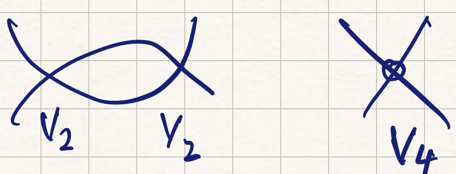
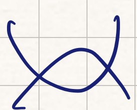
Naive dim analysis

A.M, Georgi NPB 234 (1984) 189 : track $\frac{1}{16\pi^2}$ for each loop

Gavela et al EPJC (2016) 76:485

$\pi - \pi$ scattering at p^4 :





$$I \sim \int \left(\frac{d^4p}{2\pi^4}\right) \frac{1}{p^2} \cdot \frac{1}{p^2} \cdot \frac{p^2}{f^2} \cdot \frac{p^2}{f^2} \sim \frac{p^4}{16\pi^2 f^4} \left(\log p/\mu\right)$$

$$V_4 \sim L \langle \partial_\mu \Sigma \, \partial_\mu \Sigma^\dagger \, \partial_\nu \Sigma \, \partial_\nu \Sigma^\dagger \rangle \sim L \cdot \frac{p^4 \pi^4}{f^4}$$

$L \gtrsim \frac{1}{16\pi^2}$ since L shifts by $\frac{1}{16\pi^2}$ when μ changes by order 1

[fits to L_i give numbers of this size $\sim 5\, 10^{-3}$]
Pich Rep Prog Phys 58 (1995) 563

$$\mathcal{L} = \frac{f^2}{4}\left[\langle \partial\Sigma\,\partial\Sigma^\dagger\rangle + \frac{1}{\Lambda_\chi^2}\langle \partial\Sigma\,\partial\Sigma^\dagger\,\partial\Sigma\,\partial\Sigma^\dagger\rangle + \cdots \right]$$

$$\Lambda_\chi \lesssim 4\pi f$$

holds to all orders.

General term:

$$\mathcal{L} \sim \frac{\Lambda^4}{16\pi^2}\left[\frac{\partial}{\Lambda}\right]^a \left[\frac{4\pi\phi}{\Lambda}\right]^b \left[\frac{4\pi\psi}{\Lambda^{3/2}}\right]^c \left(\frac{4\pi A}{\Lambda}\right)^d \left(\frac{g}{4\pi}\right)^e \left(\frac{\lambda}{16\pi^2}\right)^f \left(\frac{y}{4\pi}\right)^h$$

χPT: $\phi \to \pi$

$$\mathcal{L} \sim f^2\Lambda_\chi^2 \left(\frac{\partial}{\Lambda_\chi}\right)^a \left(\frac{\pi}{f}\right)^b$$

$$U \sim \exp(2i\pi/f) \sim O(1).$$

χPT valid $p \lesssim \Lambda_\chi \sim 1$ GeV

much larger than $p \lesssim f \simeq 93$ MeV

Λ_χ also the scale of resonances $m_\rho \sim 770$ GeV

Symmetry Breaking

Explicit symmetry breaking due to quark mass matrix

$$\mathcal{L} = -\bar{q}_R M^\dagger q_L - \bar{q}_L M q_R$$

$$M = \begin{bmatrix} m_u & & \\ & m_d & \\ & & m_s \end{bmatrix}$$

Treat M as a field. Then chiral symmetry is preserved provided $M \to L M R^\dagger$

$\mathcal{L}_{EFT}$: one insertion of M

$$\mathcal{L} = \underbrace{\frac{f^2}{2}}_{\text{dim } 3} B \, \mathrm{tr}(M \Sigma^\dagger + M^\dagger \Sigma)$$

$B \sim \dim 1$.

$B M^\dagger = \chi \propto$ mass matrix.

expanding in π

$$\mathcal{L} = -2B \, \mathrm{tr} \, M \pi^2$$

$$M^2_{\pi^\pm} = B(m_u + m_d) + \Delta M^2_{EM}$$

$$M^2_{K^\pm} = B(m_u + m_s) + \Delta M^2_{EM}$$

$$M^2_{K^0, \bar{K}^0} = B(m_d + m_s)$$

$\pi^0 - \eta$ matrix $\quad B \begin{bmatrix} m_u + m_d & \frac{m_u - m_d}{\sqrt{3}} \\ \frac{m_u - m_d}{\sqrt{3}} & \frac{1}{3}(m_u + m_d + 4m_s) \end{bmatrix}$

$\pi - \eta$ mixing due to $(m_u - m_d) \sim$ isospin breaking contribution to masses is <u>second order</u> in isospin breaking

$$\Delta M^2_{\pi^0} = -B\frac{(m_u - m_d)^2}{3} \frac{1}{\frac{1}{3}(4m_s - 2m_u - 2m_d)}$$

$$\Delta M^2_\eta = +$$

Neglecting this $\quad M^2_{\pi^0} = B(m_u + m_d)$

$$M^2_\eta = B\left(\frac{1}{3}m_u + \frac{1}{3}m_d + \frac{4}{3}m_s\right)$$

ΔM^2_{EM} is electromagnetic contribution and equal for (π^+, K^+) [Dashen's Thm]

1. $M^2_{GB} \propto m_q$

2. Overall scale not determined because of B

3. Can get mass ratios (Weinberg)

$$\frac{m_u}{m_d} = \frac{M^2_{K^+} - M^2_{K^0} + 2M^2_{\pi^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 0.55$$

$$\frac{m_s}{m_d} = \frac{M^2_{K^0} + M^2_{K^+} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 20.1$$

④ Gell-Mann - Okubo formula

$$4 M^2_{K^0} = 3 M^2_\eta + M^2_{\pi^0}$$

$$0.99\ \mathrm{GeV}^2 = 0.92\ \mathrm{GeV}^2$$

$$\mathcal{L}_{QCD} = -\bar{q}_L M q_R - \bar{q}_R M^\dagger q_L$$

$$e^{iW} = \int e^{i S_{QCD}}$$

$$\left.\frac{\partial W_{QCD}}{\partial M_{ij}}\right|_{M=0} = -\langle \bar{q}_{Li}\, q_{Rj} \rangle_{M=0}$$

$$\mathcal{L}_{\chi PT} = \cdots \frac{f^2}{2} B\, \mathrm{tr}(M\Sigma^\dagger + M^\dagger \Sigma)$$

$$\left.\frac{\partial W_{\chi PT}}{\partial M_{ij}}\right|_{M=0} = \frac{f^2}{2} B \langle (\Sigma^\dagger)_{ji} \rangle_{M=0} = \frac{f^2}{4} B\, \delta_{ij}$$

since $\langle \pi^2 \rangle = 0$ in massless limit $\bar{L}R + \bar{R}L$

$$\langle \bar{q}_{Li}\, q_{Rj} \rangle = -\frac{f^2}{2} B\, \delta_{ij} \qquad \langle \bar{q}_i q_j \rangle = -f^2 B \delta_{ij}$$

$$f_\pi^2 m_{\pi^0}^2 = f^2 B (m_u + m_d) = -\langle \bar{q} q \rangle (m_u + m_d)$$ Gell-Mann Oakes Renner 1968

$$\left[f_\pi^2 m_\pi^2 \sim f_a^2 m_a^2 \times O(1) \text{ constants} \right]$$

$$\bar{m} = \frac{1}{2}(m_u + m_d) \approx 3.45 {}^{+0.35}_{-0.15}\ \mathrm{MeV} \text{ at } \mu = 2\ \mathrm{GeV}.$$

$$-\langle \bar{q} q \rangle \simeq (280\ \mathrm{MeV})^3 \qquad \Lambda f^2 \sim (220\ \mathrm{MeV})^3$$

$M \to L M R^\dagger$ $\qquad$ $\det M \to \det M$

$M^\dagger \to R M L^\dagger$ $\qquad$ $(M^\dagger)^{-1} \to L (M^\dagger)^{-1} R^\dagger$

$(\det M)(M^\dagger)^{-1}$ transforms just like M.

Kaplan, A.M. PRL 56 (1986) 2004

χPT: $\quad M \to M + \lambda \det M (M^\dagger)^{-1}$

cannot tell the two apart.

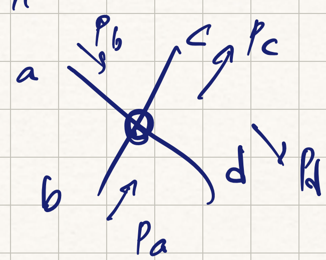
$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \qquad \det M (M^\dagger)^{-1} = \begin{bmatrix} m_d m_s & & \\ & m_u m_s & \\ & & m_u m_d \end{bmatrix}$$

so if $m_u = 0$ can get $(m_u)_{eff} \propto m_d m_s$

can tell them apart by doing a lattice computation and varying m.

$\pi\pi$ Scattering Lengths

$$\mathcal{L} = \frac{1}{3f^2} \operatorname{tr}\left[\pi, \partial_\mu \pi\right]^2 + \frac{2}{3f^2} B \operatorname{tr} M \pi^4$$



$$\pi = \pi^a T^a \qquad \pi^2 = \frac{1}{4} \pi^a \pi^a \mathbb{1}.$$

$$[\pi, \partial_\mu \pi] = i \epsilon^{abc} \pi^a \partial_\mu \pi^b T^c$$

$$\mathcal{L} = \frac{1}{3f^2}\left(-\frac{1}{2}\right)\left[(\pi\cdot\pi)(\partial\pi\cdot\partial\pi) - (\pi\cdot\partial\pi)^2\right]$$

$$+ \frac{2}{3f^2} B (m_u + m_d) \frac{(\pi\cdot\pi)^2}{16}$$

$$\mathcal{L} = \frac{1}{6f^2}\left[(\pi\cdot\partial\pi)^2 - (\pi\cdot\pi)(\partial\pi\cdot\partial\pi)\right]$$

$$+ \frac{m_\pi^2}{24f^2} (\pi\cdot\pi)^2 + \cdots$$

$$\mathcal{A} = \frac{i}{f^2}(s - m_\pi^2)\, \delta_{ab}\delta_{cd} + \frac{i}{f^2}(t - m_\pi^2)\, \delta_{ac}\delta_{bd}$$

$$+ \frac{i}{f^2}(u - m_\pi^2)\, \delta_{ad}\delta_{bc}$$

$$s = (p_a + p_b)^2 \qquad t = (p_a - p_c)^2 \qquad u = (p_a - p_d)^2$$

$\mathcal{A}$ is Bose symmetric and crossing symmetric.

$$A_{I=0} = \frac{i}{f^2}\left(3s + t + u - 5m_\pi^2\right) \qquad \propto \delta_{ab}$$

$$A_{I=1} = \frac{i}{f^2}(t - u) \qquad A_{ab} = -A_{ba}$$

$$A_{I=2} = \frac{i}{f^2}\left(t + u - 2m_\pi^2\right) \qquad A_{ab} = A_{ba}$$
$$A_{aa} = 0$$

at threshold $\quad s = 4m_\pi^2 \qquad t = u = 0$

$$A_{I=0} = \frac{i}{f^2}\left(7m_\pi^2\right)$$

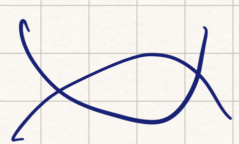
$$A_{I=1} = 0$$

$$A_{I=2} = \frac{i}{f^2}\left(-2m_\pi^2\right)$$

$$L \equiv \frac{m_\pi}{8\pi f^2} \qquad \left.\begin{aligned} a_{I=0} &= \frac{7}{4}L \\ a_{I=1} &= 0 \\ a_{I=2} &= -\frac{1}{2}L \end{aligned}\right\} \text{scattering lengths}$$

$$a = -\frac{i}{2}\,\frac{A}{16\pi m_\pi}$$

one-loop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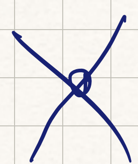
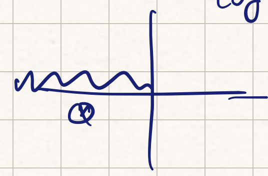


L_4

$$A \sim \frac{C}{16\pi^2} p^4 \log p^2/\mu^2 + L(\mu)\, p^4$$

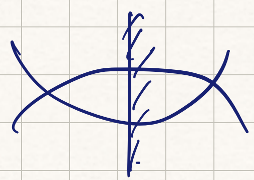
$$\mu \frac{d}{d\mu} A = 0 \Rightarrow \mu \frac{d}{d\mu} \hat{L}_i = -\frac{\hat{\Gamma}_i}{16\pi^2}$$

$p^4 \log p^2/\mu^2$ or $m^2 \log m^2/\mu^2$



called chiral logs.

$$\log(-s-i\epsilon) = \log s - \underbrace{i\pi}_{\text{fixed by unitarity}}$$



RGE: $\mu \dfrac{dL_4}{d\mu} \propto L_2^2$

$\mu \dfrac{dL_6}{d\mu} \propto L_2 L_4$

RGE running of higher order terms using lower order coefficients

No $\dfrac{dL_4}{d\mu} \propto L_4$ because of the $+2L$ in Weinberg formula for power counting.

RGE consistent with naive dim analysis estimate.