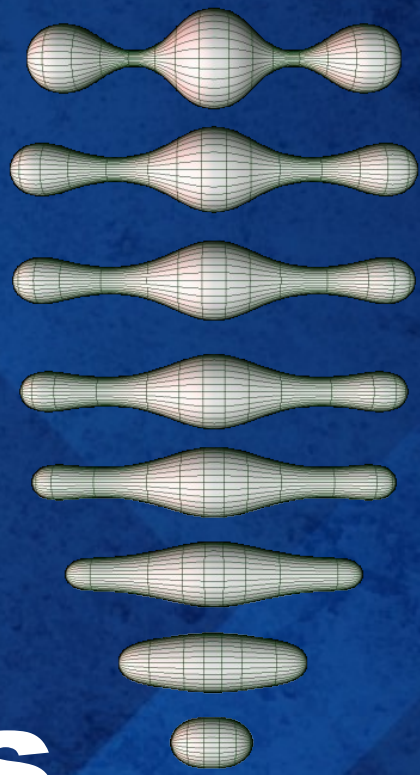
Ricci Flow Analysis of Financial Assets



Presenter:

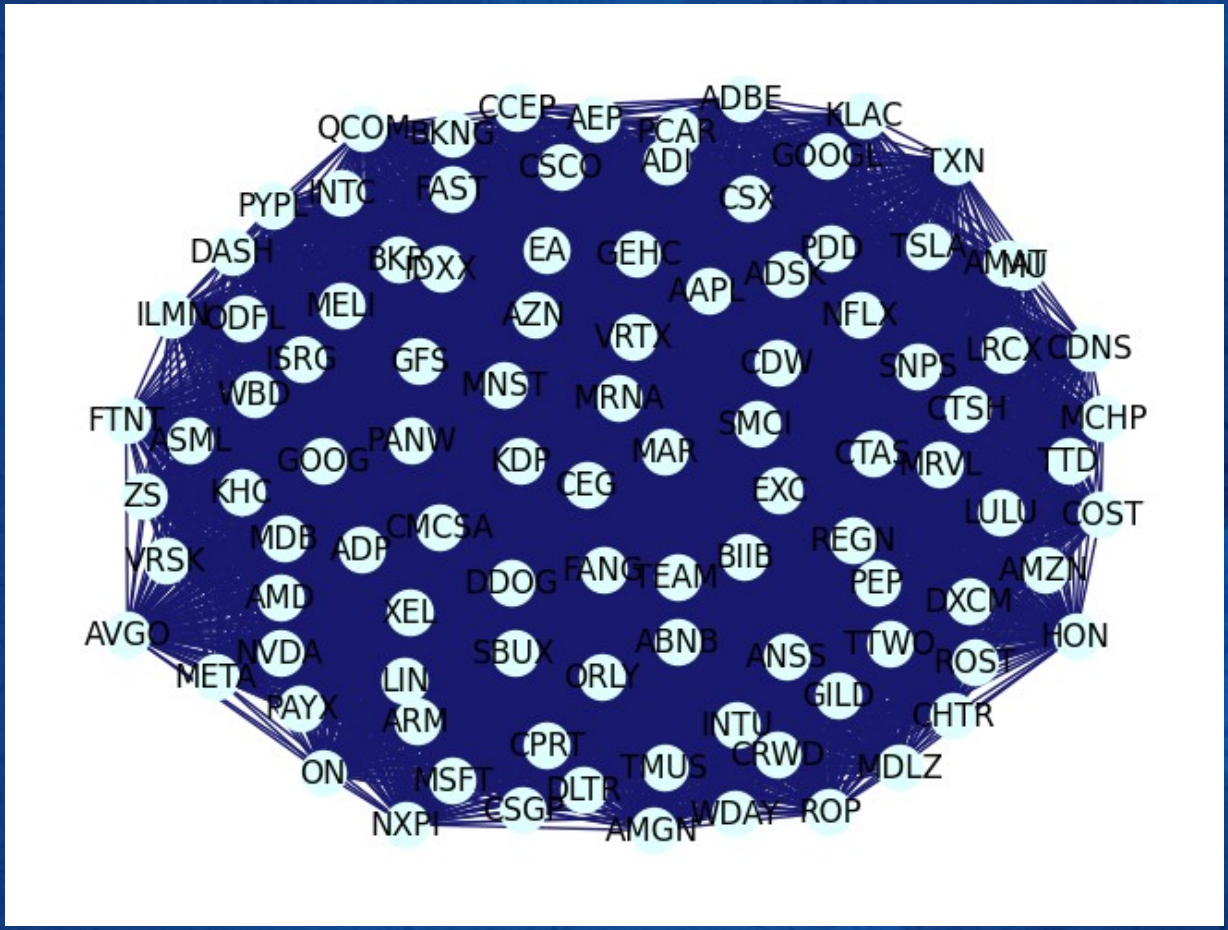
Bhargavi Srinivasan

Organization:

CNRS, LPTMS

NASDAQ100 2024

Bhargavi Srinivasan (2024)



Fully connected graph without apparent structure
Average correlation 38%

Correlation computed from 2020-01-01 to 2024-11-15, daily data from Yahoo finance

Why study stock correlations?

Complex dynamical system

- What are the economic factors driving stock prices?
- Fundamental sectors are limited and might not reflect the current reality of a company. GE, Google
- Data driven analysis can reveal factors driving company prices

Applications

- Risk measure and risk modeling
- Detecting emerging thematic risk basket
- Portfolio allocation and portfolio optimization
- Derivative pricing...

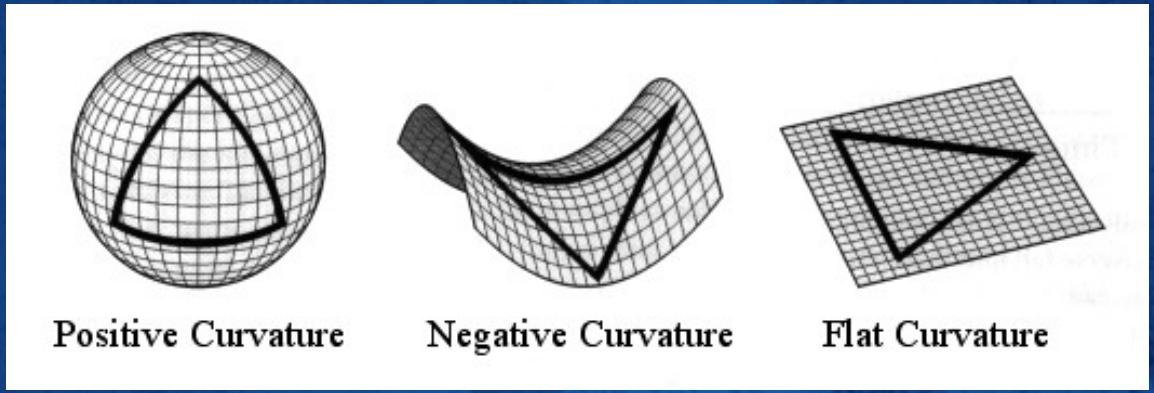
Trends in Data Analysis

- **Topological Data Analysis**
  - ***Persistent Homology / Betti numbers***

- **Geometrical Data Analysis**
  - ***Ricci curvature and flow***
    - ***Forman Ricci combinatorial***
    - ***Olliver Ricci with Optimal Transport***

What is Ricci Flow?

Riemann: 2-manifolds can be classified



- Poincaré conjecture (1904)
  What happens for 3-manifolds?
- Thurston Geometrization (1982)
- Hamilton-Ricci flow (1982)
- Perelman solution (2003)

Ricci Flow definition

$$\frac{\partial}{\partial t} g_{ij}(t) = -2 R_{ij}(t)$$

$g_{ij}(t)$ family of metrics

$R_{ij}(t)$ Ricci curvature tensor computed from $g_{ij}(t)$

+“surgery” for singularities

Ricci Flow Einstein metric

$$g_{ij}(t)=(1-2\lambda t)\,g_{ij}(0)$$

$$R_{ij}(t)=R_{ij}(0)=\lambda\, g_{ij}(0)$$

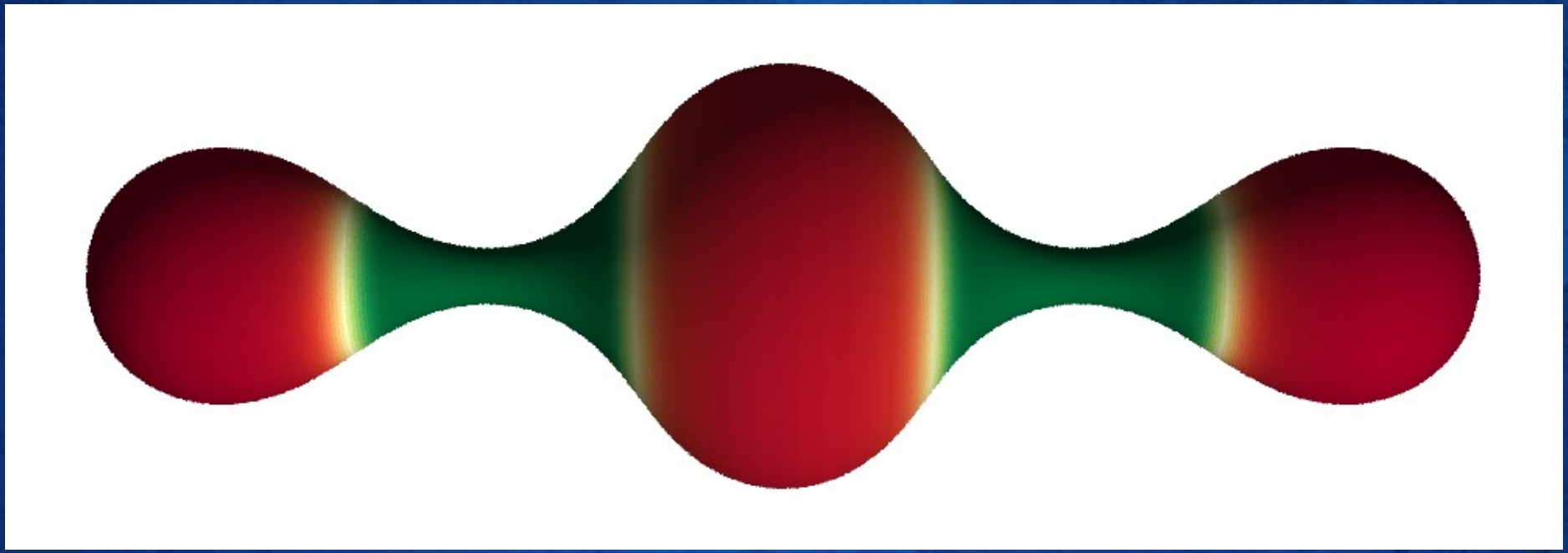
$\lambda>0:(1-2\lambda t)<1$ **Flow contracts**

! singularity at $t=(\frac{1}{2\lambda})$

$\lambda=0:(1-2\lambda t)=1$ **Ricci flat**

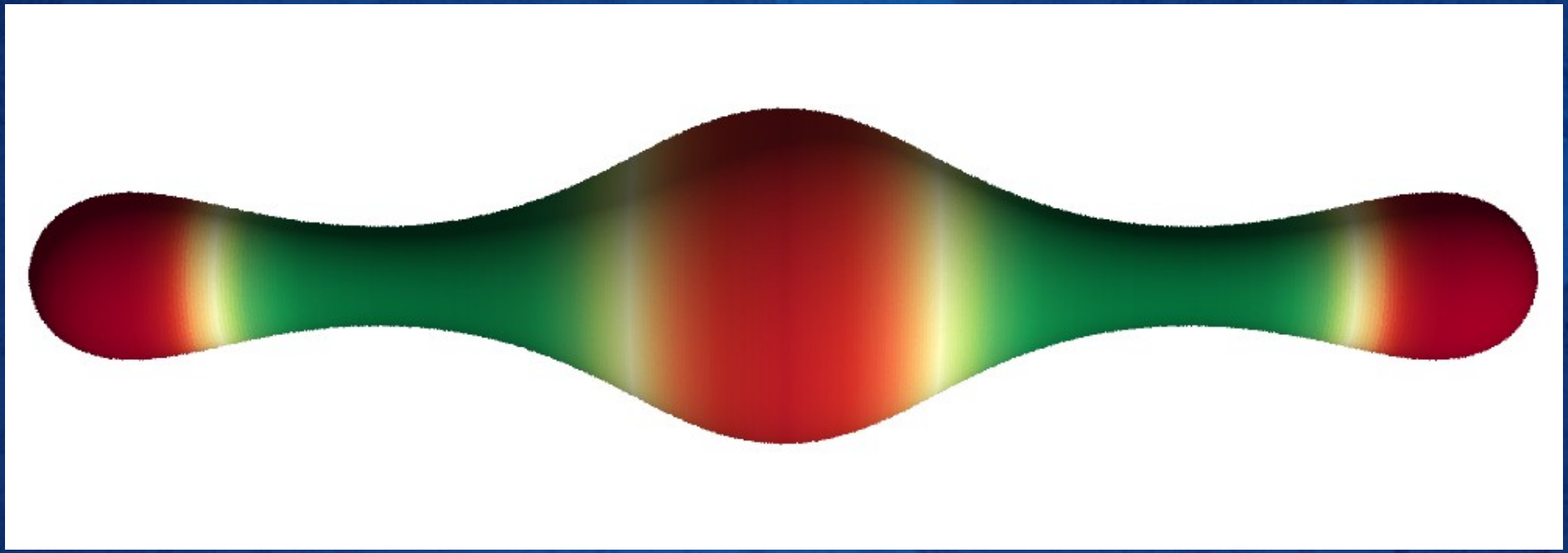
$\lambda<0:(1-2\lambda t)>1$ **Flow expands**

Getting a feel for Ricci Flow



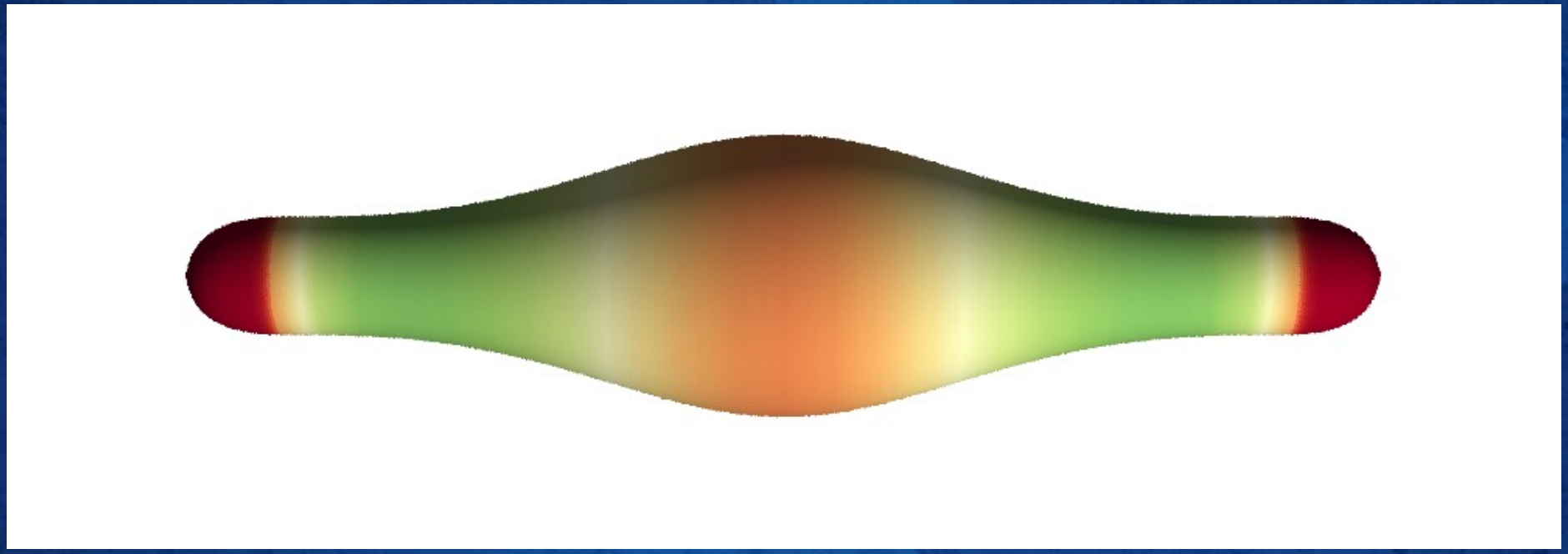
Ricci flow dynamics of a surface of revolution shape converging to a sphere
Rubenstein and Sinclair, Experimental Mathematics (2005),

Getting a feel for Ricci Flow



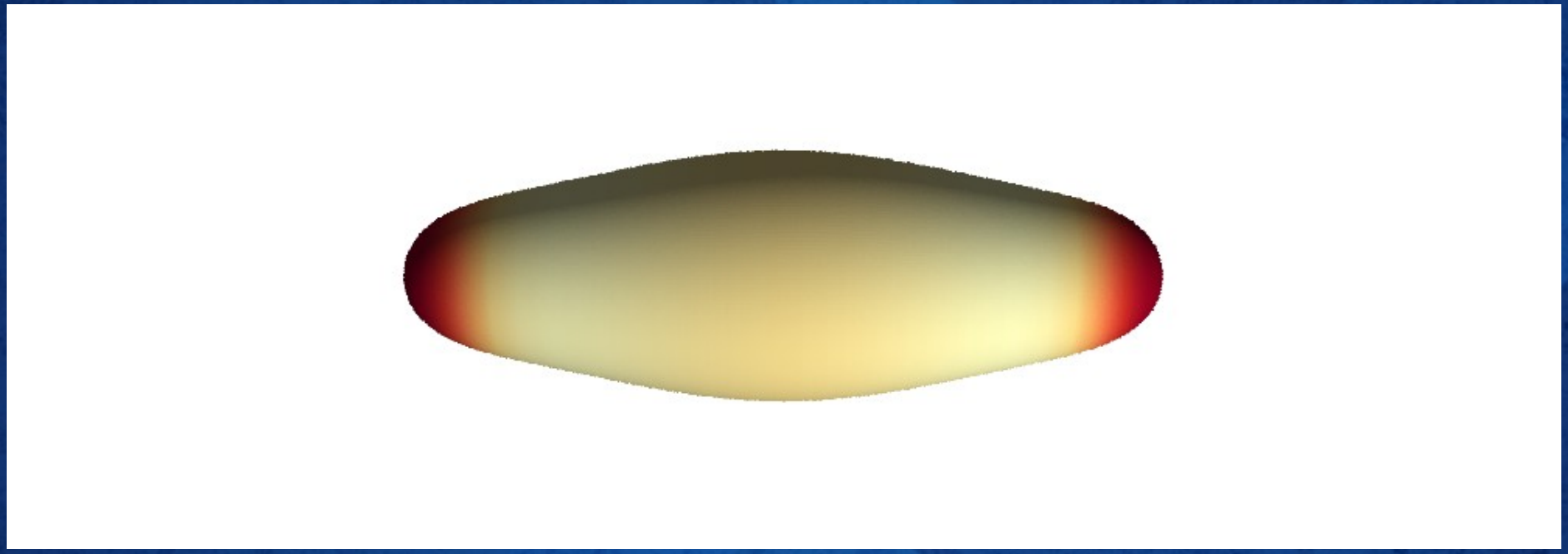
Ricci flow dynamics of a surface of revolution shape converging to a sphere

Getting a feel for Ricci Flow



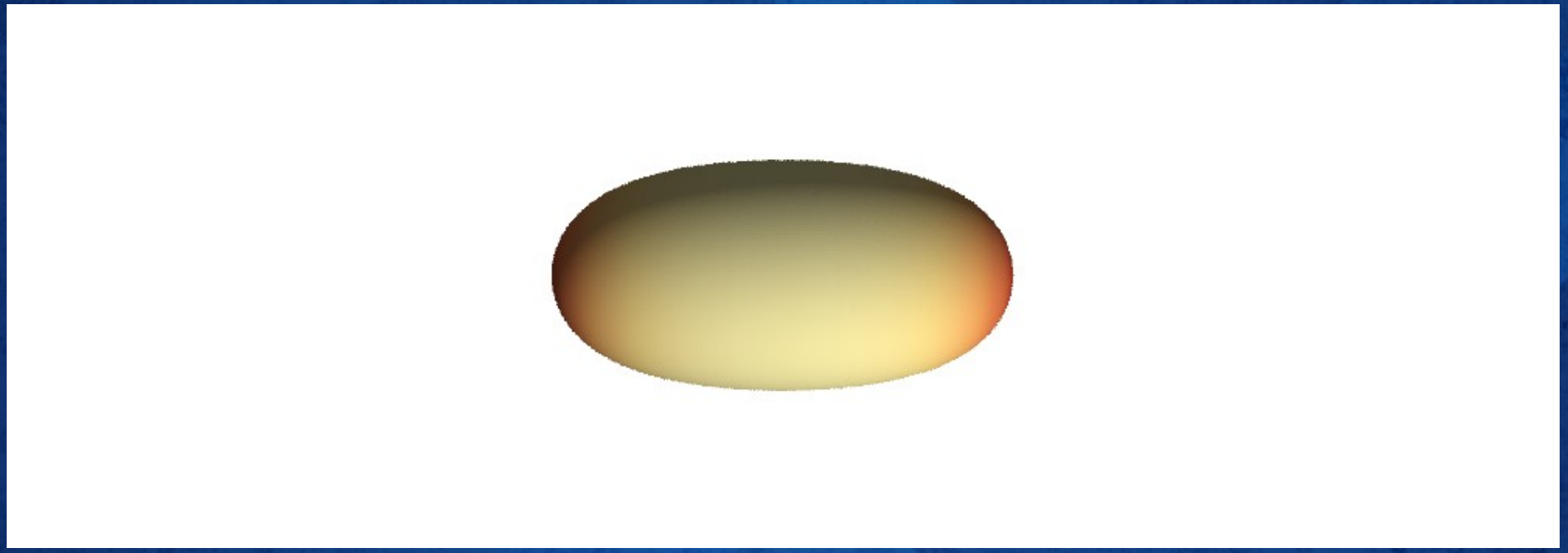
Ricci flow dynamics of a surface of revolution shape converging to a sphere

Getting a feel for Ricci Flow



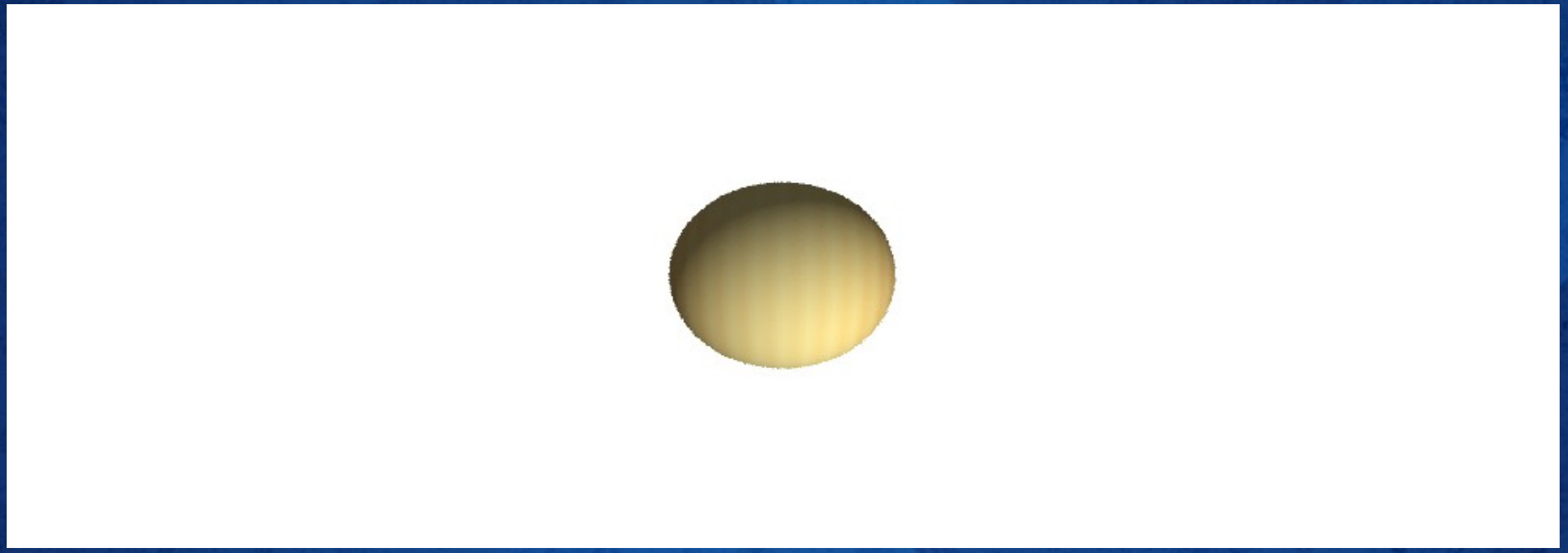
Ricci flow dynamics of a surface of revolution shape converging to a sphere

Getting a feel for Ricci Flow



Ricci flow dynamics of a surface of revolution shape converging to a sphere

Getting a feel for Ricci Flow



Ricci flow dynamics of a surface of revolution shape converging to a sphere

Ollivier-Ricci curvature

$$\kappa(x,y) = 1 - \frac{W(x,y)}{d(x,y)}$$

$\kappa(x,y)$ **Ollivier Ricci curvature**

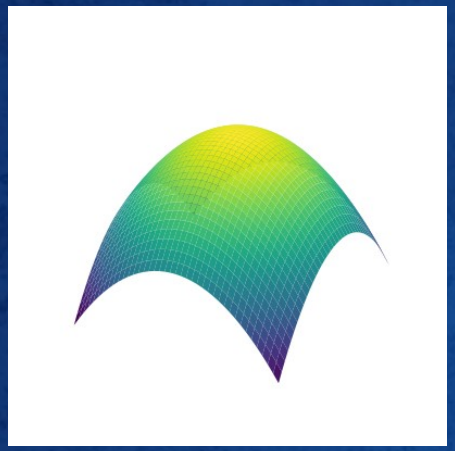
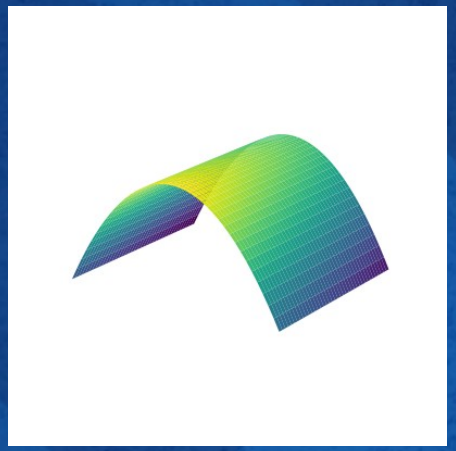
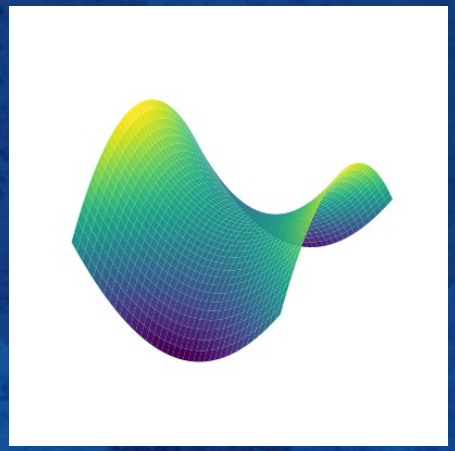
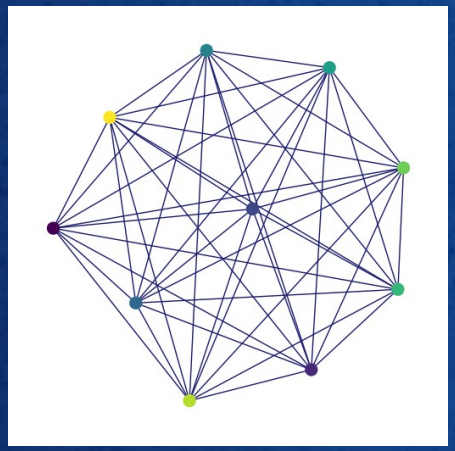
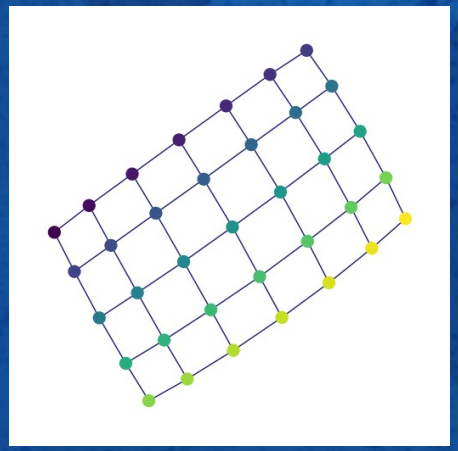
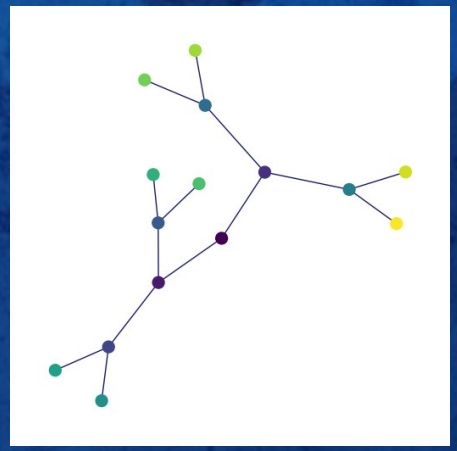
$W(x,y)$ **OT Wasserstein distance**

$d(x,y)$ **Metric distance**

→ **Defines a "coarse" curvature for metric spaces**

Y. Ollivier, J. Funct. Anal. (2009)

Ollivier-Ricci curvature

Positive/Spherical
$\kappa > 0$

Zero/Flat
$\kappa = 0$

Negative/Hyperbolic
$\kappa < 0$

Ollivier-Ricci curvature

Graph $G(V,e)$

N vertices: V_i, edge weights: e_{ij}

Edge curvature: $\kappa_{ij} = 1 - \frac{W_{ij}}{d_{ij}}$

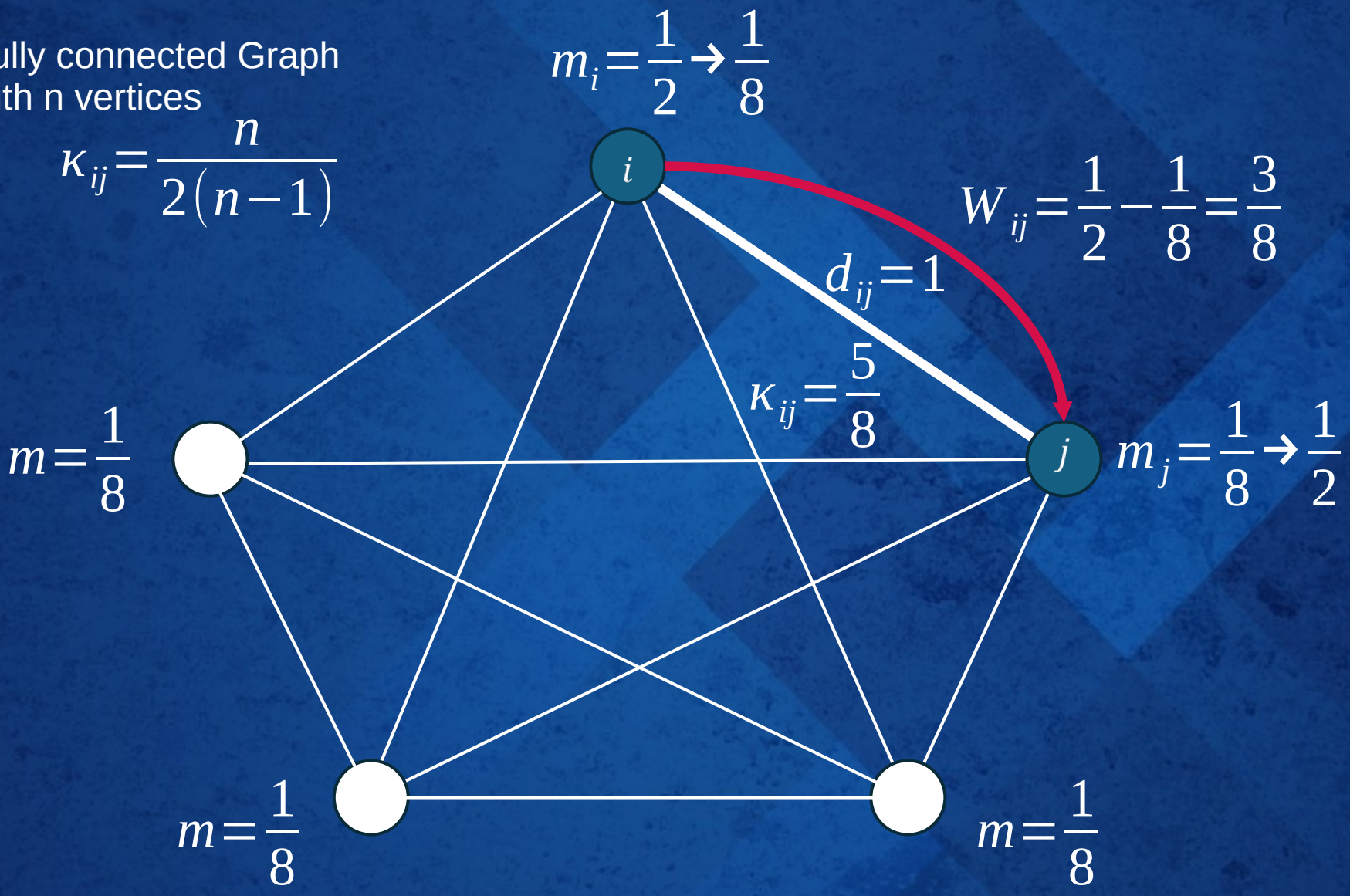
d_{ij} Distance computed from e_{ij}

W_{ij} Wasserstein OT distance

Ollivier-Ricci curvature

Fully connected Graph
with n vertices

$$\kappa_{ij} = \frac{n}{2(n-1)}$$



Graph Ricci flow

$$d_{ij}(t+1)=(1-\kappa_{ij}(t))d_{ij}(t)$$

- $\kappa>0$: Reduces edge weight
- $\kappa<0$: Increases edge weight
- $\kappa=0$: Keeps edge weight constant

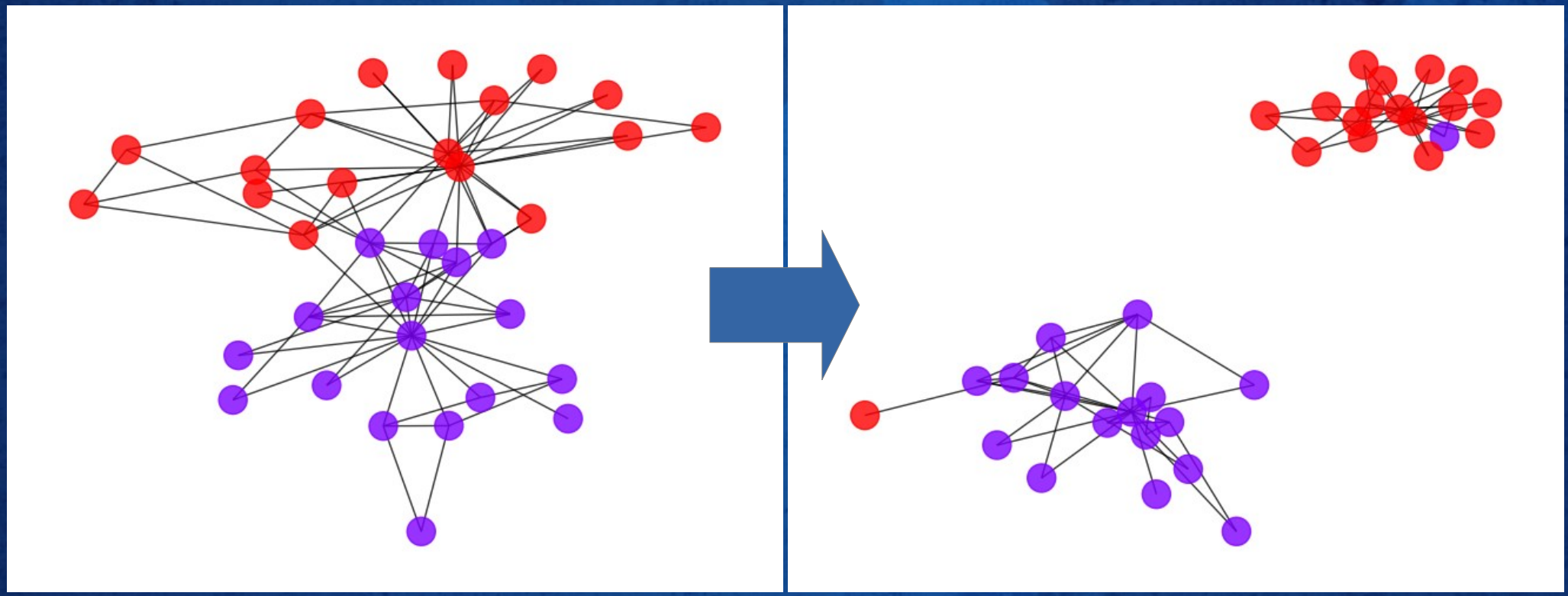
- Singulaties removed with surgery

Weber, Saucan, Jost, J Complex Netw (2017)
Ni, CC., Lin, YY., Luo, F. et al. Sci Rep (2019)
J. Jost, Riemannian Geometry and Geometric Analysis, Springer, 2011

Community detection

- **Benchmark: Zachary karate club**

Zachary, W. W., J Anthropological Research (1977)
Girwan, M., Newman, M. E. J., PNAS (2002)



Clustering stocks

- **N stocks** $1 \le i \le N$
- $P_i(t)$ **Price of stock i at time t**
- $r_i(t) = \frac{P_i(t) - P_i(t-1)}{P_i(t-1)}$ **Return of i at t**
- C_{ij} **Correlation between i and j**

Pearson correlation coefficient

$$C_{ij} = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{\left(\langle r_i^2 \rangle - \langle r_i \rangle^2\right)\left(\langle r_j^2 \rangle - \langle r_j \rangle^2\right)}}$$

Clustering stocks

- Distance from correlation

$$d_{ij} = \sqrt{2\left(1 - C_{ij}\right)}$$

- Defines a graph:
  - Vertex for each stock i
  - edge weights $e_{ij} = d_{ij}$

- Stocks are positively correlated

  fully connected graph

10 Stocks Toy Model

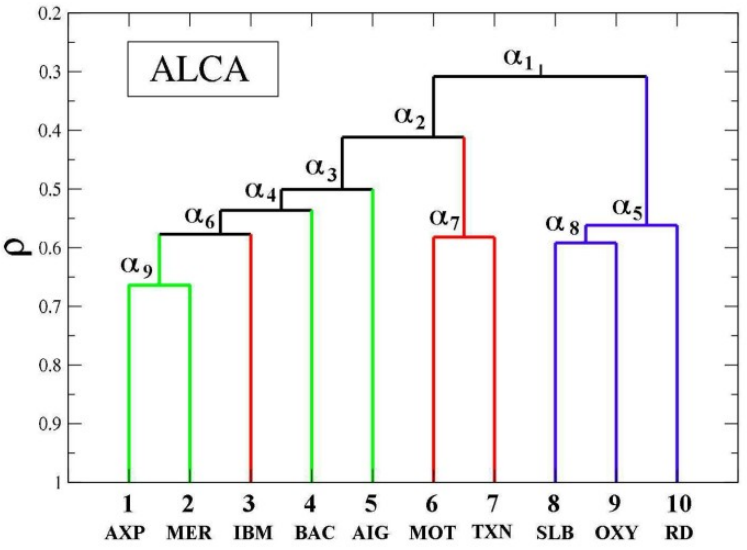
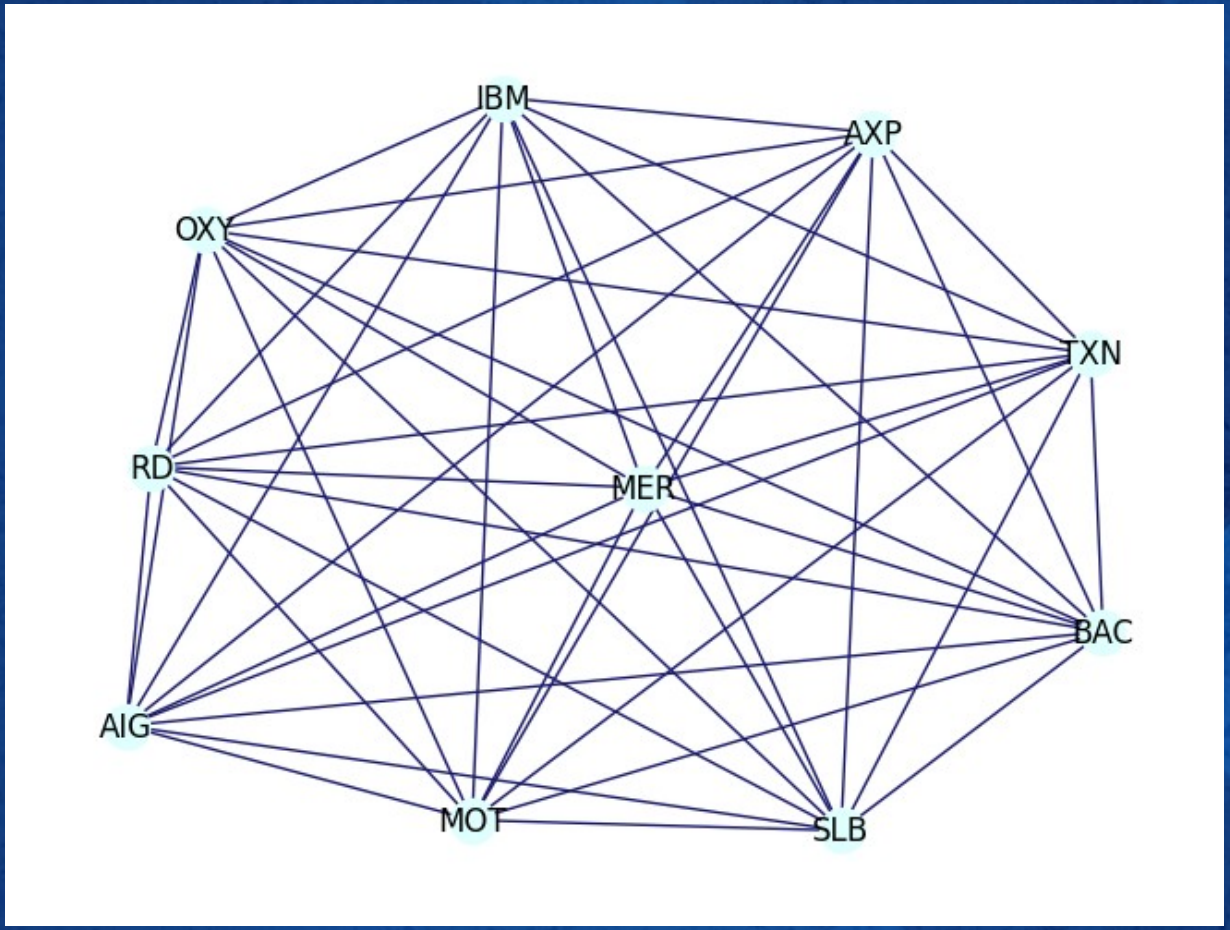


Fig. 1. Average linkage cluster analysis. Illustrative example of a hierarchical tree associated to a system of $N = 10$ stocks (tick symbols label stocks at the bottom of the hierarchical tree. Each element of the system is also labeled with an integer number). The color of line indicates the primary economic sector of the stock, red for technology, blue for energy and green for financial. The labels of the nodes of the hierarchical tree are used in the discussion of the hierarchically nested factor model of Section 3.

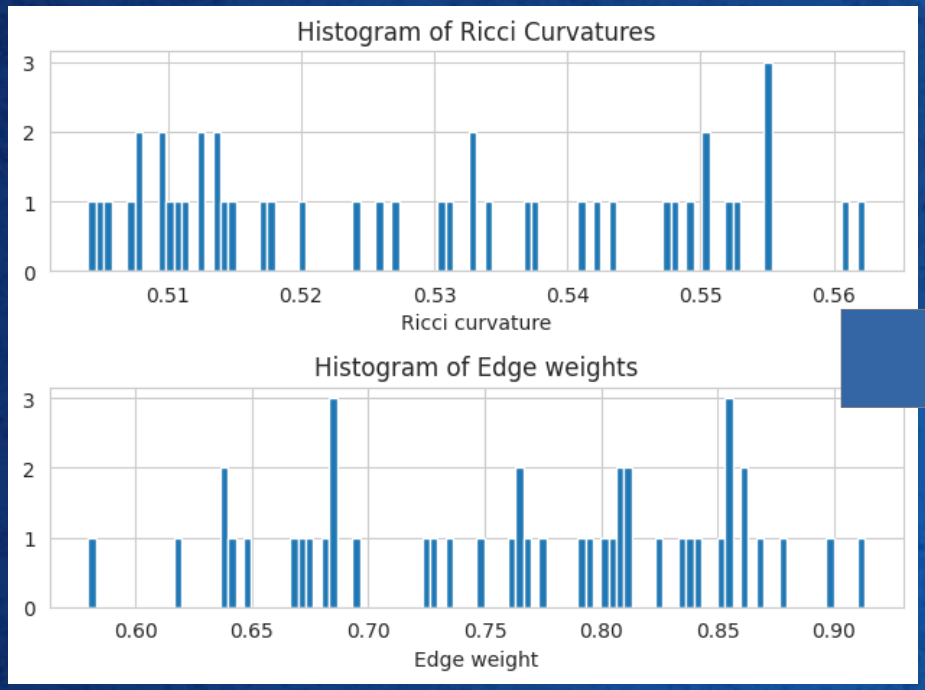
10 stocks correlation example
Tumminello, Lillo, Mantegna (2008), Mantegna (1999)

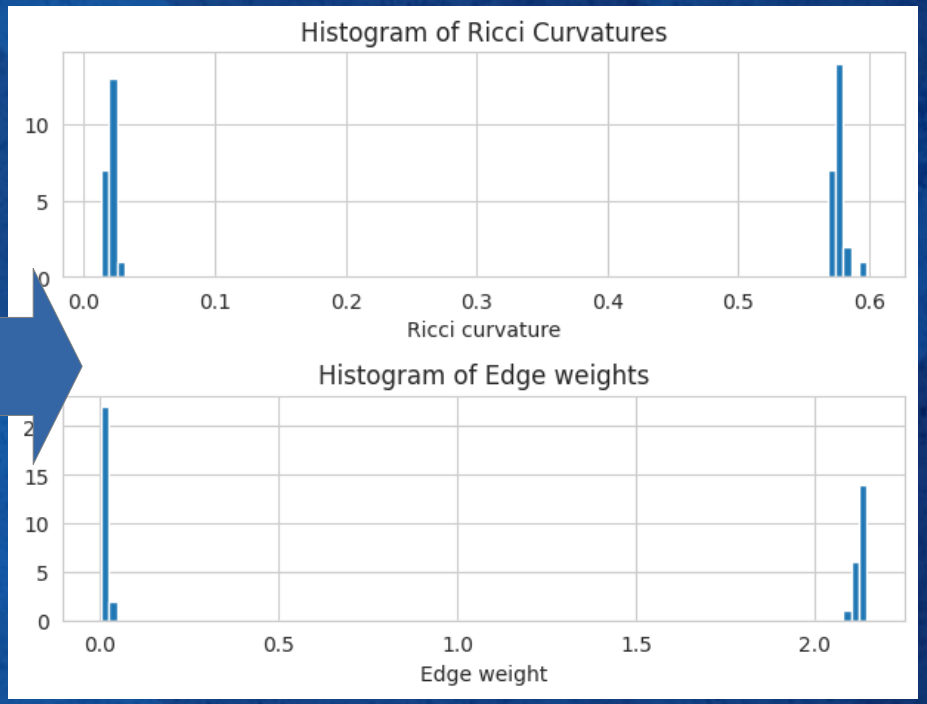
Toy Model Ricci Flow



Bhargavi Srinivasan (2024) 10 stocks correlation example
Average correlation 40%

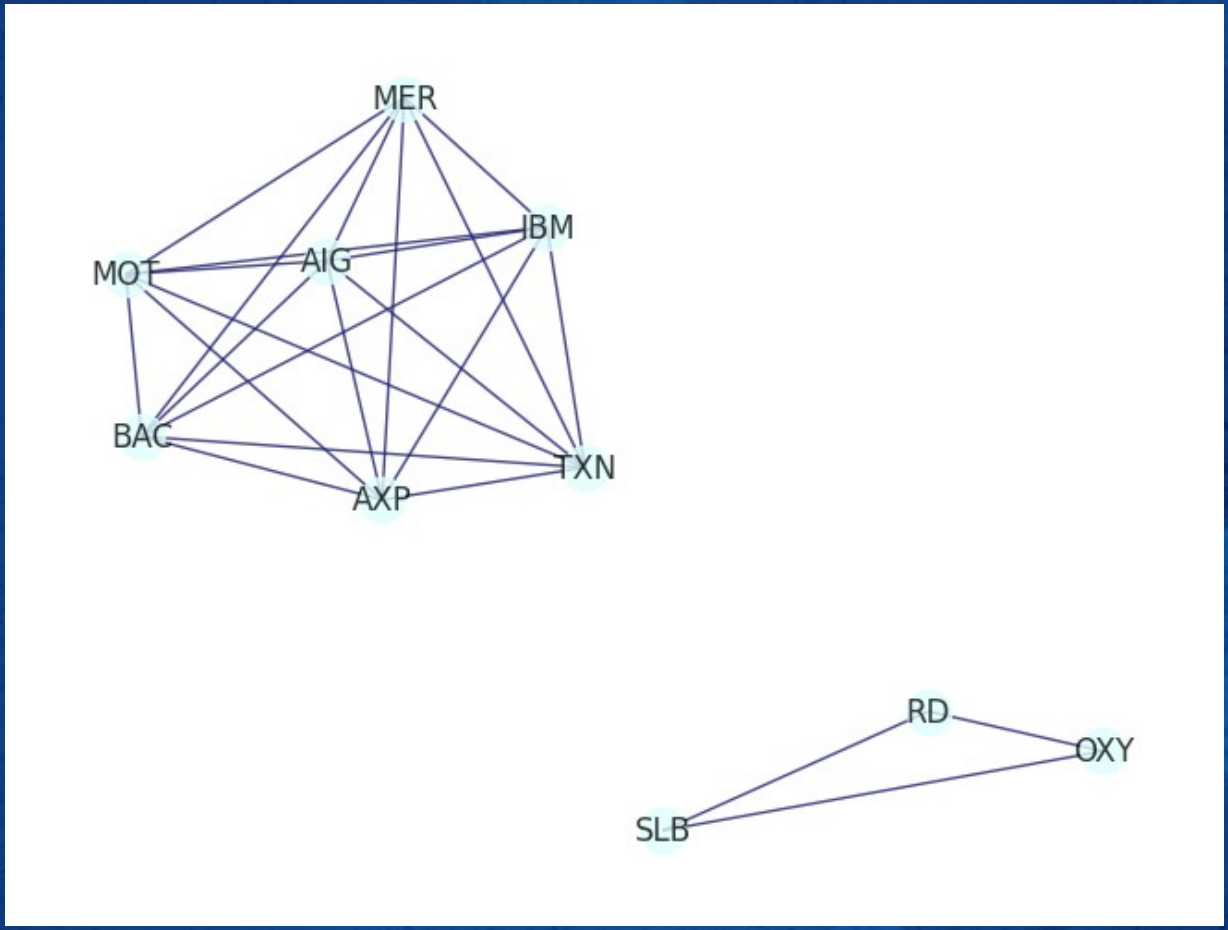
Ricci Flow 10 iterations





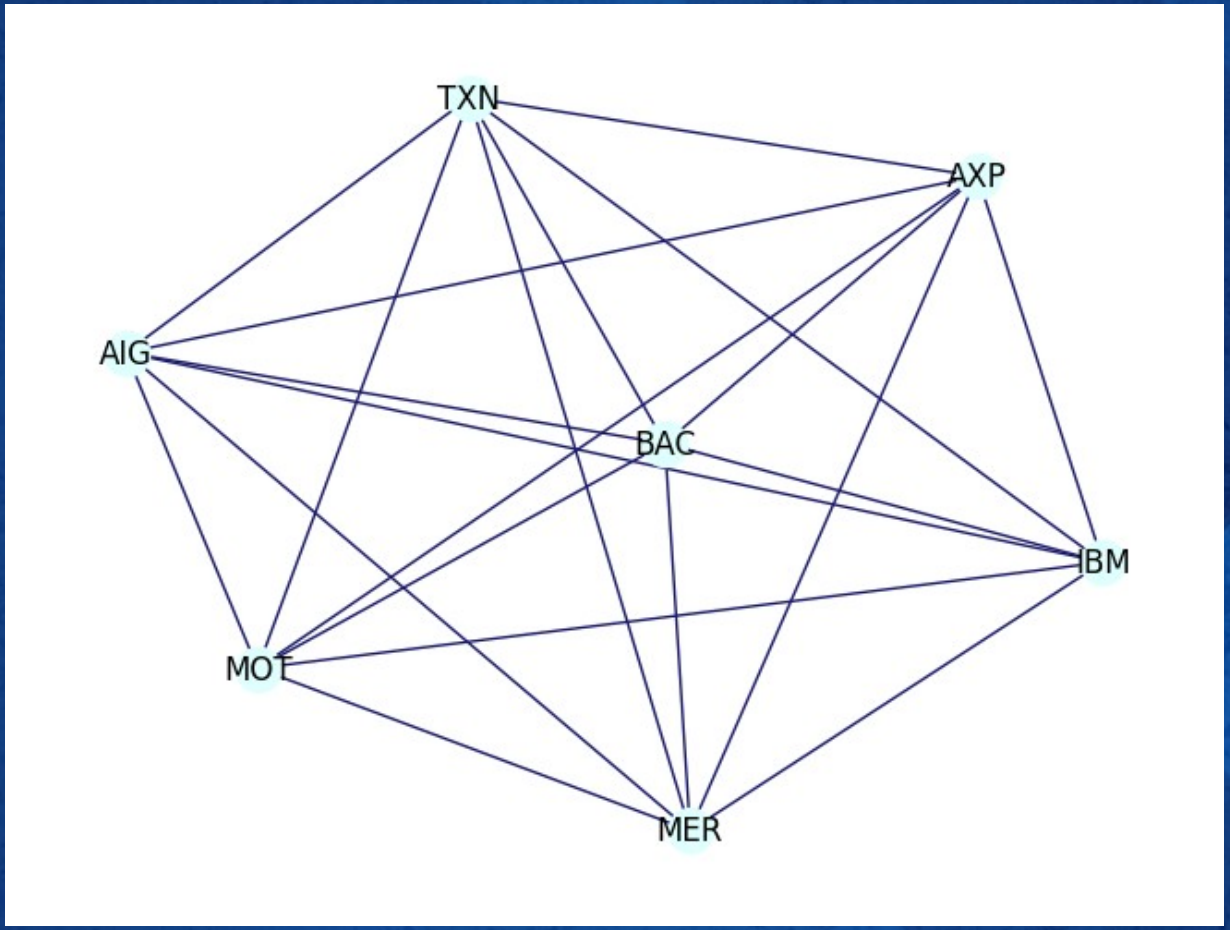
Surgery: remove edges>1

Toy Model level 1



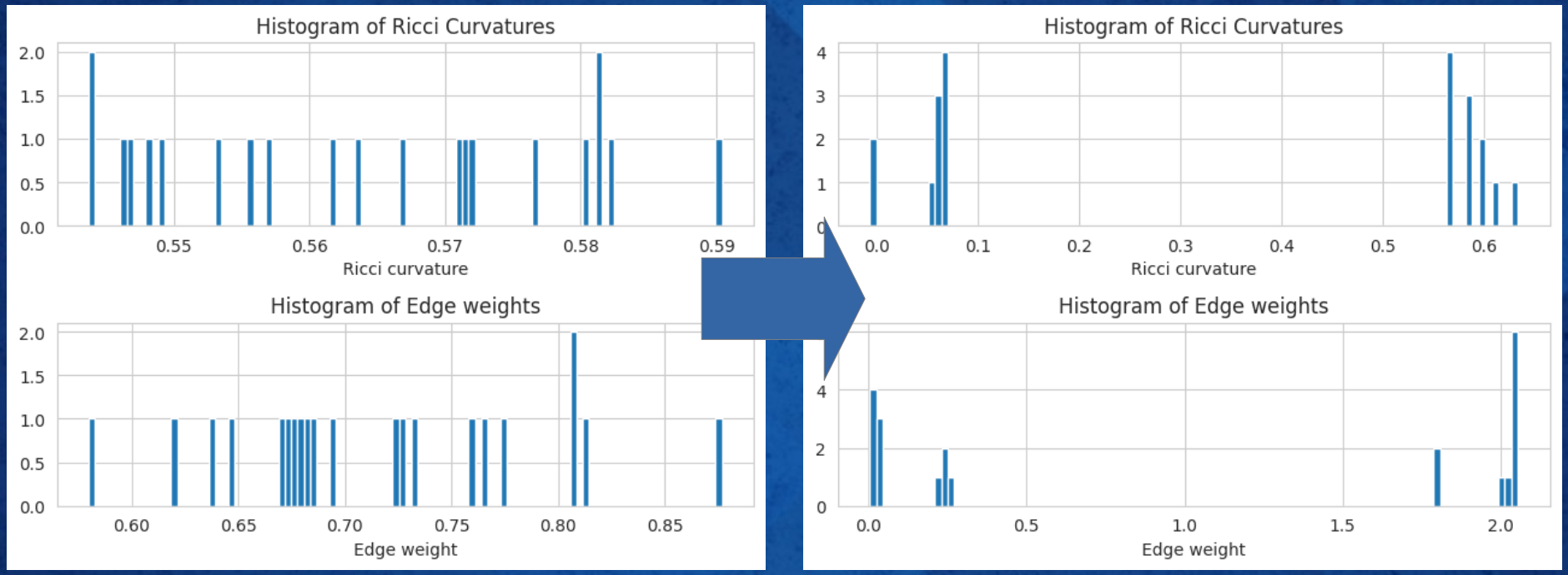
The energy sector names OXY,RD,SLB separates,
Bhargavi Srinivasan (2024)

Toy Model level 2



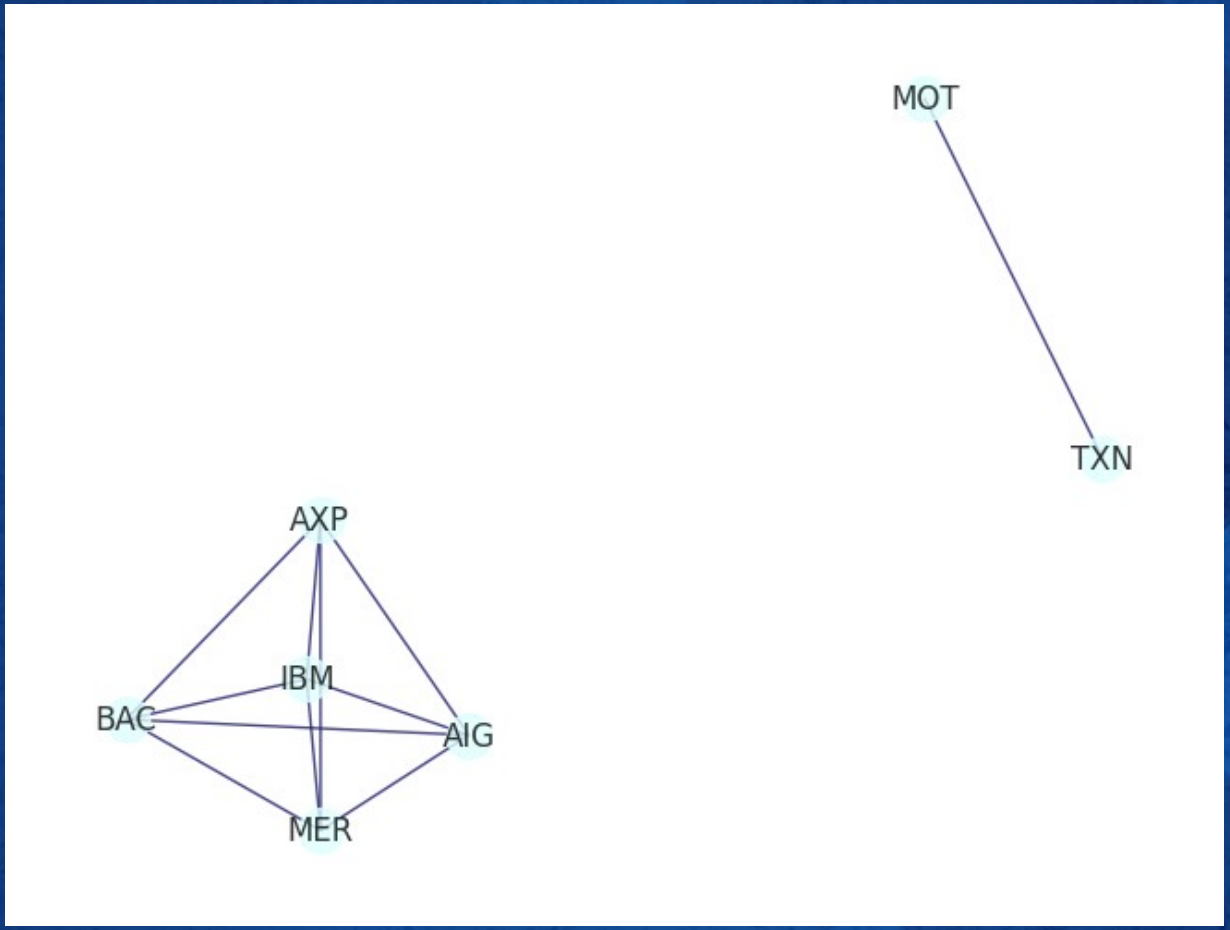
Proceed with the 7 remaining names

Ricci Flow 10 iterations



Surgery: remove edges>1

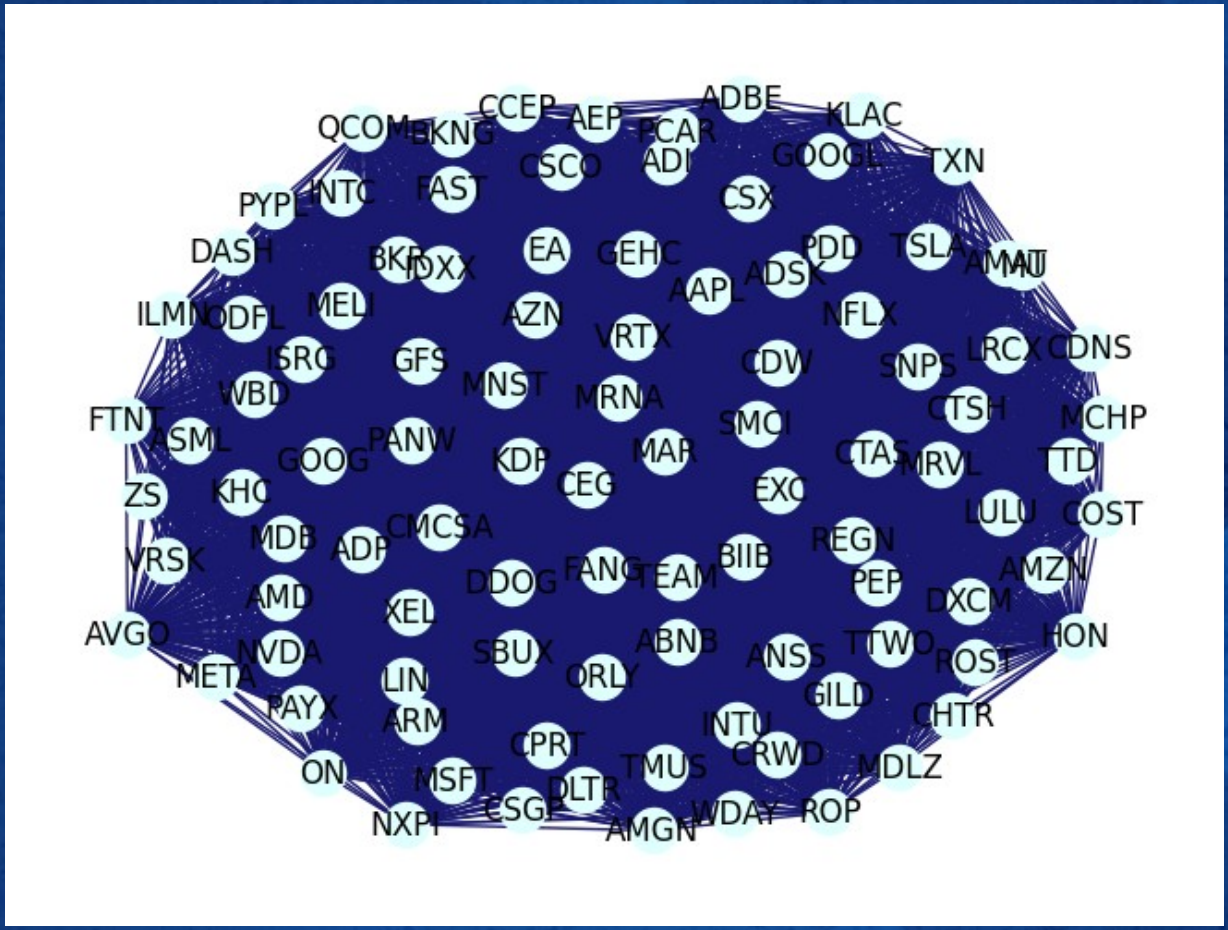
Toy Model level 2



Semiconductors MOT,TXN separates
Bhargavi Srinivasan (2024)

NASDAQ100 2024

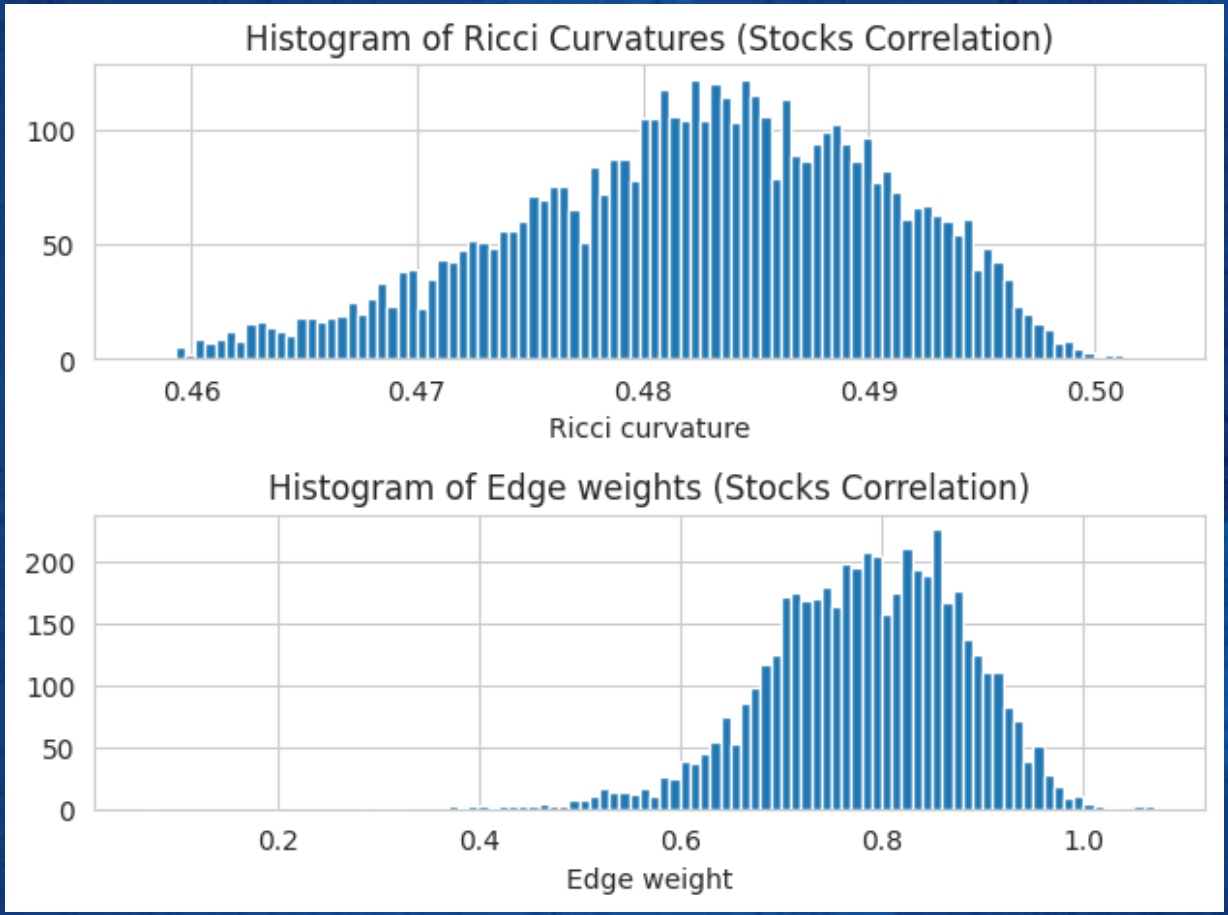
Bhargavi Srinivasan (2024)



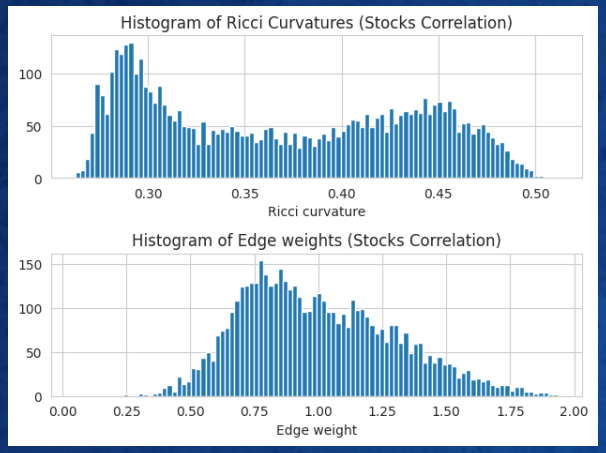
Fully connected graph without apparent structure
Average correlation 38%

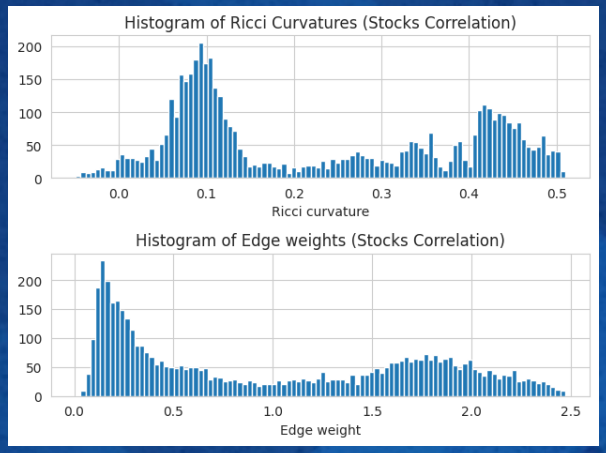
Correlation computed from 2020-01-01 to 2024-11-15, daily data from Yahoo finance

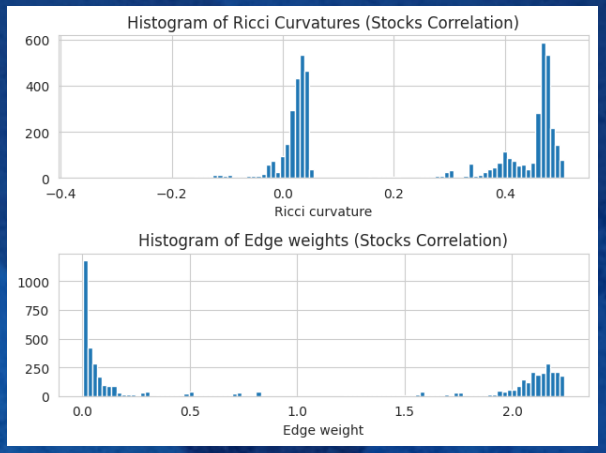
NASDAQ100 curvatures

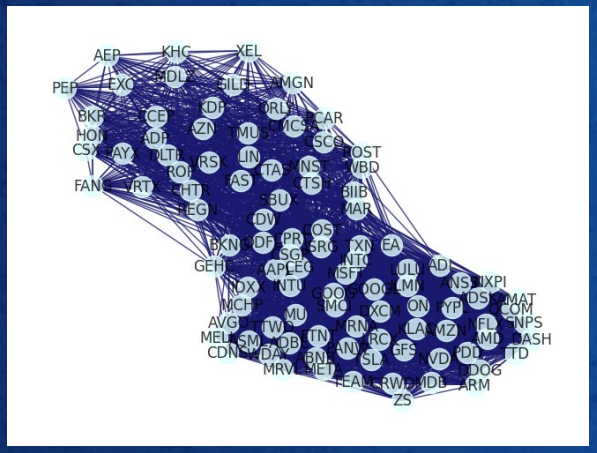


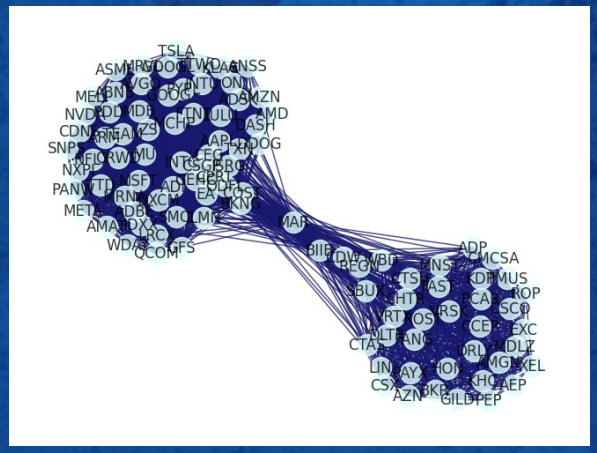
NASDAQ Ricci Flow level 1

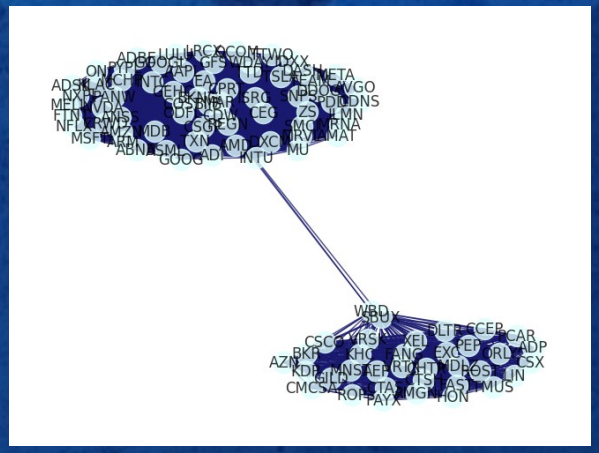










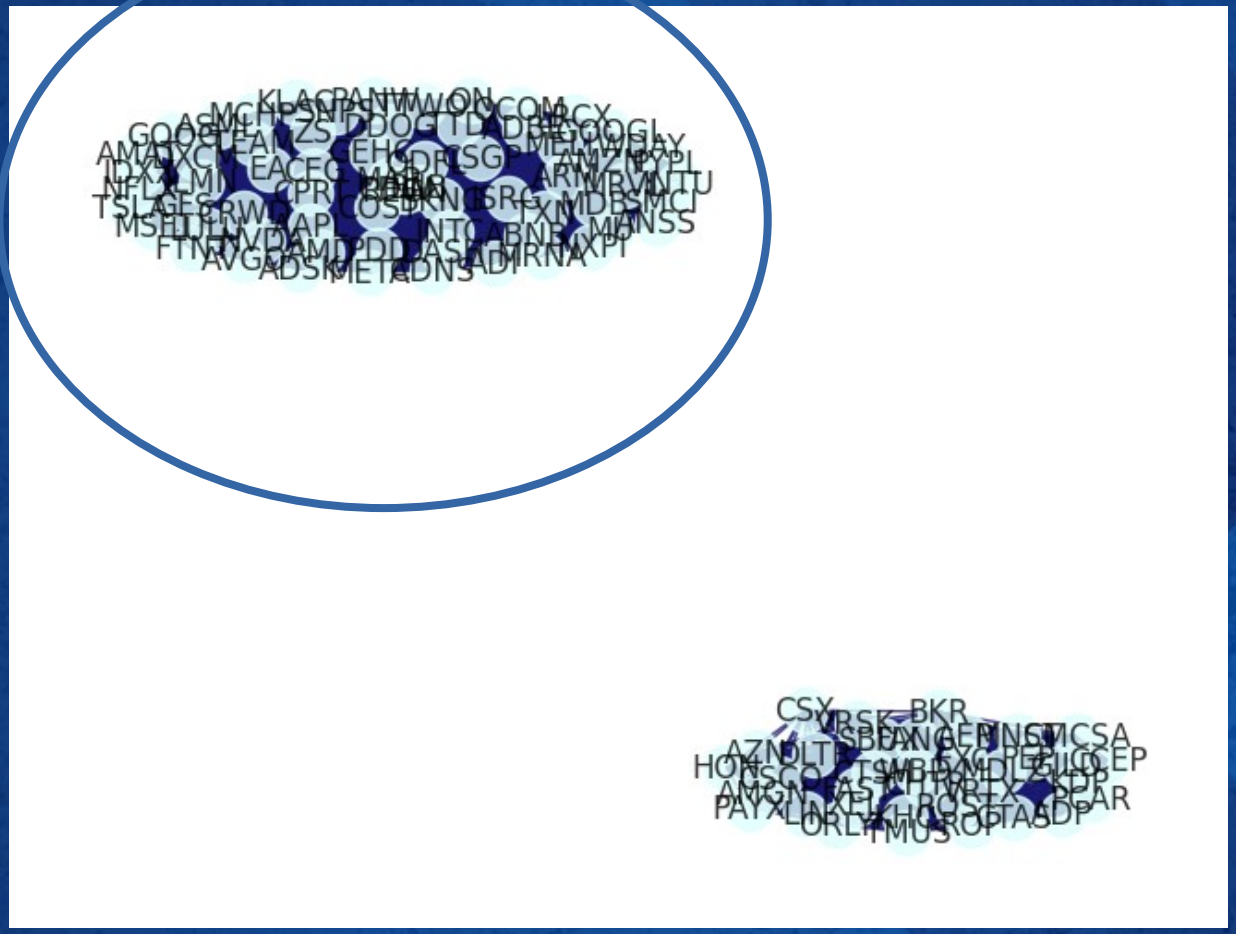


5 iterations

10 iterations

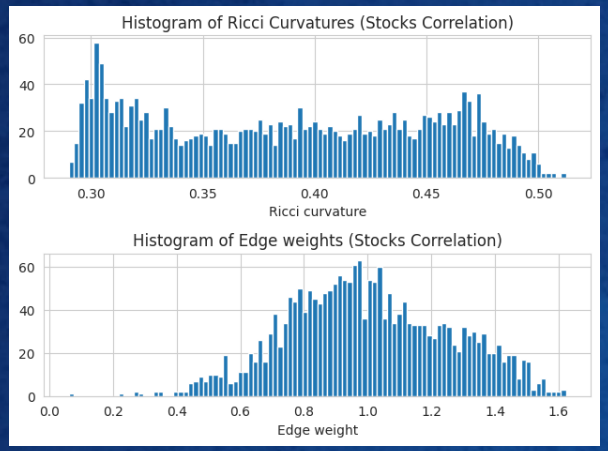
15 iterations

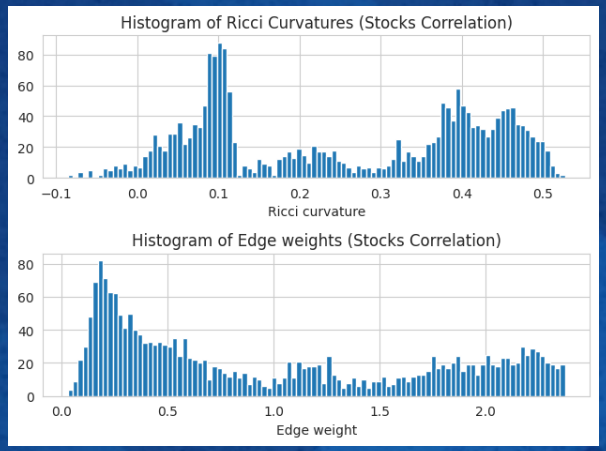
NASDAQ level 1

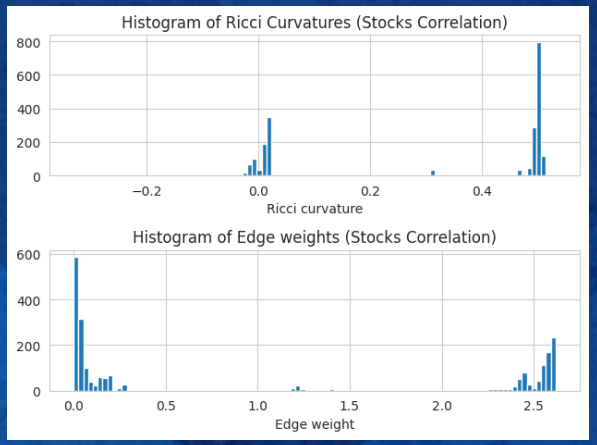


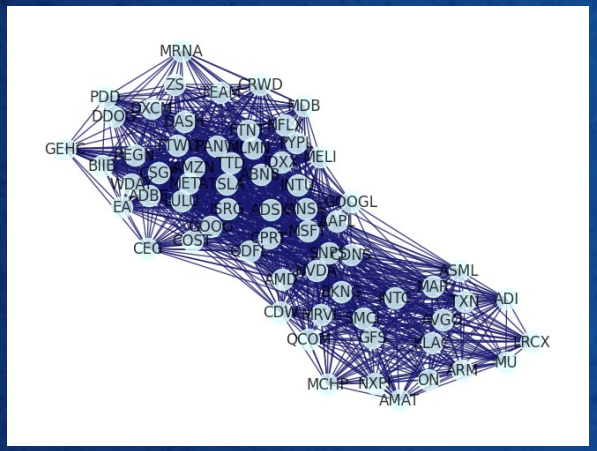
Level 1 after surgery: 2 separate groups
Bhargavi Srinivasan (2024)

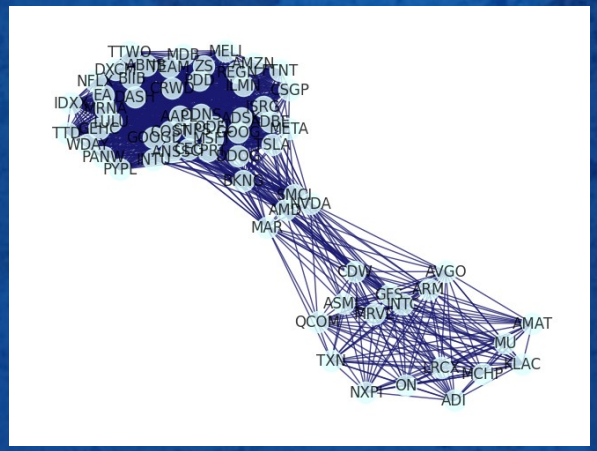
NASDAQ Ricci Flow level 2

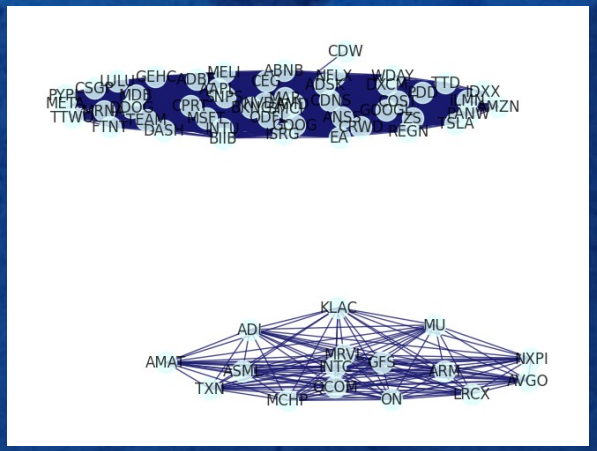










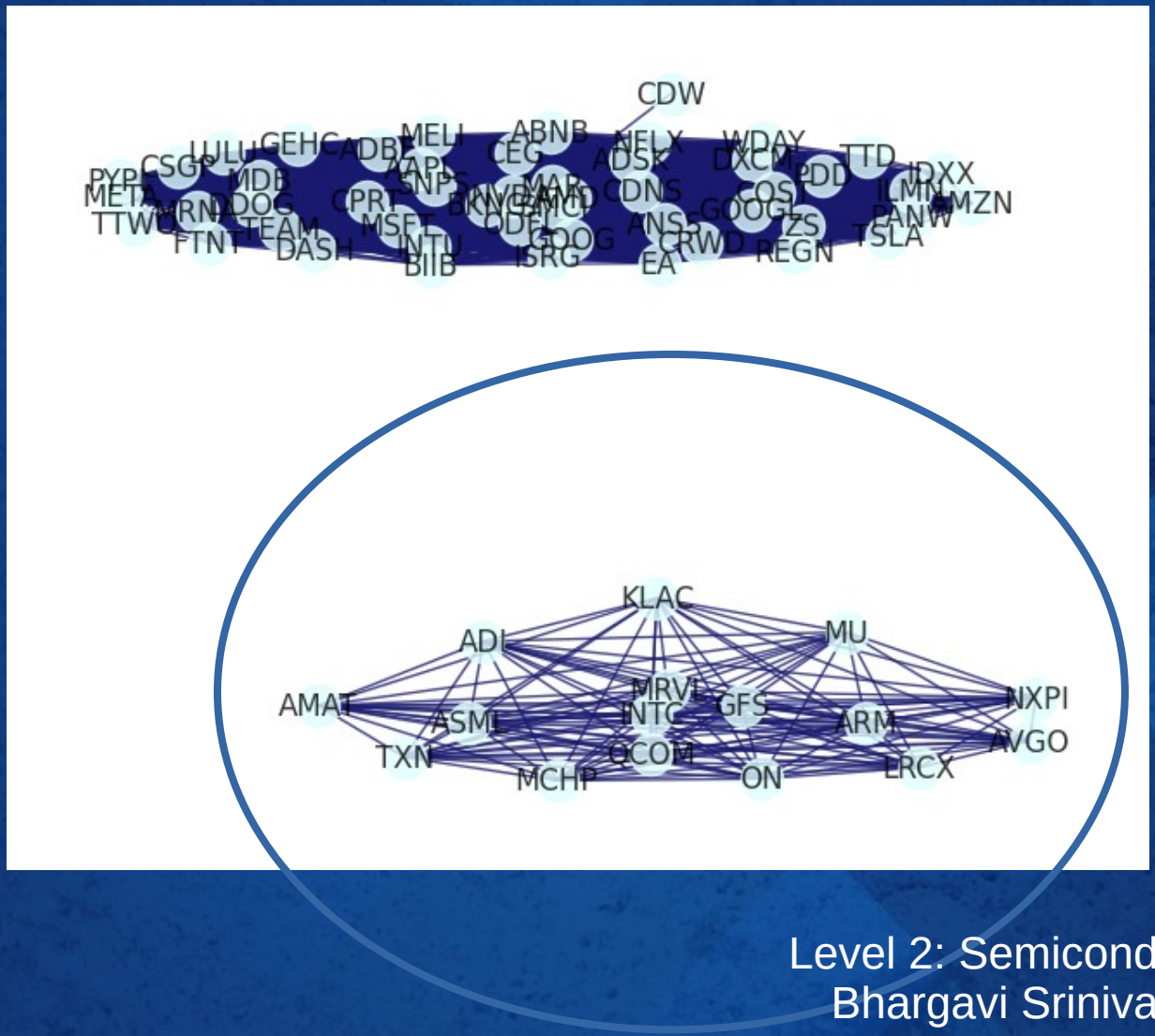


5 iterations

10 iterations

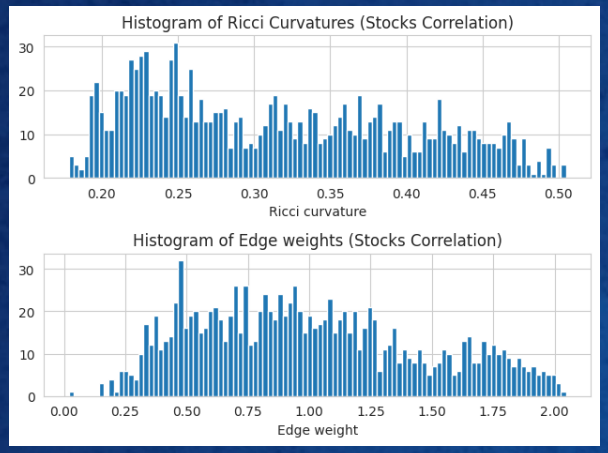
15 iterations

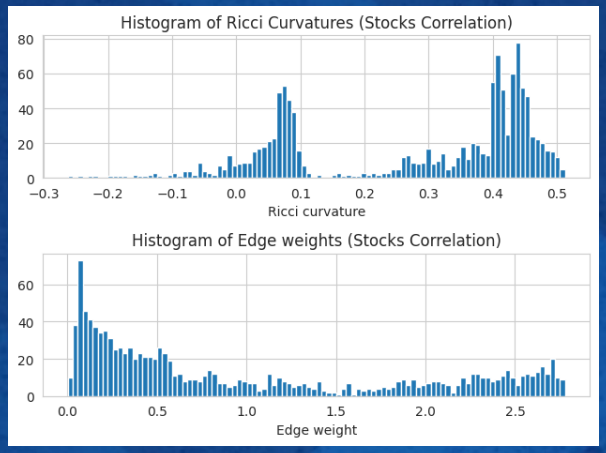
NASDAQ level 2

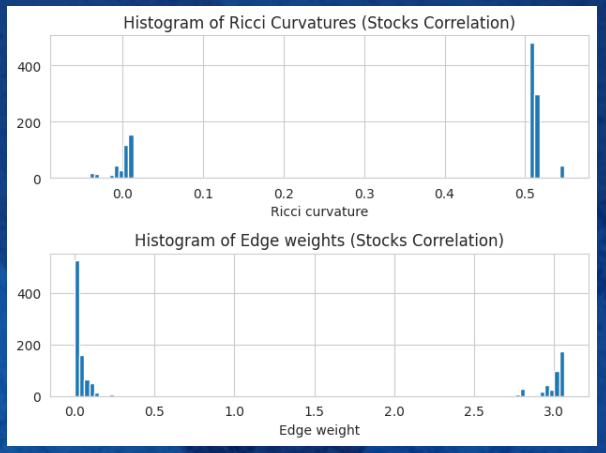


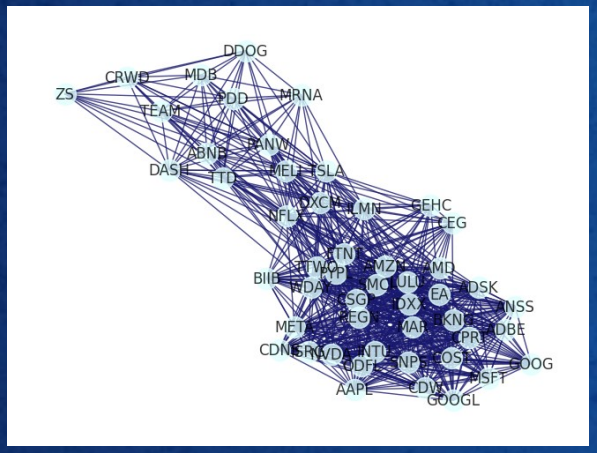
Level 2: Semiconductor cluster
Bhargavi Srinivasan (2024)

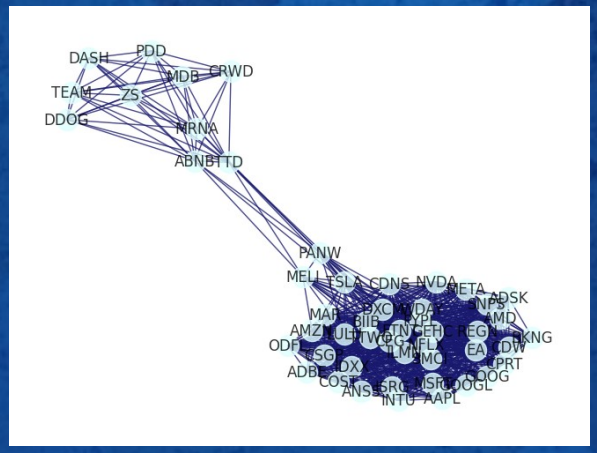
NASDAQ Ricci Flow level 3

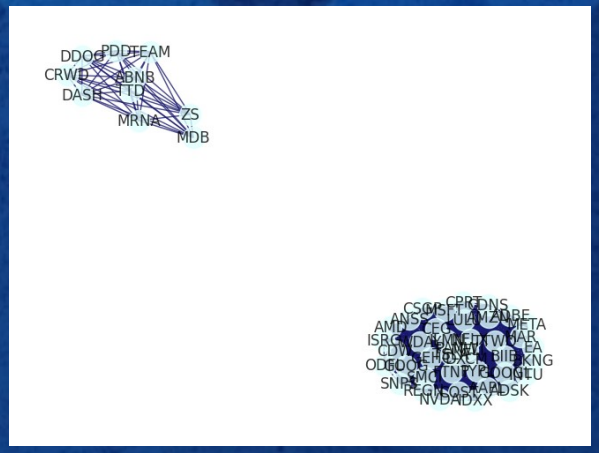










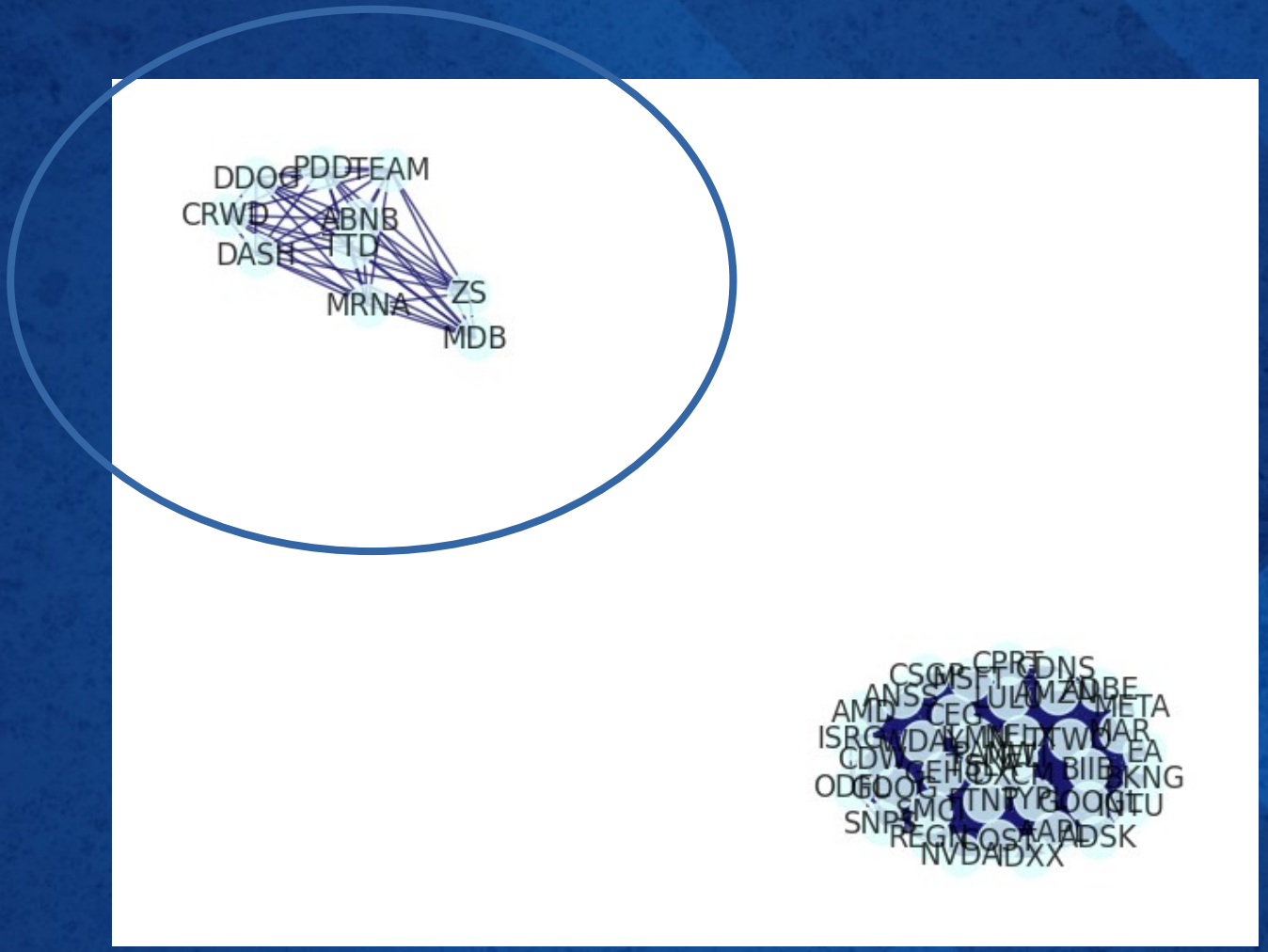


10 iterations

15 iterations

20 iterations

NASDAQ level 3



Level 3: Web retail
Worst performers (Moderna) out of index Fri Dec 13
Bhargavi Srinivasan (2024)

Future directions

- Curvature and crashes
- Supply chain fragility
- OT algo: finite temperature, dynamical OT