ICTS Lectures on SMEFT, HEFT, and the Geometry of Field Space

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Plan

EFT for BSM

Why? Want to systemetize experimental search for indirect signals of BSM.

Tool of choice is EFT. (See also "primary operators")

* Model dependent! (Model agnostic)

Power counting is a UV hypothesis.

How should we build the EFT?

Work with $v=0$ (SMEFT) or $v \neq 0$ (HEFT)?

Must specify

1) Dofs
2) Symmetries
3) Power counting

$v=0$	$v \neq 0$
1) H	1) $h, \vec{\pi}$
2) $SU(2) \times U(1)$	2) $U(1)_{EM}$
3) Mass dimension	3) Derivatives?

Let us simplify our lives and assume custodial symmetry

$\Rightarrow SU(2) \times U(1) \to SU(2)_L \times SU(2)_R \cong O(4)$

Custodial sym is only approximate in SM

Explicitly broken by gauging $U(1)_Y \subset SU(2)_R$ and due to fermion mass splittings

SMEFT

Focus on scalar sector. Let $\vec{\varphi}$ be fundamental of $O(4)$:

$$\vec{\varphi} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix} \qquad \text{w/} \quad \vec{\varphi} \to O\,\vec{\varphi} \quad \text{under } O(4)$$

(O is 4×4 orthogonal matrix.)

Identify $\quad H = \dfrac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_4 + i\varphi_3 \end{pmatrix}$

$$\mathcal{L}_{SMEFT} = A(|H|^2)\,|\partial H|^2 + \frac{1}{2} B(|H|^2)\left[\partial(|H|^2)\right]^2 - \tilde{V}(|H|^2) + \mathcal{O}(\partial^4)$$

w/ $A, B, \tilde{V}$ are real analytic at origin $|H| = 0$.

Geometrically, φ_i are Cartesian coordinates.

HEFT

Goldstones of $O(4)/O(3)$ $\vec{\pi}$ ← transform non-linearly

Singlet scalar field h

Define $$\vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1-n_1^2-n_2^2-n_3^2} \end{pmatrix}$$

Under $O(4)$ $h \to h$ and $\vec{n} \to O\vec{n}$

$\vec{n}(\vec{\pi}) \in S^3$ is 4-component unit vector w/ $\vec{n}\cdot\vec{n} = 1$.

The constrained vector $\vec{n}$ transforms linearly.

The rotations in the 12, 13, and 23 planes act linearly on (n_1, n_2, n_3) and leave n_4 invariant. However, if one does eg a 14 rotation (infinitesimal)

$$\delta n_1 = \theta n_4, \quad \delta n_2 = 0, \quad \delta n_3 = 0, \quad \delta n_4 = -\theta n_4$$

Then the transformation of the unconstrained π fields is $\delta\pi_1 = \theta\sqrt{v^2 - \vec{\pi}\cdot\vec{\pi}}$, $\delta\pi_{2,3} = 0$

$\Rightarrow$ non-linear.

$$\mathcal{L}_{HEFT} = \frac{1}{2}\left[\mathbb{K}(h)\right]^2(\partial h)^2 + \frac{1}{2}\left[vF(h)\right]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

w/ $\mathbb{K}$, F, V are real analytic about the physical vacuum $h=0$.

Geometrically, HEFT is like polar coordinates.

* Ultimately, HEFT is description used to do physical calculations, since need to work in physical vacuum.

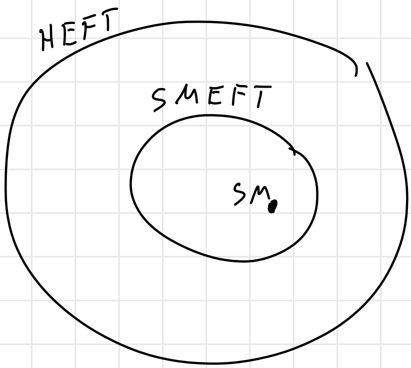
Remember EFT requires truncation of power counting expansion.

Compare $\tilde{V}(H)$ up to dim 6 and $V(h)$ up to 6 fields

$$\tilde{V}(H) = -\mu^2 |H|^2 + \lambda |H|^4 + \frac{1}{\Lambda^2} |H|^6$$

$$V(h) = m^2 h^2 + c_3 h^3 + c_4 h^4 + c_5 h^5 + c_6 h^6$$

Clearly HEFT has larger parameter space then SMEFT.



If we parametrize BSM searches w/ SMEFT, are we potentially missing anything?

Motivates understanding the relationship between HEFT and SMEFT.

Note: preference is to work w/ SMEFT since that is already hard enough.

Also much more natural from model building perspective.

Assume no obstruction to mapping between SMEFT and HEFT:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_4 + i\varphi_3 \end{pmatrix} \quad \text{and} \quad \vec{\varphi} = (v_0 + h)\,\vec{n}$$

How to determine v_0? ← Revisit

(Note v sets gauge boson masses, etc)

Let's write some $O(4)$ symmetric objects setting $v = v_0$ for simplicity:

$$|H|^2 = \frac{1}{2}\vec{\varphi}\cdot\vec{\varphi} = \frac{1}{2}(v+h)^2$$

$$|\partial H|^2 = \frac{1}{2}(\partial\vec{\varphi})^2 = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(v+h)^2(\partial\vec{n})^2$$

$$(\partial|H|^2)^2 = (\vec{\varphi}\cdot\partial\vec{\varphi})^2 = (v+h)^2(\partial h)^2$$

The using This, we can write (Exercise)

$$\mathcal{L}_{HEFT} = \frac{1}{2}\left[K(h)\right]^2(\partial h)^2 + \frac{1}{2}\left[v\,F(h)\right]^2(\partial\vec{n})^2 - V(h) + \ldots$$

$$= \frac{v^2F^2}{2|H|^2}|\partial H|^2 + \frac{1}{2}(\partial|H|^2)^2\frac{1}{2|H|^2}\left(K^2 - \frac{v^2F^2}{2|H|^2}\right)$$

$$- \tilde{V}(|H|^2) + \ldots$$

(Notice non-analiticity.)

Field Redefinition Invariance of S-Matrix

We can extract S-matrix elements from connected correlation functions using the LSZ reduction formula, which schematically takes the form

$$A_n \sim \int \left[\prod d^4x_i \, e^{ip_i\cdot x_i}\right] D^{-1}_{x_1y_1} \cdots D^{-1}_{x_ny_n} \langle \varphi(y_1)\ldots\varphi(y_n)\rangle_{conn} \Big|_{J=0}$$

where $D^{-1}_{x_1y_1}$ are the inverse propagators from $x_1 \to y_1$.

We compute the correlation functions from the path integral:

$$Z[J] = \int \mathcal{D}\varphi \exp\left(i S'[\varphi] + i\int d^4x\, \varphi(x) J(x)\right)$$

Writing $Z[J] = \exp i W[J]$ we have

$$\langle \varphi(x_1)\ldots\varphi(x_n)\rangle_{connected} = (-i)^n \frac{\delta^n (iW)}{\delta J(x_1)\ldots\delta J(x_n)}$$

Now, lets do a field redef:

$$\varphi(x) = F(\varphi'(x))$$

The new Lagrangian is

$$\mathcal{L}(\varphi) = \mathcal{L}(F(\varphi')) = \mathcal{L}'(\varphi')$$

The path integral becomes

relabeled integration variable

$$Z'[J] = \int \mathcal{D}\phi' \, e^{iS(\mathcal{L}'(\phi') + J\phi')} = \int \mathcal{D}\varphi \, e^{iS(\mathcal{L}'(\varphi) + J\varphi)}$$

We can compare this with the original path int after making the field redef:

$$Z[J] = \int \mathcal{D}\varphi' \left| \frac{\delta F}{\delta \varphi'} \right| e^{iS(\mathcal{L}'(\varphi') + JF(\varphi'))}$$

The Jacobian $\left| \frac{\delta F}{\delta \varphi'} \right| = 1$ in dim reg (scaleless int)

Relabeling $\phi' \to \varphi$, we have

$$Z[J] = \int \mathcal{D}\varphi \, e^{iS(\mathcal{L}'(\varphi) + JF(\varphi))}$$

The only difference between Z and Z' is the coupling to the source: $JF(\varphi)$ vs. $J\varphi$

Clearly the connected correlators will not be the same. However, all the S-matrix depends on are the poles in the correlation functions. This is what the factors of D^{-1} extract.

All that is required to determine the propagator is the interpolating field formula: $\langle p | \varphi(x) | 0 \rangle \neq 0$.

As long as we ensure that this condition holds for φ and F, then we have a good interpolating field in both cases.

Setting the wave function renormalization to 1, we know $\langle p|\varphi(x)|0\rangle = e^{ip\cdot x} \neq 0$

So we must restrict ourselves to field redefs of the form $\phi = F(\varphi') = \varphi' + f(\varphi')$

Then $\langle p|F(\varphi')|0\rangle = \langle p|\phi'|0\rangle + \cancel{\langle p|f(\varphi')|0\rangle}^{\,0}$

$$= e^{ip\cdot x} \neq 0$$

For this class of field redefs, the S-matrix is unchanged. Specifically, $f(\varphi')$ is a local analytic expansion in fields and derivatives.

Finally, note that field redefinitions can mix terms at different order in power counting.

Ex: Let $\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2$. Define $F(\varphi') = \varphi' + \frac{\varphi'^3}{\Lambda^2}$

Show that one still gets a free theory for $\varphi'\varphi' \to \varphi'\varphi'$ scattering at tree-level. (Do this)

Field Redefinitions and EFT

An EFT Lagrangian is a local expansion in terms of fields and derivatives.

We can only make field redefinitions of the form $\tilde{\varphi}^i = \varphi^j F^{ij}(\varphi)$ where F is a real analytic function of the fields (F has a convergent Taylor expansion about $\varphi = 0$.) and $F^{ij}(0) = \delta^{ij}$.

This implies that we are working with a real analytic manifold.

However, polar coordinates obscure the analyticity of the origin as we now explain:

Consider a 2d manifold $\mathbb{R}^2$. Define polar coordinates that map all points except the origin to (r, θ) w/ line element

$ds^2 = dr^2 + r^2 d\theta^2$.

Now consider two Cartesian-like charts $C1$ w/ (x_1, y_1) and C_2 w/ (x_2, y_2)

Away from the origin, we have invertable and analytic relations $x_1 = r\cos\theta$, $y_1 = r\cos\theta$. and $x_2 = (r+r^2)\cos\theta$, $y_2 = (r+r^2)\cos\theta$

So we can relate them to each other:

$$x_2 = x_1\left(1+\sqrt{x_1^2+y_1^2}\right)$$

$$y_2 = y_1\left(1+\sqrt{x_1^2+y_1^2}\right)$$

These are not real analytic at the origin.

This non-analyticity manifests when computing the components of the metric:

$$ds^2 = dr^2 + r^2 d\theta^2 = dx_1^2 + dy_1^2$$

$$= \frac{1}{x_1^2+x_2^2}\left[\frac{(x_2\,dx_2 + y_2\,dy_2)^2}{1+4\sqrt{x_2^2+y_2^2}} + \frac{(x_2\,dy_2 - y_2\,dx_2)^2}{\left(1+\sqrt{1+4\sqrt{x_1^2+x_2^2}}\right)^2}\right]$$

This is in exact analogy with what can go wrong when mapping from HEFT → SMEFT.

Want to distinguish this "unphysical" non-analyticity from "physical" ones. A tool to do this is to look for physical singularities on the manifold using curvature invariants.

For example, take $ds^2 = dr^2 + T(r)\, d\theta^2$

Then we can compute the Ricci scalar

$$R(r) = \frac{(T')^2}{2T^2} - \frac{T''}{T}$$

For the flat space case, $T = r^2$, and we have

$R = 0 \Rightarrow$ no physical singularities

If we had e.g. $T = r \Rightarrow R(r) = 1/2r^2 \Rightarrow R(0) \to \infty$,

so this case has a physical obstruction

at the origin.

Applying these ideas to our EFT, take the following example

Claim (Exercise)

$$\mathcal{L}_H = \frac{1}{2}\left(1 + \frac{\sqrt{2|H|^2}}{v} + \frac{|H|^2}{2v^2}\right)|\partial H|^2 + \frac{1}{4v^2}\left(\frac{v}{\sqrt{2|H|^2}} + \frac{3}{4}\right)\frac{1}{2}\left(\partial |H|^2\right)^2 - \tilde{V}_H$$

but sending $h \to h_1 = h + \frac{1}{4v} h^2$

$$\Rightarrow \mathcal{L}_{H_1} = |\partial H_1|^2 - \tilde{V}_{H_1}$$

Field redefs of h can completely obscure the analytic properties in terms of H.

Electroweak Symmetry Restoration

How do we identify the point on the field space manifold where EW symmetry is restored? AJM showed that this corresponds to identifying an "O(4) invariant fixed point" on the manifold. This is a point $\vec{\varphi}_0$ where $O\vec{\varphi}_0 = $ zero where O is an $O(4)$ transformation. Clearly if a linear rep exists s.t. $\vec{\phi} \to O\vec{\phi}$ then $\vec{\varphi} = $ zero is such a fixed point.

To show the converse is true, assume that a set of coordinates exist that transform under $O(4)$ that contains an $O(4)$ invariant fixed point. Then the Coleman, Wess, Zumino "Linearization Lemma" tells us that a set of coordinates exist in the neighborhood of the fixed point that transform linearly under $O(4)$.

So now we know that the existance of an $O(4)$ invariant point on the manifold implies that we can write the coordinates in a linear representation (a necissary condition to have SMEFT). How do we identify such a point from

$$\mathcal{L}_{HEFT} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}v^2 F(h)^2 (\partial \vec{n})^2 + O(4) \text{ sym terms}$$

Note that $\partial \vec{n}$ is only invariant under $O(3)$ transformations. So we are looking for a point h_* such that $F(h_*) = 0$. Then we can identify $h_* = -v$. This allows us to find a possible SMEFT point within the HEFT framework.

Field Space Geometry

Start with a set of coordinates (fields) φ^i

Under a coordinate change (aka a field redefinition without derivatives)

$\varphi^i = \varphi^i(\tilde{\varphi})$ then $\partial_\mu \varphi^i$ transforms as

$$\partial_\mu \tilde{\varphi}^i = \left(\frac{\partial \tilde{\varphi}^i}{\partial \varphi^j}\right) \partial_\mu \varphi^j$$ due to the chain rule.

Check this (e.g. for polynomial field redefs)

Similarly, a tensor transforms according to its index structure. E.g.

$$\tilde{T}^{ij} = \left(\frac{\partial \tilde{\varphi}^i}{\partial \varphi^k}\right)\left(\frac{\partial \tilde{\varphi}^j}{\partial \varphi^l}\right) T^{kl} \quad \text{and} \quad \tilde{T}_{ij} = \left(\frac{\partial \varphi^k}{\partial \tilde{\varphi}^i}\right)\left(\frac{\partial \varphi^l}{\partial \tilde{\varphi}^j}\right) T_{kl}$$

Infinitesmal line element in field space is

$$ds^2 = G_{ij}(\varphi)\, d\varphi^i d\varphi^j$$

We call $G_{ij}(\varphi)$ the field space metric

The metric has the following properties:

- Transforms as a 2-tensor
- Symmetric: $G_{ij} = G_{ji}$
- Non-singular: No row or column can be the zero vector.

Write a generic 2 derivative scalar field Lagrangian in the form

$$\mathcal{L}_{kin} = \frac{1}{2} G_{ij}(\varphi)\, \partial_\mu \varphi^i \partial^\mu \varphi^j$$

G_{ij} is symmetric ✓

G_{ij} is non-singular since there must be a non-zero kinetic term for each scalar field ✓

G_{ij} transforms as a 2-tensor?

Under a transformation $\varphi \to \tilde{\varphi}(\varphi)$

Then $$\mathcal{L}_{kin} = \underbrace{\frac{1}{2} G_{ij}(\varphi) \frac{\partial \varphi^k}{\partial \tilde{\varphi}^i} \frac{\partial \varphi^l}{\partial \tilde{\varphi}^j}}_{\equiv\ \tilde{G}_{kl}(\tilde{\varphi})} \partial_\mu \tilde{\varphi}^k \partial^\mu \tilde{\varphi}^l$$

So it transforms as a tensor ✓

and $$\tilde{\mathcal{L}}_{kin} = \mathcal{L}_{kin} \Big|_{\substack{\varphi \to \tilde{\varphi} \\ G \to \tilde{G}}}$$

What about derivatives of scalar functions (e.g V)

$$V(\varphi) \to V(\tilde{\varphi})$$

$$\frac{\partial V}{\partial \varphi^i} = \frac{\partial \tilde{\varphi}^j}{\partial \varphi^i} \frac{\partial V}{\partial \tilde{\varphi}^j}$$ ← tensor

tensor ↙ not tensor ↙

$$\frac{\partial^2 V}{\partial \varphi^i \partial \varphi^j} = \frac{\partial \tilde{\varphi}^k}{\partial \varphi^i} \frac{\partial \tilde{\varphi}^l}{\partial \varphi^j} \frac{\partial^2 V}{\partial \tilde{\varphi}^k \partial \tilde{\varphi}^l} + \frac{\partial^2 \tilde{\varphi}^k}{\partial \varphi^i \partial \varphi^j} \frac{\partial V}{\partial \tilde{\varphi}^k}$$

$\Rightarrow$ Need to covariantize

Introduce Christoffel symbols

$$\Gamma^i_{jk} = \frac{1}{2} G^{il} \left(\partial_j G_{kl} + \partial_k G_{lj} - \partial_l G_{jk} \right)$$

Then covariant derivative ∇_i acts on V as

$$\nabla_i V = \frac{\partial}{\partial \varphi^i} V \,, \quad \nabla_i \nabla_j V = \frac{\partial^2 V}{\partial \varphi^i \partial \varphi^j} - \Gamma^k_{ij} \frac{\partial V}{\partial \varphi^k}$$

Geometry of HEFT

To identify the metric in HEFT, write

$$\mathcal{L}_{HEFT} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}\left[v F(h)\right]^2 \left(\delta^{ij} + \frac{n^i n^j}{1-n^2}\right)(\partial^\mu n^i)(\partial_\mu n^j)$$

w/ $\vec{n} = \left(n_1, n_2, n_3, \sqrt{1-\sum_i n_i^2}\right)$

Then we identify the components of the metric:

$$g_{hh} = K^2 \, , \quad g_{h\pi_i} = 0 \qquad \Big| \qquad g^{hh} = 1/K^2$$

$$g_{\pi_i \pi_j} = F^2\left(\delta_{ij} - \frac{\pi_i \pi_j}{v^2 - \vec{\pi}\cdot\vec{\pi}}\right) \qquad \Big| \qquad g^{ij} = \frac{1}{F^2}\left(\delta_{ij} - \frac{\pi_i \pi_j}{v^2}\right)$$

Turn the GR crank... (show this)

$\Rightarrow$ Ricci scalar curvature

$R = 6(\mathcal{K}_h + \mathcal{K}_\pi)$ w/ $\mathcal{K}$ are sectional curvatures

$$R_{\pi_i h h \pi_j} = -g_{hh}\, g_{\pi_i \pi_j} \mathcal{K}_h \, , \quad R_{\pi_i \pi_k \pi_l \pi_j} = (g_{il}\, g_{kj} - g_{ij}\, g_{kl})\mathcal{K}_\pi$$

$$\mathcal{K}_h = -\frac{1}{K^2}\left[\frac{F''}{F} - \frac{K'}{K}\frac{F'}{F}\right] \, , \quad \mathcal{K}_\pi = \frac{1}{(vF)^2}\left[1 - \frac{(vF')^2}{K^2}\right]$$

$$\nabla^2 V = \left(\frac{1}{K}\partial_h\right)^2 V + 3\frac{F'}{FK}\left(\frac{1}{K}\partial_h\right)V$$

Geometrizing Amplitudes

Notation

$$T_{\alpha_1\cdots\alpha_m,\beta_1\cdots\beta_n} \equiv \frac{\partial}{\partial\varphi^{\beta_n}}\cdots\frac{\partial}{\partial\varphi^{\beta_1}}T_{\alpha_1\cdots\alpha_m}$$

$$T_{\alpha_1\cdots\alpha_m;\beta_1\cdots\beta_n} \equiv \frac{\nabla}{\nabla\varphi^{\beta_n}}\cdots\frac{\nabla}{\nabla\varphi^{\beta_1}}T_{\alpha_1\cdots\alpha_m}$$

$$\text{E.g. } T_{\alpha_1\cdots\alpha_m;\beta} = T_{\alpha_1\cdots\alpha_m,\beta} - \sum_{i=1}^{m} T_{\alpha_1\cdots\underbrace{\hat\alpha_i\rho}_{\text{replace } \alpha_i \text{ with } \rho}\cdots\alpha_m}\Gamma^{\rho}_{\alpha_i\beta}$$

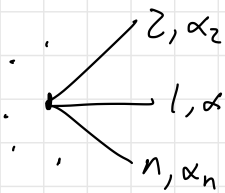
Taylor expand metric and potential about vacuum

$$\mathcal{L} = \frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{n!}\bar g_{\alpha\beta,\gamma_1\cdots\gamma_n}(\partial_\mu\varphi^\alpha)(\partial^\mu\varphi^\beta)\varphi^{\gamma_1}\cdots\varphi^{\gamma_n} - \sum_{n=0}^{\infty}\frac{1}{n!}\bar V_{,\gamma_1\cdots\gamma_n}\varphi^{\gamma_1}\cdots\varphi^{\gamma_n}$$

Note $\bar V_{,\alpha\beta} = \bar g_{\alpha\beta}\, m_\alpha^2$ (bar denotes evaluated at vacuum)

Feynman rules

$$\alpha \longrightarrow \beta = \frac{i\bar g^{\alpha\beta}}{p^2 - m_\alpha^2}$$



$$= -i\bar V_{,\alpha_1\cdots\alpha_n} - i\sum_{1\le i<j\le n} p_i\cdot p_j\, \bar g_{\alpha_i\alpha_j,\alpha_1\cdots\hat\alpha_i\cdots\hat\alpha_j\cdots\alpha_n}$$

omit

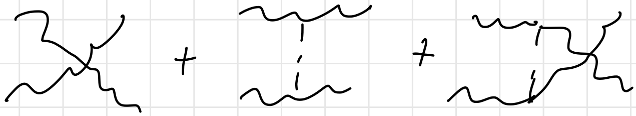
High energy Goldstone / Higgs scattering

$\pi\pi \to hh$

$$\Rightarrow \mathcal{M} = -s\,\bar{\mathcal{R}}_h + \mathcal{O}(g^4, t/s)$$

(compute these)

$\pi\pi \to \pi\pi$



$$\Rightarrow \mathcal{M} = s\,\bar{\mathcal{R}}_\varphi + \mathcal{O}(g^4, t/s)$$

Manifestly invariant under field redefs!

A useful trick to compute amplitudes is to do a field redefinition to normal coordinates at the vacuum. This implies that the metric is flat at this point on the manifold, so $\nabla \to \partial$. Additionally, the derivative of the metric vanishes

$\Rightarrow$ 3-pt vertex with derivative couplings vanishes.

Criteria for SMEFT

How can we develop a criterion for SMEFT that is robust against field redefinition ambiguities? Clearly we should try to frame the question in terms of field space geometry.

We already have one necissary condition, which is there must exist an $O(4)$ fixed point on the field space manifold h_*.

Here we want to understand if a SMEFT expansion exists at that point (analytic expression in "Cartesian coordinates")

The logic of the argument is as follows:

1) Write the most general SMEFT (up to 2-derivative order)
2) Canonically normalize the h kinetic term. This fixes the choice of field basis.

3) Rephrase the basis specific criteria in terms of curvature invariants, so that the criteria can be applied in any basis.

1) First, notice that when we map from SMEFT to HEFT :

$$\mathcal{L}_{SMEFT} = A|\partial H|^2 + \frac{1}{2}B(\partial|H|^2)^2 - \tilde{V}$$
$$= \frac{1}{2}(A+(v+h)^2B)(\partial h)^2 + \frac{1}{2}(v+h)^2A(\partial\vec{n})^2 - V$$

$\Rightarrow K = \sqrt{A+(v+h)^2B}$ & $vF = (v+h)\sqrt{A}$

where $A, B, \tilde{V}$ are real analytic functions of $|H|^2 = (v+h)^2/2$, $A(0)=0$ and $V'(v^2/2)=0$

This Lagrangian has the following properties

1) $F(h_* = -v) = 0$, $V'(h=0) = 0$

2) $K(h_*)$, $F(h_*)$, $V(h_*)$ are real analytic functions of h

3) Expanding about $h = h_*$, K & V are even and F is odd in $(h-h_*) = (v+h)$

Also, $A(0) = 1 \Rightarrow K(h_*) = vF'(h_*) = 1$.

Note that condition (3) is not field redefinition invariant. So this set of criterion are necissary but not sufficient to guarantee that SMEFT exists.

2) We want to fully fix the HEFT basis.

A natural choice is to canonically normalize the kinetic term. Let

$$v_1 + h_1 = Q(v+h) = \int_0^{v+h} dt\, \overline{K}(t) \Rightarrow dh_1 = \overline{K}\, dh_2$$

Claim that this fully fixes the freedom to redefine the fields h & $\vec{n}$, see "Is SMEFT Enough" Sec 4.1.2 for argument.

The HEFT Lagrangian in this basis is

$$\mathcal{L}_{HEFT} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(v F(h))^2 (\partial \vec{n})^2 - V(h) + \cdots$$

3) To geometrize these basis dependent statements, we use the map

$$\left.\begin{aligned} & F^{(2k)}(h_*) = 0 \quad \forall k \in \mathbb{N} \\ & v' F(h_*) = 1 \\ & V^{(2k+1)}(h_*) = 0 \quad \forall k \in \mathbb{N} \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} & F(h_*) = 0 \\ & D^{\mu_1} D_{\mu_1} \cdots D^{\mu_n} D_{\mu_n} R\big|_{h_*} < \infty \quad \forall n \in \mathbb{N} \\ & D^{\mu_1} D_{\mu_1} \cdots D^{\mu_n} D_{\mu_n} V\big|_{h_*} < \infty \quad \forall n \in \mathbb{N} \end{aligned}\right.$$

See Appendix B of "Is SMEFT Enough?" for derivation.

This motivates the approximate "Leading Order Criteria" for the existence of SMEFT:

a) $F(h)$ has a zero at some h_*

b) $\underline{K}$, F, and V have convergent Taylor expansions about h_*

c) The scalar curvature R is finite at h_*

When is HEFT Required?

Let's explore what can cause the criteria $R|_{h_*} < \infty$ to fail.

Ex: Integrate out scalar singlet at tree-level

The UV model is

$$\mathcal{L}_{UV} = |\partial H|^2 + \frac{1}{2}(\partial S')^2 - V$$

w/ $V = -\mu_H^2 |H|^2 + \lambda_H |H|^4 + \frac{1}{2}\left(m^2 + \varkappa |H|^2\right) S'^2 + \frac{1}{4}\lambda_S S'^4$

Integrate out S' using its EOMs

The Effective $\mathcal{L}$ is then (Check this)

$$\mathcal{L}_{Eff} = |\partial H|^2 - \frac{\varkappa^2}{8\lambda_S(m^2 + \varkappa |H|^2)}\left(\partial |H|^2\right)^2$$

$$+ \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{4\lambda_S}\left(m^2 + \varkappa |H|^2\right)^2$$

Note this is <u>not</u> an EFT since it contains all orders in H. EFT requires truncation.

Next, express this in terms of h and $\vec{n}$ to find

$$K(h) = \sqrt{1 - \frac{\varkappa^2 (v+h)^2}{2\lambda_S(2m^2 + \varkappa(v+h)^2)}}, \quad vF = v + h$$

$$V = -\frac{1}{2}\mu_H^2 (v+h)^2 + \frac{1}{4}\lambda_H (v+h)^4 - \frac{1}{16\lambda_S}\left(2m^2 + \varkappa(v+h)^2\right)^2$$

From here we can derive

$$R = \frac{6}{(v+h)\bar{K}^3}(\partial_h \bar{K}) + \frac{6}{(v+h)^2}\left(1 - \frac{1}{\bar{K}^2}\right) \qquad \leftarrow \begin{array}{l} \partial_h F = 1/v \\ \partial_h^2 F = 0 \end{array}$$

$$= 3\lambda_d^2 \frac{2\lambda_d(v+h)^2(\lambda_d - 2\lambda_s) - 16m^2\lambda_s}{\left(\lambda_d(v+h)^2(\lambda_d - 2\lambda_s) - 4m^2\lambda_s\right)^2}$$

Let's check all the criteria:

Clearly $F(h_* - v) = 0$, and there are no obstructions to Taylor expanding any terms about h_*.

Then we can evaluate $R(h_*) = \frac{-3\lambda_d^2}{m^2\lambda_s}$ is finite.

So we can expand $\mathcal{L}_{Eff}$ in H to find

$$\mathcal{L}_{SMEFT} = |\partial H|^2 + \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{4\lambda_s}(m^2 + \lambda_d |H|^2)^2 - \frac{1}{8}\frac{\lambda_d^2}{\lambda_s m^2}(\partial |H|^2)^2 + \text{dim } 8$$

What about the UV theory with $m^2 = 0$?

We have $R\big|_{m^2 \to 0} = \frac{6\lambda_d}{(\lambda_d - 2\lambda_s)(v+h)^2} \xrightarrow{h \to v} \infty$

So the criteria fails and HEFT is required! See "Is SMEFT Enough?" for more examples (loop level, fermions, ...).

Physical Interpretation of HEFT

We have learned that there are essentially two ways HEFT can fail:

1) The field space manifold does not contain an $O(4)$ invariant fixed point. This has the physical interpretation that there is an additional source of EW symmetry breaking that does not go to zero when the Higgs vev vanishes.
2) The fixed point exists but the curvature (or its covariant derivatives) diverges at the fixed point. This has the interpretation that we have integrated out a particle that gets all of its mass from the Higgs. Therefore, there is a BSM massless state at the $O(4)$ fixed point and SMEFT does not have all the necissary dofs.

The lesson is an intuitive one: SMEFT fails if the BSM physics is "non-decoupling". Then it is not possible to match onto SMEFT, and one must match onto HEFT.

In fact, we showed that these arguments can be used to show that HEFT violates perturbative unitarity at a scale $\mathcal{O}(4\pi v)$ when the EFT is modeling a BSM state that gets all (or most) of its mass from v.
(see "Unitarity violation and the Geo of HEFT")

Practicle Criterion for HEFT

One should match onto HEFT when integrating out a state whose mass is near or below the weak scale.

The point is that while SMEFT may exist, the expansion might converge so slowley that it is not practically useful.

See Sec 8 of "Is SMEFT Enough?"

Outlook

How should we organize indirect searches for BSM physics?

- If we want to use EFT, we have to decide if we want to assume that the new physics is of the "decoupling" type or not, i.e., Should we use SMEFT? Then it is natural to ask if there are any "non-decoupling" UV completions that are consistent with the data. We studied this (w/ Ian Banta) and named these types of new particles "loryons." Some param space is still viable!
- We can also try to organize searches in a more bottom up approach. This is the idea of "primaries".
- My personal view is that the only thing that matters is to not miss the signs of new physics!

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