Spin-Orbit Coupled Dirac Fermions On Honeycomb Lattice

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APS Satellite Meeting
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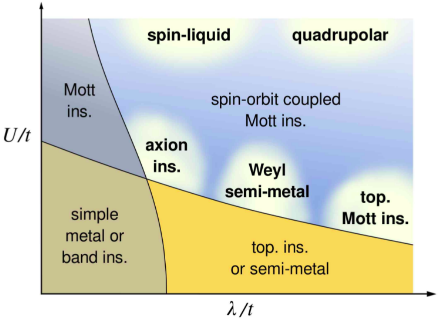
Symmetry realization in low-energy sector and SOC

- SOC and interactions can give rise to interesting phases of matter.



(Ref : Witczak-Krempa, Chen, Kim, Balents, 2014)

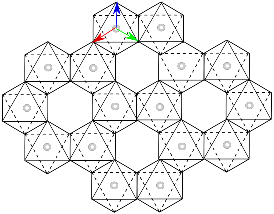
Figure: Schematic phase diagram in presence of SOC(λ) and Coulomb interaction(U)

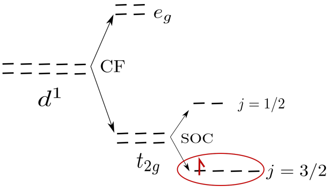
- Symmetries can act non-trivially on the low-energy degrees of freedom.

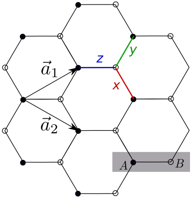
Outline

- Implementation of symmetries on the low-energy degrees of freedom
- Different phases in presence of interactions.

d^1 Honeycomb Materials







$$H = -\frac{t}{\sqrt{3}} \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \psi^\dagger(\mathbf{r}) U_{\mathbf{r}\mathbf{r}'} \psi(\mathbf{r}') + \frac{U}{2} \sum_{\mathbf{r}} \psi^\dagger \psi (\psi^\dagger \psi - 1) \quad (1)$$

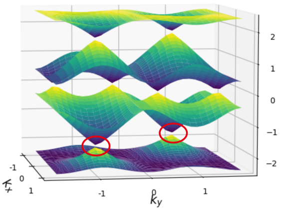
Has emergent global $SU(4)$ *symmetry.*

Candidate materials are : α-$ZrCl_3$, α-$HfCl_3$, $A_2M'O_3$ ($A = Na, Li$ and $M' = Nb, Ta$ *etc.*)

Ref : Yamada, Oshikawa, Jackeli, PRL 121, 097201 (2018)

Low-energy theory

- Low energy DOF are Dirac fermions.



$$\mathcal{S}_0 = \int d^3x \; \bar{\chi} \left[-i \left(\mathbb{I}_4 \otimes \gamma_\mu \right) \partial_\mu \right] \chi \tag{2}$$

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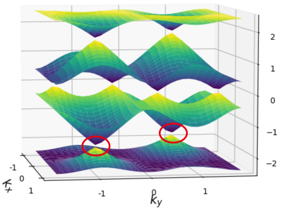

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- $>$ Symmetry transformations
  $>$ Effect of interactions

$$H_{\text{int}} = \int d^2\mathbf{x} d^2\mathbf{x}' \; V_{ijkl}(\mathbf{x} - \mathbf{x}') \chi_i(\mathbf{x})^\dagger \chi_j(\mathbf{x}')^\dagger \chi_k(\mathbf{x}) \chi_l(\mathbf{x}')$$

Symmetries

- Lattice symmetries :

$$S : \chi(\mathbf{x}) \xrightarrow{S} \Omega_S^f \otimes \Omega_S^c \; \chi(S^{-1}\mathbf{x}) \tag{3}$$

$$E.g., \qquad \mathcal{I} : \chi(\mathbf{x}) \xrightarrow{\mathcal{I}} (i\Sigma_{45}) \otimes (i\gamma_0) \; \chi(-\mathbf{x}) \tag{4}$$

- Time reversal :

$$\mathcal{T} : \chi(\mathbf{x}) \xrightarrow{\mathcal{T}} \Omega_{\mathcal{T}}^f \otimes \Omega_{\mathcal{T}}^c K \; \chi(\mathbf{x}) \tag{5}$$

- Emergent SU(8) :

$$\chi(\mathbf{x}) \to \exp\left(-i\lambda_a \mathcal{P}_a\right) \chi(\mathbf{x}), \tag{6}$$

with

$$\mathcal{P}_a = \left\{ \underbrace{\zeta_i}_{chiral\ space} , \underbrace{\Sigma_j}_{flavor\ space} , \Sigma_i \zeta_j \right\} \tag{7}$$

Mean-field phases

- The semi-metal phase

$$\mathcal{S}_0 = \int d^2\mathbf{x}d\tau\ \bar{\chi}\left[-i\gamma_\mu\partial_\mu\right]\chi \tag{8}$$

is stable in presence of weak interactions

$$H_{\text{int}} = \int d^2\mathbf{x}d^2\mathbf{x}'\ V_{ijkl}(\mathbf{x}-\mathbf{x}')\chi_i(\mathbf{x})^\dagger\chi_j(\mathbf{x}')^\dagger\chi_k(\mathbf{x})\chi_l(\mathbf{x}'). \tag{9}$$

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- In case of strong interaction, a fermion bilinear

$$\Delta_a = \langle -i\bar{\chi}\mathcal{P}_a\chi \rangle \tag{10}$$

can condense and gap out the fermions.
- There are 64 of such bilinears.
- These have the slowest decaying correlations (Hermele, Senthil, Fisher, 2005)

Mean field phases

- Now, we look at how these bilinears(Δ_a) transform under the action of lattice symmetries.
- Space group has 1,2 and 3-dimensional representations

$$
\begin{aligned}
&64\ bilinears \\
&= 2 \cdot A_{1g} \oplus 2 \cdot A_{2g} \oplus 1 \cdot A_{1u} \oplus 1 \cdot A_{2u} \oplus 2 \cdot E_g \oplus 3 \cdot E_u \\
&\quad \oplus 4 \cdot T_{1g} \oplus 4 \cdot T_{2g} \oplus 4 \cdot T_{1u} \oplus 4 \cdot T_{2u} \\
&= 2 \cdot A^e_{1g} \oplus 2 \cdot A^o_{2g} \oplus 1 \cdot A^o_{1u} \oplus 1 \cdot A^o_{2u} \\
&\quad \oplus 1 \cdot E^e_g \oplus 1 \cdot E^o_g \oplus 2 \cdot E^o_u \\
&\quad \oplus 3 \cdot T^e_{1g} \oplus 1 \cdot T^o_{1g} \oplus 1 \cdot T^e_{2g} \oplus 3 \cdot T^o_{2g} \\
&\quad \oplus 2 \cdot T^e_{1u} \oplus 2 \cdot T^o_{1u} \oplus 2 \cdot T^e_{2u} \oplus 2 \cdot T^o_{2u}
\end{aligned} \tag{11}
$$

SU(4) symmetric phases

Four SU(4) symmetric masses:

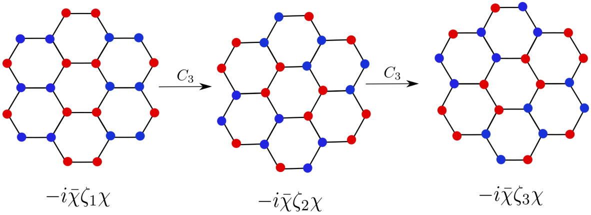
$$4 = 1 \oplus 3 \tag{12}$$

$$\underbrace{-i\bar{\chi}\chi,}_{QH} \quad \underbrace{-i\bar{\chi}\zeta_1\chi, -i\bar{\chi}\zeta_2\chi, -i\bar{\chi}\zeta_3\chi}_{Stripy\ CDW} \tag{13}$$

- The quantum Hall mass produces a Chern-Simons term :

$$\mathcal{L}_{CS} = \pm\, i\frac{N_f}{4\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda, \quad (N_f = 4) \tag{14}$$

- CDW :



Comparison with Graphene

SU(2) symmetric phases in graphene with electrons are

$$4 = 1 \oplus 1 \oplus 2 \tag{15}$$

- QH
- Staggered charge density wave
- Kekulé

SU(4) breaking phases

- Consider the mass

$$-i\bar{\chi}\Sigma_{45}\chi, \tag{16}$$

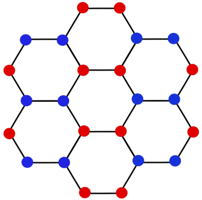
  - $\Sigma_{45} \sim (J_x J_y J_z + J_y J_z J_x + J_z J_x J_y) - 15i/8$.
  - Does not break break lattice symmetries.
  - Breaks $SU(4)_F \to U(1)_{45} \times SO(4)$.
  - Develops mutual CS terms in the action

$$\mathcal{S}[A_c, A_o] = -i \int d^2\mathbf{r} d\tau \; \overline{\chi}(\mathbf{r}) \left[\not{D} - m_0 \Sigma_{45} \right] \chi(\mathbf{r}) \tag{17}$$

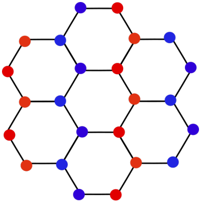
$$S_{CS} = i \frac{N_f}{2\pi} \; sgn(m_0) \; \int d^3x \; \epsilon^{\mu\nu\lambda} (A_c)_\mu \partial_\nu (A_o)_\lambda \tag{18}$$

SU(4) breaking phases

- **Density waves :**



Stripy density wave in Σ_{12}



Zig-zag density wave in Σ_{35}

Here,

$$\Sigma_{12} = \frac{7}{3}\left(S_z - \frac{4}{7}S_z^3\right)$$

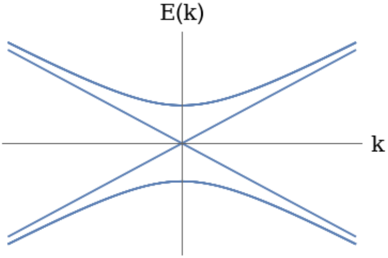
$$\Sigma_{35} = \frac{2}{3\sqrt{3}}\left[(S_x^2 S_z + S_x S_z S_x + S_z S_x^2) - (S_y^2 S_z + S_y S_z S_y + S_z S_y^2)\right]$$

- 15 topological phases corresponding to different ways of breaking flavor space symmetry SU(4).

- Partially gapless phases.

Partially Gapless Phases

- Some masses have partially gapless spectrum.



- These are density waves with semi-metal character.

- Need to understand the gaplessness, symmetry protection and phase transitions between gapless phases.

Summary

- Due to SOC, lattice symmetries act non-trivially on flavor space.
- Emergent SU(8) symmetry in low-energy theory.
- Because of large symmetry, we have different density waves and different QH phases.
- Because of non-trivial symmetry implementation, there are masses with partially gapless spectrum.

Future Directions

- Study the phase transitions.
- Study the superconducting instabilities.

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- *Thank You*

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