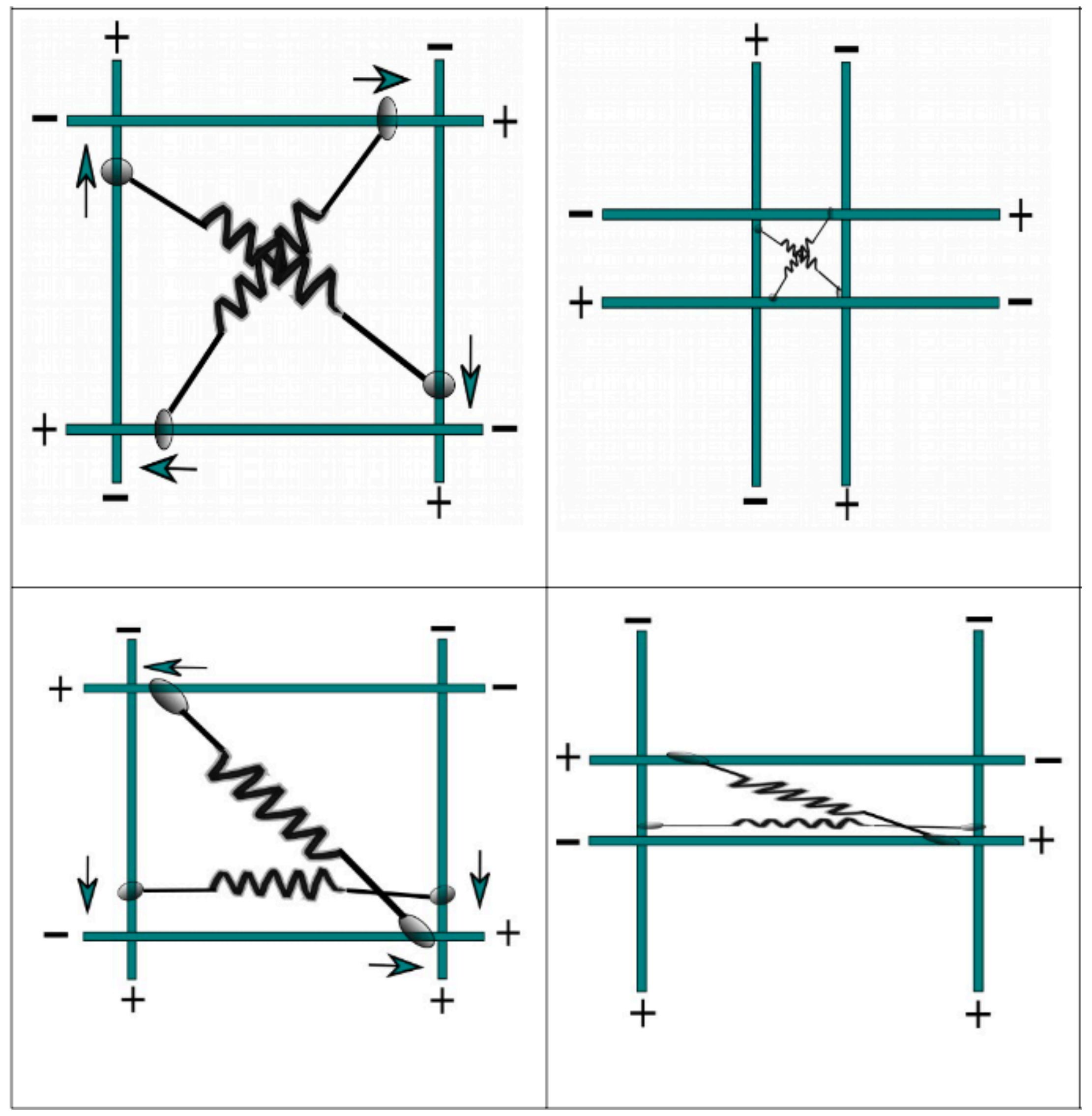
Stresslets: force generators and force sensors

activity — force generators

catch or slip bonds (strain dependent binding-unbinding) — force sensors

Examples: Contractile Myosin-Actin Complex, Extensile Kinesin-Microtubule Complex

Actomyosin Complexes

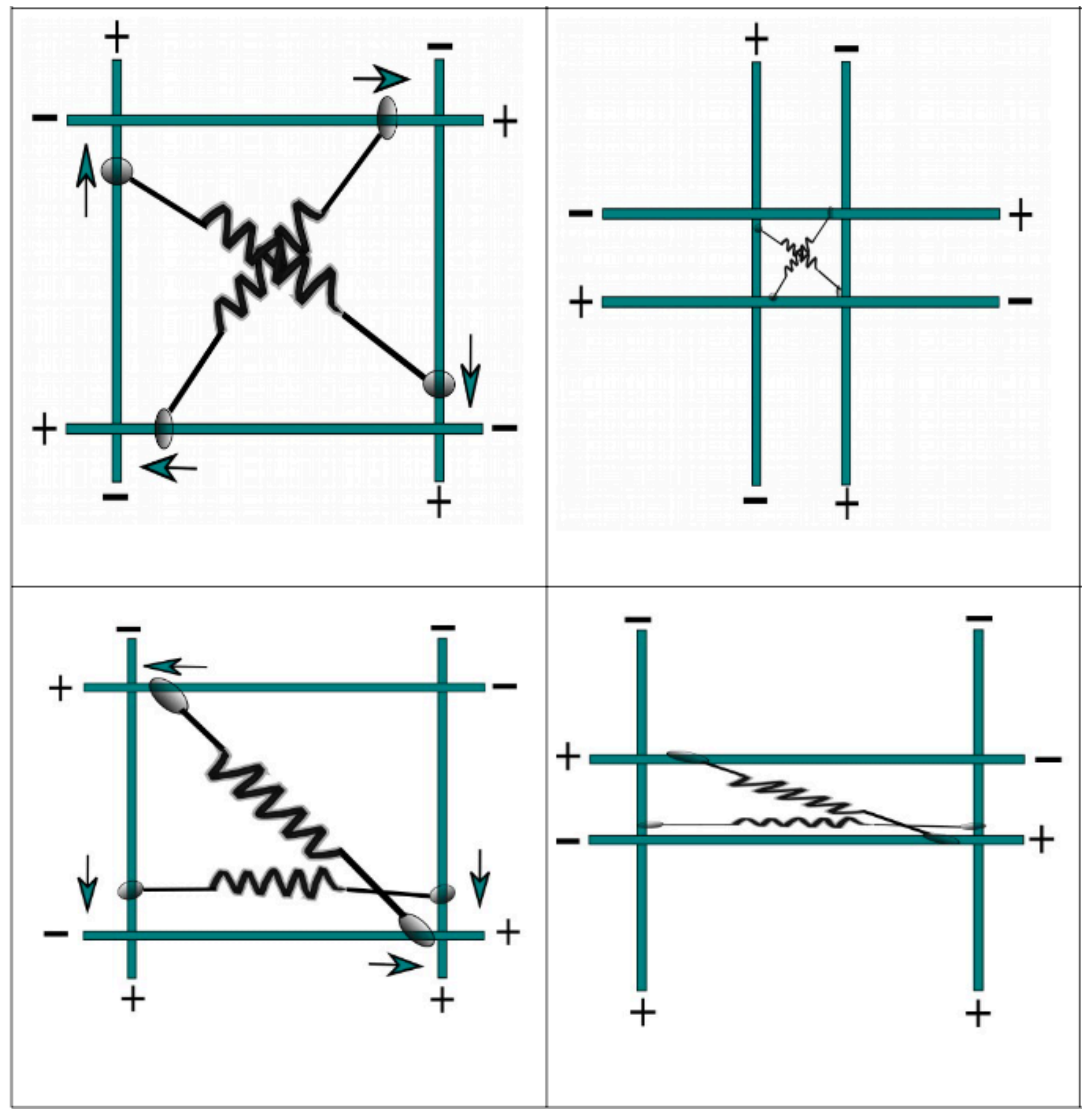


Stresslets: force generators and force sensors

↑ activity (under force generators)

↑ catch or slip bonds (strain dependent binding-unbinding) (under force sensors)

Actomyosin Complexes



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localizations of molecular <u>stresslets</u>

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patterning of <u>stress</u> at the 'macroscale' of the cell

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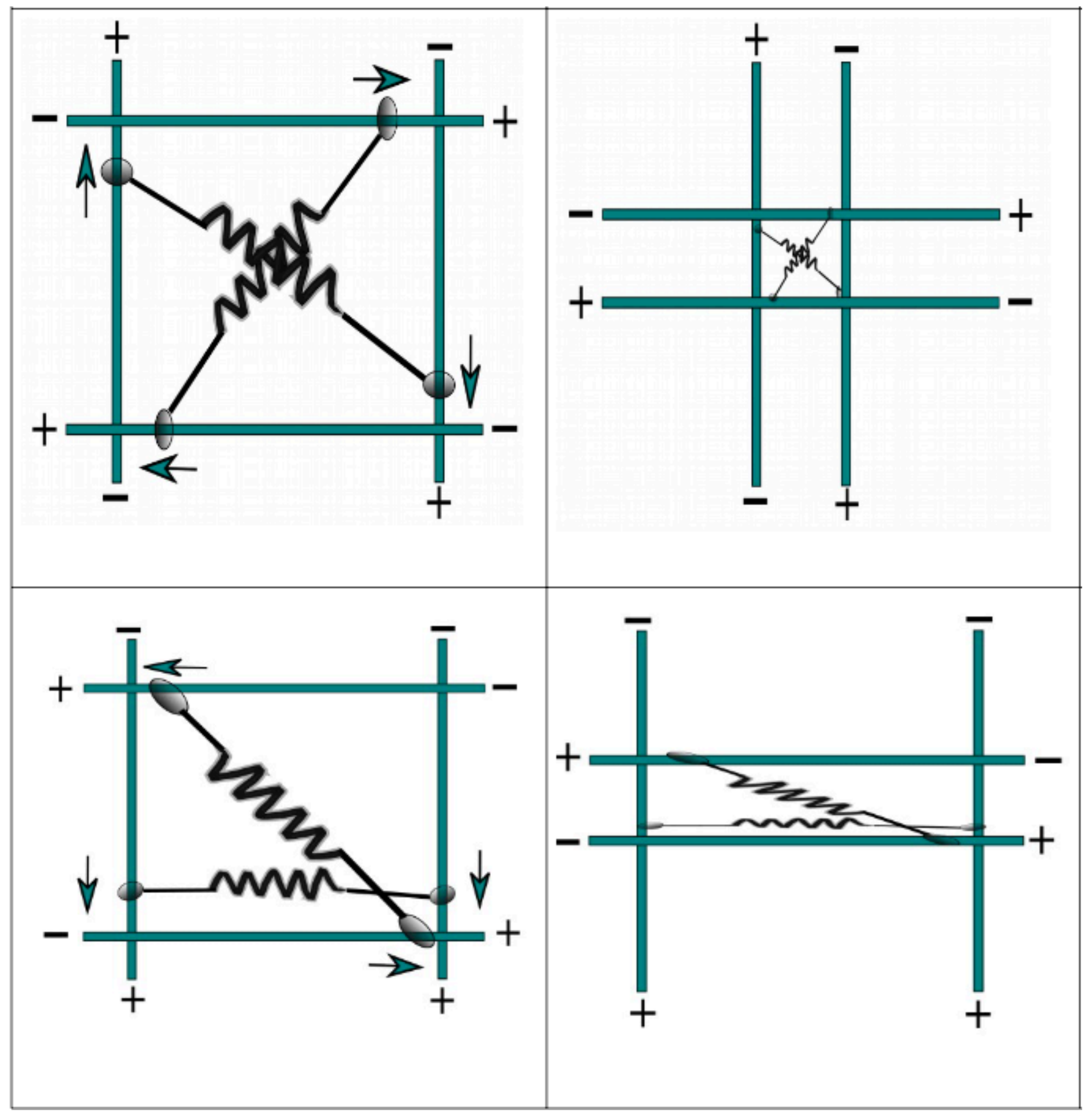

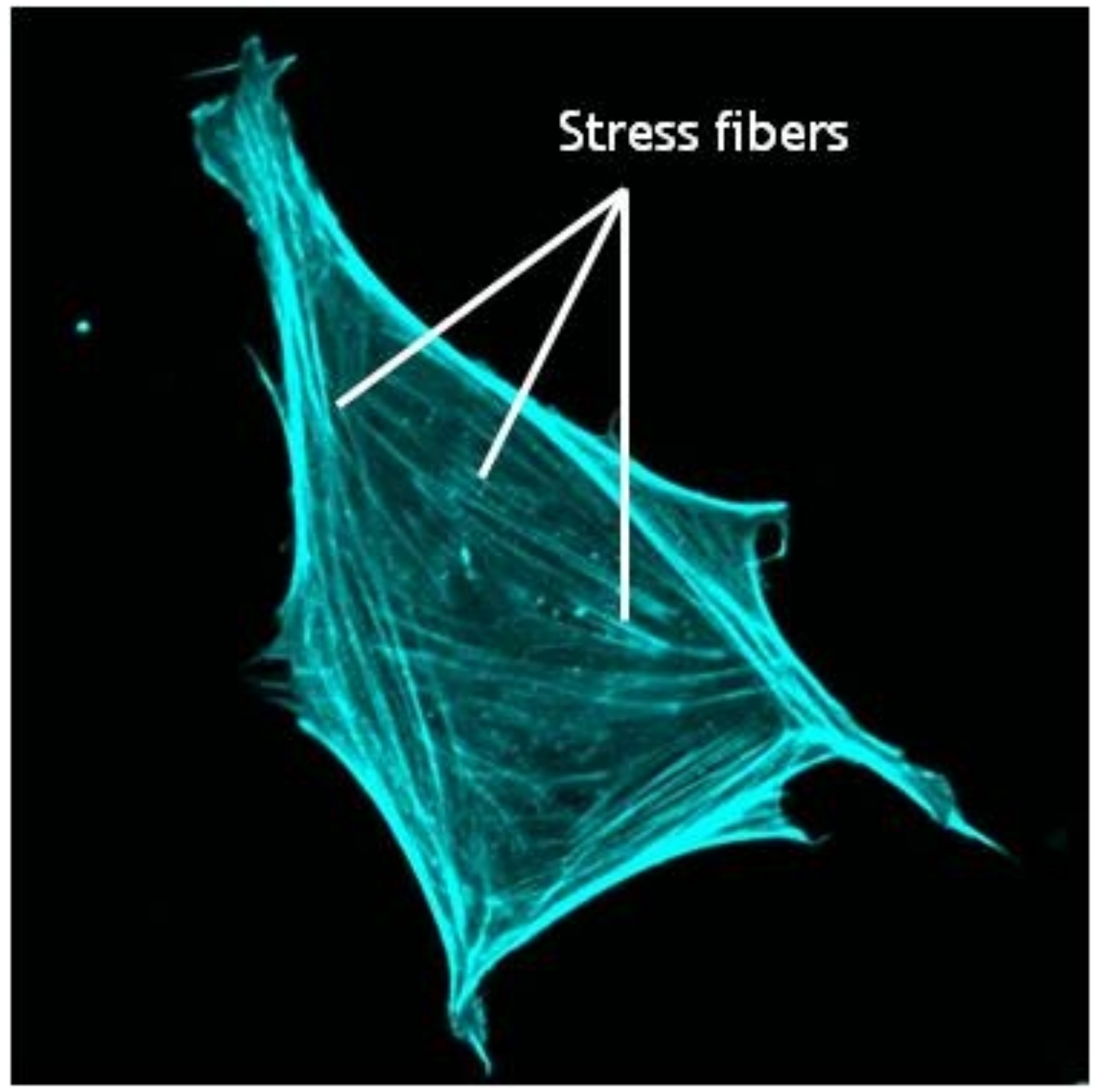

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Spatiotemporal Segregation of a Uniform Mixture of Stresslets

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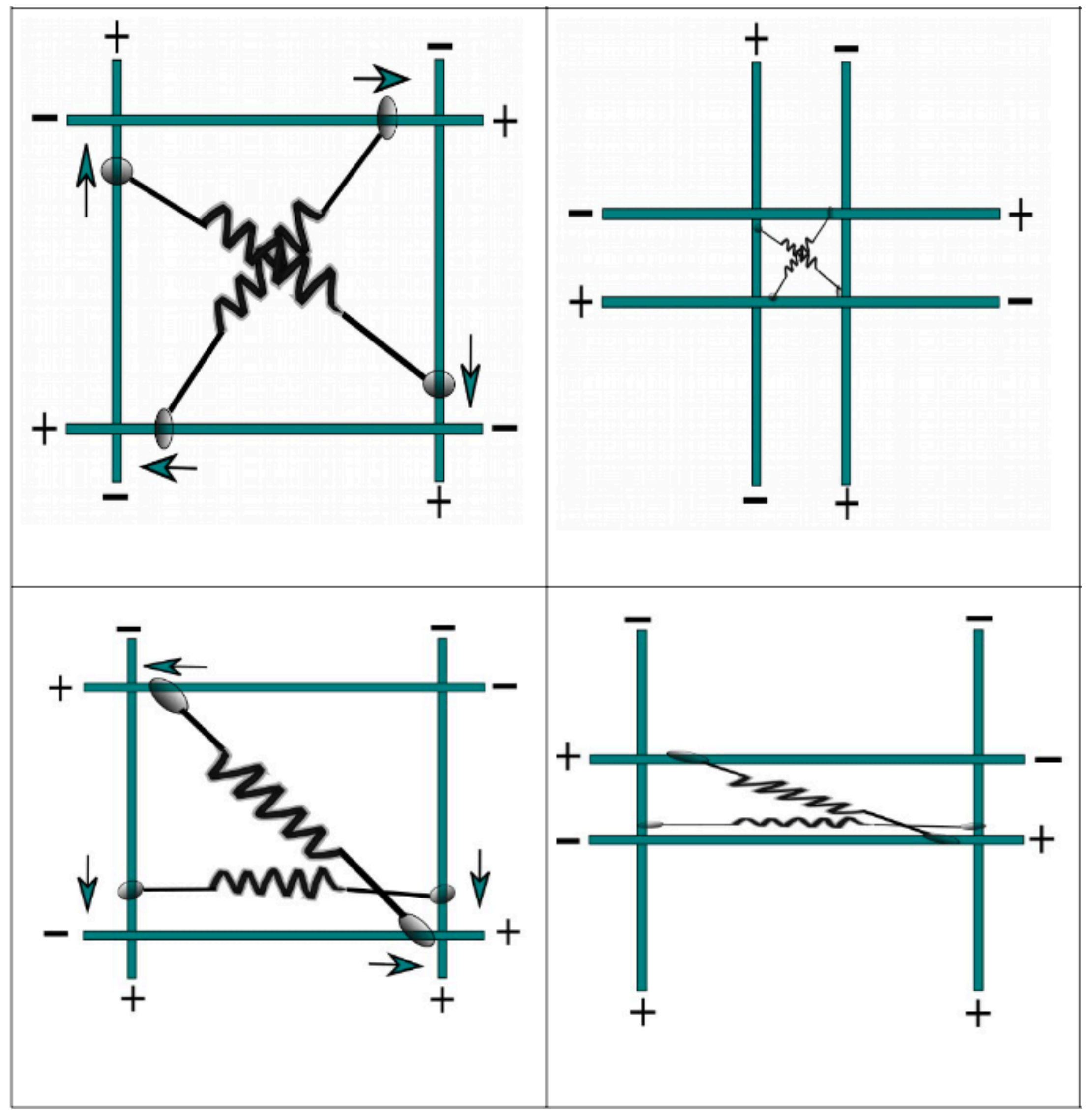

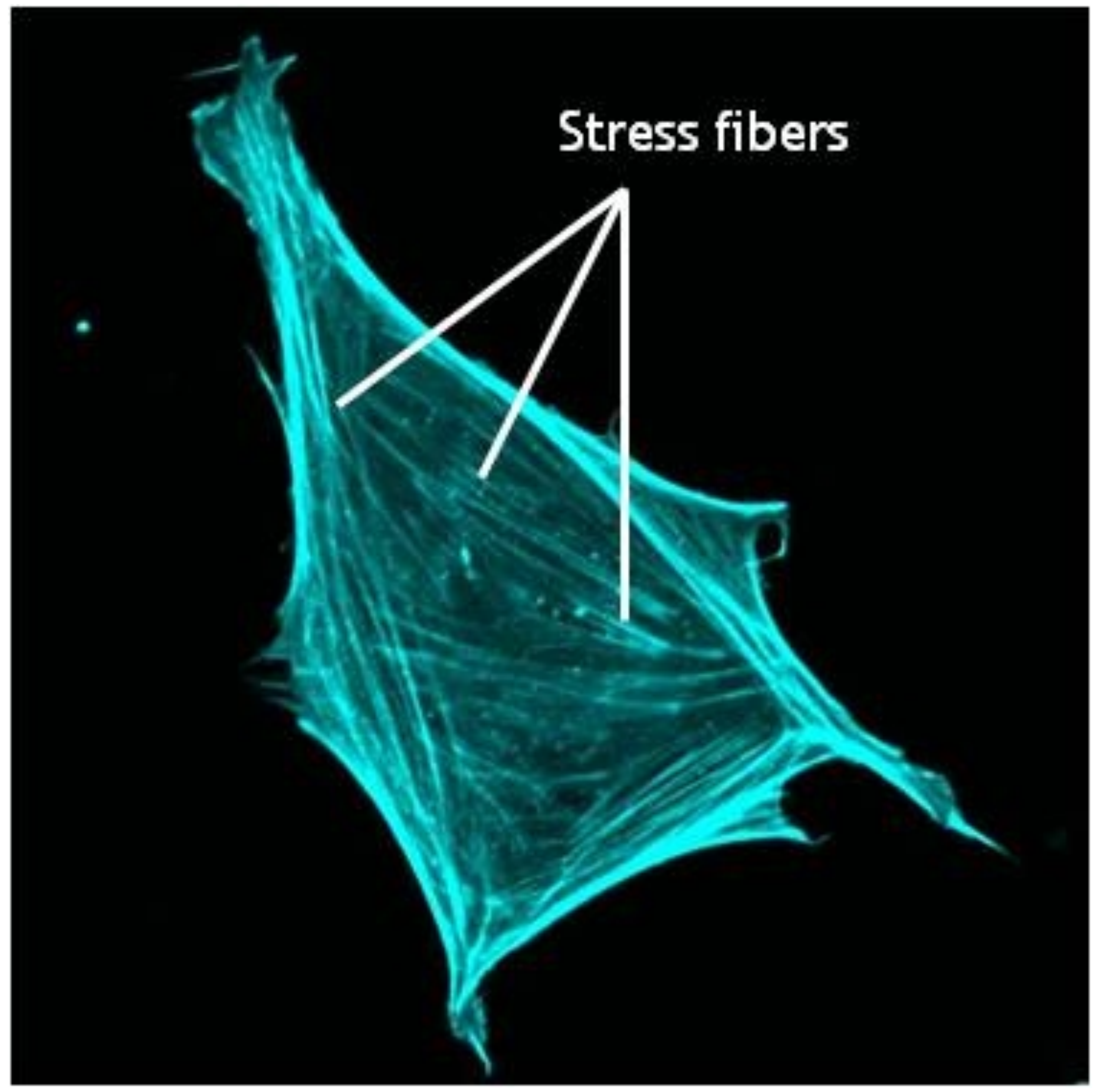

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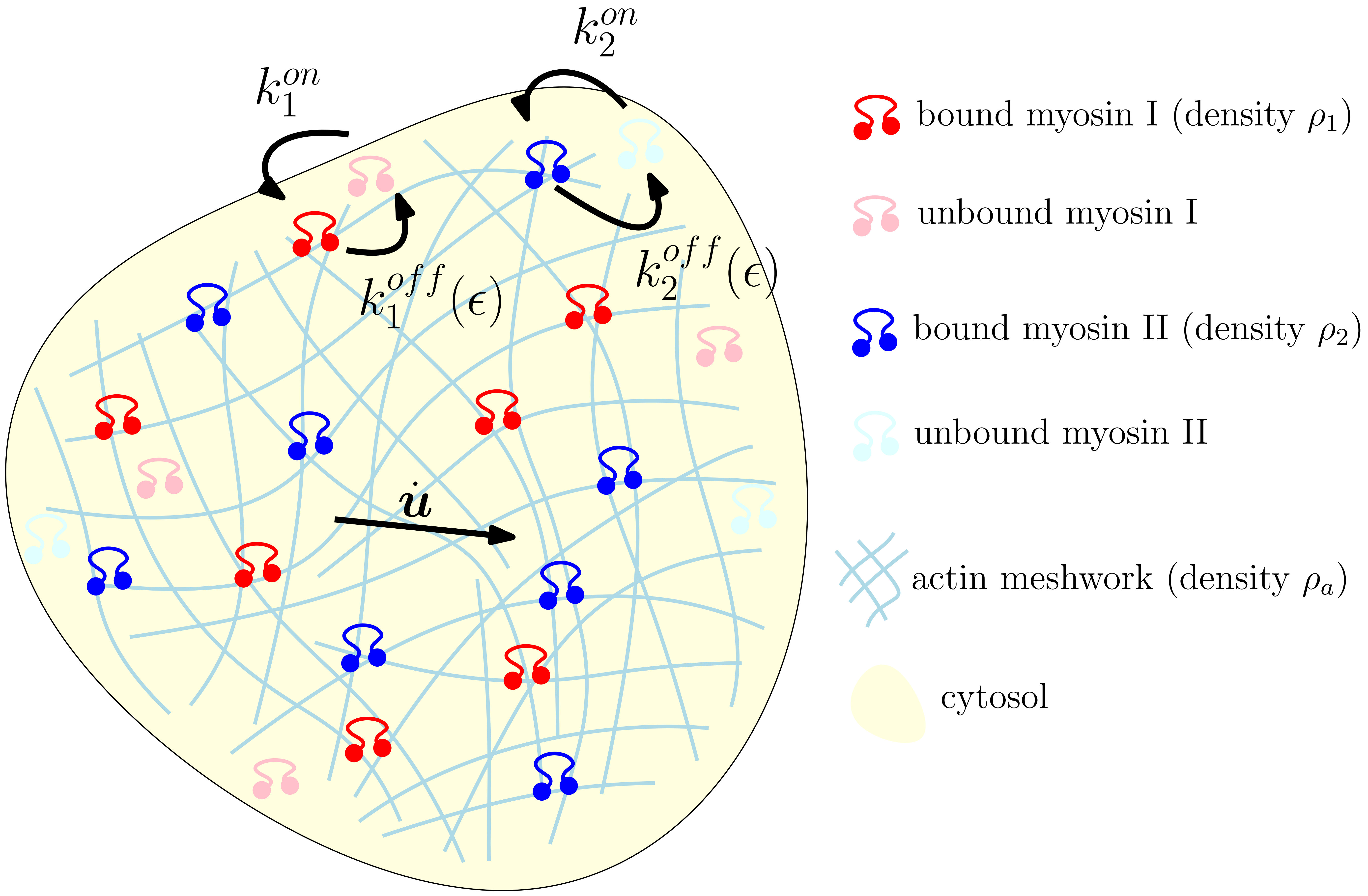
Hydrodynamic Model



Hydrodynamic Model

Hydrodynamics of the Meshwork

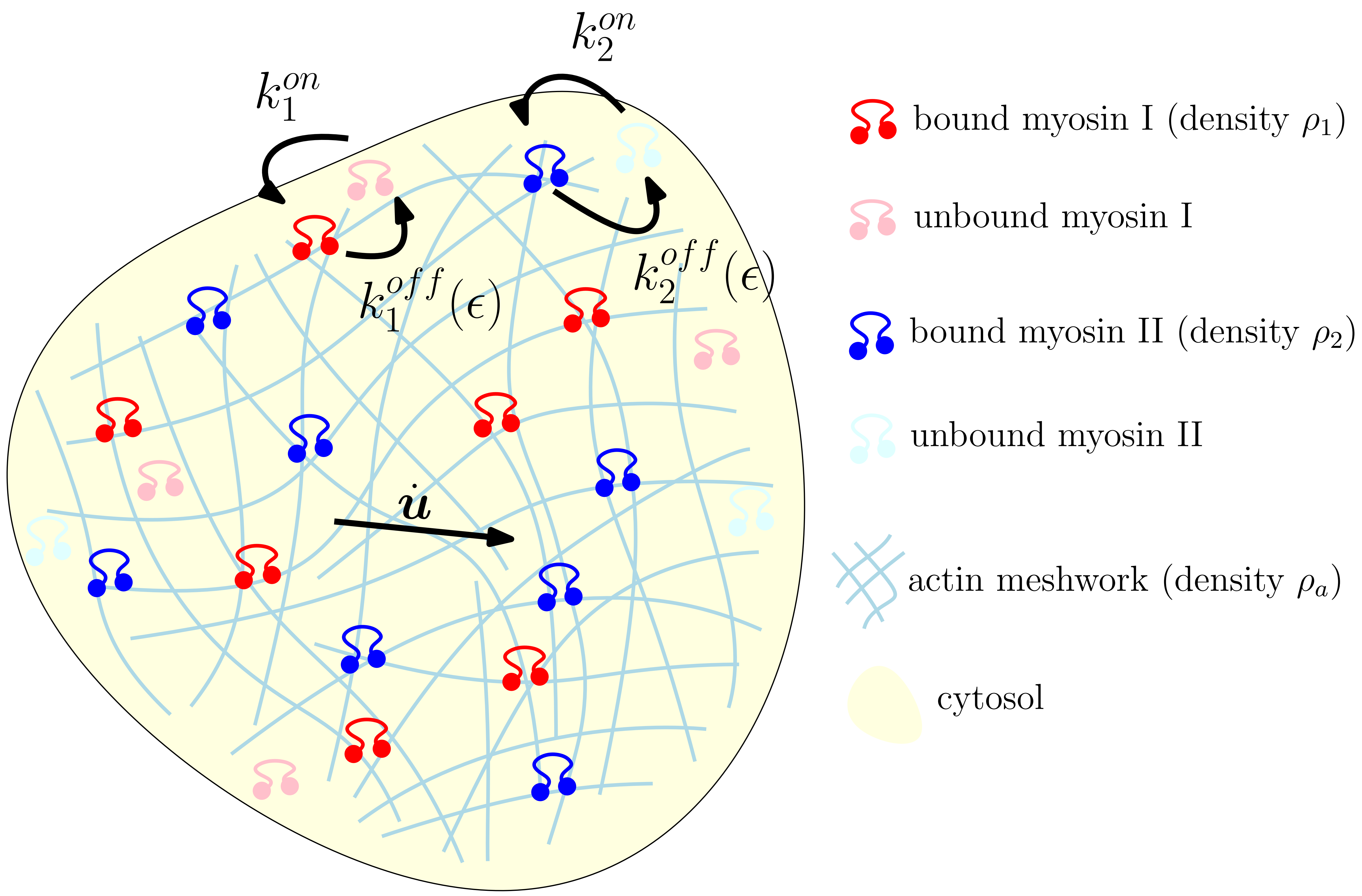
$$\Gamma\, \dot{\boldsymbol{u}} = \nabla \cdot \boldsymbol{\sigma}$$

$$\dot{\rho}_a + \nabla \cdot (\rho_a\, \dot{\boldsymbol{u}}) = 0 \quad \Rightarrow \quad \delta\rho_a := \rho_a - \rho_a^0 \propto -\mathrm{tr}\boldsymbol{\epsilon}$$

$$\boldsymbol{\sigma} = \frac{\delta F}{\delta \boldsymbol{\epsilon}} \;+\; \chi(\rho_a)\,(\zeta_1\rho_1 + \zeta_2\rho_2)\,\Delta\mu\,\mathbf{I} \;+\; \eta\,\dot{\boldsymbol{\epsilon}}$$

elastic (passive) — active — viscous (passive)

$$F = \int \left(\frac{1}{2}\mathbb{C}[\boldsymbol{\epsilon}]\cdot\boldsymbol{\epsilon} + C\,\delta\rho_a\,\mathrm{tr}\boldsymbol{\epsilon} + \frac{A}{2}\delta\rho_a^2\right) dx$$



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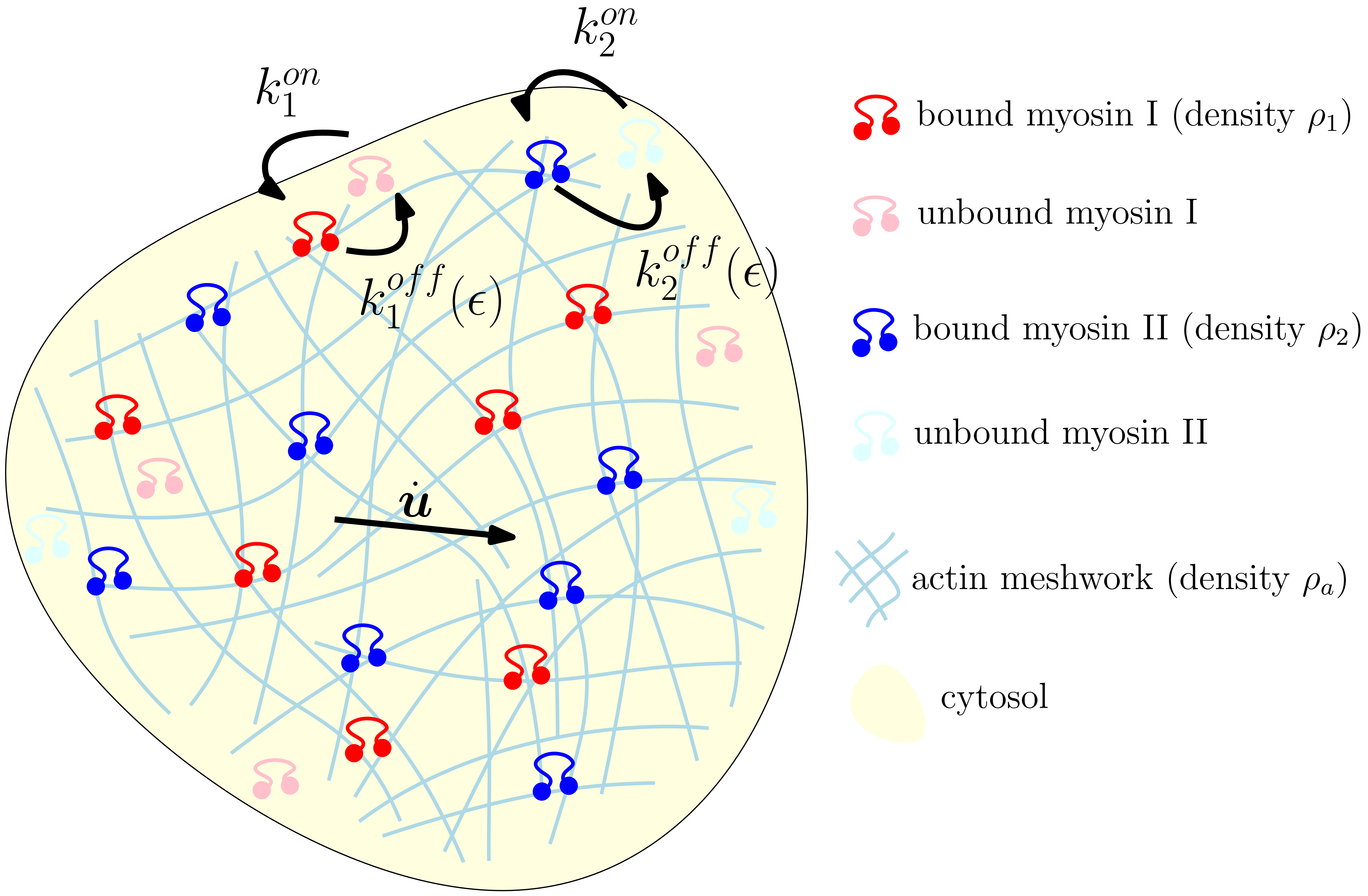

Hydrodynamics of the Stresslets

$$\dot{\rho}_1 + \nabla\cdot(\rho_1\,\dot{\boldsymbol{u}}) = D\,\nabla^2\rho_1 + k_1^{on}\,\rho_a - k_1^{off}(\boldsymbol{\epsilon})\,\rho_1$$

$$\dot{\rho}_2 + \nabla\cdot(\rho_2\,\dot{\boldsymbol{u}}) = D\,\nabla^2\rho_2 + k_2^{on}\,\rho_a - k_2^{off}(\boldsymbol{\epsilon})\,\rho_2$$

$$k_1^{off}(\boldsymbol{\epsilon}) = k_{10}^{off}\,e^{\alpha_1\,\mathrm{tr}\boldsymbol{\epsilon}}$$

$$k_2^{off}(\boldsymbol{\epsilon}) = k_{20}^{off}\,e^{\alpha_2\,\mathrm{tr}\boldsymbol{\epsilon}}$$

$\alpha_{1,2} > 0$: catch bond

$\alpha_{1,2} < 0$: slip bond

Governing Hydrodynamic Equations

scalar version, non-dimensionalized

$$\epsilon = \partial_x u \qquad \rho := \frac{\rho_1 + \rho_2}{2}, \qquad \phi := \frac{\rho_1 - \rho_2}{2}$$

Segregation Order Parameter

$$\dot{u} = \partial_x \sigma,$$

$$\dot{\rho} + \partial_x(\rho\,\dot{u}) = D\,\partial^2_{xx}\rho + \left(1 - \frac{C}{A}\,\epsilon\right) - \left(k_1 + k_3\,\epsilon + o(\epsilon)\right)\rho - \left(k_2 + k_4\,\epsilon + o(\epsilon)\right)\phi$$

$$\dot{\phi} + \partial_x(\phi\,\dot{u}) = D\,\partial^2_{xx}\phi + k_5\left(1 - \frac{C}{A}\epsilon\right) - \left(k_1 + k_3\,\epsilon + o(\epsilon)\right)\phi - \left(k_2 + k_4\,\epsilon + o(\epsilon)\right)\rho$$

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scalar version, non-dimensionalized

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$$\sigma = \sigma_0 + \tilde{B}\,\epsilon + B_2\,\epsilon^2 + B_3\,\epsilon^3 + \dot{\epsilon} + o(\epsilon^3)$$

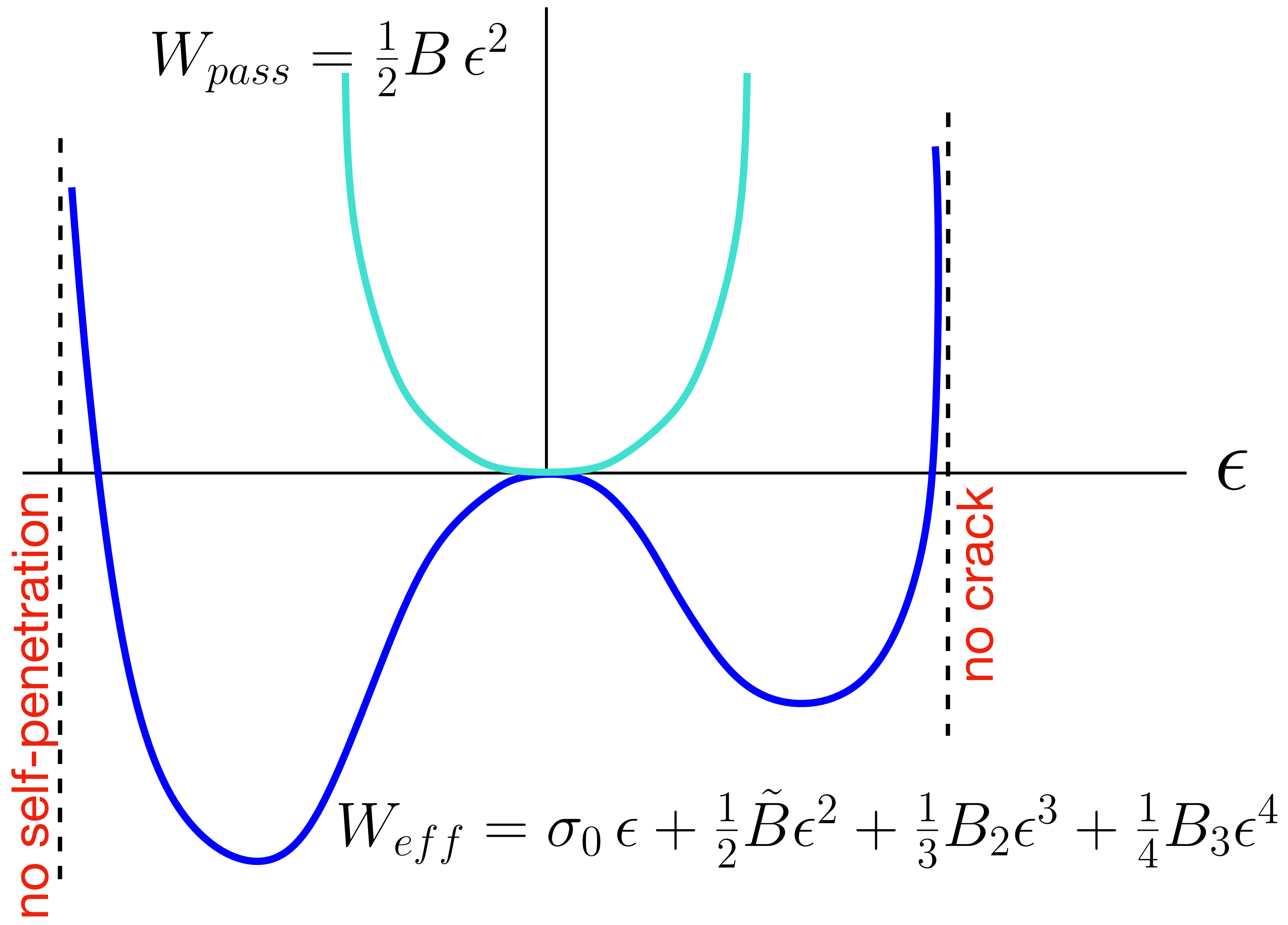
$$\sigma_0 := 2\,\chi(\rho_a^0)\big(\zeta_{\rm avg}\,\rho + \zeta_{\rm rel}\,\phi\big),$$ active back stress

$$\tilde{B} := B - \frac{C^2}{A} - 2\,\chi'(\rho_a^0)\,\frac{C}{A}\big(\zeta_{\rm avg}\,\rho + \zeta_{\rm rel}\,\phi\big),$$ activity renormalised linear elastic modulus

$$B_2 := \chi''(\rho_a^0)\left(\frac{C}{A}\right)^2\big(\zeta_{\rm avg}\,\rho + \zeta_{\rm rel}\,\phi\big),$$

$$B_3 := -\frac{\chi'''(\rho_a^0)}{3}\left(\frac{C}{A}\right)^3\big(\zeta_{\rm avg}\,\rho + \zeta_{\rm rel}\,\phi\big).$$

active non-linear elastic moduli (B_2, B_3)



Linear Stability of the homogeneous unstrained uniform steady state

$$u_0 = 0 \qquad \phi_0 = 0 \qquad \rho_0 = 1/k_1$$

$$\begin{bmatrix} \delta\dot{\hat{u}}(t,q) \\ \delta\dot{\hat{\rho}}(t,q) \\ \delta\dot{\hat{\phi}}(t,q) \end{bmatrix} = \begin{bmatrix} -\frac{\tilde{B}_0\, q^2}{1+q^2} & \frac{2\,\zeta_{\text{avg}}\, iq}{1+q^2} & \frac{2\,\zeta_{\text{rel}}\, iq}{1+q^2} \\ -\left(\frac{C}{A} + \frac{k_3}{k_1} - \frac{\tilde{B}_0 q^2}{(1+q^2)k_1}\right) iq & -Dq^2 - k_1 + \frac{2q^2\,\zeta_{\text{avg}}}{(1+q^2)k_1} & -k_2 + \frac{2q^2\,\zeta_{\text{rel}}}{(1+q^2)k_1} \\ -\left(\frac{k_2}{k_1}\frac{C}{A} + \frac{k_4}{k_1}\right) iq & -k_2 & -Dq^2 - k_1 \end{bmatrix} \begin{bmatrix} \delta\hat{u}(t,q) \\ \delta\hat{\rho}(t,q) \\ \delta\hat{\phi}(t,q) \end{bmatrix}$$

General solution for distinct eigenvalues: $\sum_{i=1}^{3} c_i e^{\lambda_i(q)\,t}\,\mathbf{V}_i(q)$

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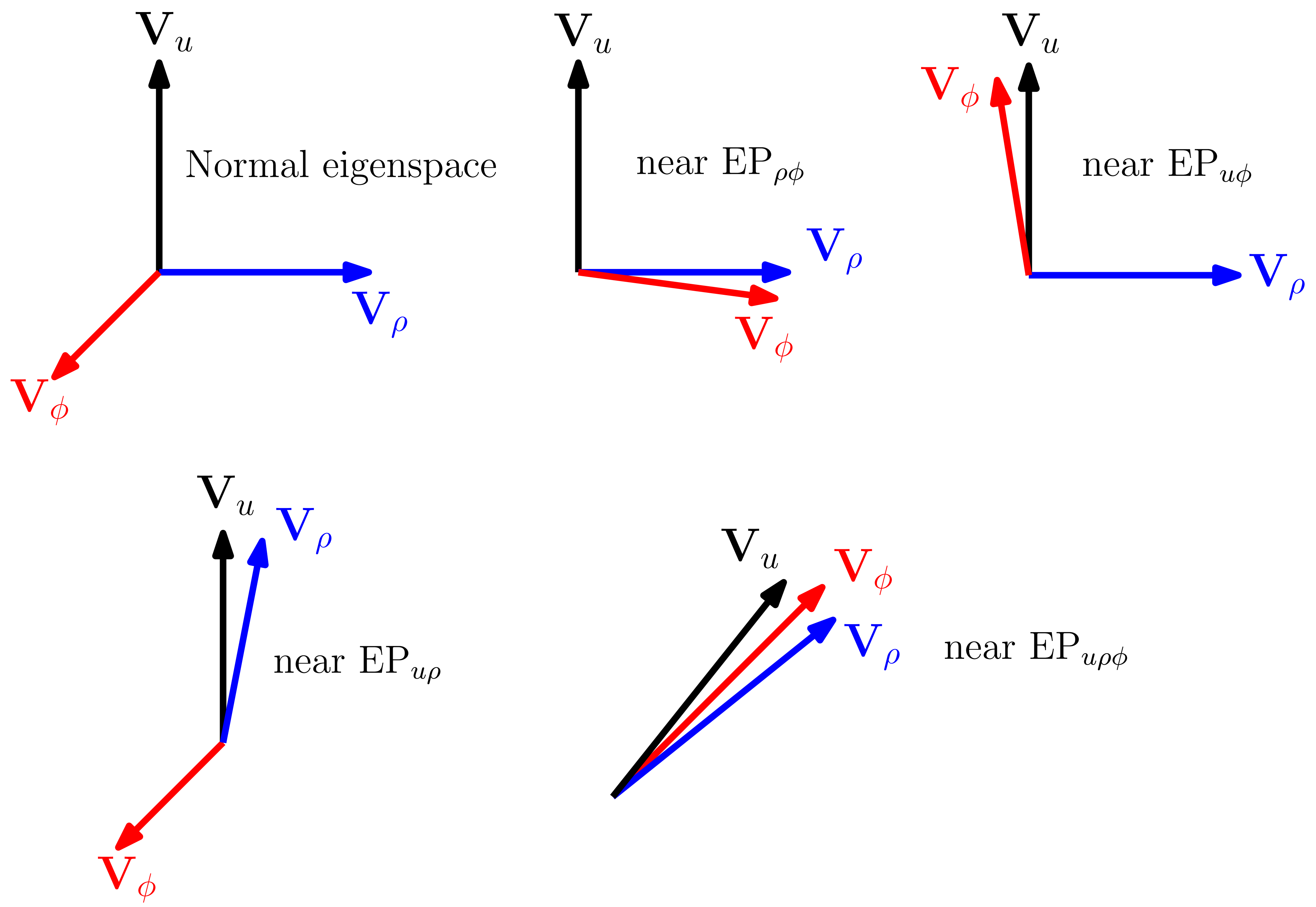
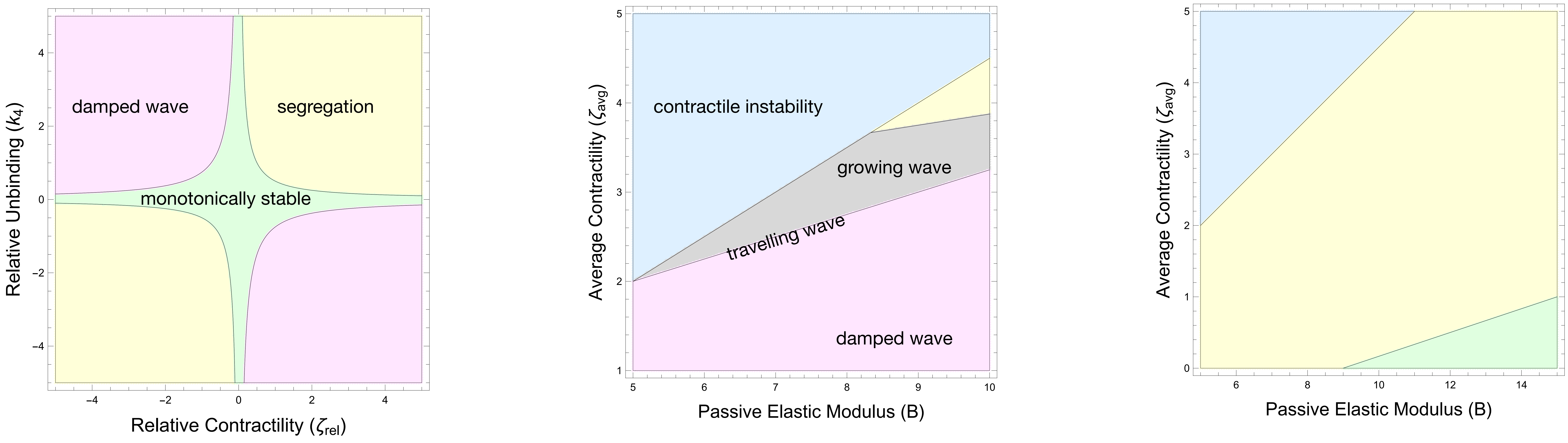


Non-Hermitian Dynamics:

Non-orthogonal Eigenvectors,

Exceptional Point where some of the Eigenvalues and Eigenvectors coalesce

Phase Diagram from Linear Stability Analysis



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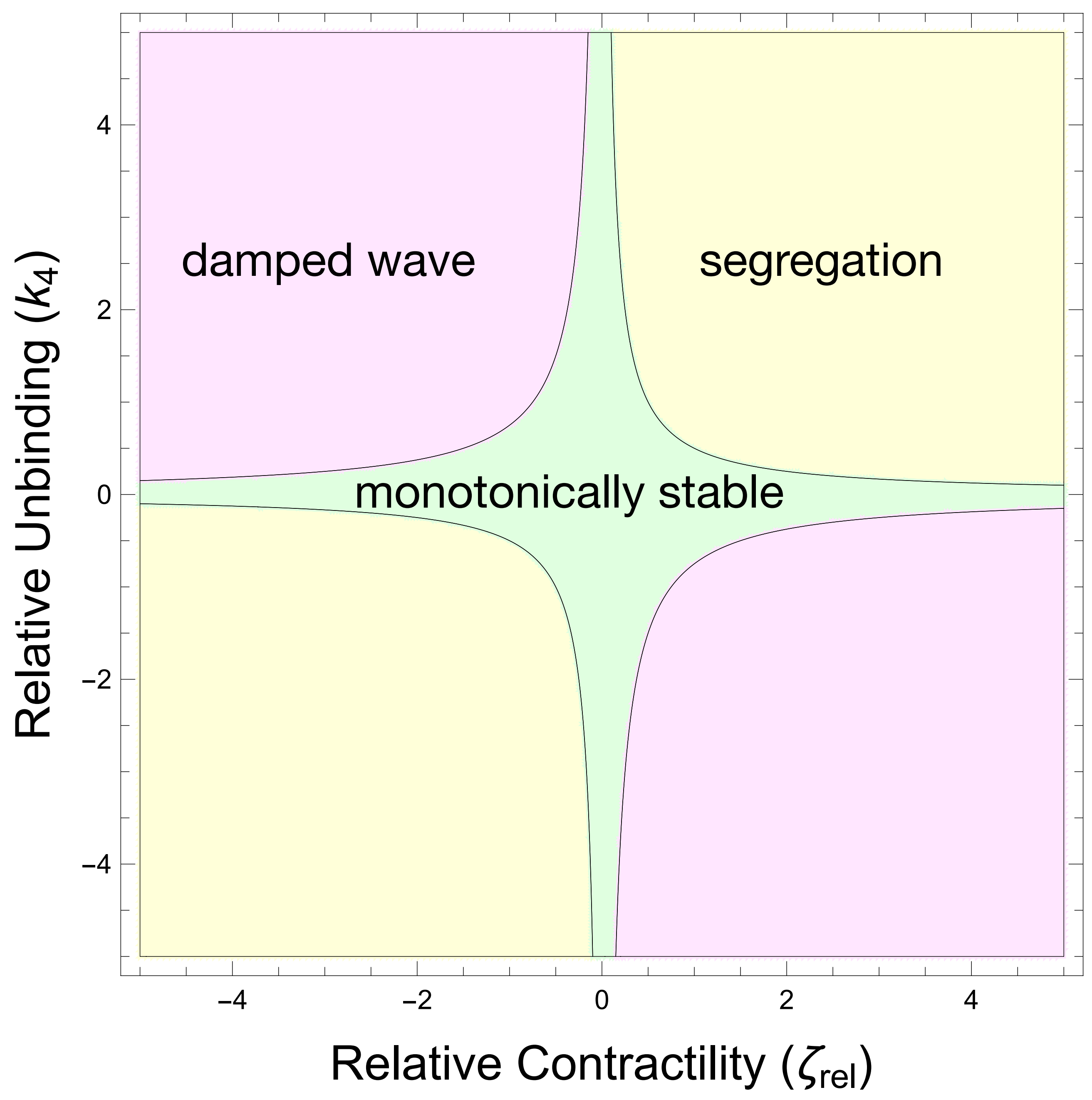
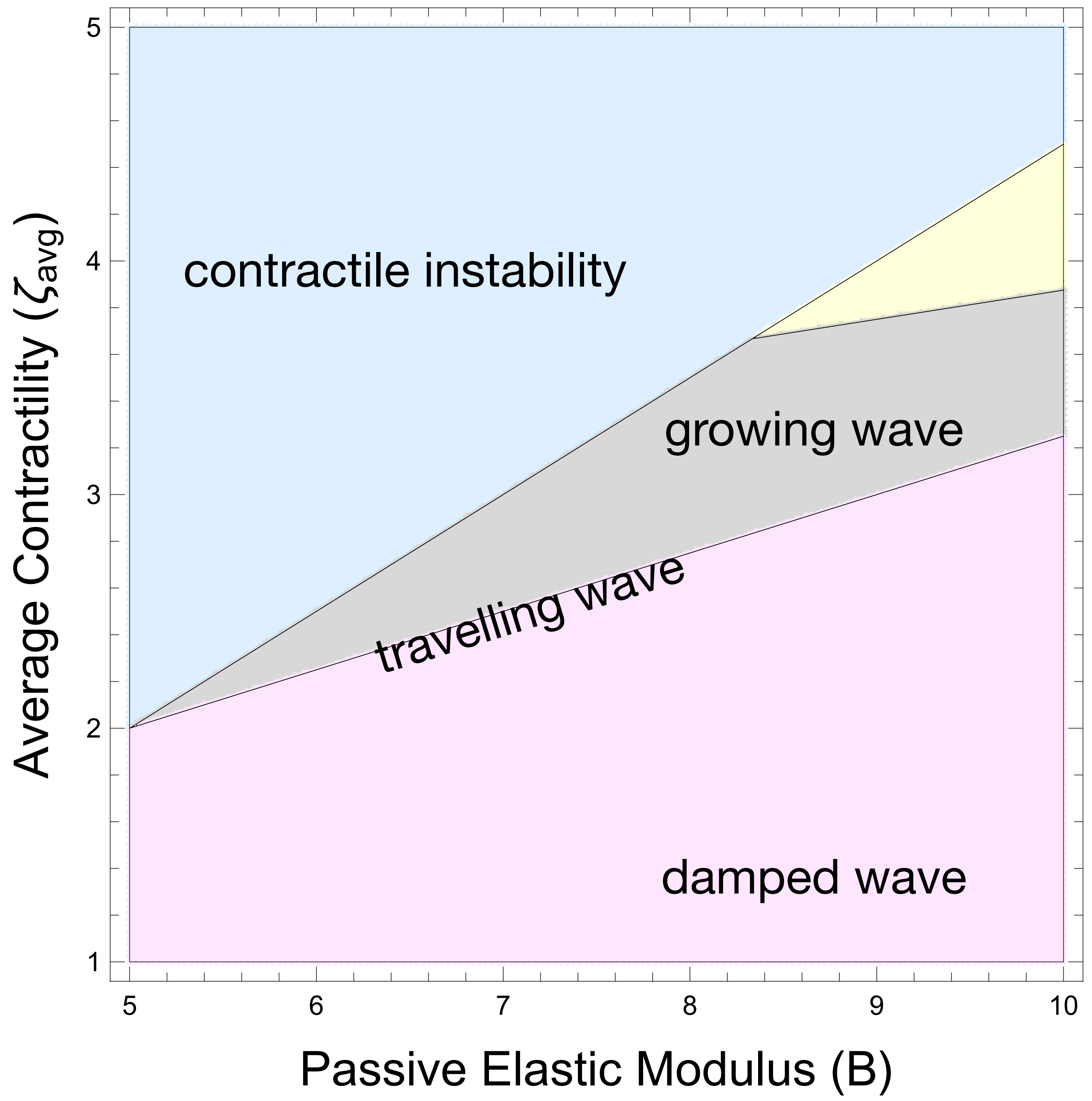
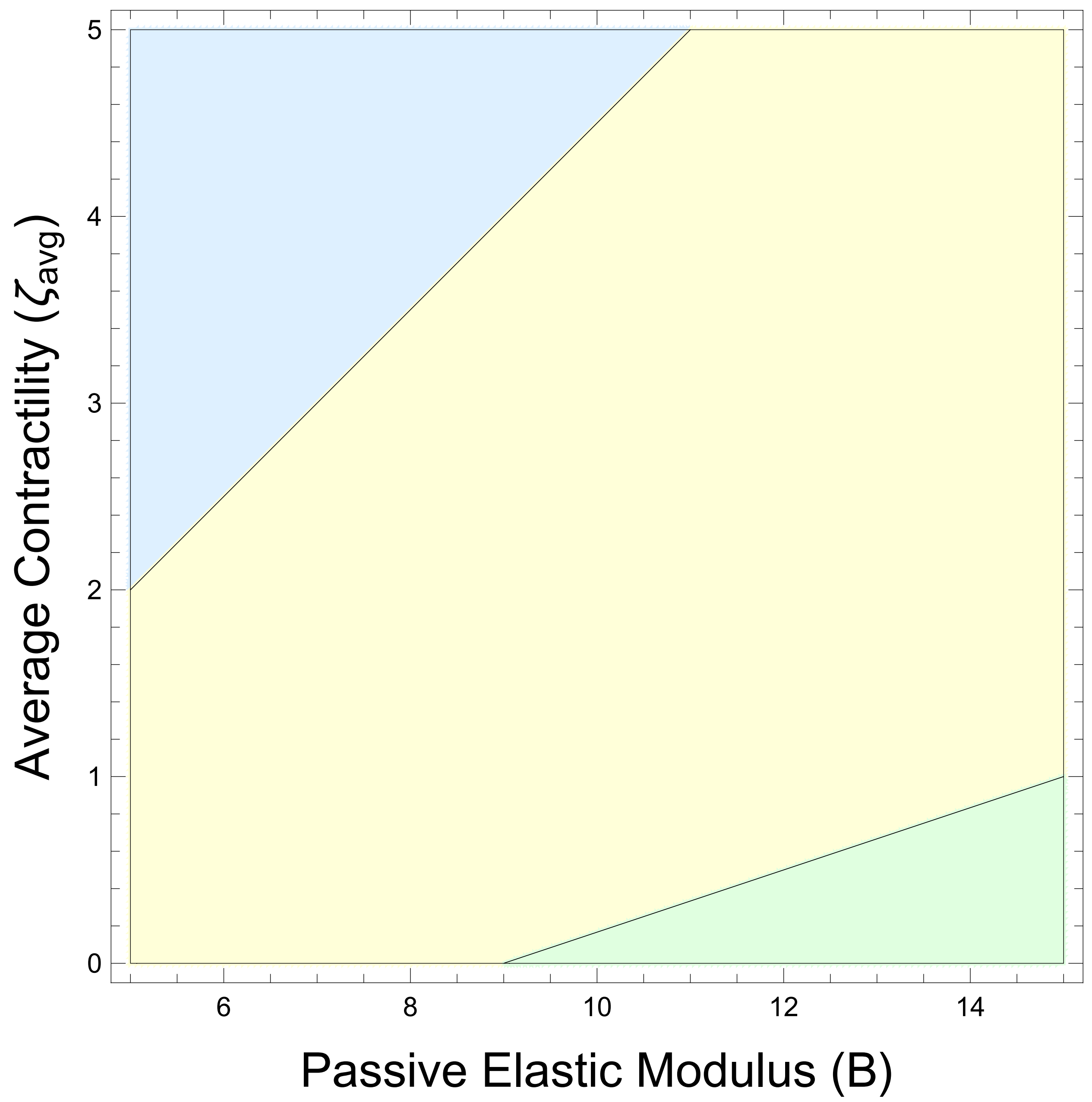
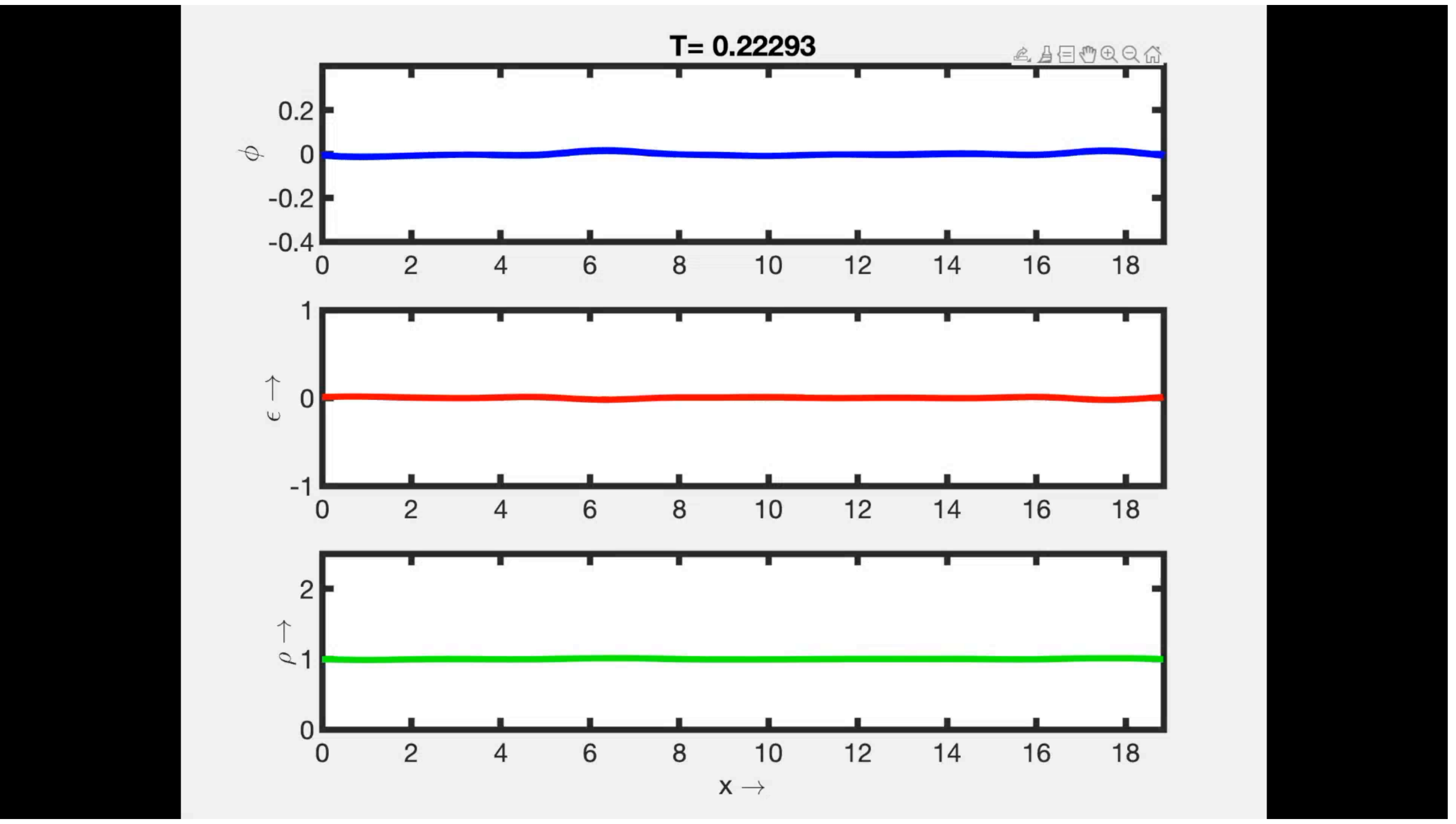




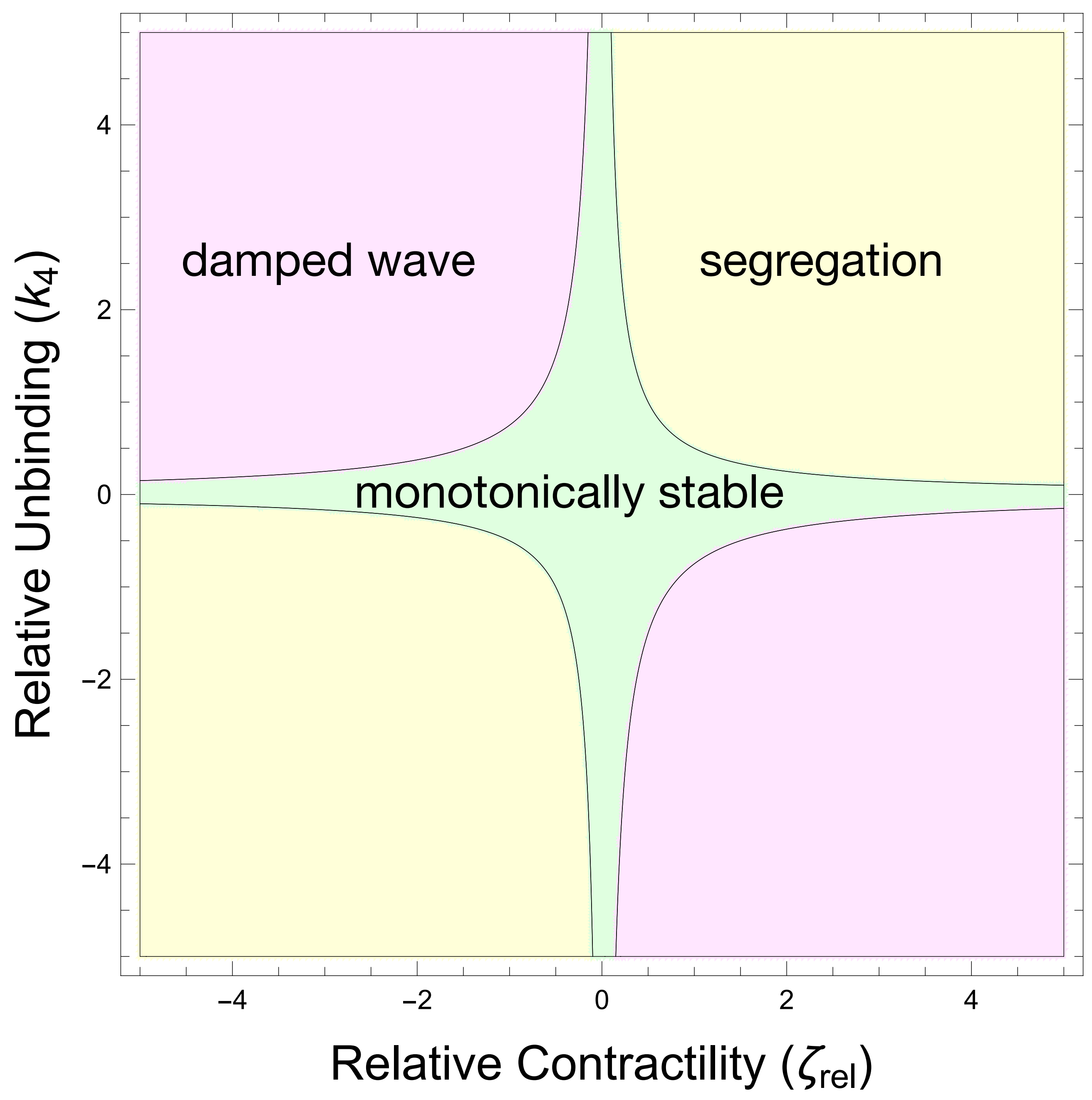


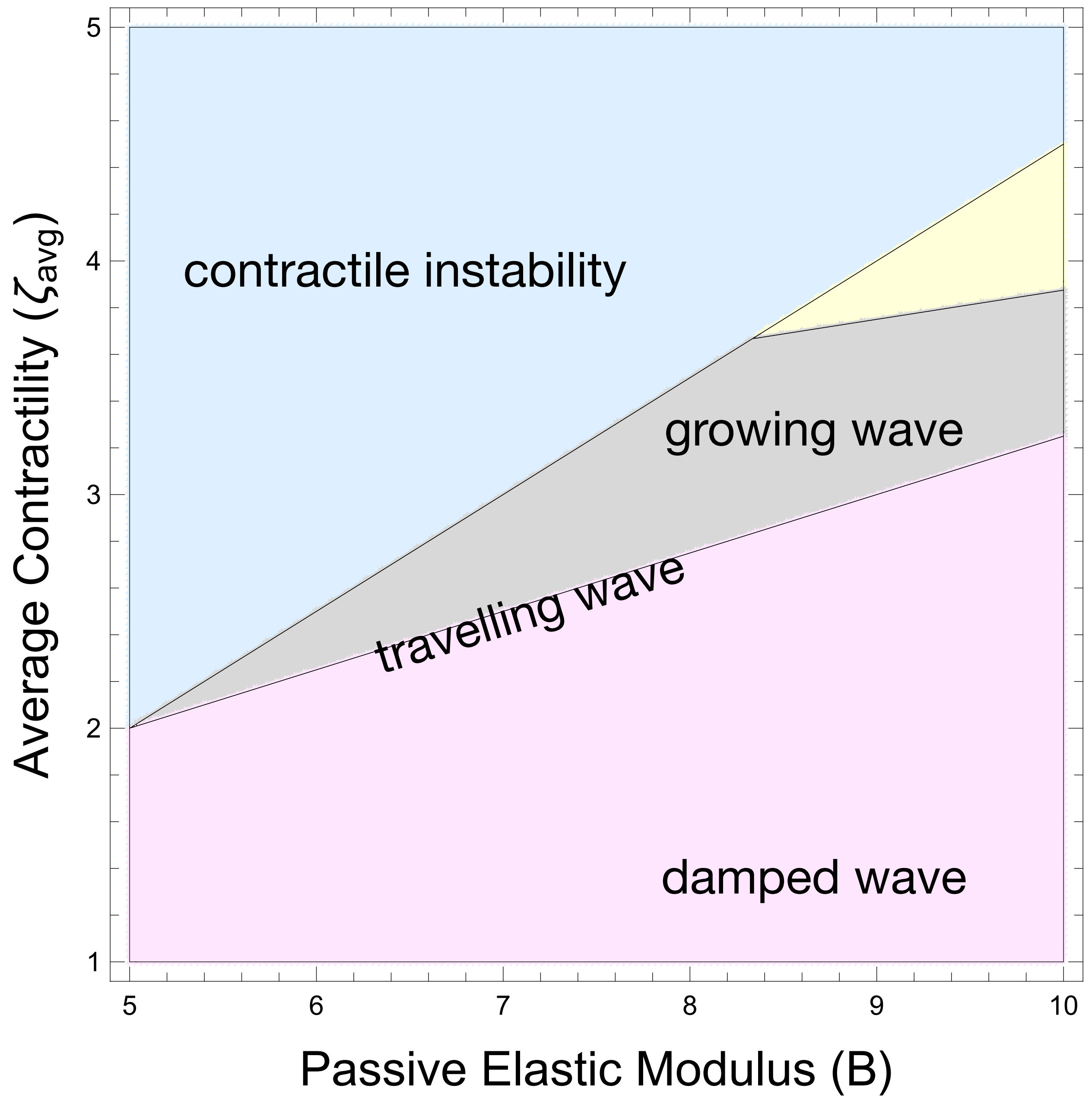


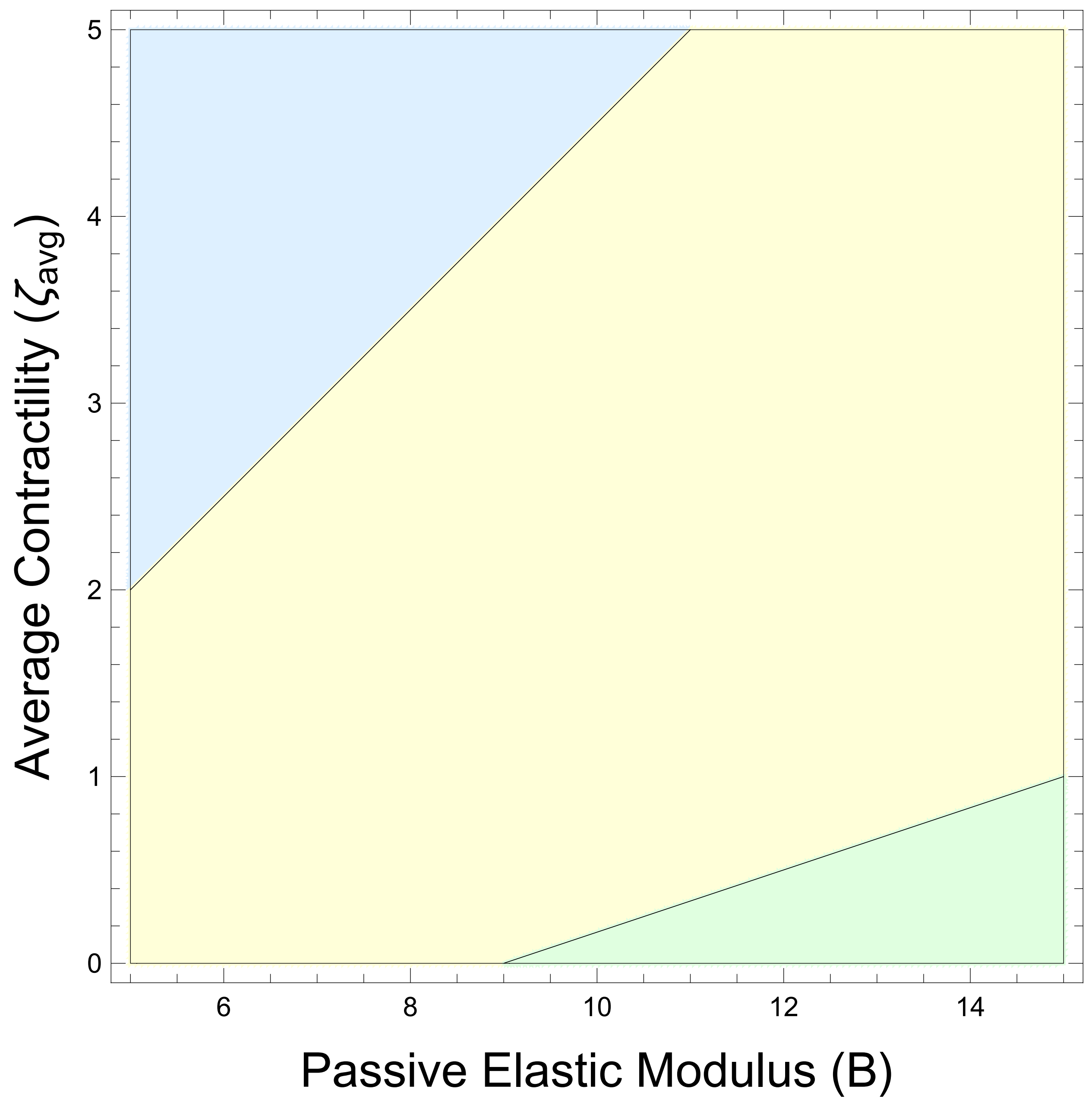
Segregation

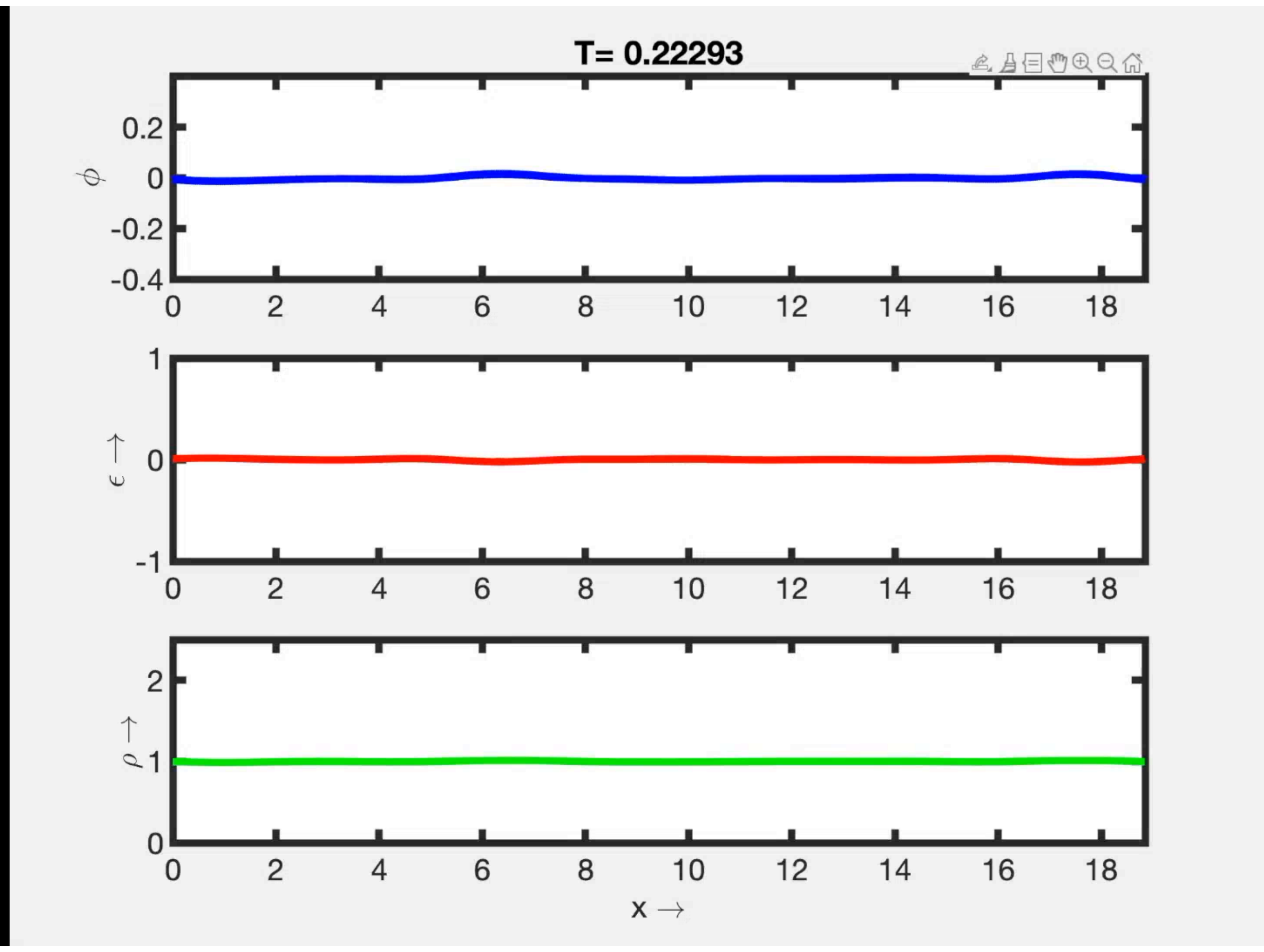
stronger stresslets unbind slower

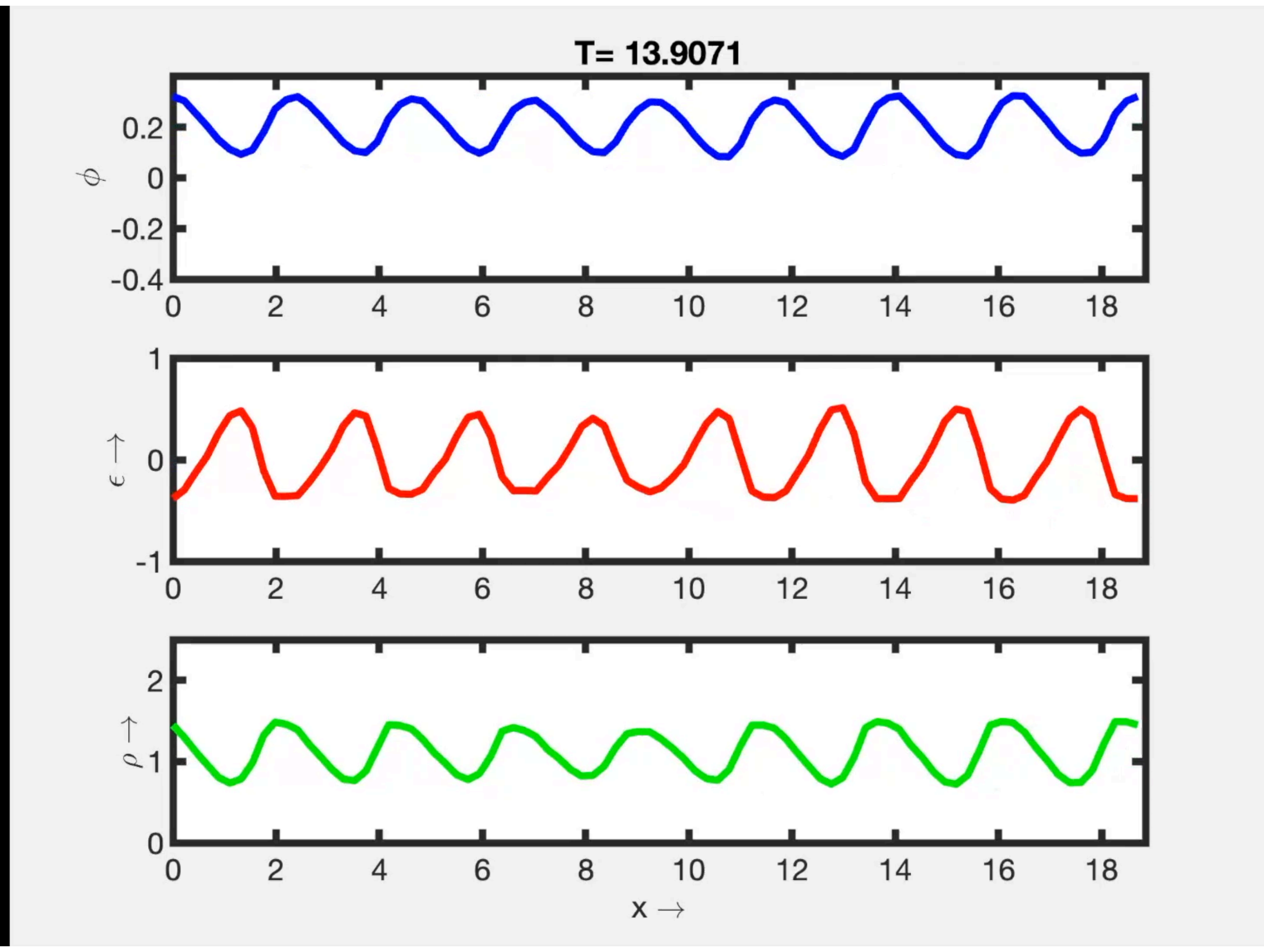
Phase Diagram from Linear Stability Analysis











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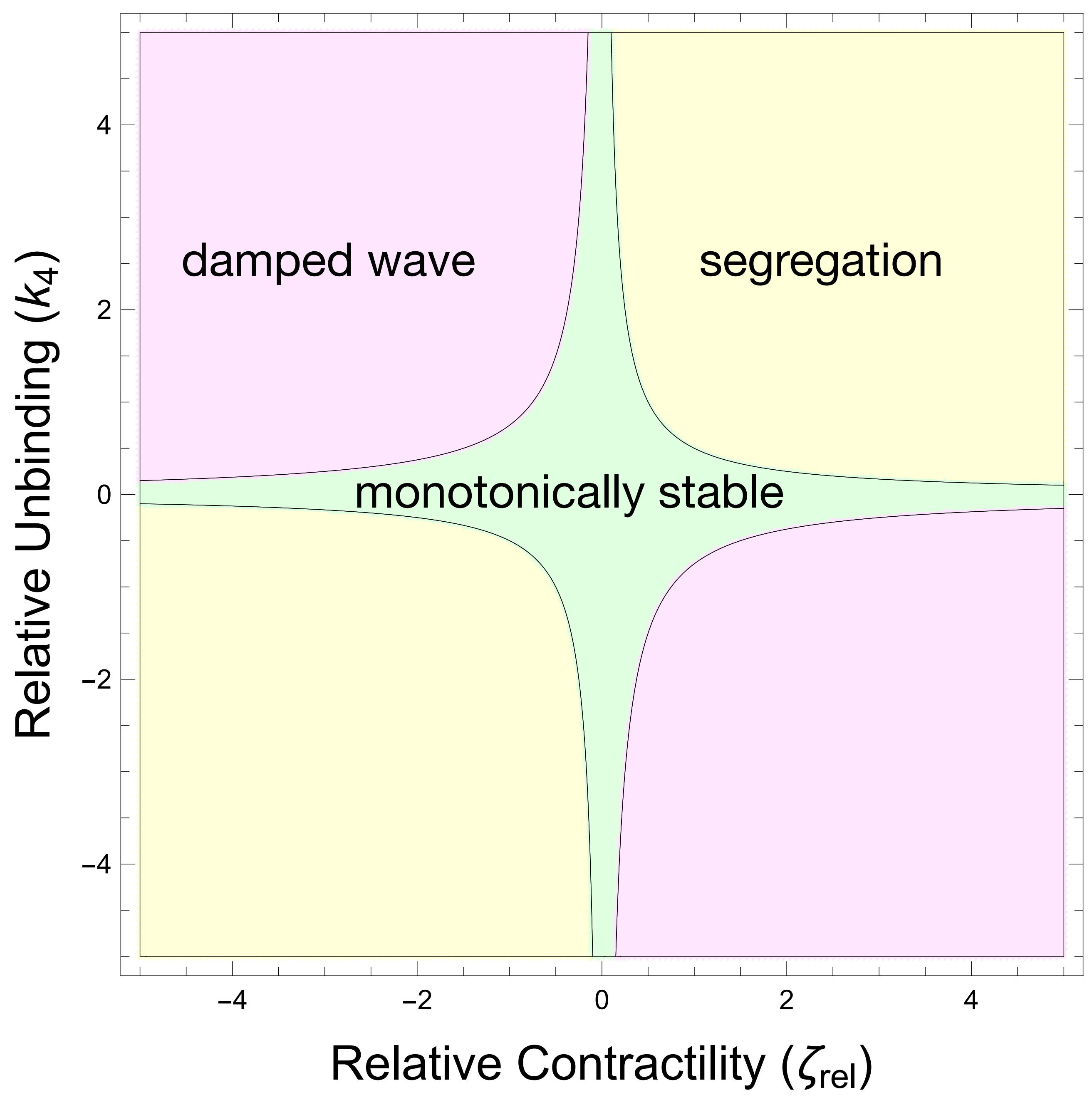
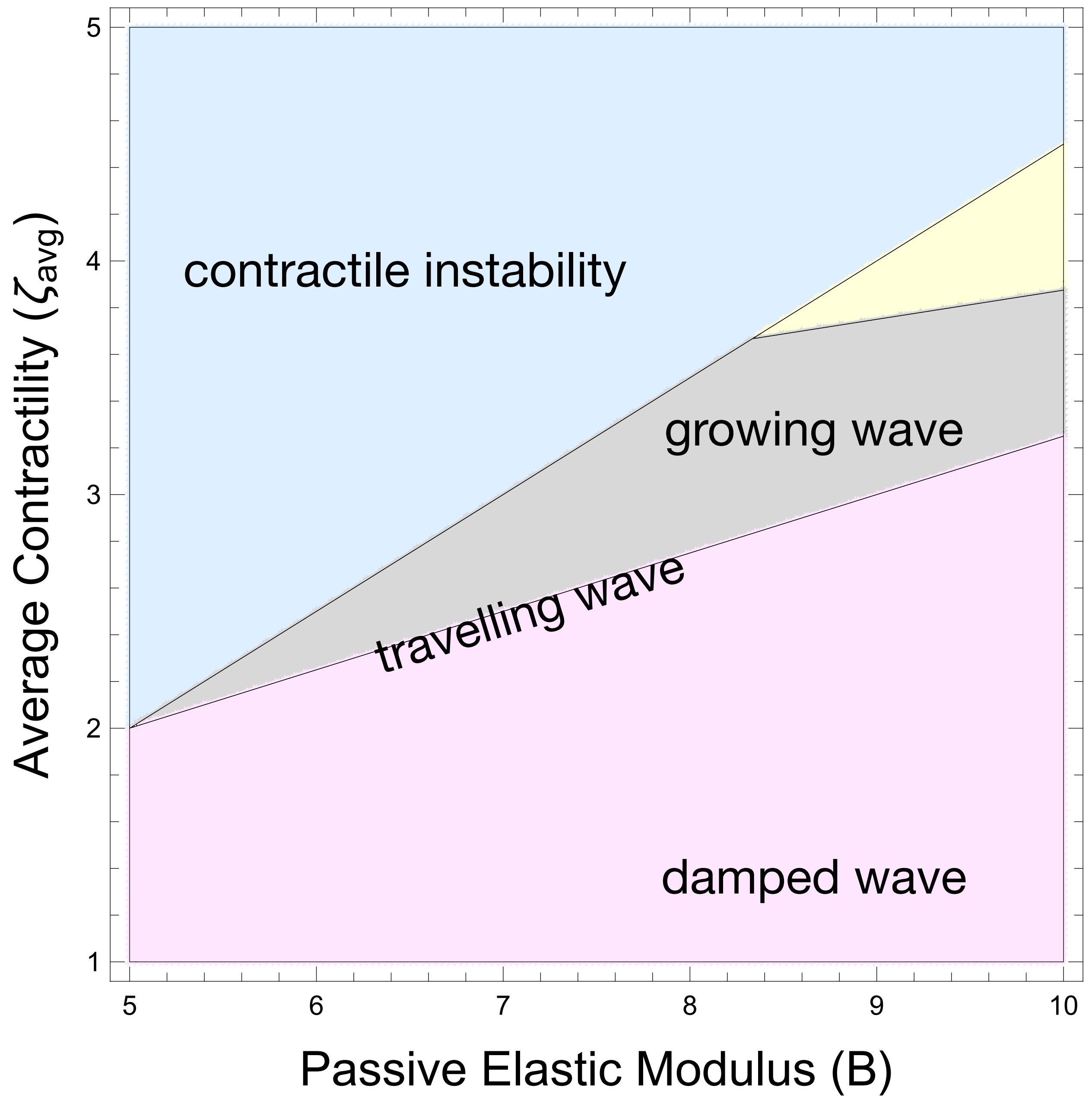
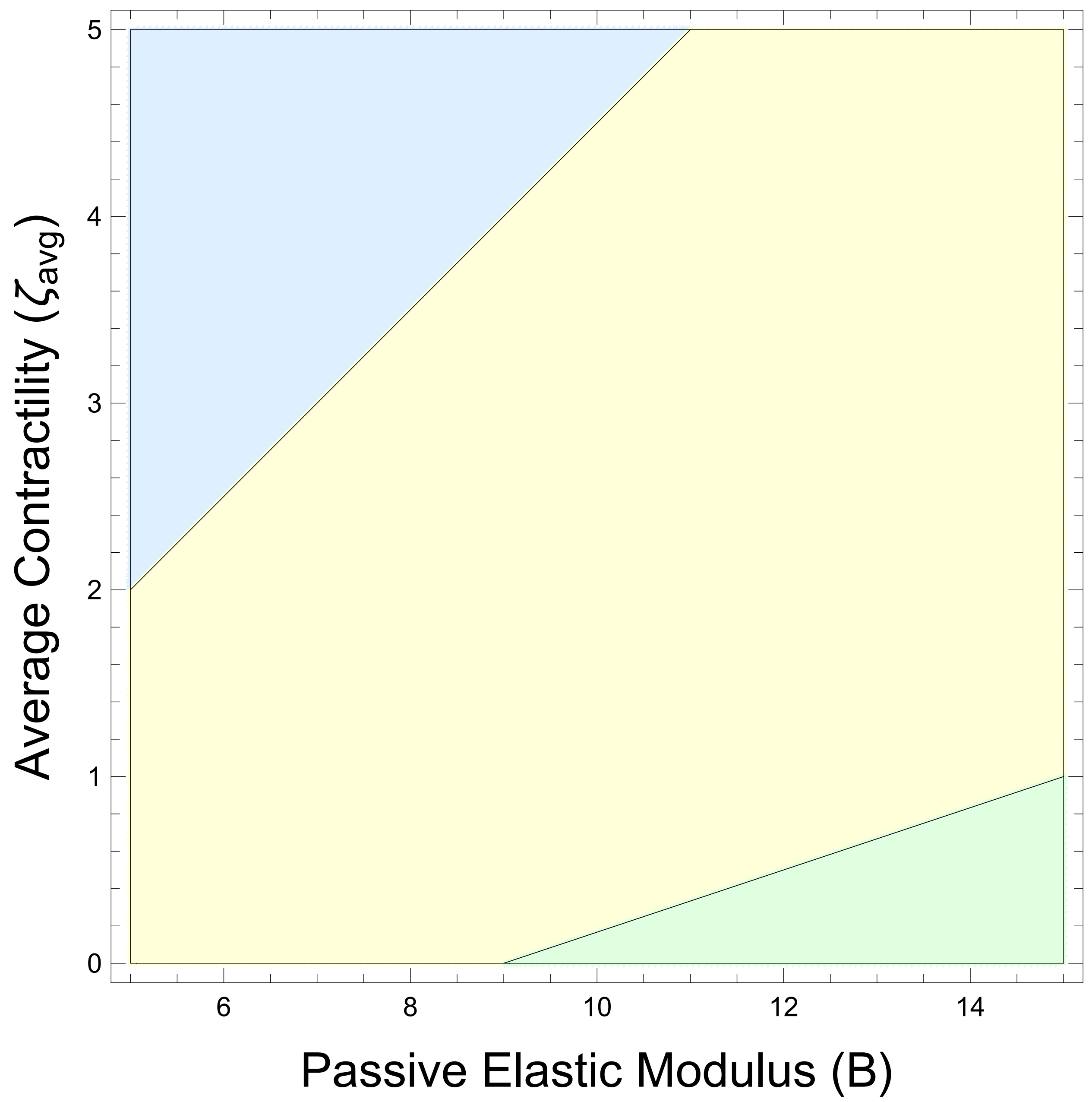
Travelling Wave

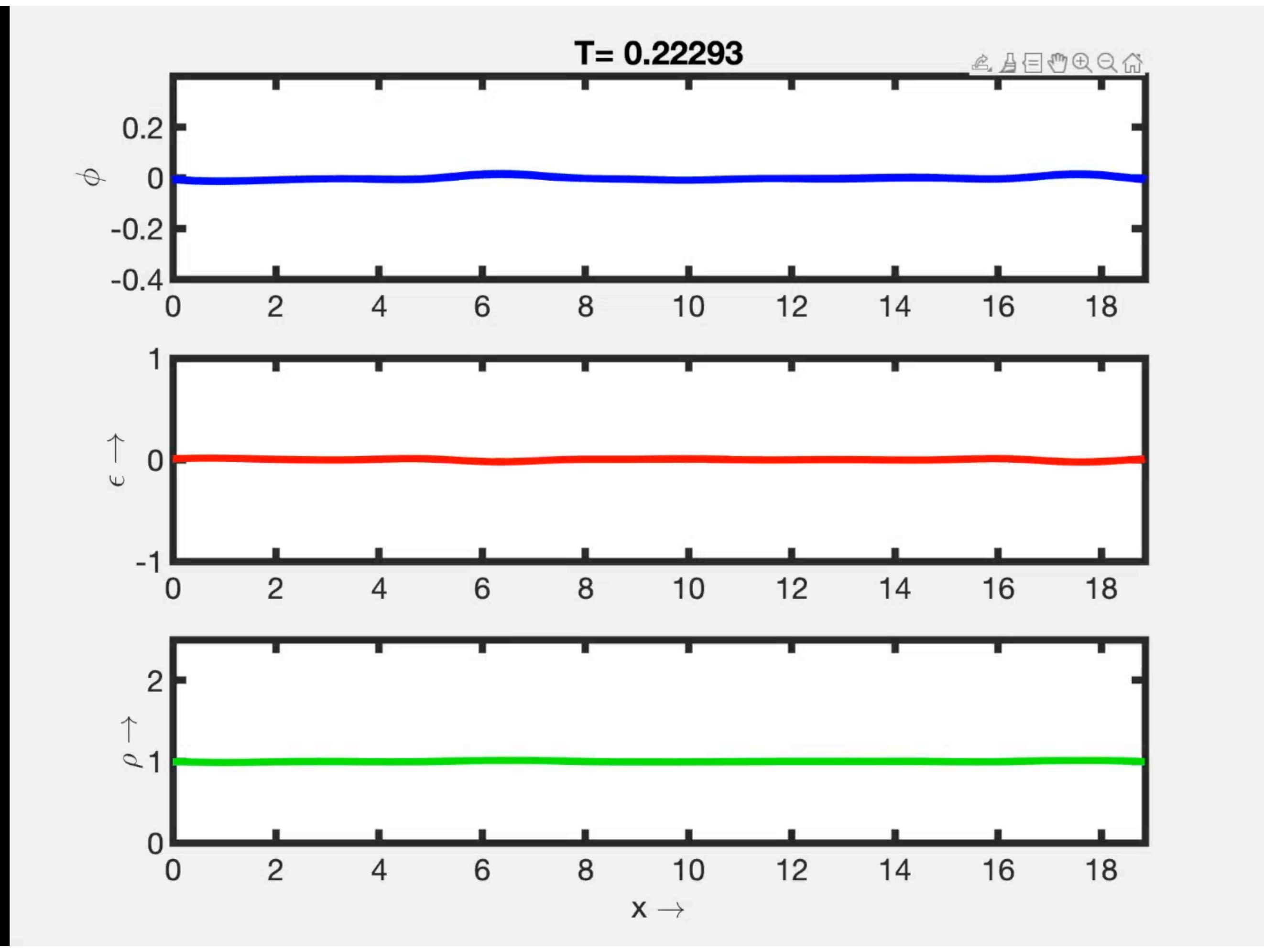
stronger stresslets unbind faster

Phase Diagram from Linear Stability Analysis









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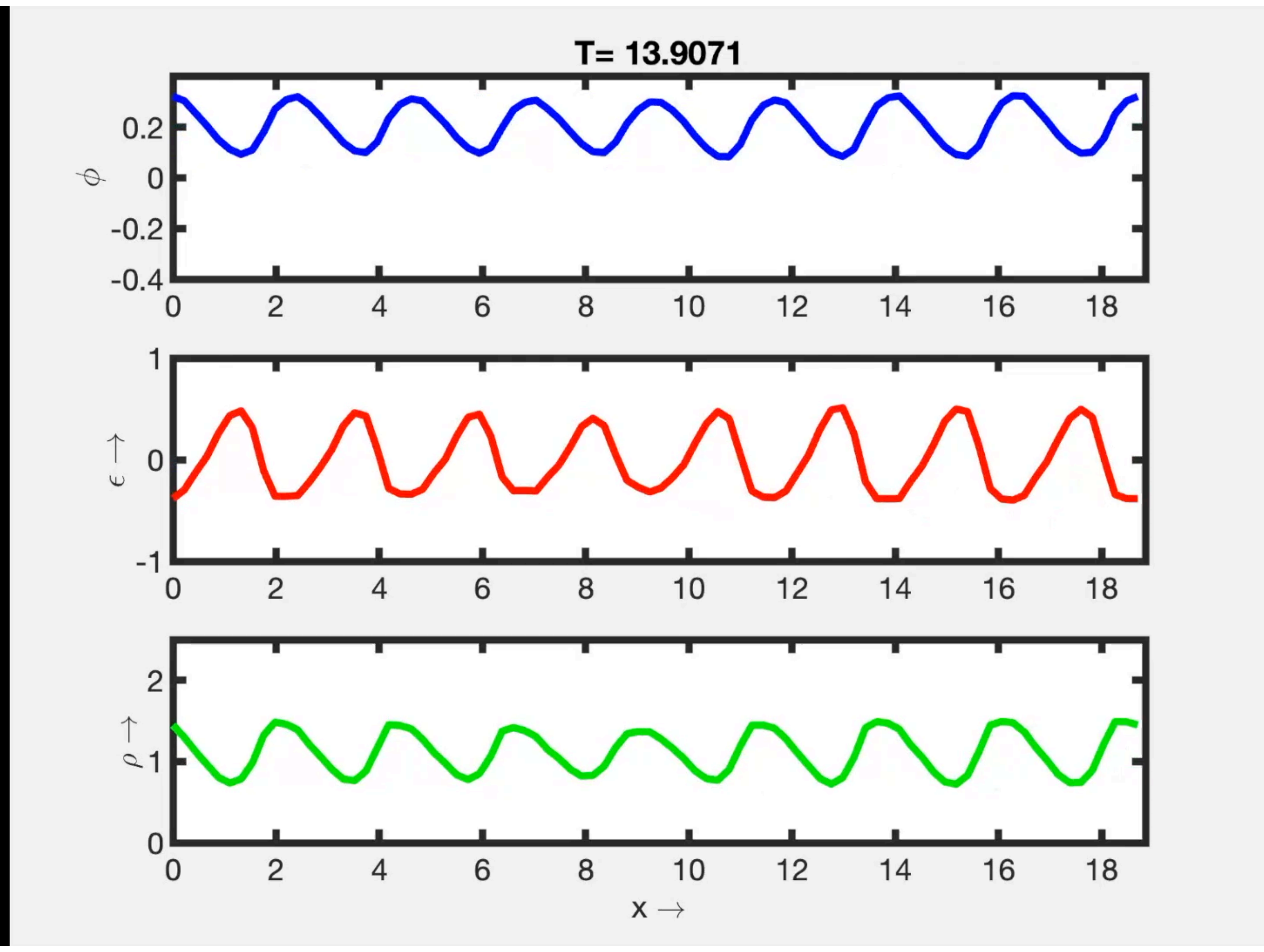

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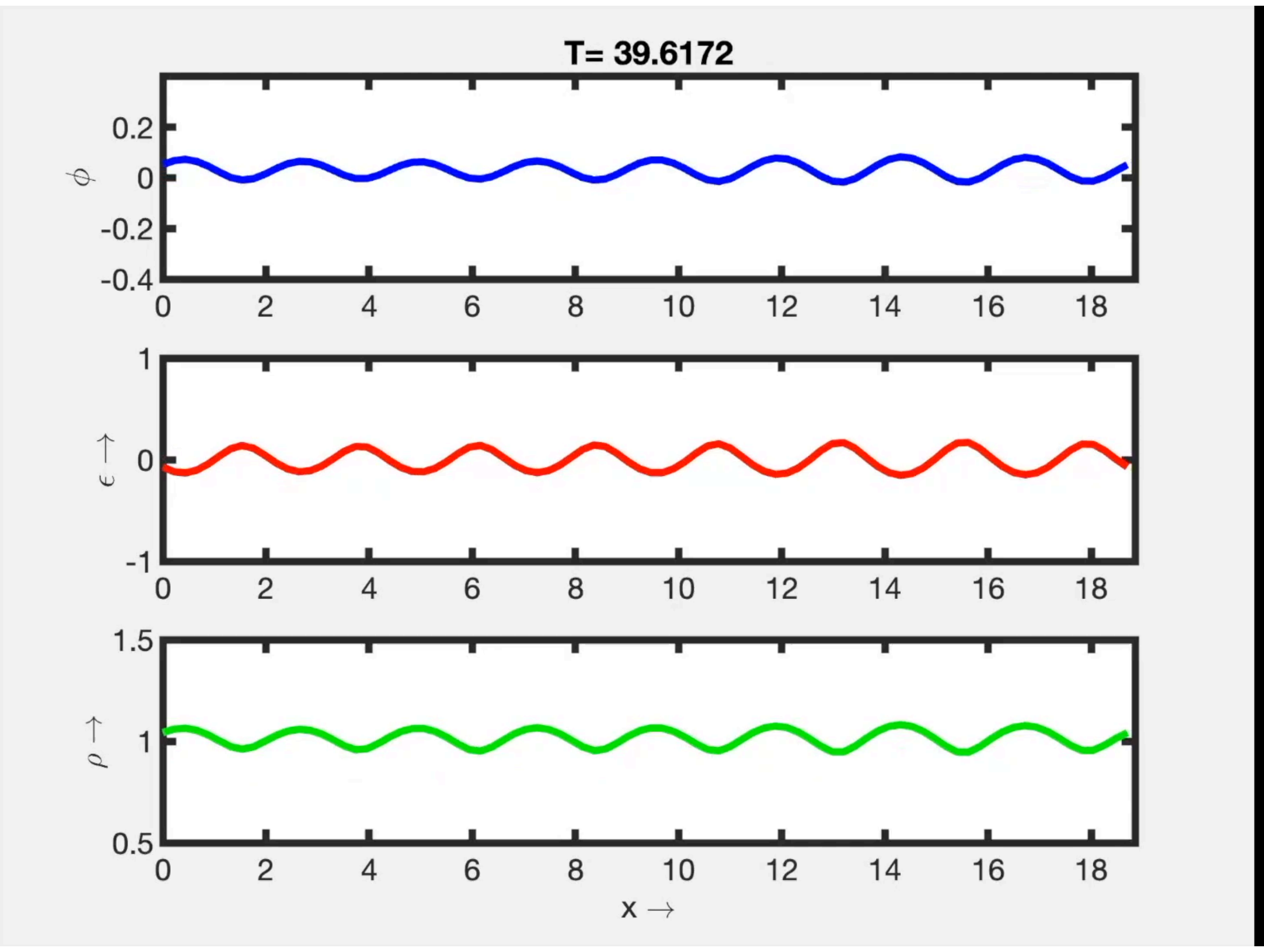


Swap

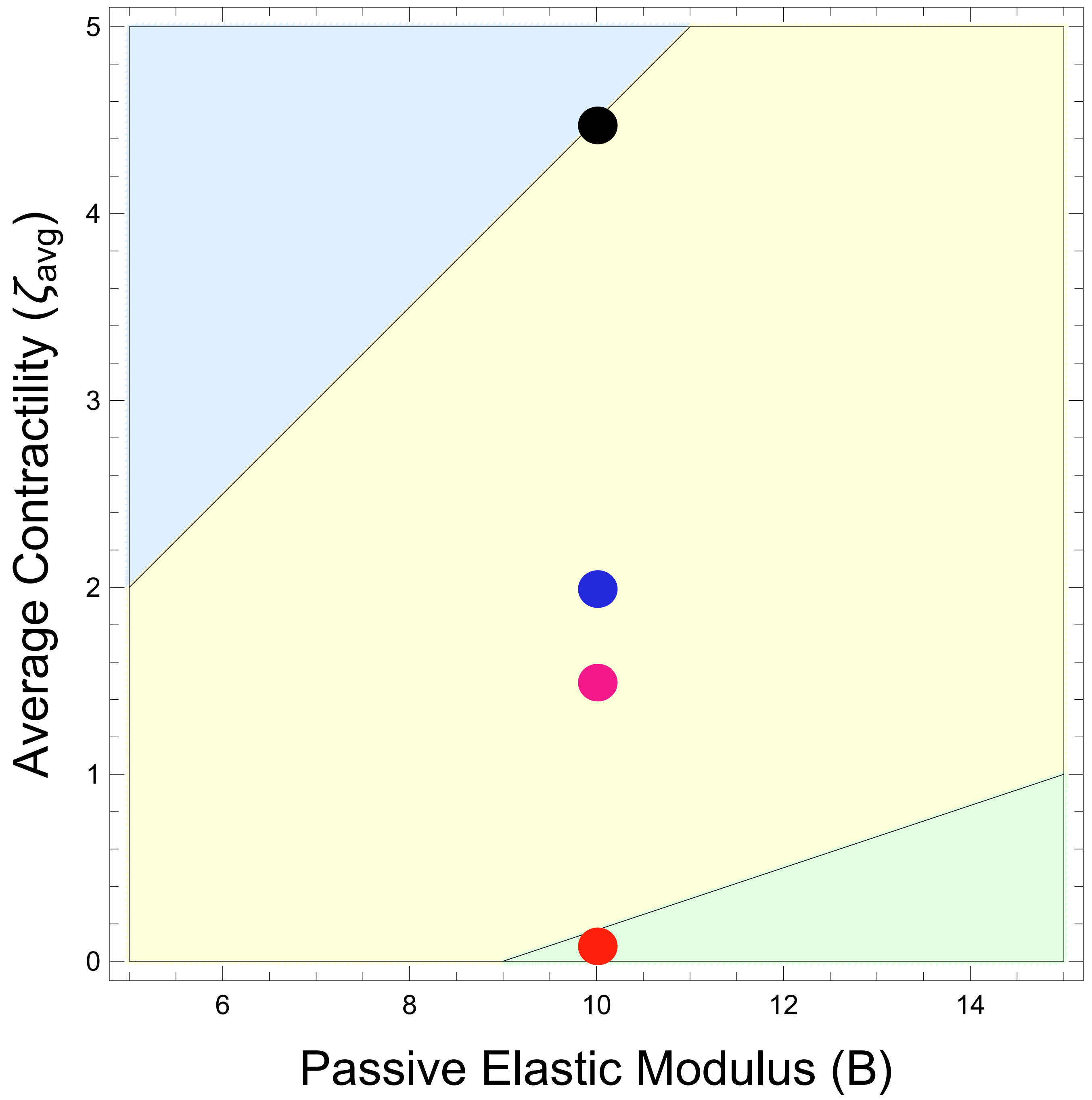
stronger stresslets unbind before c.i.

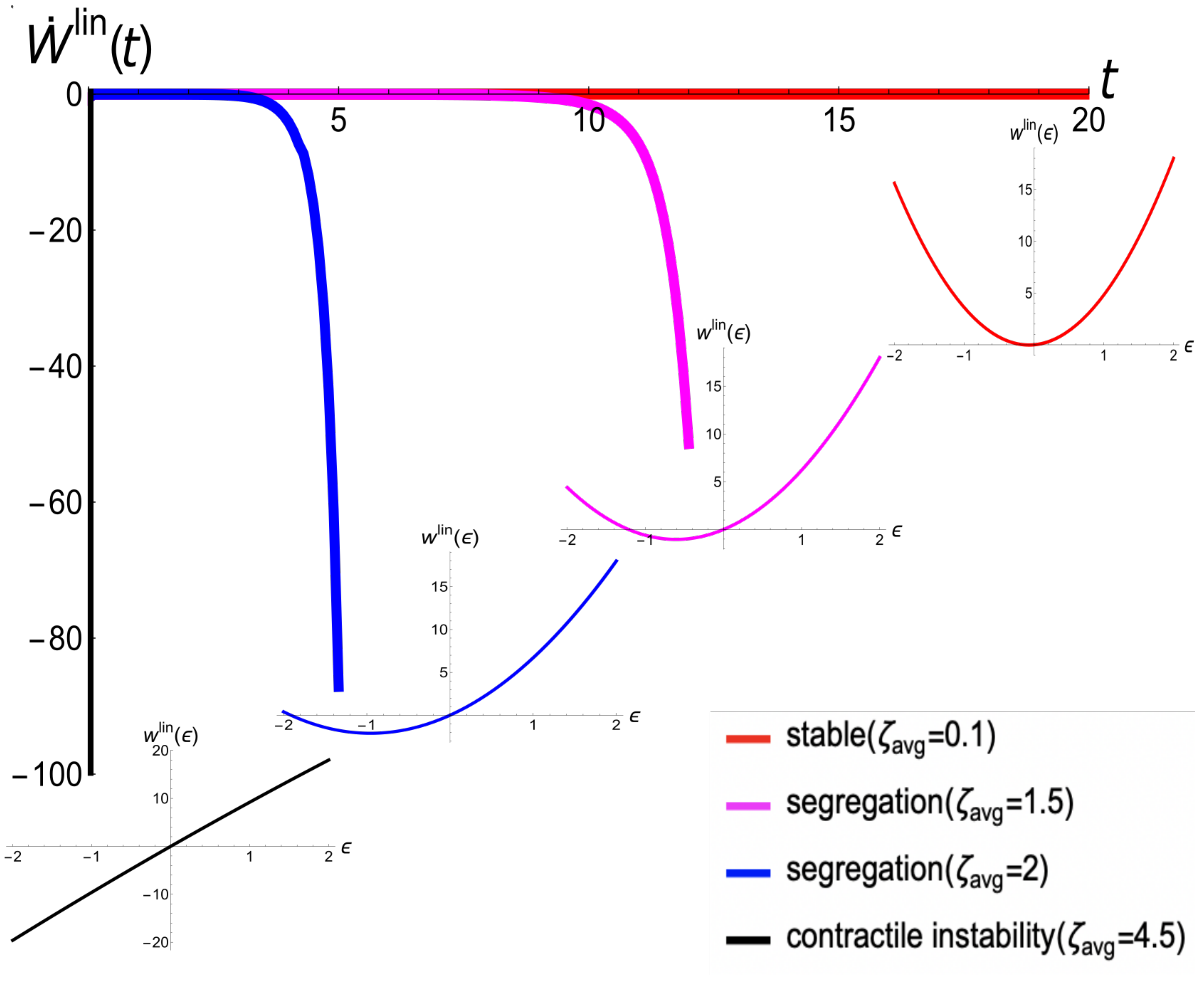
Driving Force for 'Linear' Segregation: Elastic Stress Dissipation

Effective Strain Energy Density

$$w^{lin} := \sigma_0(\rho, \phi)\, \epsilon + \frac{1}{2}\tilde{B}(\rho, \phi)\, \epsilon^2.$$

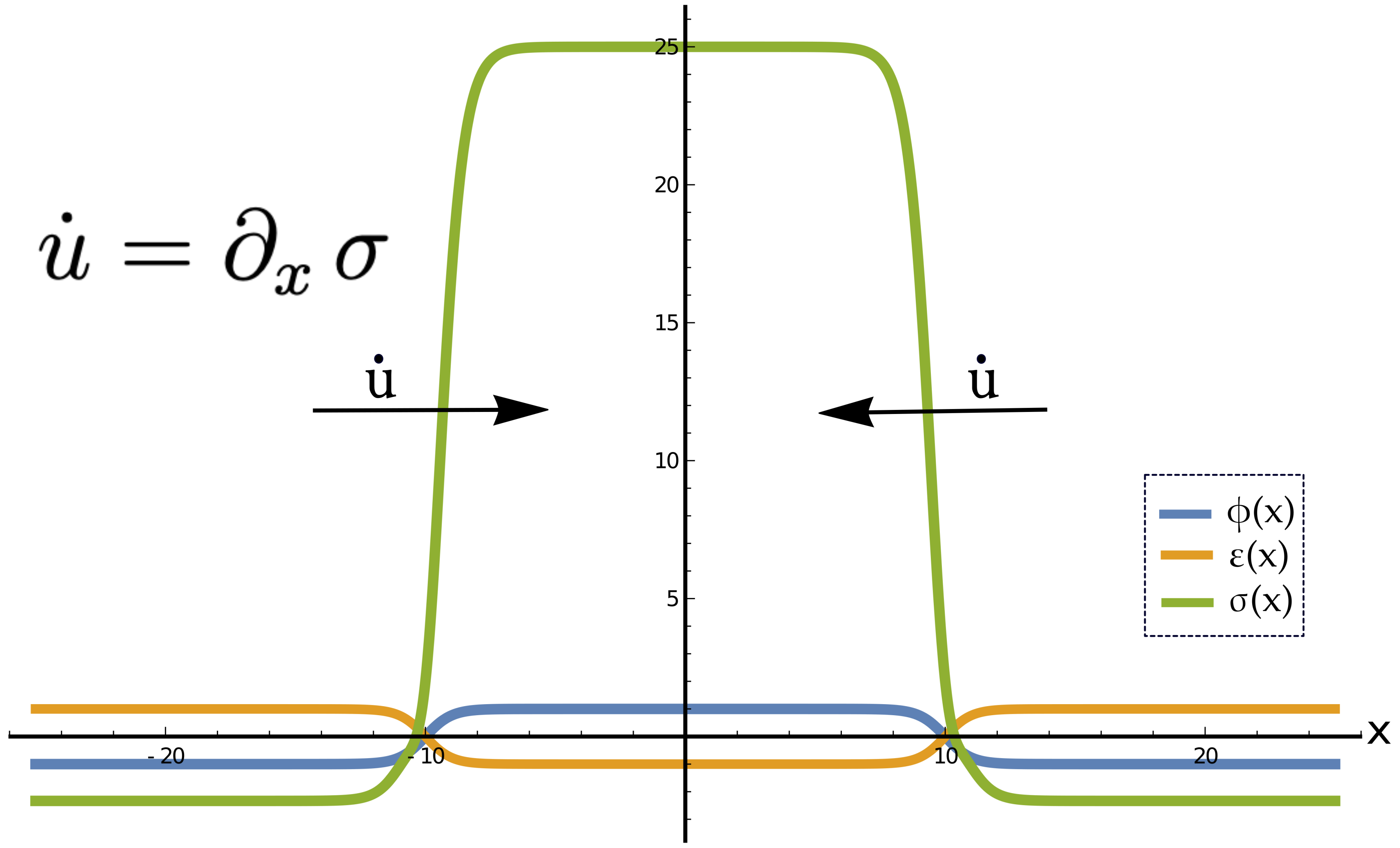
$$\dot{W}^{lin}(t) \quad := \quad \int_{-a}^{a} \dot{w}^{lin}\, dx$$

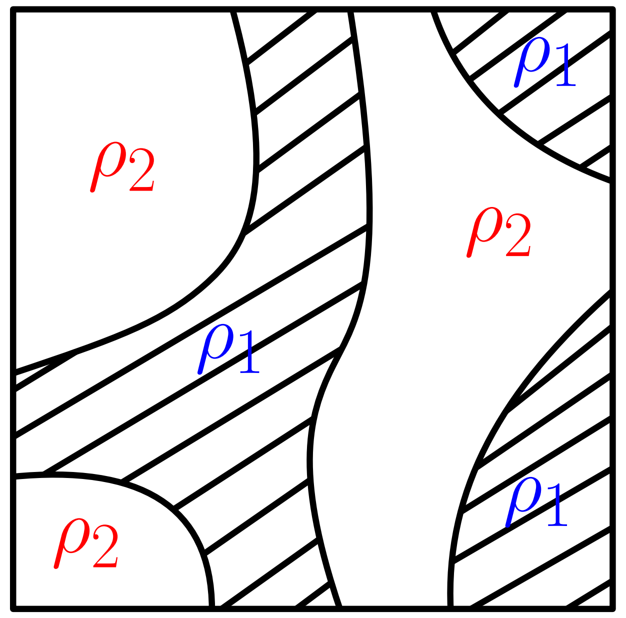




Late Time State
of the Segregated Domains

$$\dot{u} = \partial_x \sigma$$





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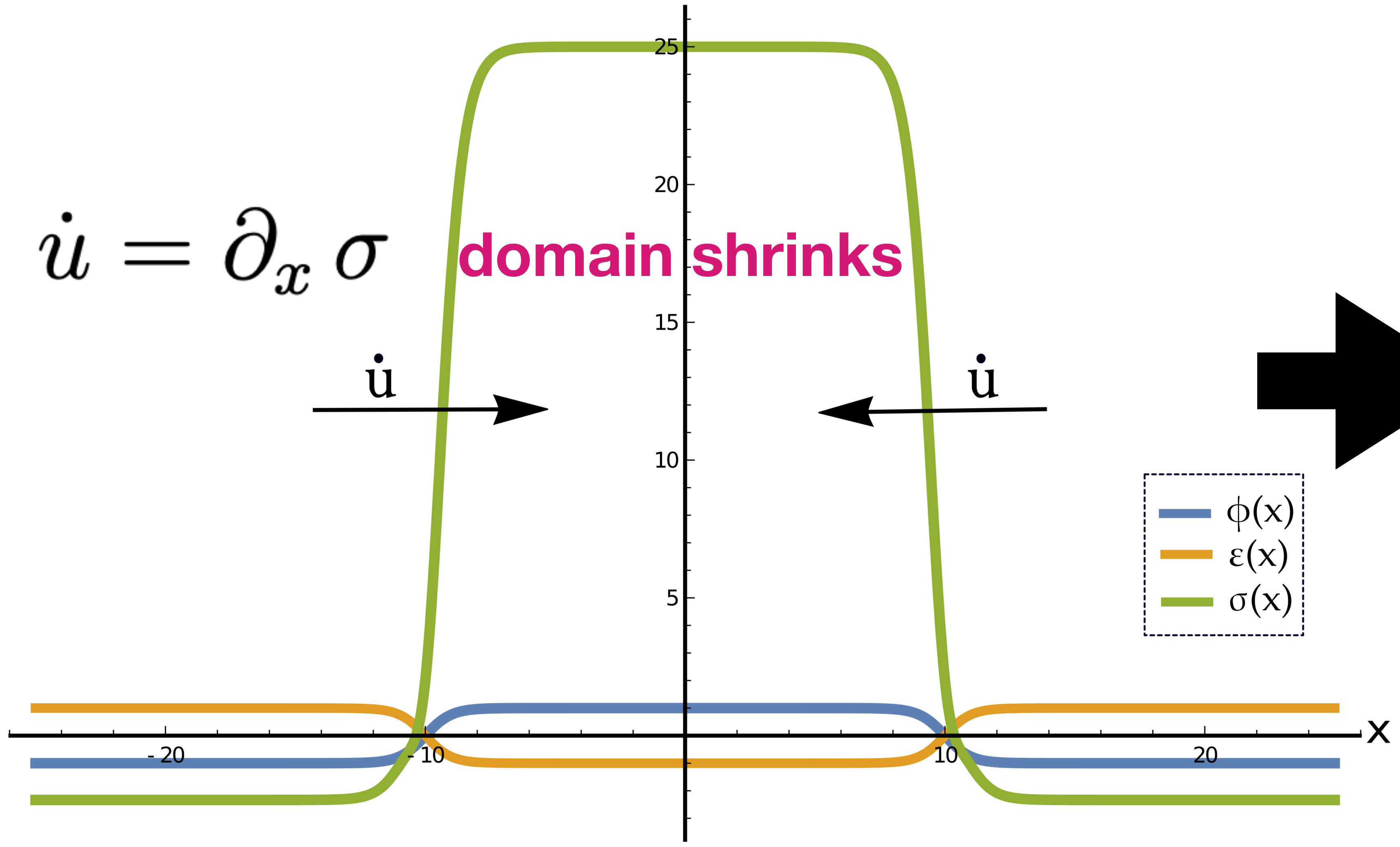
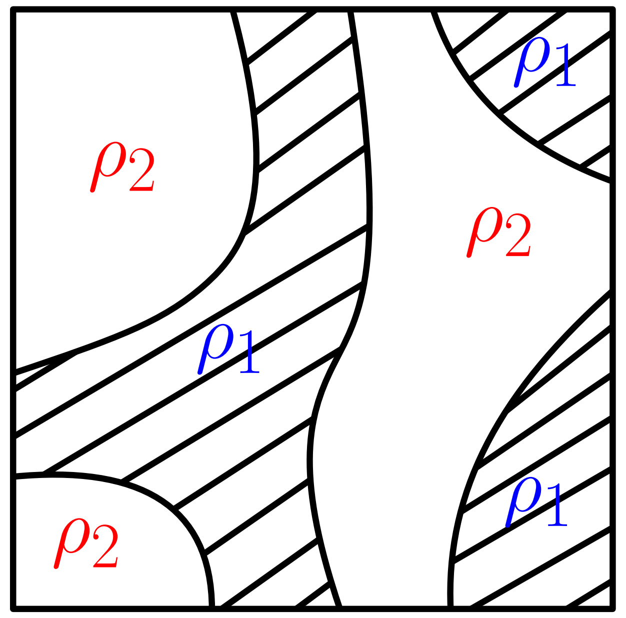
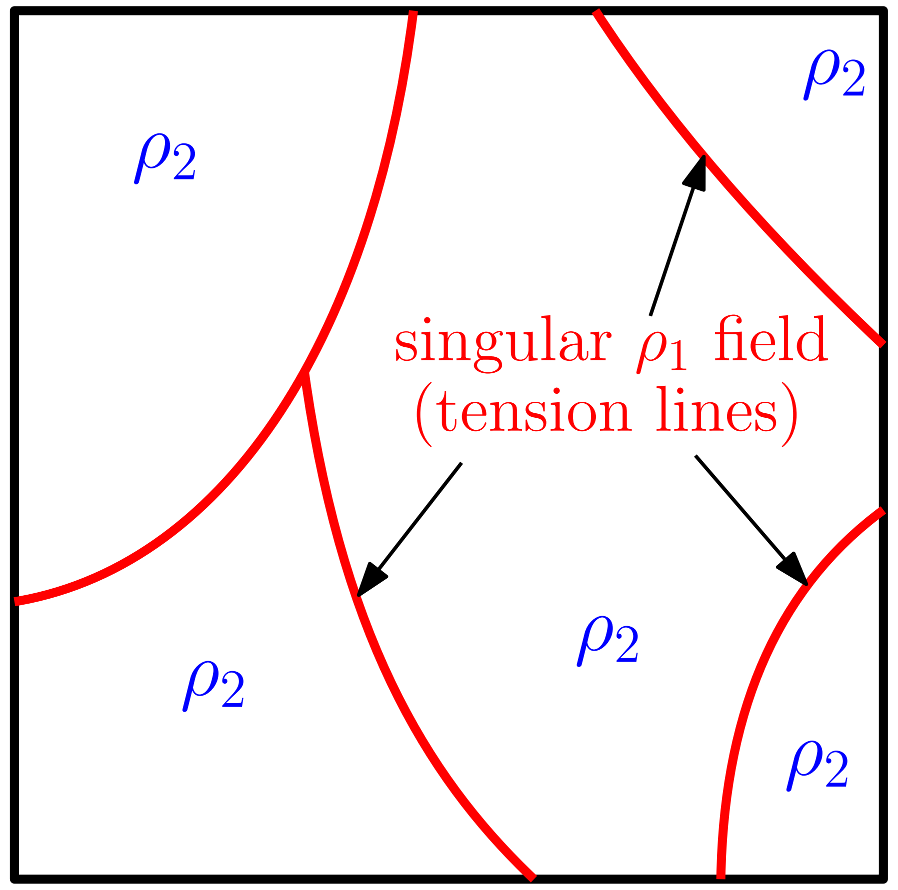
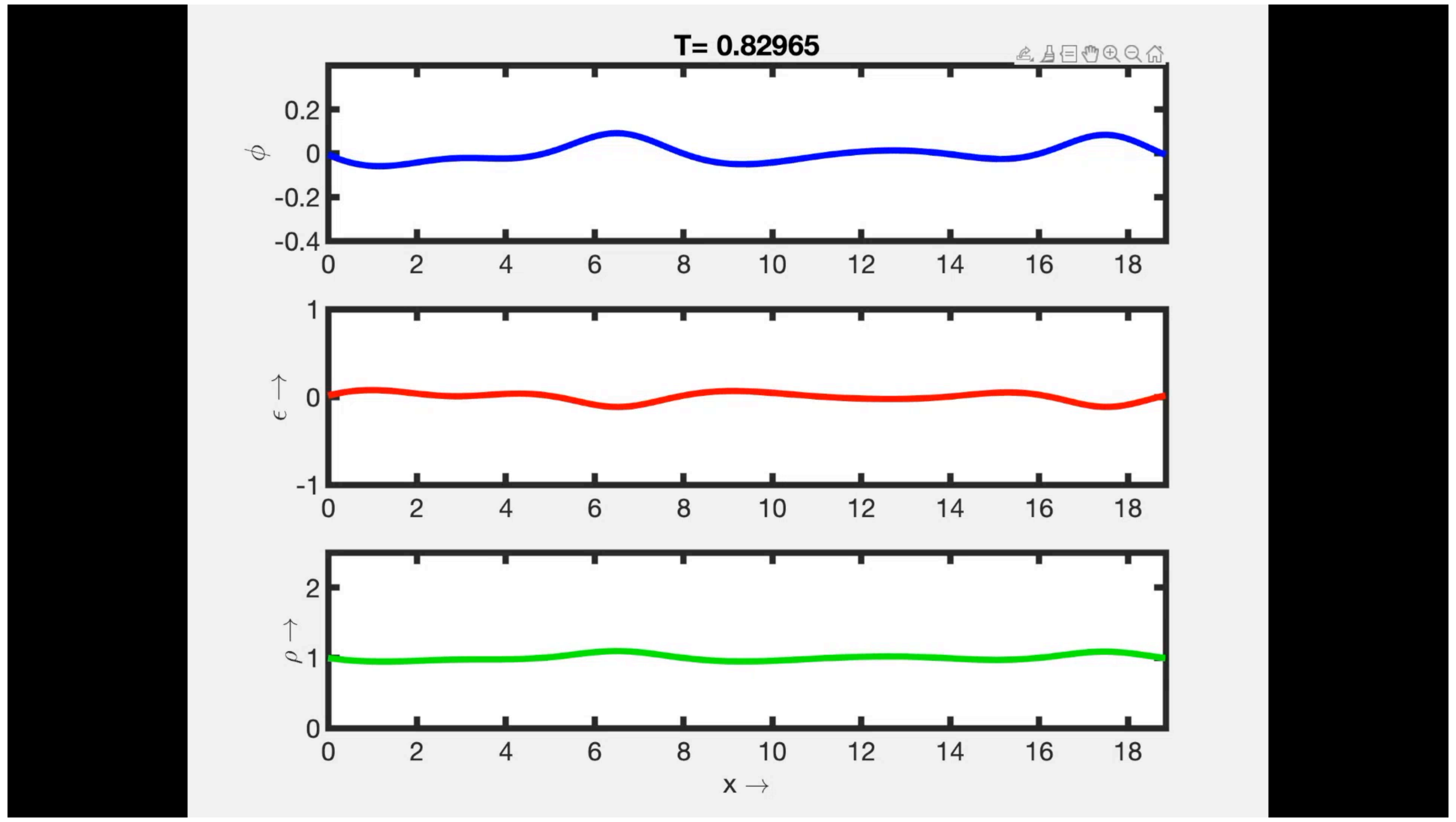




anchored boundary condition



Singular Structures of Stronger Contractile Stresslets



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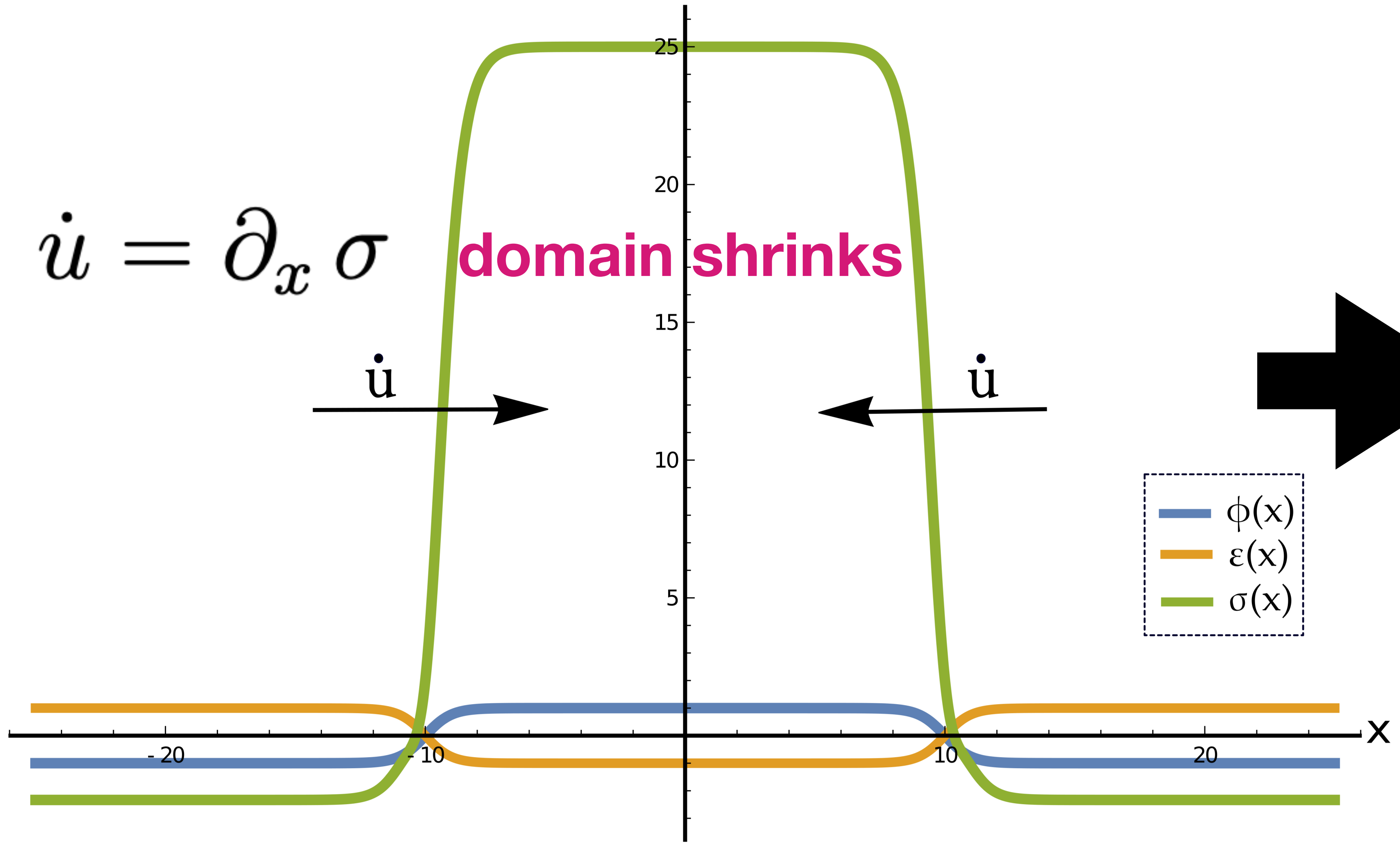

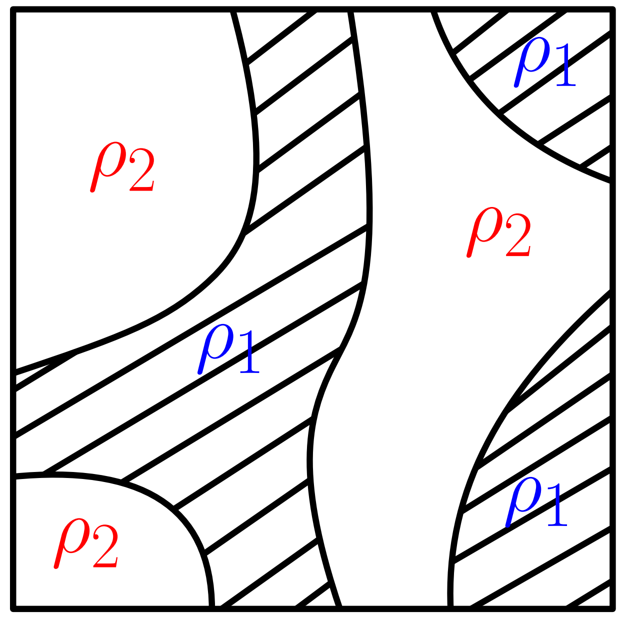

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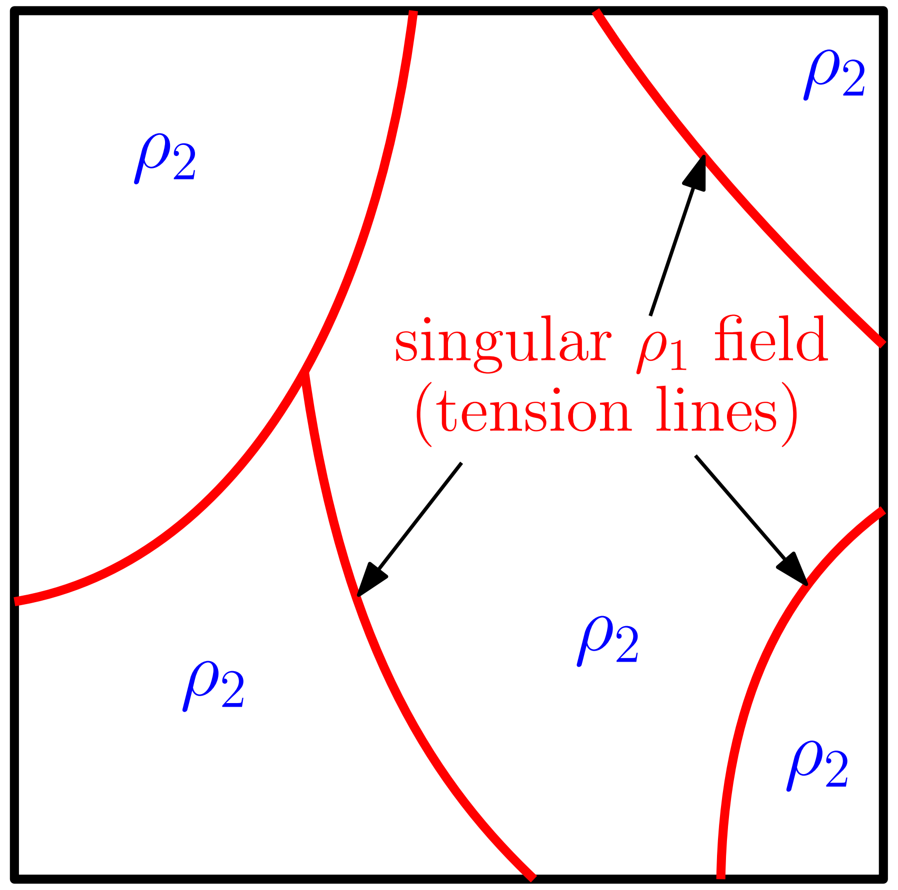

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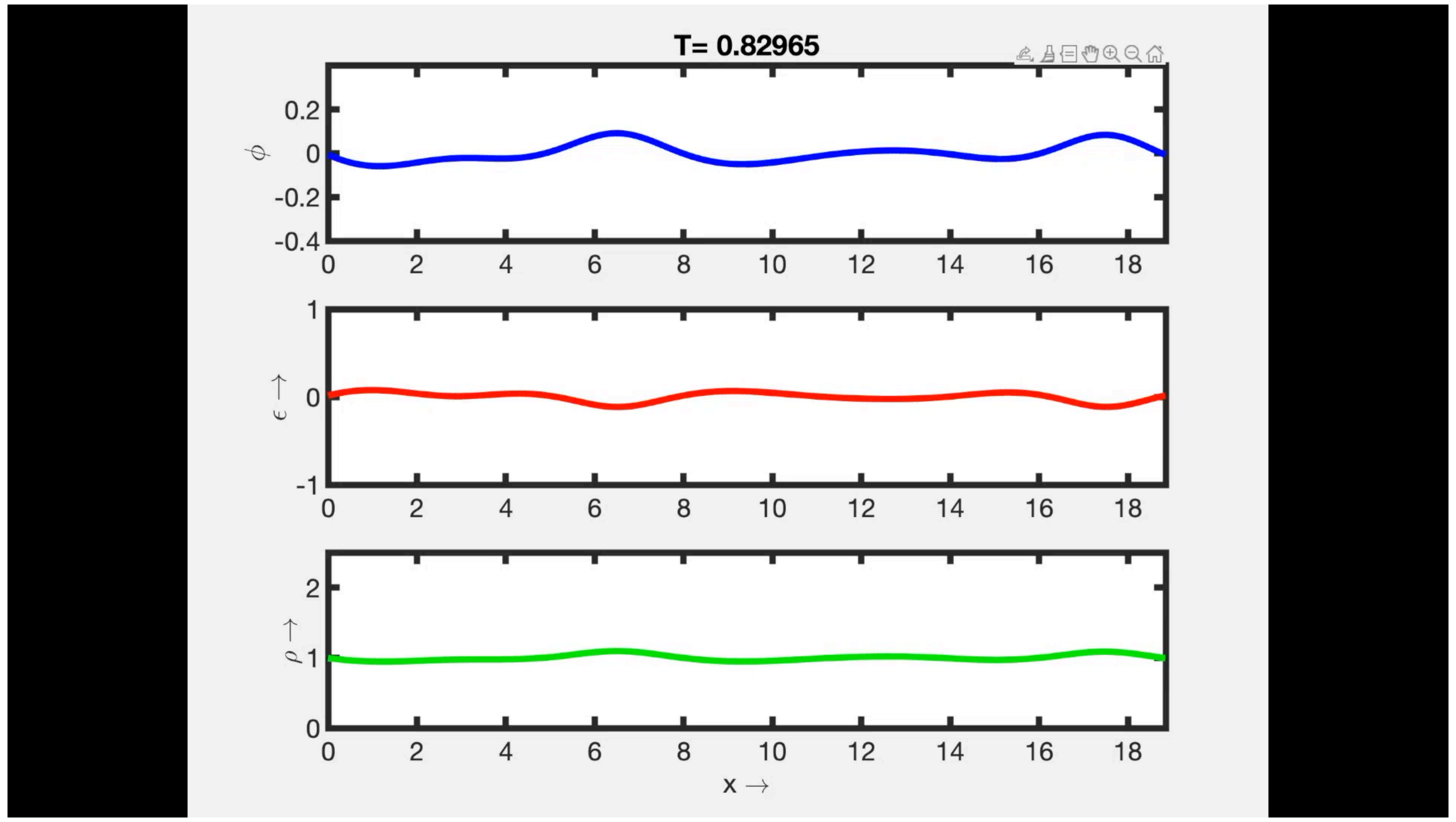
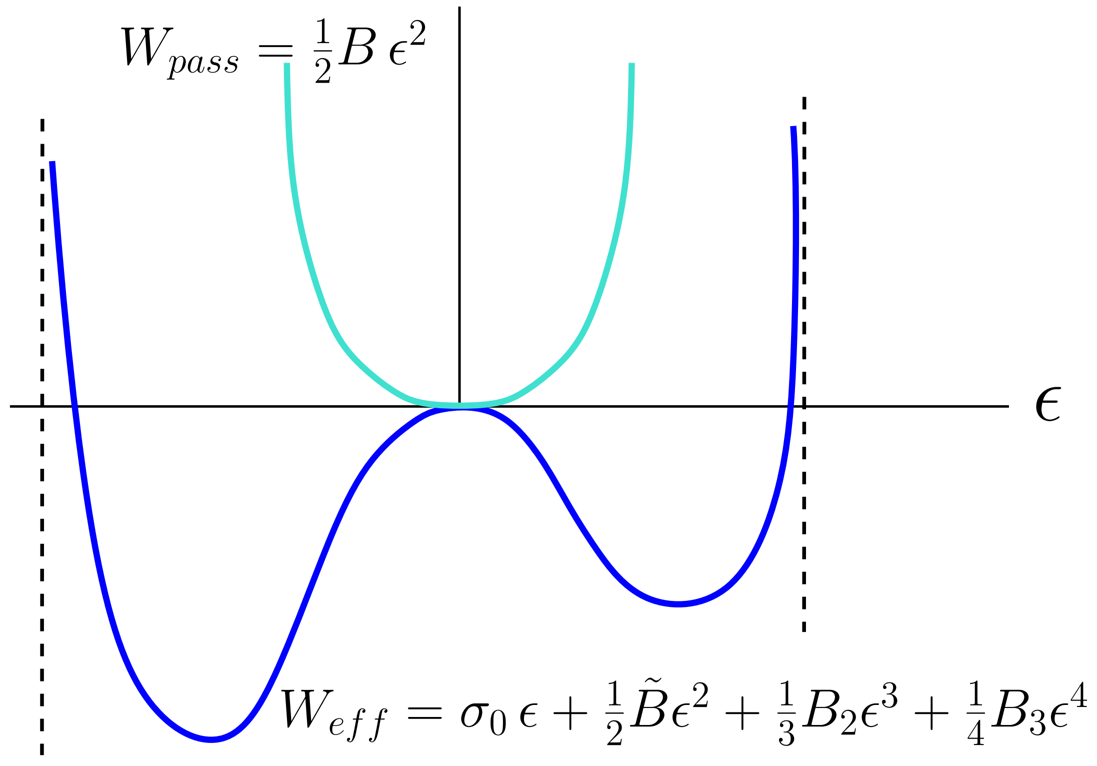
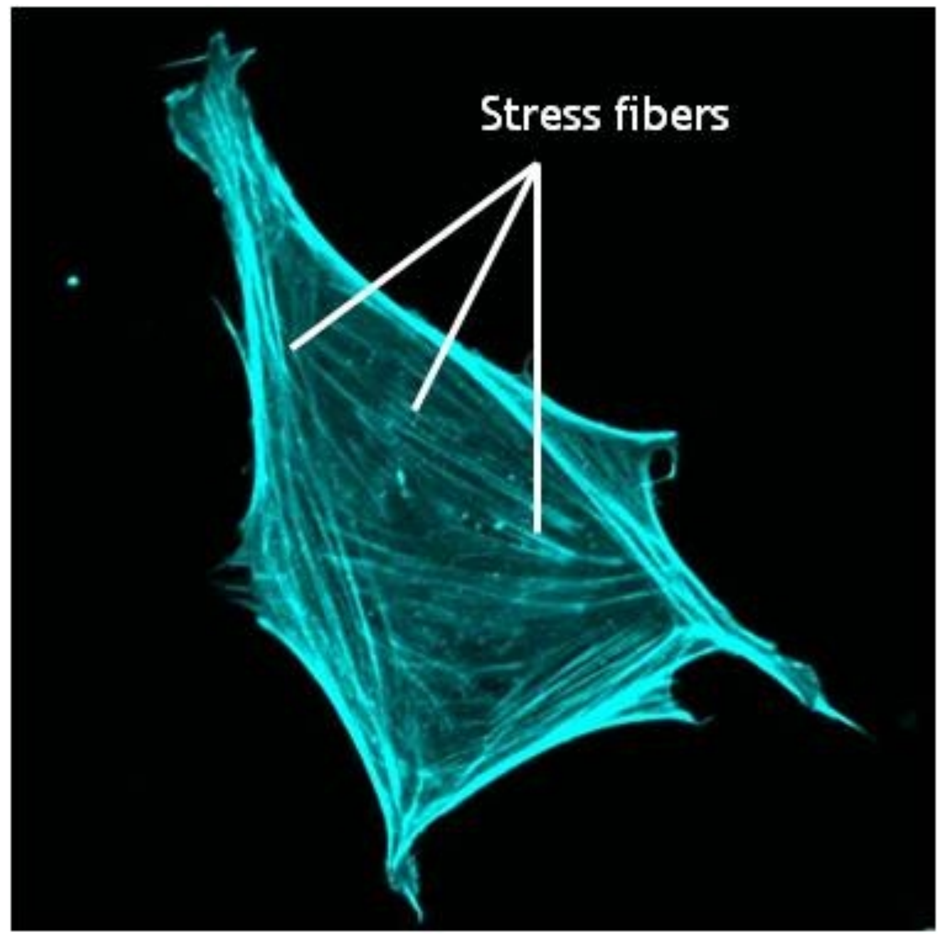




Physical singularity is suppressed by Steric Hindrance

Stress Concentration on Points, Lines, Surfaces



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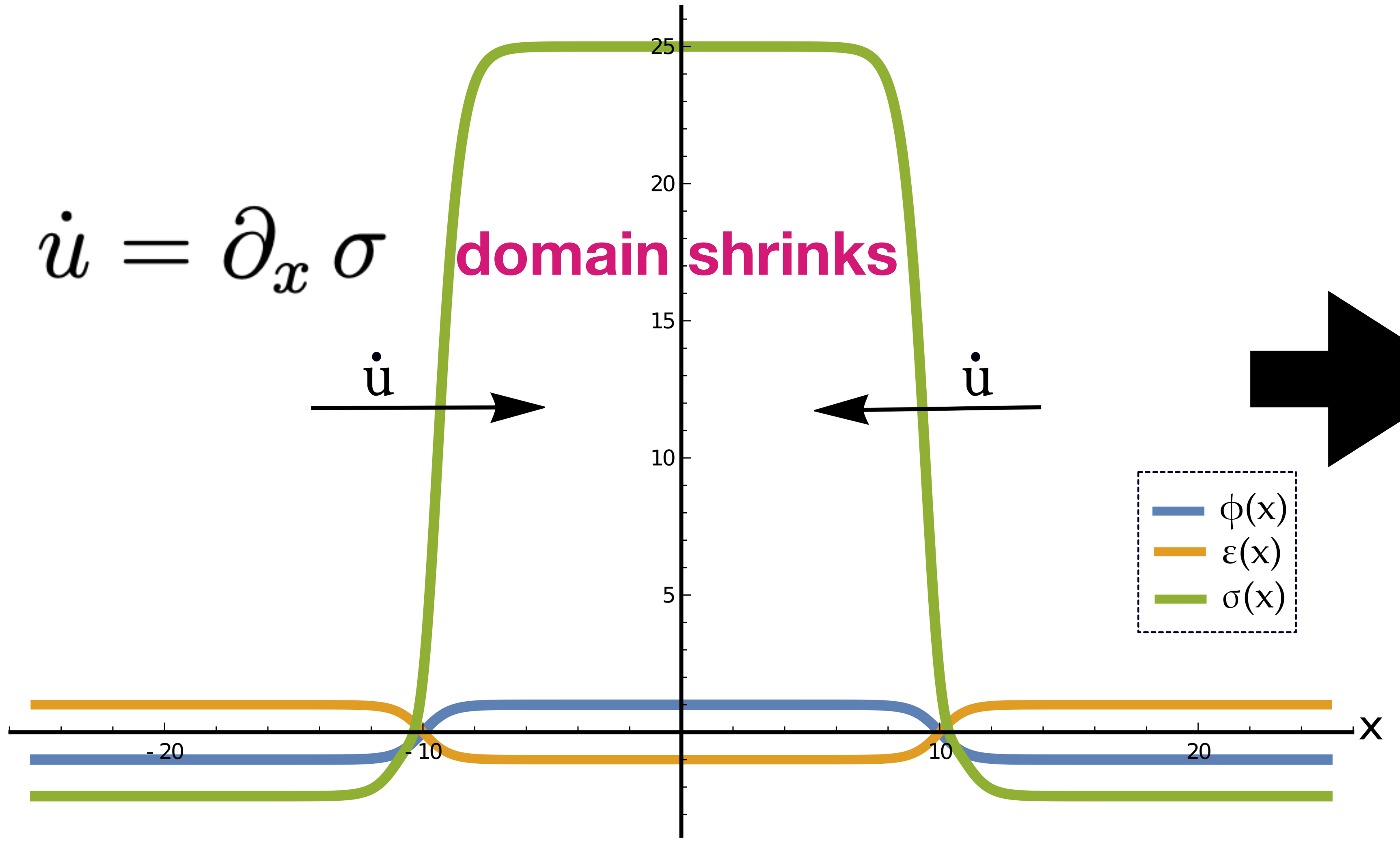

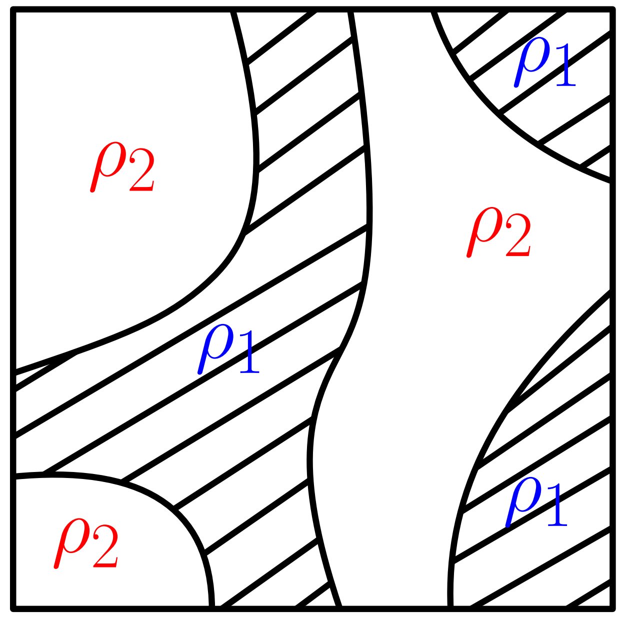

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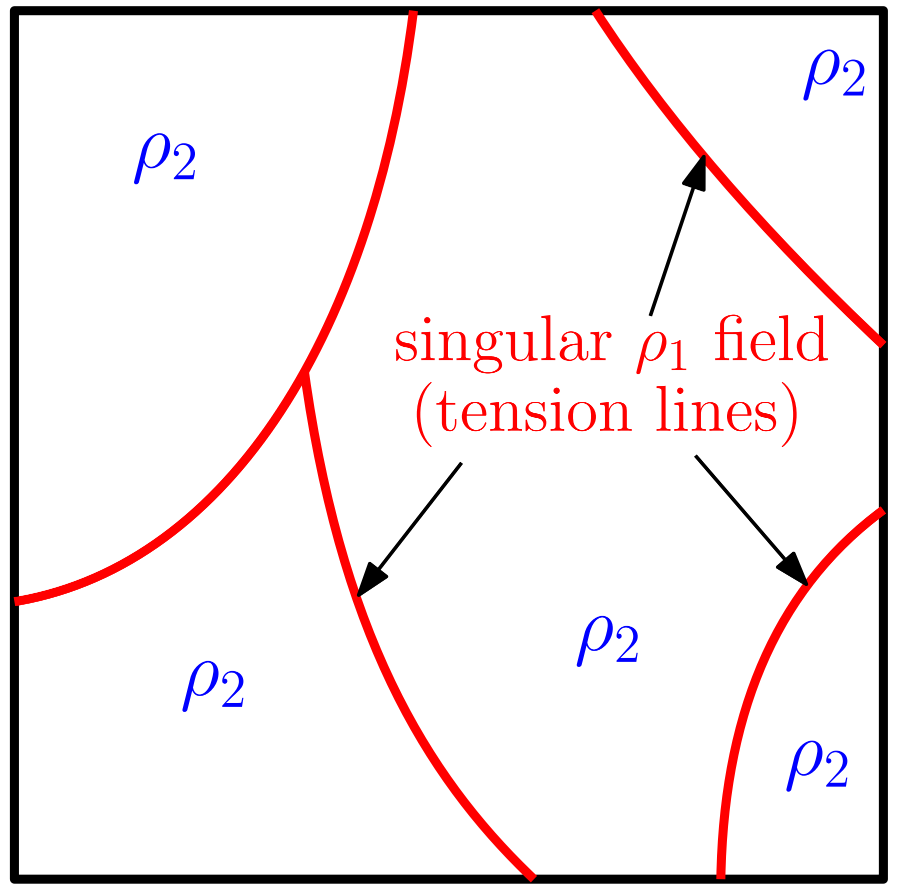

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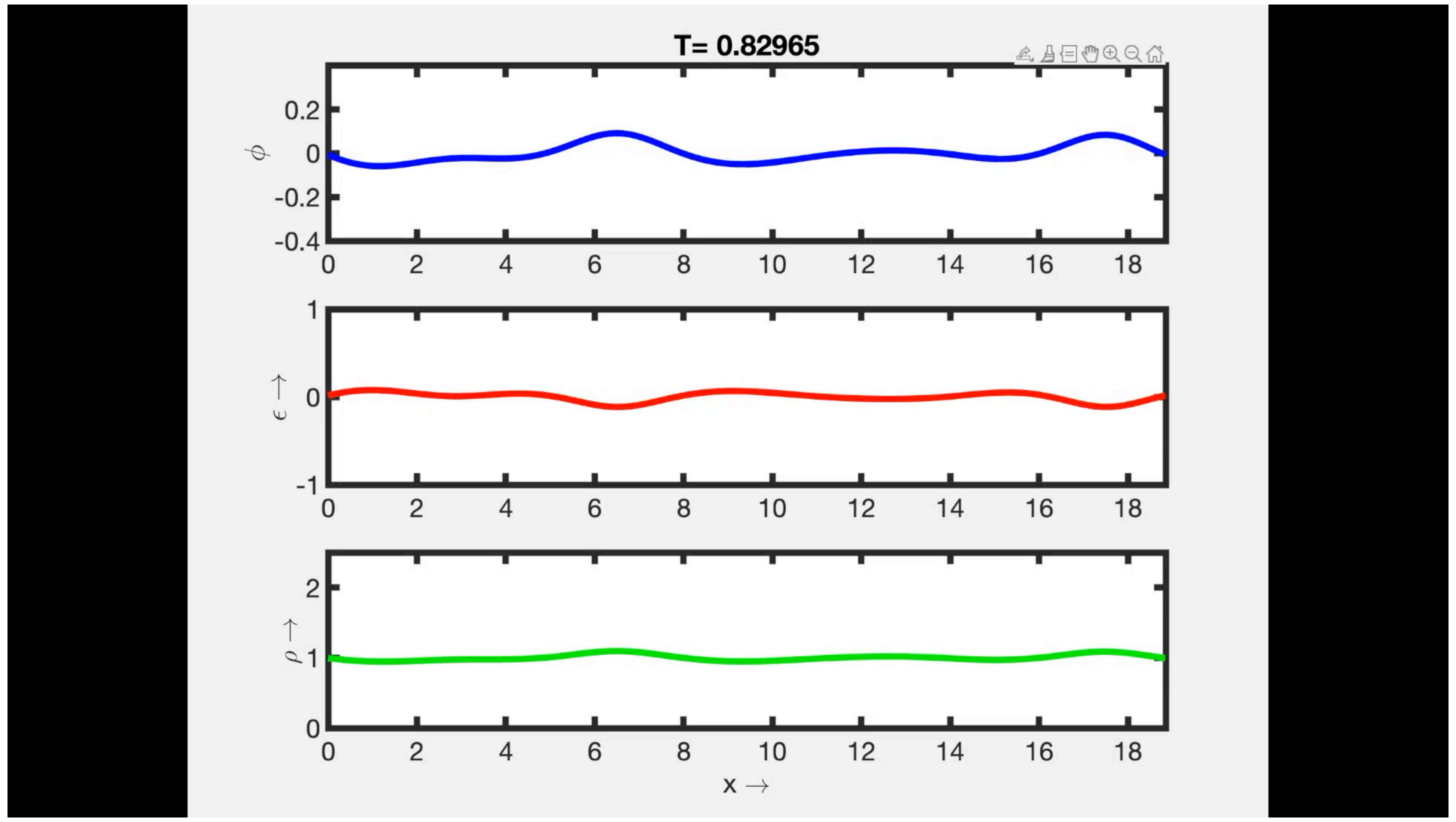

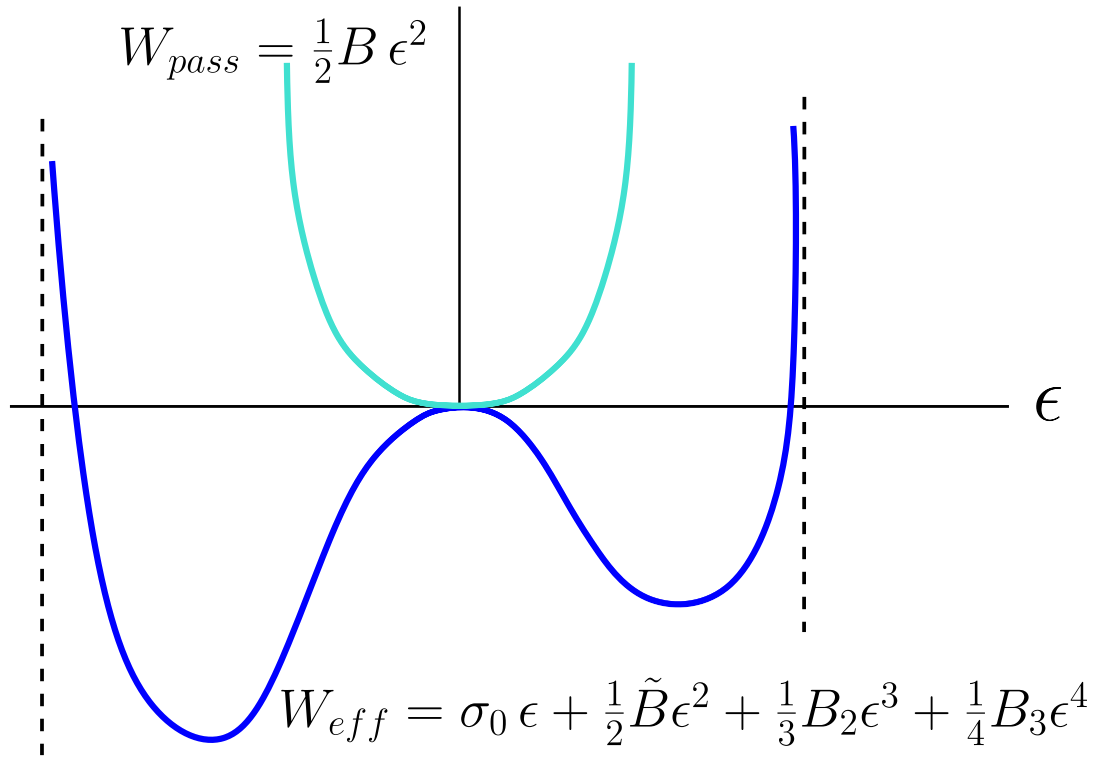

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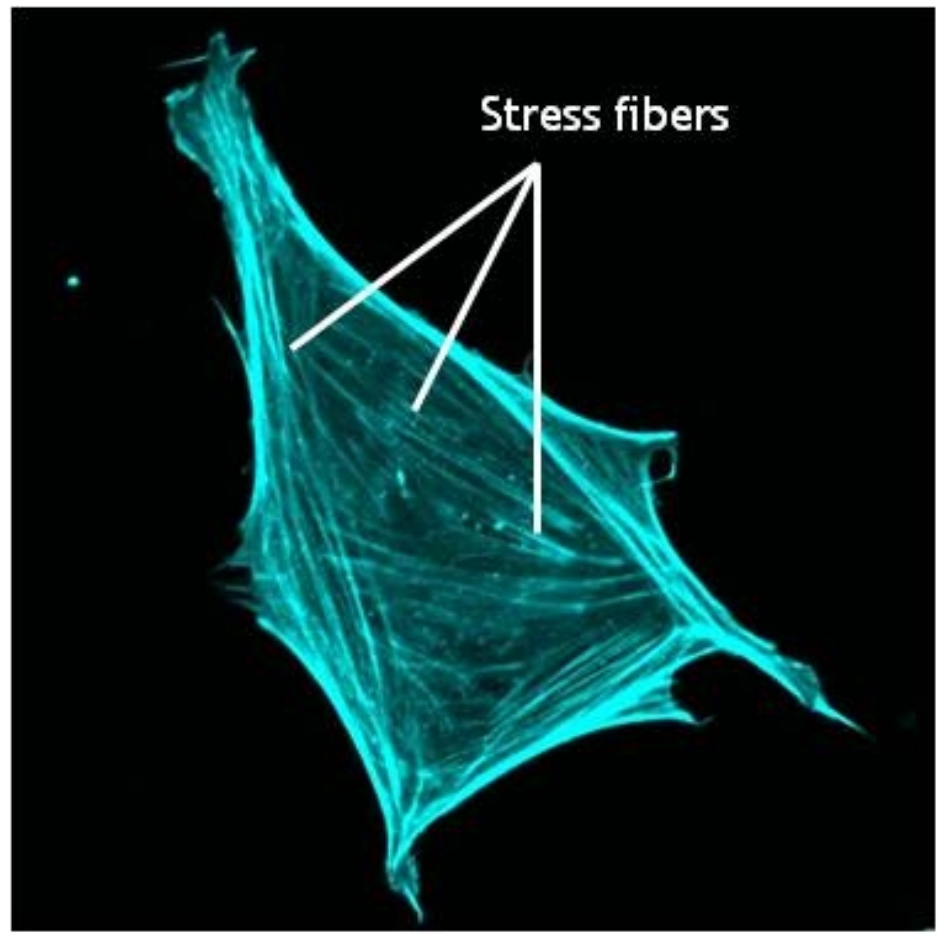
Stress Concentration on Points, Lines, Surfaces

$$[\![p^e]\!] = \gamma H - [\![p^a]\!]$$ **(active Young-Laplace law)**



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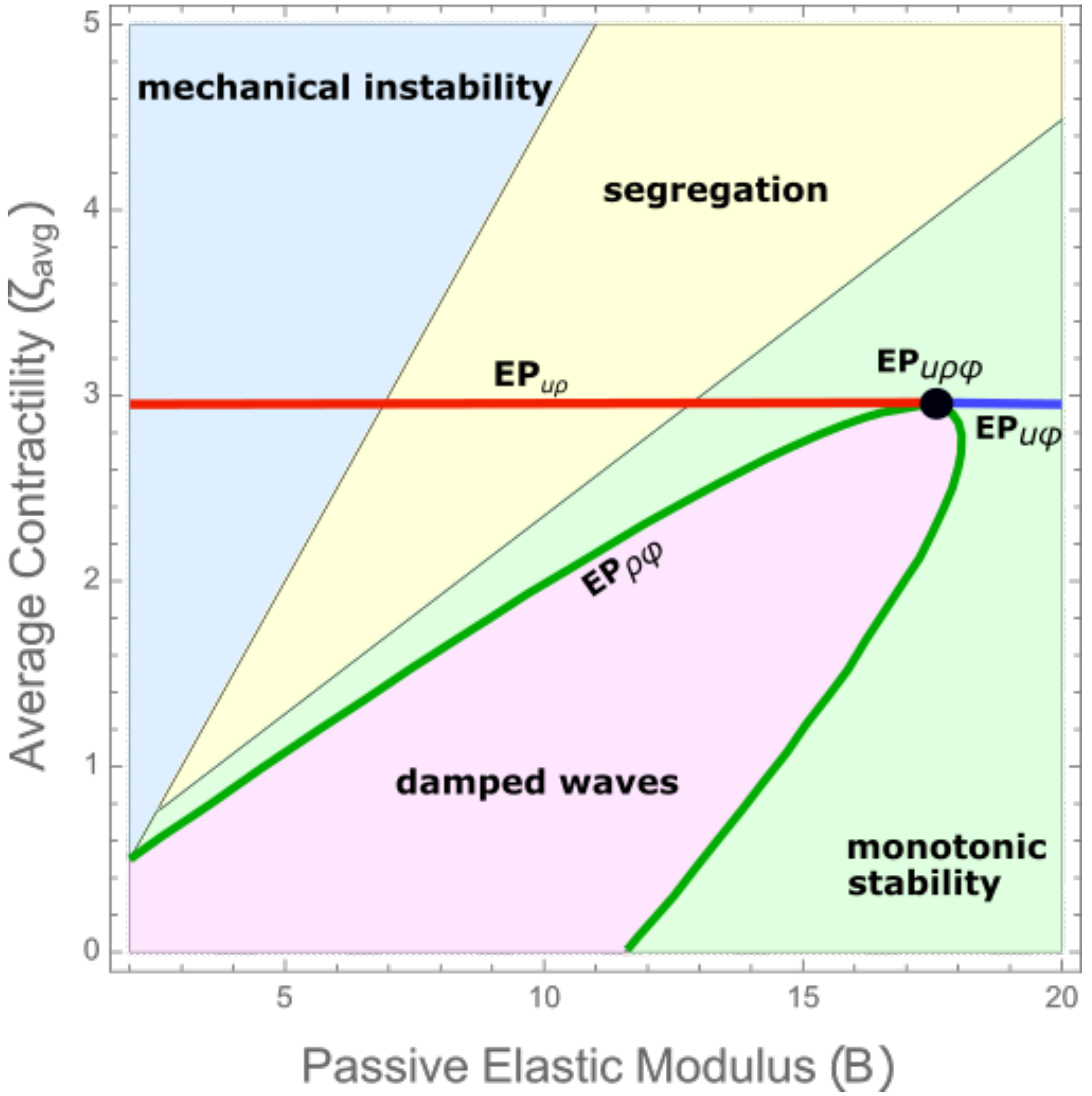
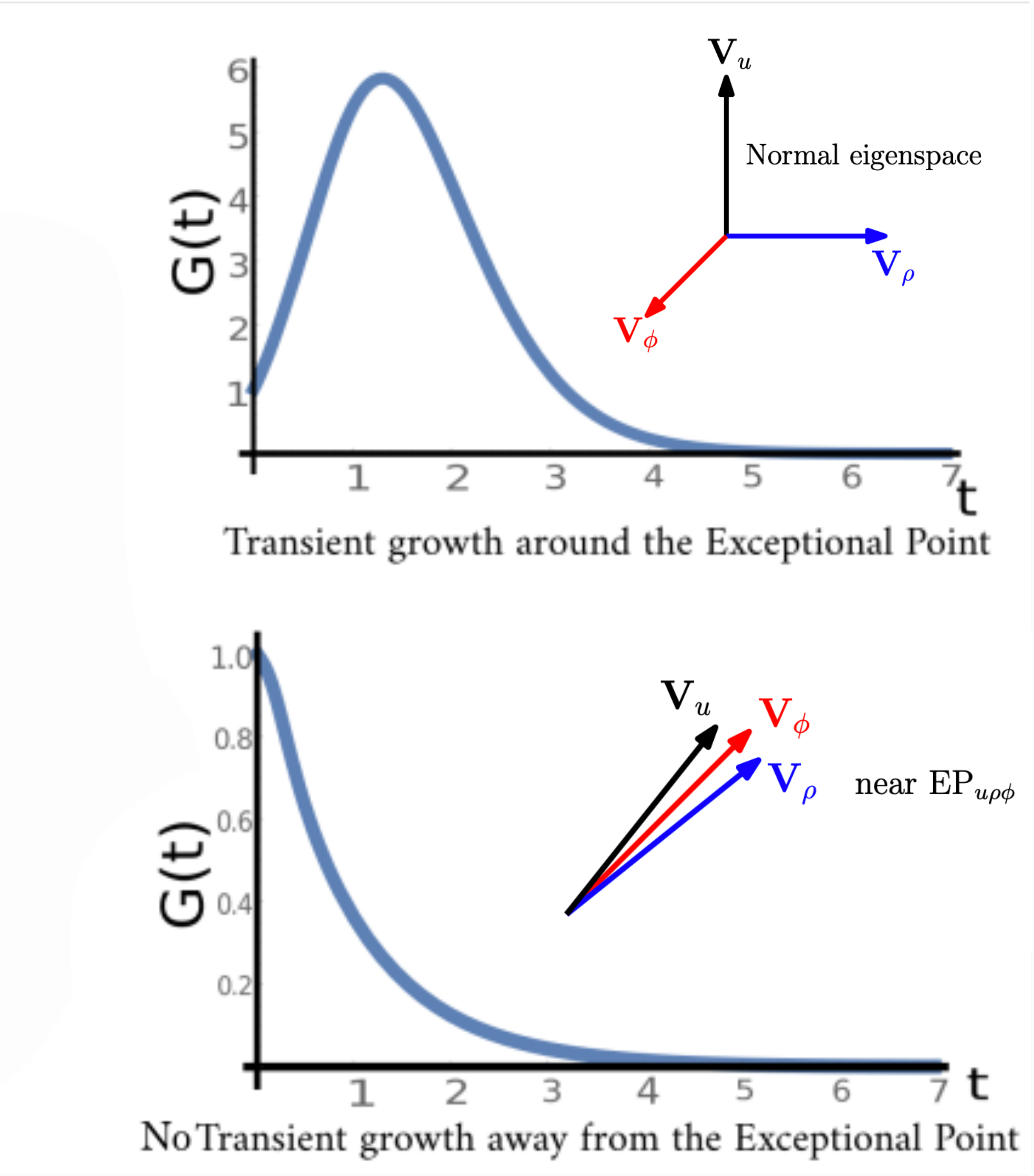
Exceptional Points, Transient Growth, Bypass Transition



$$G(t) := \sup_{\mathbf{w}(\cdot,0)\neq\mathbf{0}} \frac{||\mathbf{w}(\cdot,t)||^2}{||\mathbf{w}(\cdot,0)||^2} = \sup_{\mathbf{w}(\cdot,0)\neq\mathbf{0}} \frac{||\exp(\mathbf{M}t)\mathbf{w}(\cdot,0)||^2}{||\mathbf{w}(\cdot,0)||^2} = ||\exp(\mathbf{M}t)||^2 = ||\mathbf{V}\exp(\mathbf{\Lambda}t)\mathbf{V}^{-1}||^2$$



Eigenvalue-based analysis is misleading near EPs

P J Schmid, *Nonmodal Stability Theory*, Annu. Rev. Fluid Mech., **39**, 2007

Instead of static singular structures, *bypass transition* to other phases are possible, e.g., moving singularities

Summary

Activity and Strain-dependent Turnover lead to Spatiotemporal Segregation of the Uniform Mixture of Stresslets

Elastic Stress Dissipation drives Segregation in the Linear Regime

Nonlinear Feedback leads to Stress Singularities on Lower Dimensional Structures, akin to Stress Fibers

Exceptional Points lead to Bypass Transition

Thank You for Your Attention!