Primordial gravitational waves as probe of dark matter and leptogenesis in GWs missions

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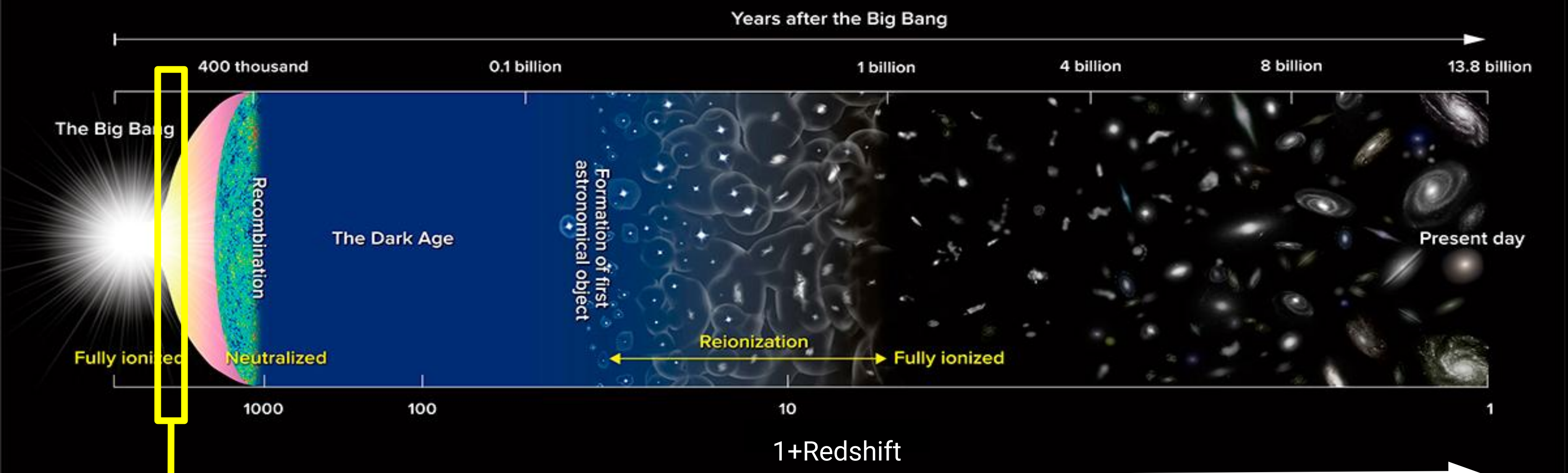
Based on:
arXiv: 2405.06741
JHEP 12(2024)150

Collaborators:
A. Ghoshal, S. Pal

Hearing beyond the standard model with cosmic sources of Gravitational Waves

ICTS Bengaluru



past → *present*

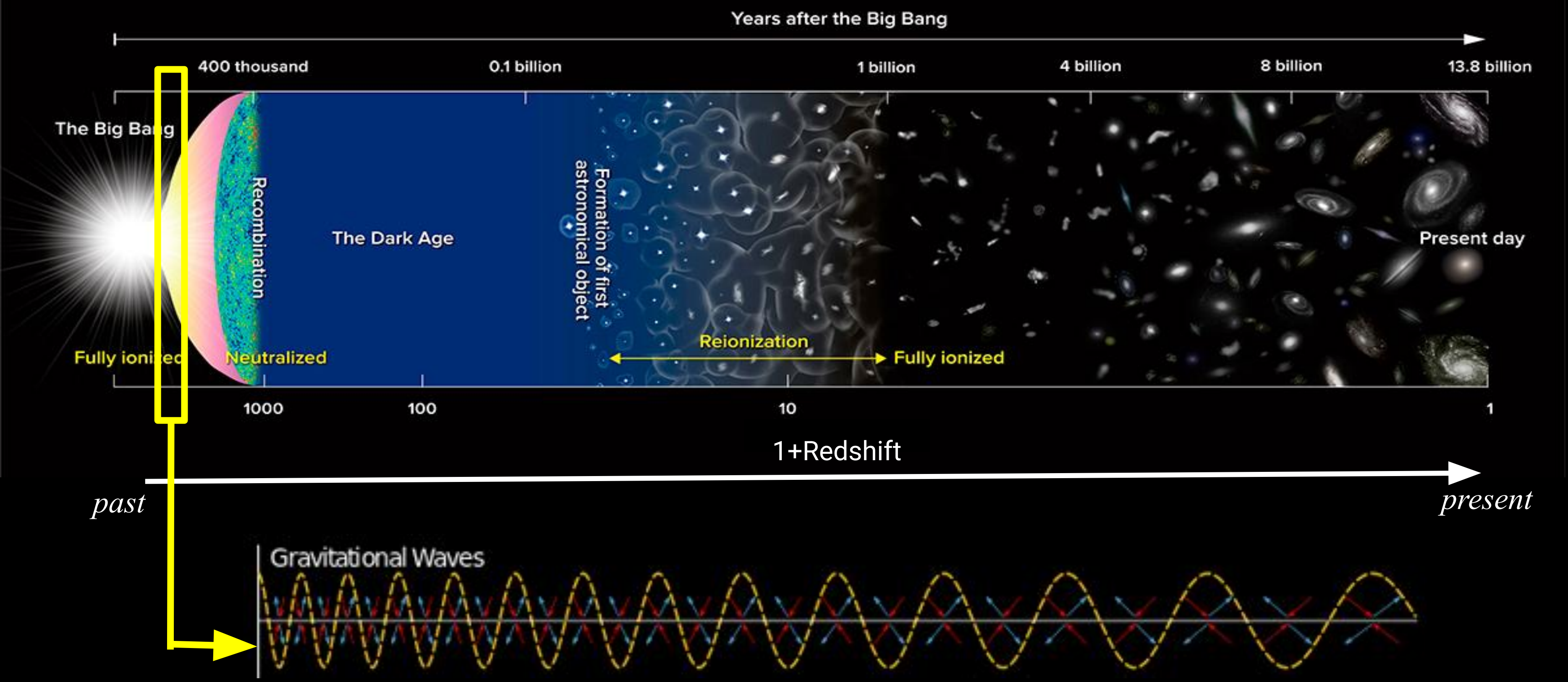
Primordial fluctuations → *Curved metric*

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}(\mathcal{R}, h_{ij})$$

Tensor perturbation

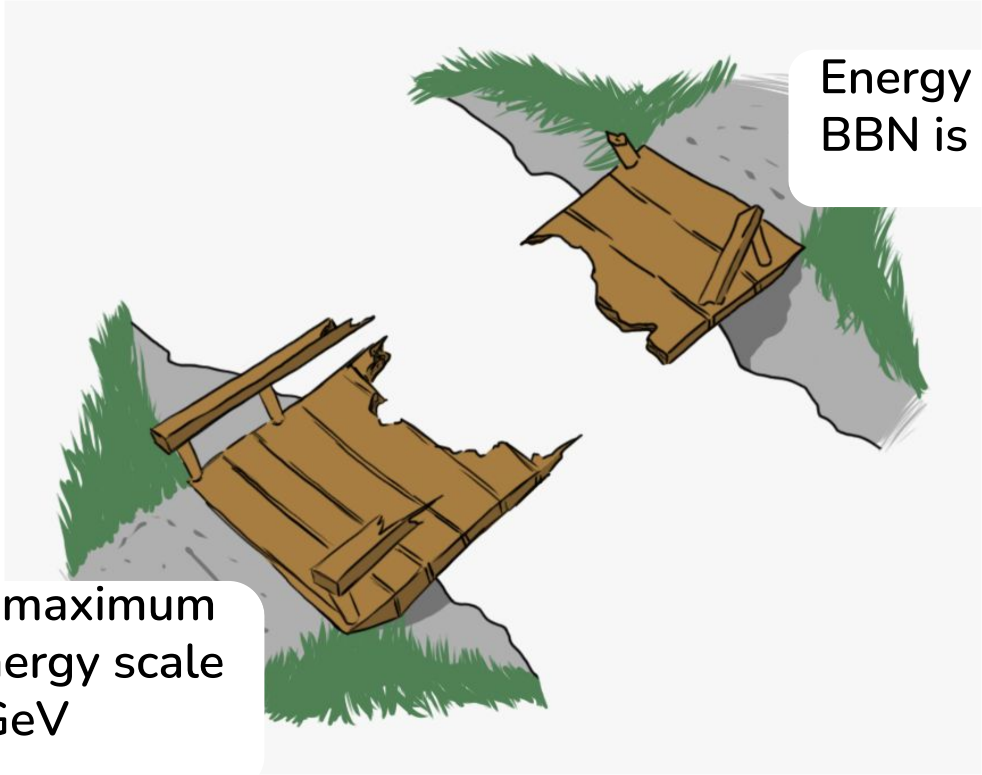
$$\langle h_{ij}^{*} \cdot h_{ij} \rangle$$

GW spectrum



- *Primordial GWs carry the information of cosmic inflation.*
- *Travel uninterrupted, from past to present.*

Wait…we have some difficulties in probing the early Universe!

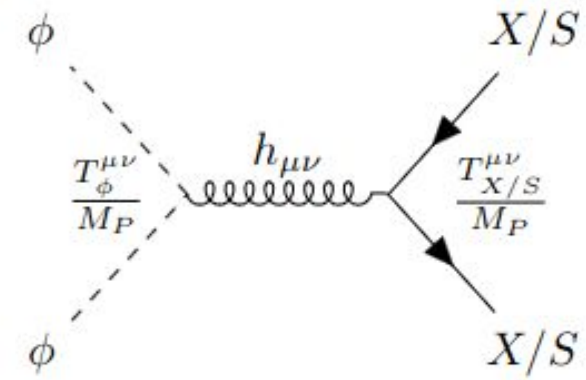


Energy scale of BBN is ~1 MeV

CMB predicts maximum inflationary energy scale ~10^{14} GeV

Reheating may be a possible solution ...

Gravitational Reheating



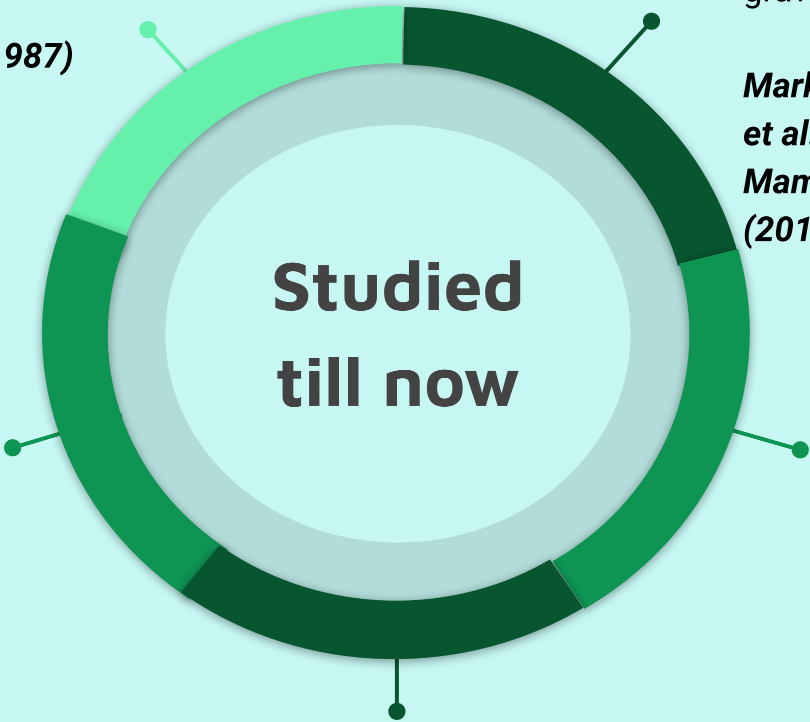
- ❑ No evidence of DM from direct and indirect search till now, except gravitational interaction.
- ❑ No direct coupling between inflaton and daughter particles. But always couples to gravity.
- ❑ LHC can not probe very high scale whereas GWs can carry these signatures.

Gravitational production of particles

Ford (1987)

Gravitational production of DM and gravitational leptogenesis

Markkanen et al. (2015), Artymowski et al.(2017), Hashiba et al. (2018), Mambrini et al. (2021), Adshead et al. (2017), Barman et al. (2022)



Signature at the PGWs

Artymowski et al.(2017), Haque et al.(2022), Barman et al. (2022), Haque et al.(2023)

Gravitational reheating and several inflationary scenarios

Haque et al. (2022), Barman et al.(2024), Dorsch et al.(2024)

Gravitational production of other cosmological relics

Barman et al.(2023)

What we have studied …

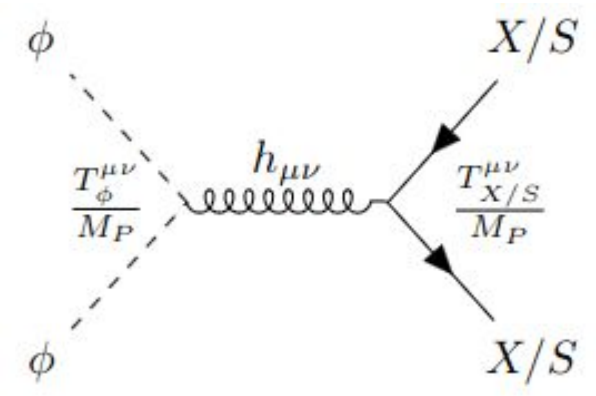
- Probed the mass scale of DM using the interferometric missions with assessing high SNR .
- Probed the scale of gravitational leptogenesis.
- Assessed the uncertainties on the parameters for the upcoming GW missions.

A.Ghoshal, DP, S.Pal (*arXiv: 2405.06741*)
JHEP 12(2024)150

Outline of the discussion …

- ➢ Signatures of the parameters at the GW detectors.
- ➢ Assessing the signal-to-noise ratio for each of the detectors.
- ➢ Forecast analysis on the parameters for the GW missions.
- ➢ Estimating the uncertainties of the parameters using MCMC analysis.
- ➢ Probing the scale of DM and gravitational leptogenesis.

Gravitational Reheating



Inflation Potential $V(\phi) \sim \phi^n$

Behaviour of the potential at the proximity of its minima

Equation of state, $\omega \approx (n-2)/(n+2)$

Model parameters

Considering
non-minimal coupling → $\mathcal{L}_{non-min} = \xi|\mathrm{h}|^2 R/M_P^2$

Eq. Ref.: *Barman et al., JHEP 12 (2022) 072*

Model parameters

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Tensor spectral index, n_T

$$P_T(k) = \underbrace{A_T}_{\frac{2H_{\rm inf}^2}{\pi^2 M_P^2}} \left(\frac{k}{k_0}\right)^{n_T}$$

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$$A_T = \frac{2H_{inf}^2}{\pi^2 M_P^2}$$

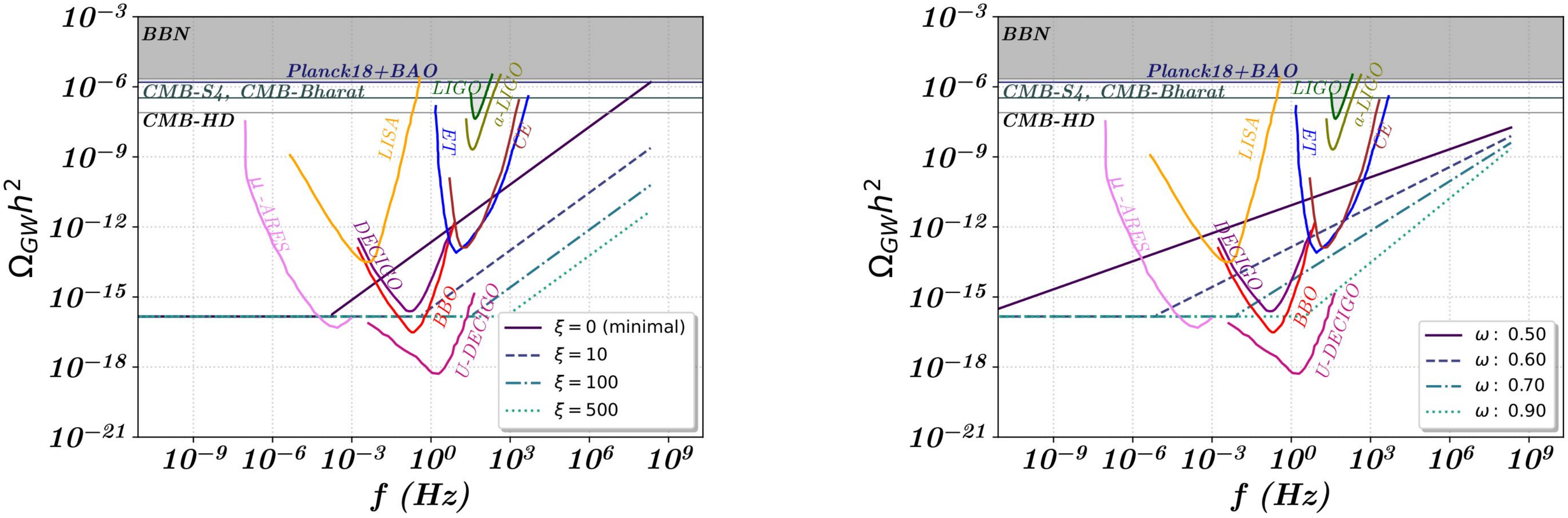
Parameters:

$\{\omega, \xi, n_T, H_{inf}\}$

Eq. Ref.: *Haque et al., Phys.Rev.D 107 (2023) 4, 043531*

GW spectrum for today

$$\Omega_{GW}(k)h^2 \simeq \begin{cases} \Omega_R h^2 P_{\rm T}(k)\frac{4\mu^2}{\pi}\left[\Gamma\left(\frac{5+3\omega}{2+6\omega}\right)\right]^2\left(\frac{k}{2\mu k_{\rm re}}\right)^{\frac{6\omega-2}{3\omega+1}} & \text{for } k_{\rm RH} < k \leq k_{\rm end} \\ \Omega_R h^2 P_{\rm T}(k) & \text{for } k \leq k_{\rm RH}, \end{cases}$$



ω=0.8

ξ=10

Eq. Ref.: *Haque et al., Phys.Rev.D 107 (2023) 4, 043531*

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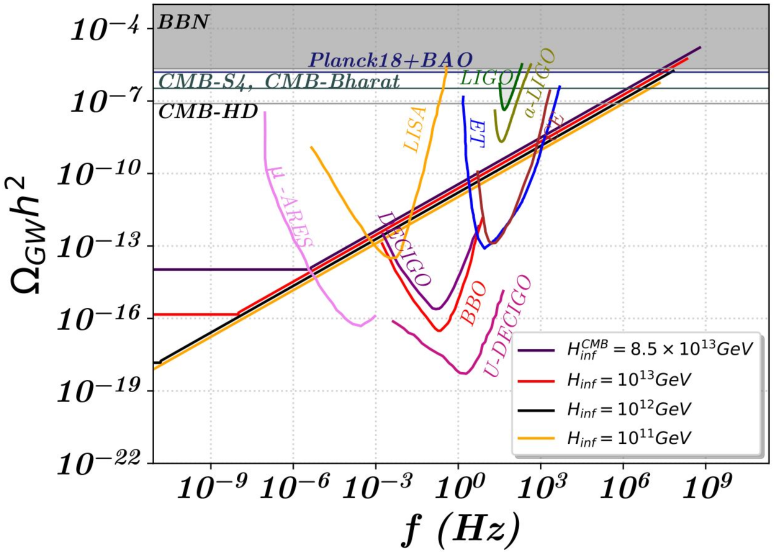
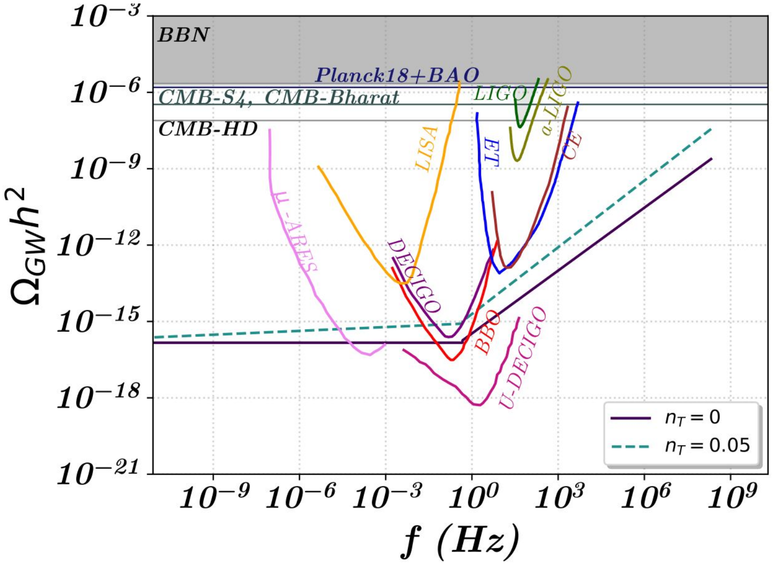


ω=0.65



ξ=10

Signal-to-noise ratio

SNR serves as a crucial tool in analysing the detectional prospect of signal

$$\mathrm{SNR} \equiv \sqrt{\tau \int_{f_{\rm min}}^{f_{\rm max}} df \left(\frac{\Omega_{\rm GW}(f, \{\theta\}) h^2}{\Omega_{\rm n}(f) h^2} \right)^2}$$

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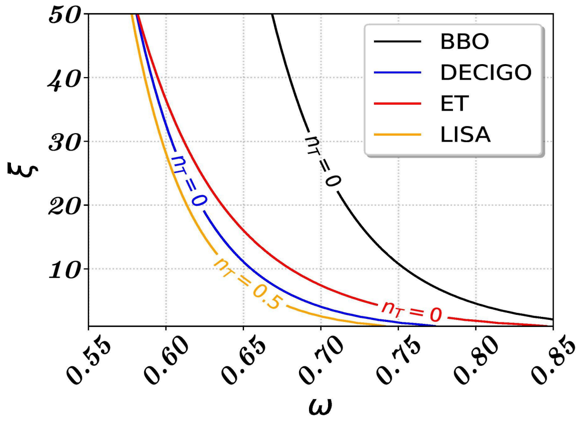

τ = 4 years

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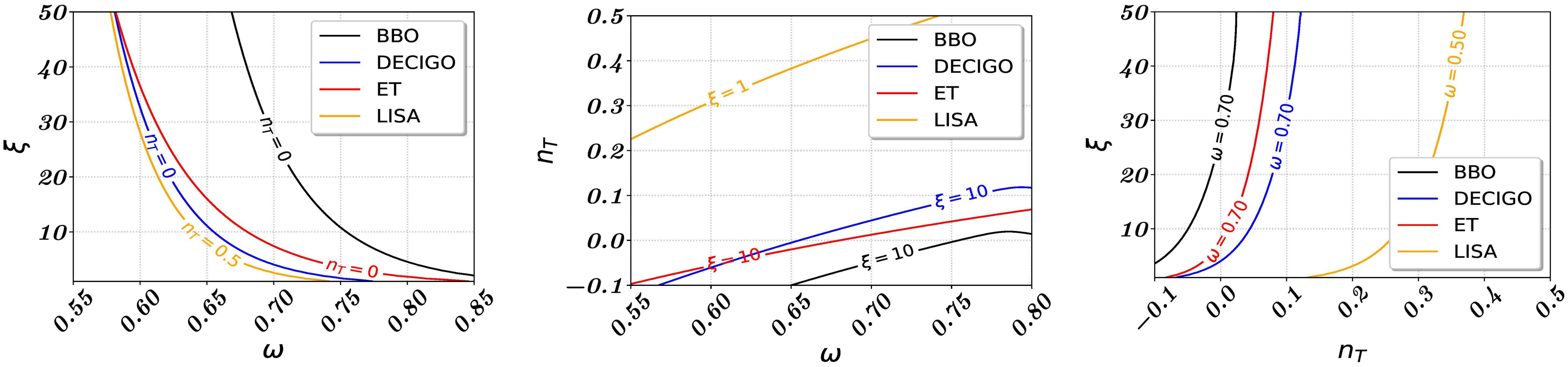

τ = 4 years

Proceeding further ...

estimation of uncertainties on the parameters for the future GW missions

Fisher matrix analysis

- Does not shift the mean value.
- Estimate only the error on the parameters.

MCMC analysis

- Shifts the mean value.
- Estimate both mean value and error on it, providing combined parameter estimation.

Fisher matrix analysis

Likelihood function for GW signal is defined as

$$\mathscr{L}(\theta) = \prod_{b=1}^{N_b} \sqrt{\frac{n_b}{2\pi\Omega_{\rm n}(f_b)^2}} \exp\left(-\frac{n_b\left(\Omega_{\rm sig}(f_b,\theta) - \Omega_{fid}(f_b)\right)^2}{\Omega_{\rm n}(f_b)^2}\right)$$

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$[(f_b - f_{b-1})\tau]$

GW spectra of the signal

Noise spectra for the detectors

Ω_{sig} for the fiducial values of the parameters

Fisher matrix analysis

Likelihood function for GW signal is defined as

$$\mathscr{L}(\theta) = \prod_{b=1}^{N_b} \sqrt{\frac{n_b}{2\pi\Omega_n(f_b)^2}} \exp\left(-\frac{n_b\left(\Omega_{\rm sig}(f_b,\theta) - \Omega_{fid}(f_b)\right)^2}{\Omega_n(f_b)^2} \right)$$

Chi-squared distribution

$$\mathcal{L}(\theta) \equiv ln\left(\mathscr{L}(\theta)\right)$$

Fisher matrix

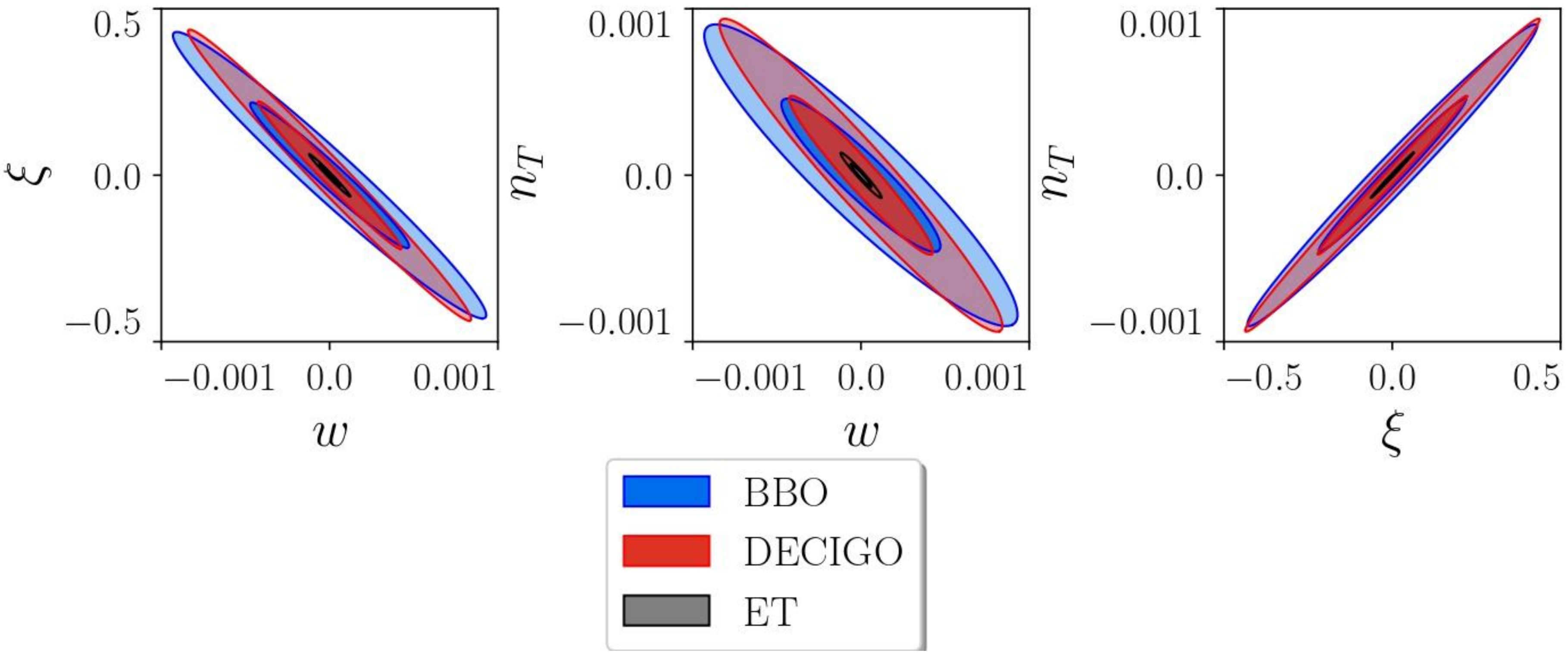
$$F_{ij} = \left\langle -\frac{\partial^2 \mathcal{L}(\theta)}{\partial\theta_i \partial\theta_j} \right\rangle$$

Using chi-square

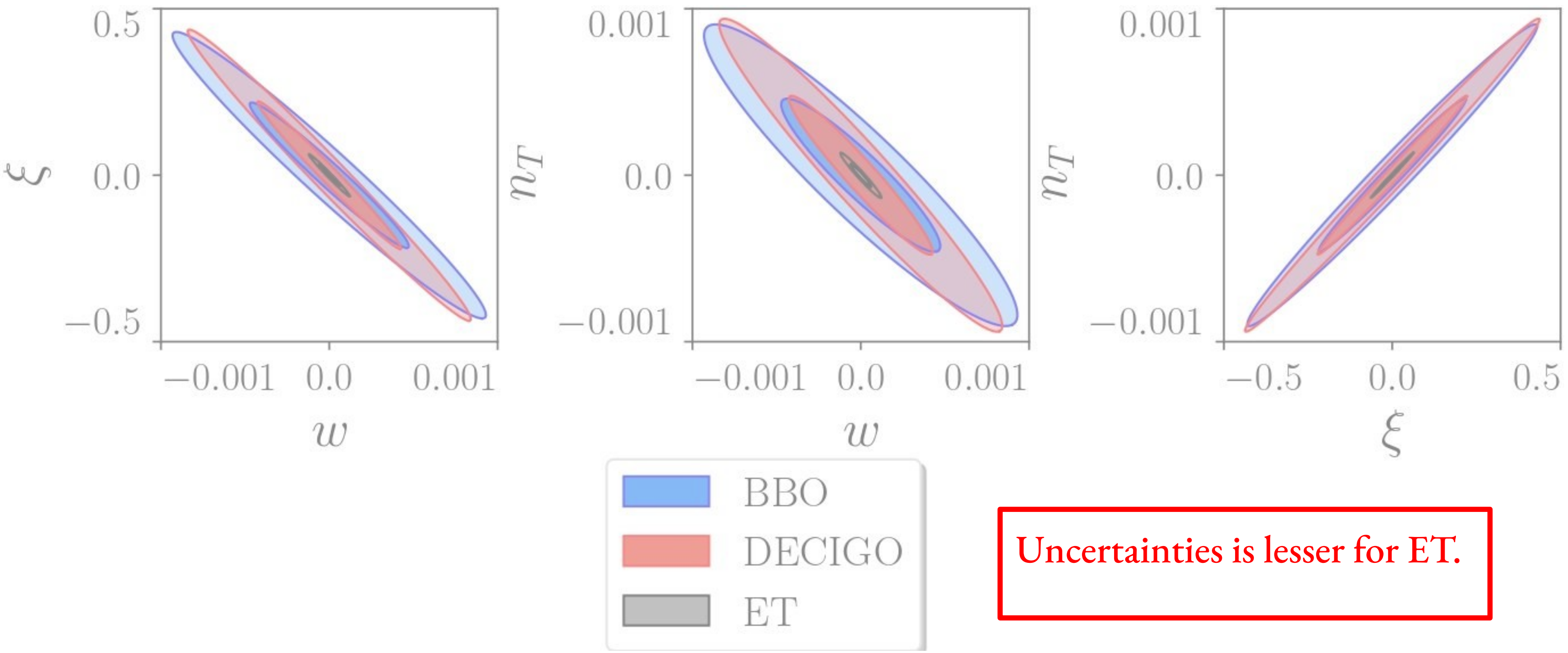
$$F_{ij} = \tau \sum_{b=1}^{N_b} \frac{2\Delta f_b}{\Omega_n^2} \frac{\partial \Omega_{\rm sig}}{\partial\theta_i} \frac{\partial \Omega_{\rm sig}}{\partial\theta_j}$$

$$[C_{ij}] = [F_{ij}]^{-1}$$

Fisher matrix analysis



Fisher matrix analysis



Uncertainties is lesser for ET.

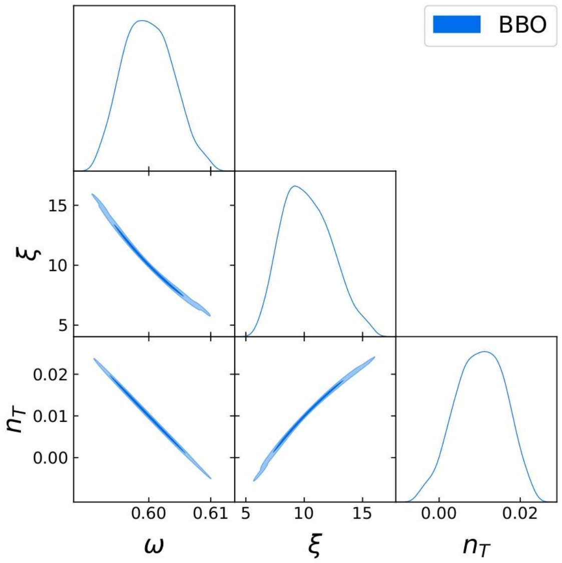
Markov chain Monte Carlo analysis

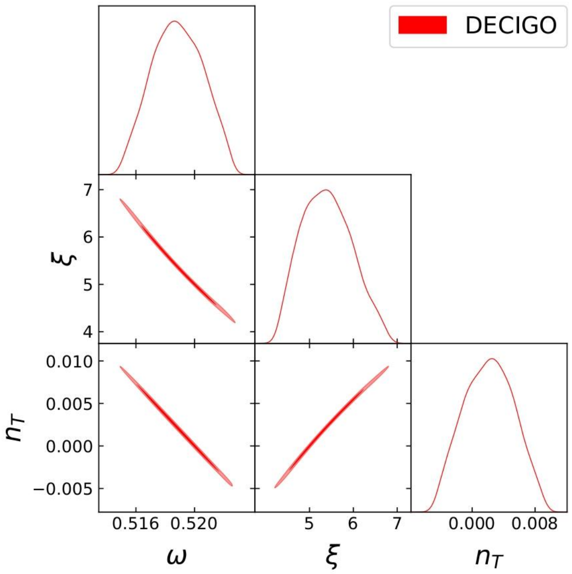
Likelihood function for GW signal is defined as

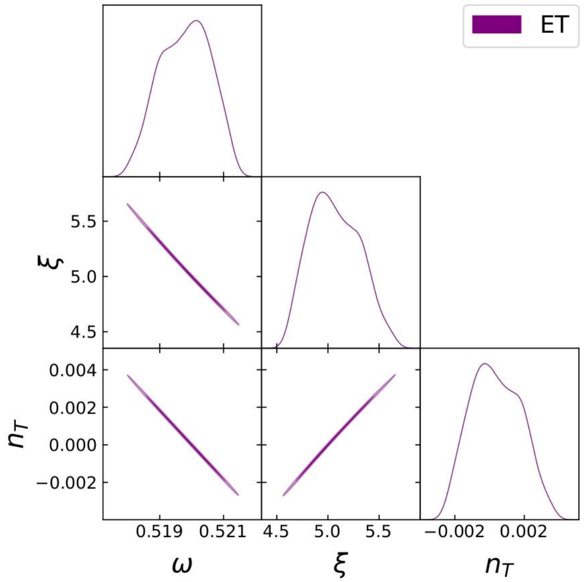
$$\mathscr{L}(\theta) = \prod_{b=1}^{N_b} \sqrt{\frac{n_b}{2\pi\Omega_{\mathrm{n}}(f_b)^2}} \exp\left(-\frac{n_b \left(\Omega_{\mathrm{sig}}(f_b, \theta) - \Omega_{fid}(f_b)\right)^2}{\Omega_{\mathrm{n}}(f_b)^2} \right)$$

Parameters: $\{\omega, \xi, n_T\}$

Markov chain Monte Carlo analysis







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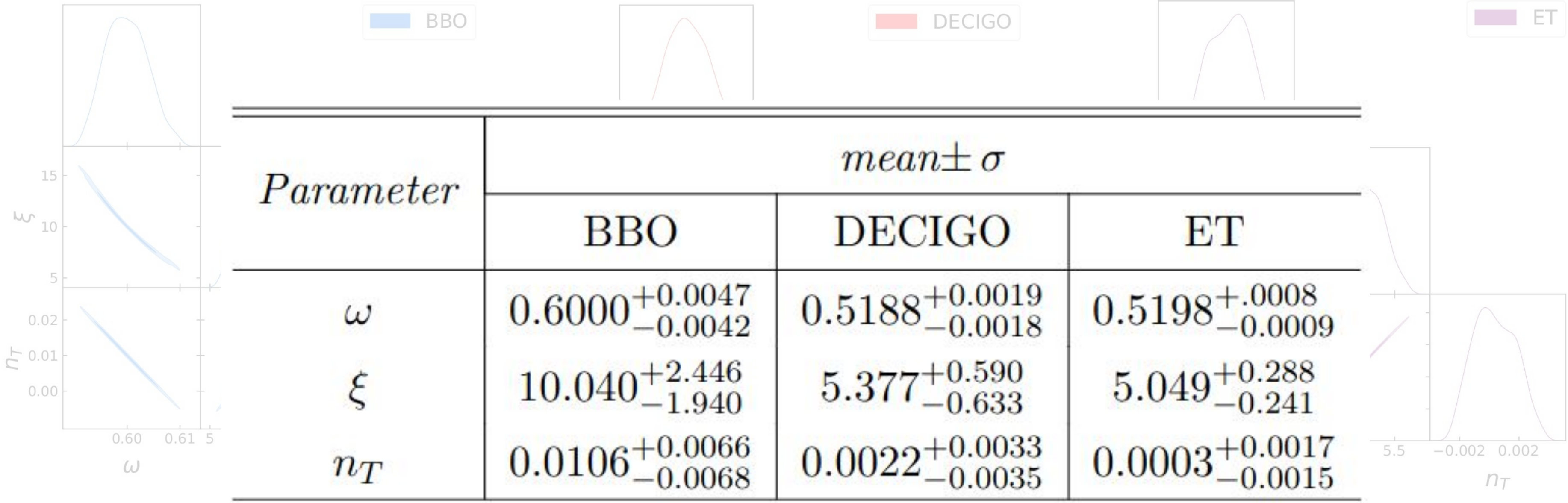
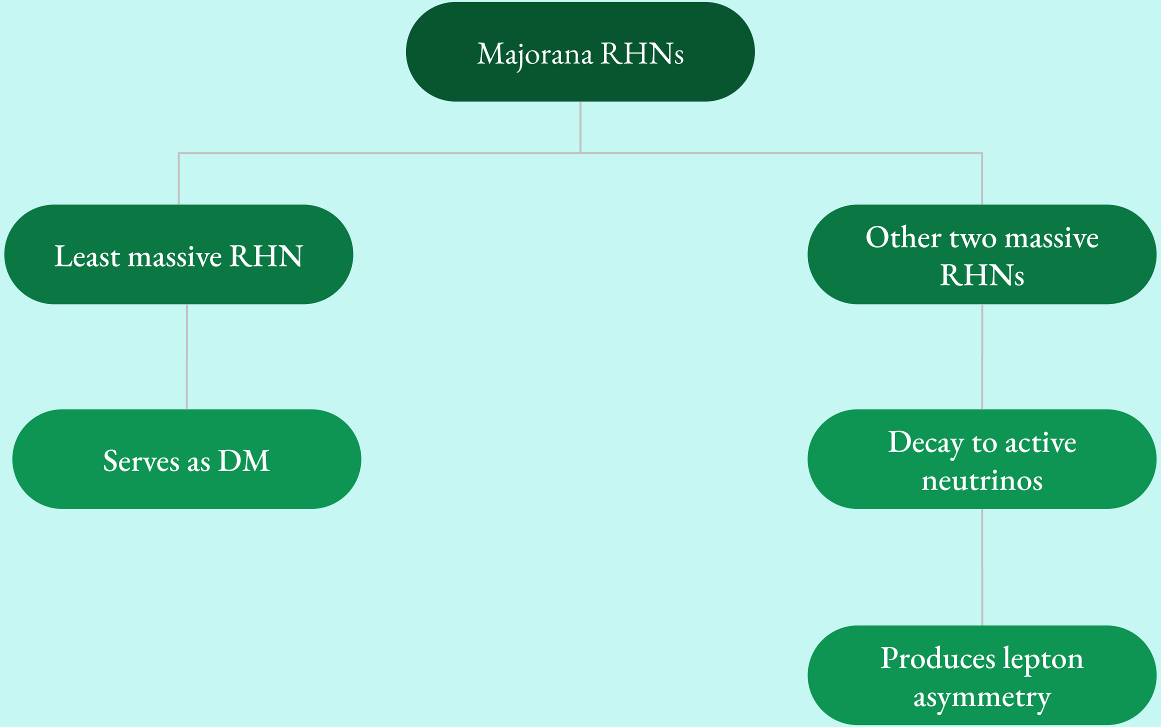


Parameter	*mean* ± σ		
	BBO	DECIGO	ET
ω	$0.6000^{+0.0047}_{-0.0042}$	$0.5188^{+0.0019}_{-0.0018}$	$0.5198^{+.0008}_{-0.0009}$
ξ	$10.040^{+2.446}_{-1.940}$	$5.377^{+0.590}_{-0.633}$	$5.049^{+0.288}_{-0.241}$
n_T	$0.0106^{+0.0066}_{-0.0068}$	$0.0022^{+0.0033}_{-0.0035}$	$0.0003^{+0.0017}_{-0.0015}$

ET is most promising.

Gravitational production of DM



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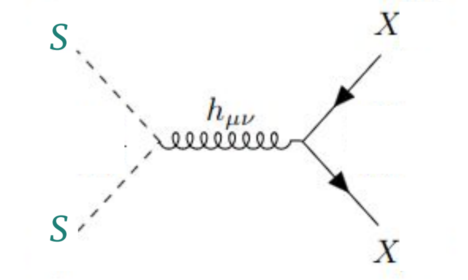
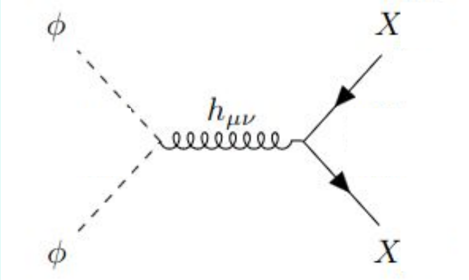


- Least massive RHN serves as DM
- DM can be generated via two possible ways:
    - from the thermal bath of SM

$$\Omega^{T}_{\rm DM}\, h^2 \simeq 1.6 \times 10^8\, \frac{g_0\, \beta_{1/2}}{g_{\rm RH}}\, \frac{M_{\rm DM}}{\rm GeV}\, \frac{\left(\frac{\pi^2 g_{\rm RH}}{30}\right)^{-\frac{5}{6}-\frac{5}{3n}} (7-4n)^2\,(n+2)}{6\sqrt{3}\,(n+5)(n-1)(5n-2)} \left(\frac{T_{\rm RH}}{M_P}\right)^{\frac{5n-20}{3n}} \left(\frac{\rho_{\rm end}}{M_P^4}\right)^{\frac{n+5}{3n}}$$

    - from the direct scattering of inflaton



$$\frac{\Omega^{\phi}_{\rm DM} h^2}{0.12} = \frac{\Sigma^{(n)}_{\rm DM}}{2.4^{\frac{8}{n}}}\, \frac{(n+2)}{n(n-1)} \left(\frac{10^{-11}}{\lambda}\right)^{\frac{2}{n}} \left(\frac{10^{40}\,{\rm GeV}^4}{\rho_{\rm RH}}\right)^{\frac{1}{4}-\frac{1}{n}} \times \left(\frac{\rho_{\rm end}}{10^{64}\,{\rm GeV}^4}\right)^{\frac{1}{n}} \left(\frac{M_{\rm DM}}{1.1\times 10^{7+\frac{6}{n}}\,{\rm GeV}}\right)^3$$

Eq. Ref.: *Barman et al., JHEP 12 (2022) 072*

Gravitational production of DM

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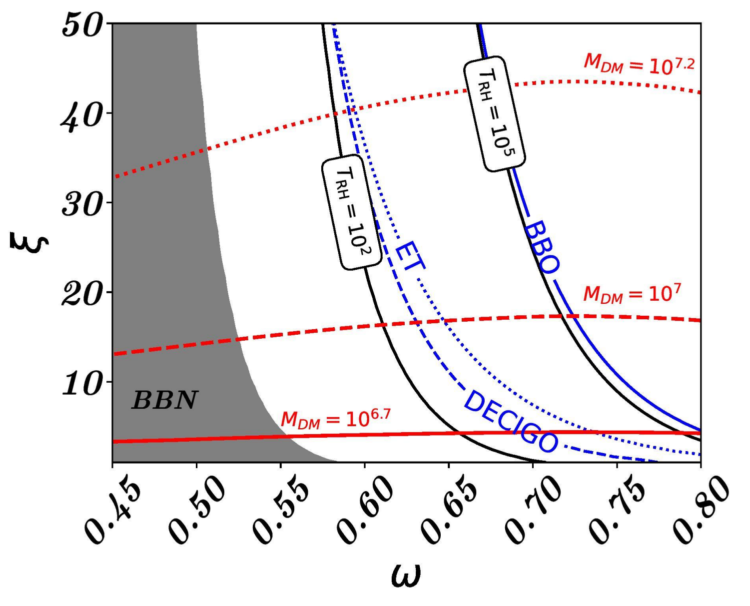

BBO, DECIGO, ET, for instance, are able to probe DM mass for 5×10^{6} GeV < M_{DM} with SNR > 10 for 4 years of observations.

Gravitational leptogenesis

- Other two RHNs produce lepton asymmetry (Y_L). This lepton asymmetry can be converted into baryon asymmetry (Y_B) as :

$$\mathbf{Y_B = (28/79)\, Y_L}$$

- Y_B can be expressed as :

$$Y_B = \frac{28}{79}\epsilon\,\frac{n_{\mathcal{N}}^{\phi}(T_{\rm RH})}{s}$$
$$\simeq 3.5\times10^{-4}\,\delta_{\rm eff}\left(\frac{m_\nu}{0.05\,{\rm eV}}\right)\left(\frac{M_{\mathcal{N}}}{10^{13}\,{\rm GeV}}\right)\frac{n_{\mathcal{N}}^{\phi}(T_{\rm RH})}{s(T_{\rm RH})}$$

- Observed value of $\mathbf{Y_B = 8.7\times10^{-11}}$ **GeV**

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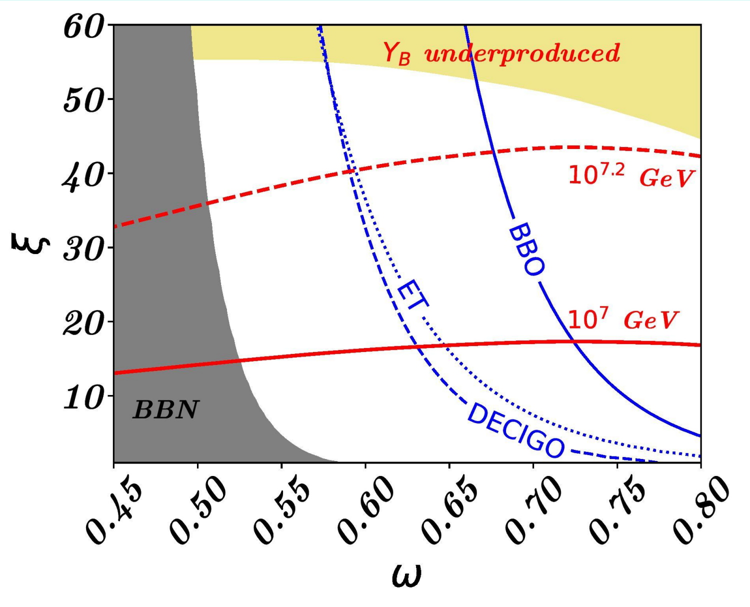
Gravitational leptogenesis

- Observed value of $Y_B = 8.7 \times 10^{-11}$ GeV

BBO, DECIGO, ET, for instance, are able to probe **gravitational leptogenesis for $M_N = 8\times10^{12}$ GeV** with DM mass for **5×10^6 GeV < M_{DM} < 1.6×10^7 GeV**, with SNR > 10 for 4 years of observations.

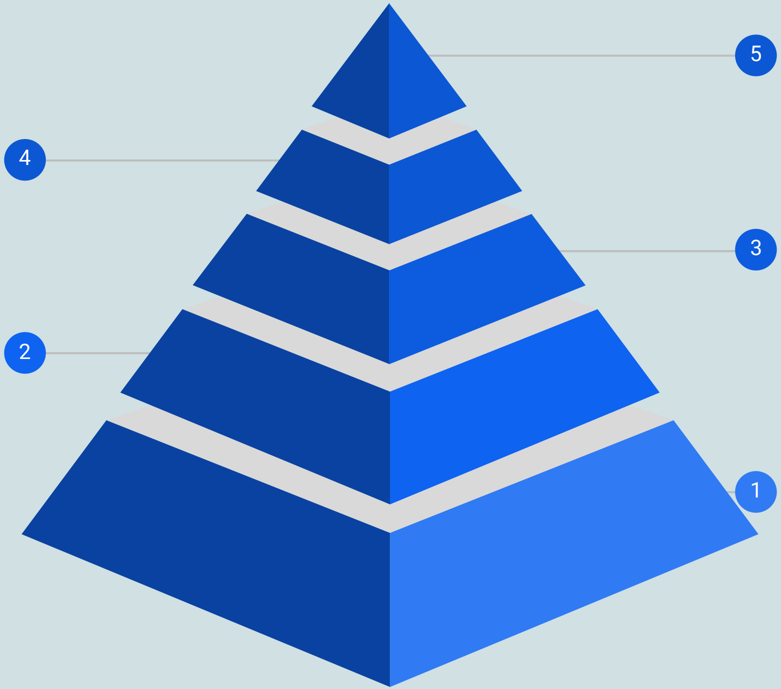


MCMC analysis

4 We have shown combined parameter analysis and the correlation is unaltered as it is in Fisher analysis. The error on the parameter is minimum for ET.

SNR

2 We estimate the SNR for the detectors. We have shown that the BBO indicates its heightened sensitivity for detecting the signal. The study reveals a negative correlation between ω and ξ.



Probing DM and leptogenesis

5 Lightest RHN serves as DM. We find that BBO, DECIGO, ET, for instance, are able to probe DM mass for 5×10^6 GeV < M_{DM} < 1.6×10^7 GeV with a SNR> 10, along with the baryon asymmetry due to gravitational leptogenesis for heavy RHN mass M_N to be around 8×10^{12} GeV.

Fisher forecast

3 The relative uncertainties on ω and ξ are less than 10%, which is for SNR>10. We have illustrated that correlation is stronger for ET whereas it is lesser for LISA.

Detection of PGWs

1 We have presented the signature of EOS, non-minimal coupling, tensor spectral index and H_{inf} at the GW detectors (BBO, DECIGO, ET, LISA).

A.Ghoshal, DP, S.Pal (*arXiv: 2405.06741*)
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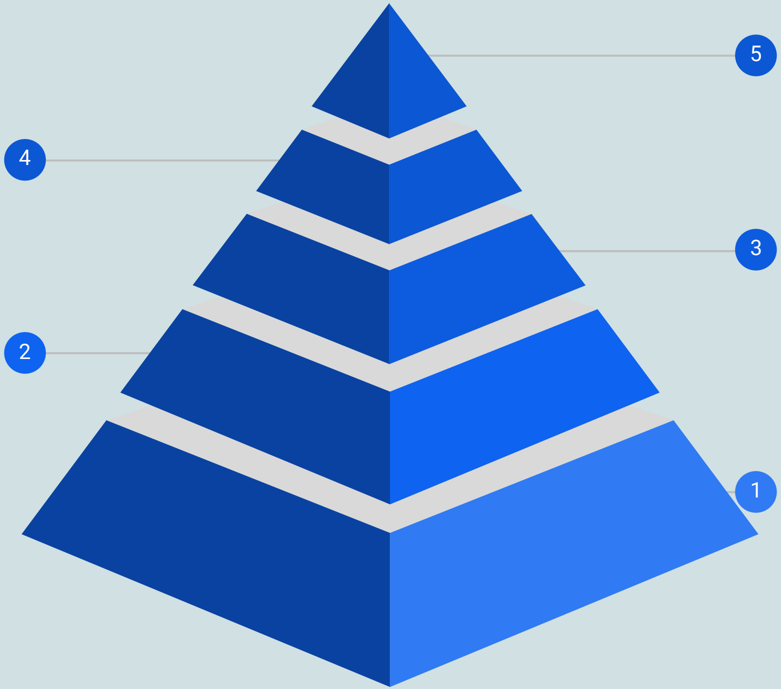

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Thank you!!

Reserve Slides...

A new opportunity…

- ❑ PGWs travel uninterrupted way.
- ❑ Very high mass scale can be probed … not be probed in LHC.
- ❑ To probe the nature of dark matter, mass range of dark matter, axions.
- ❑ To probe the scale of leptogenesis.

… A unique observational window to investigate the particle physics models.

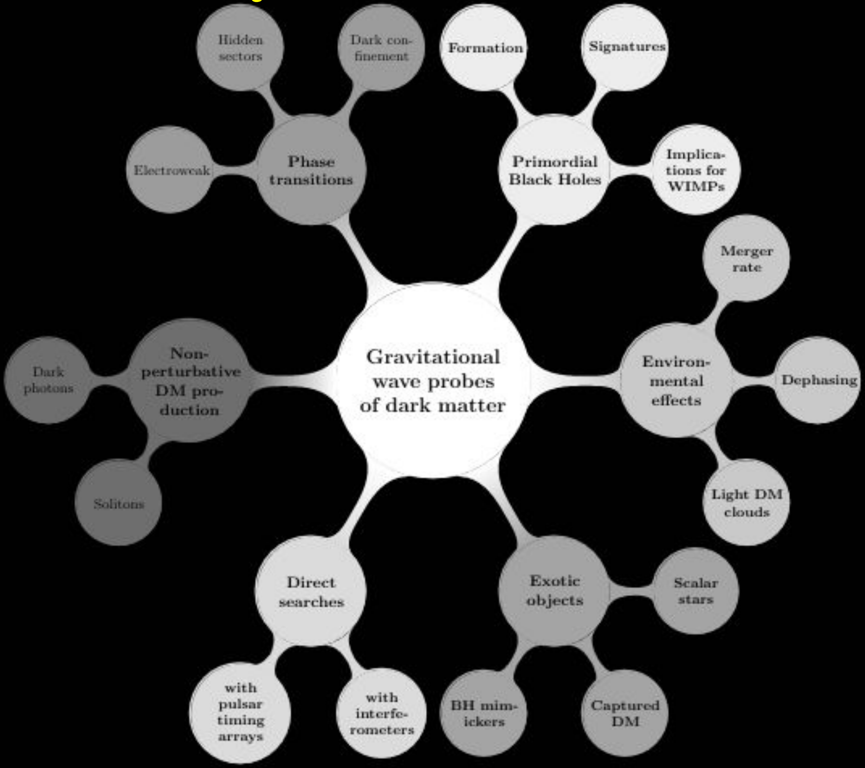


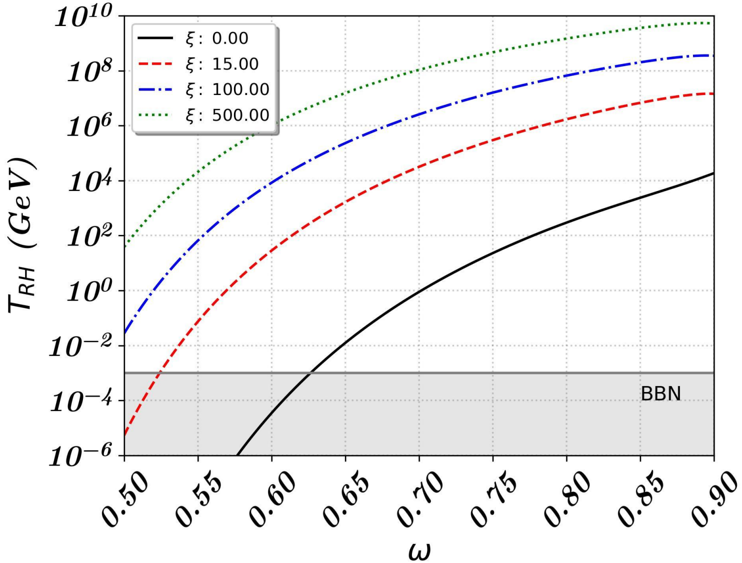
Fig. Ref.: *G. Bertone et al. SciPost Phys. Core 3, 007 (2020)*

Model parameters

Considering
non-minimal coupling → $\mathcal{L}_{non-min} = \xi |h|^2 R / M_P^2$

Reheating Temperature

$$T_{\rm RH}^4 = \frac{30}{\pi^2 g_{\rm RH}} M_P^4 \left(\frac{\rho_{\rm end}}{M_P^4}\right)^{\frac{4n-7}{n-4}} \left(\frac{\alpha_n^\xi \sqrt{3}\,(n+2)}{8n-14}\right)^{\frac{3n}{n-4}}$$



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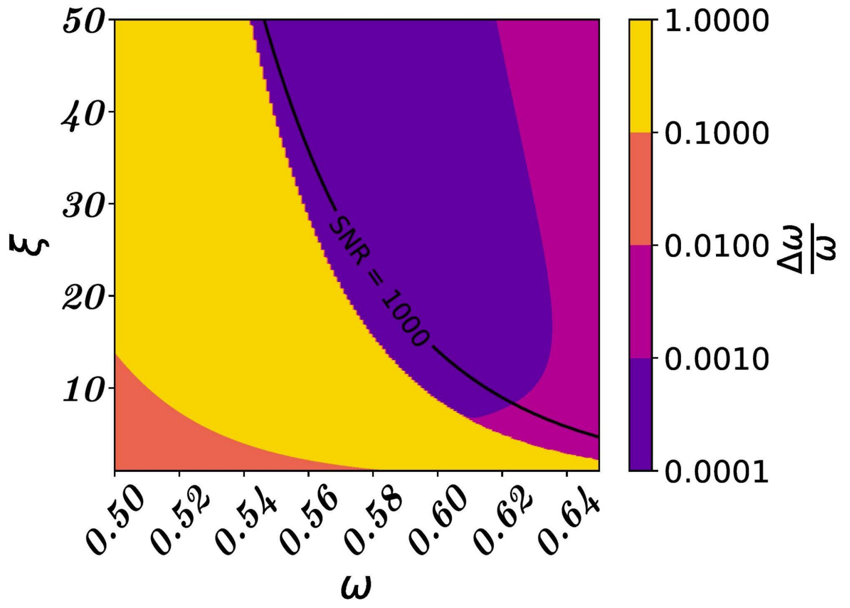
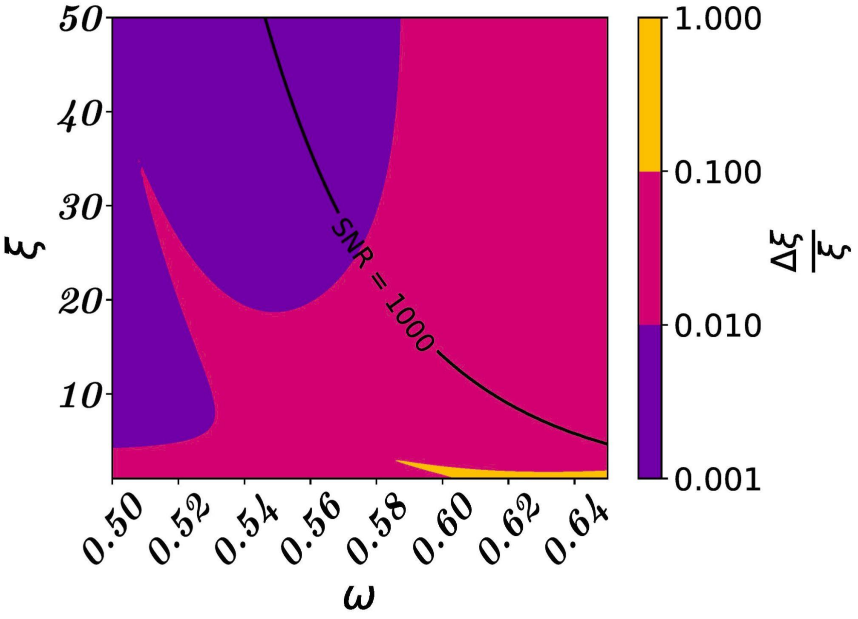
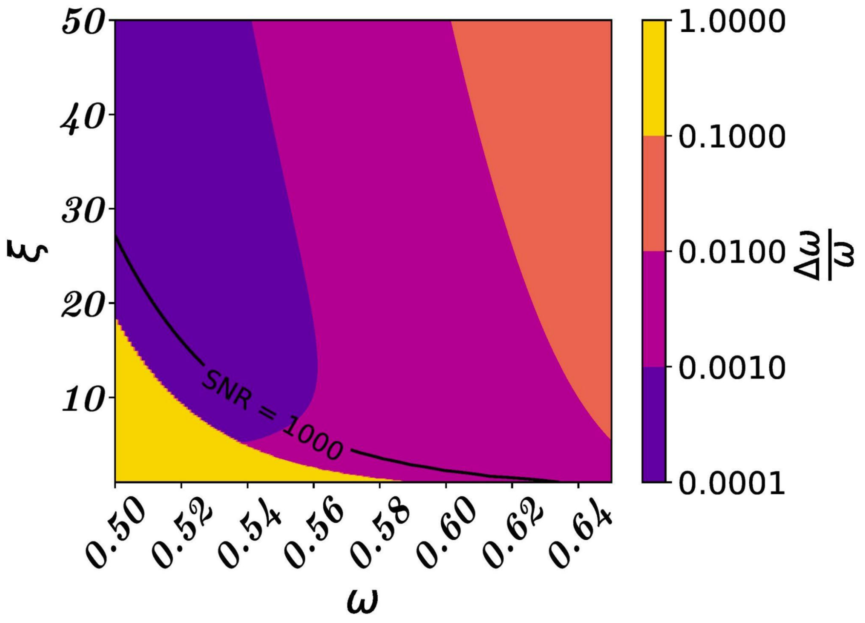
Fisher matrix analysis

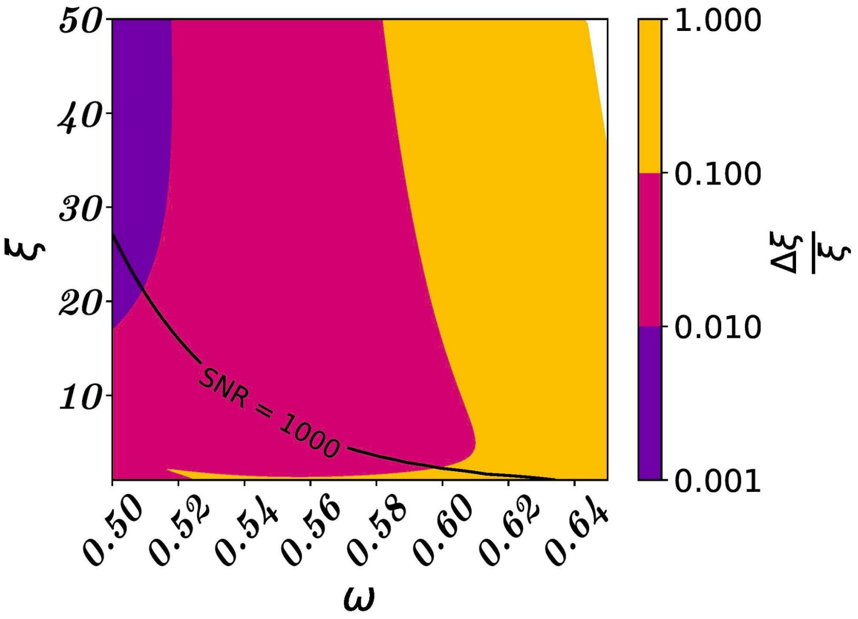




For BBO

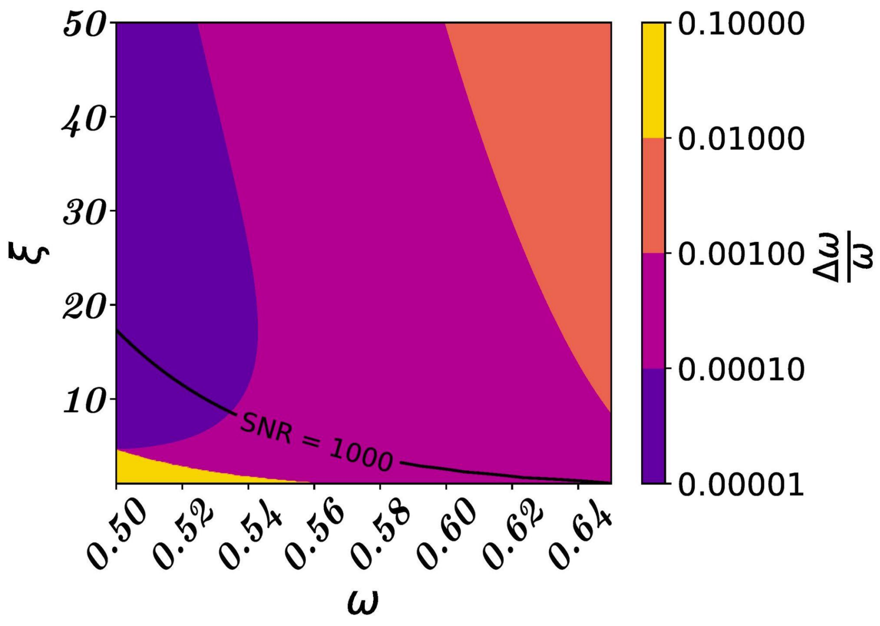
Fisher matrix analysis

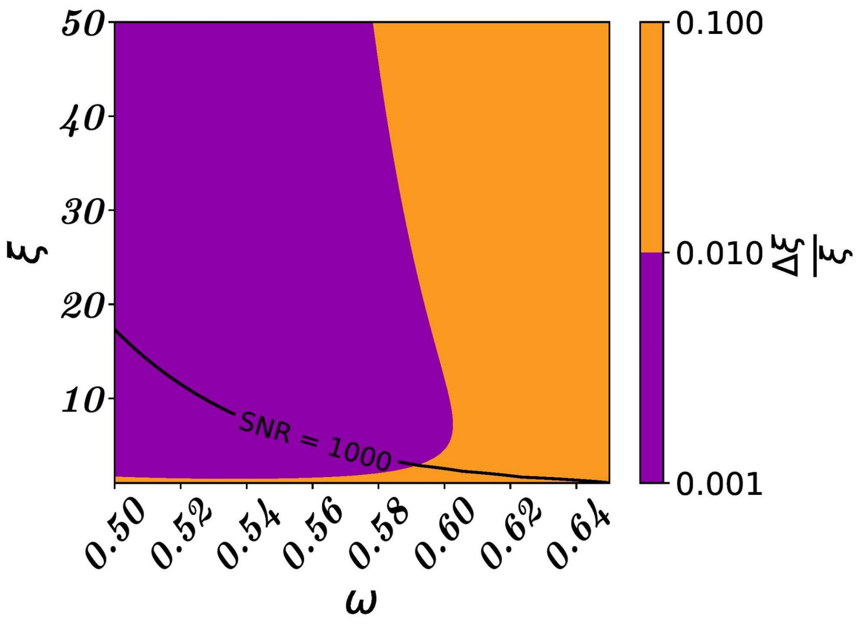




For DECIGO

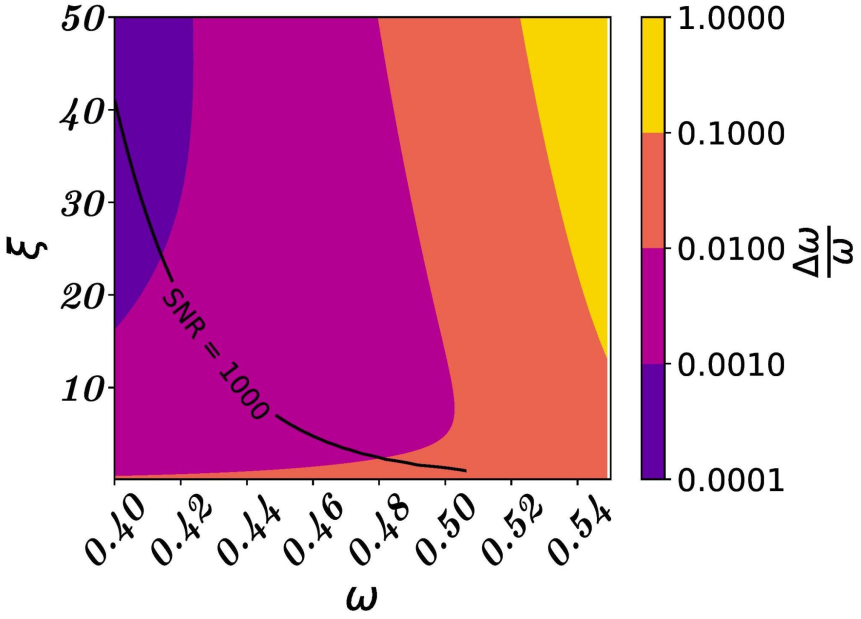
Fisher matrix analysis

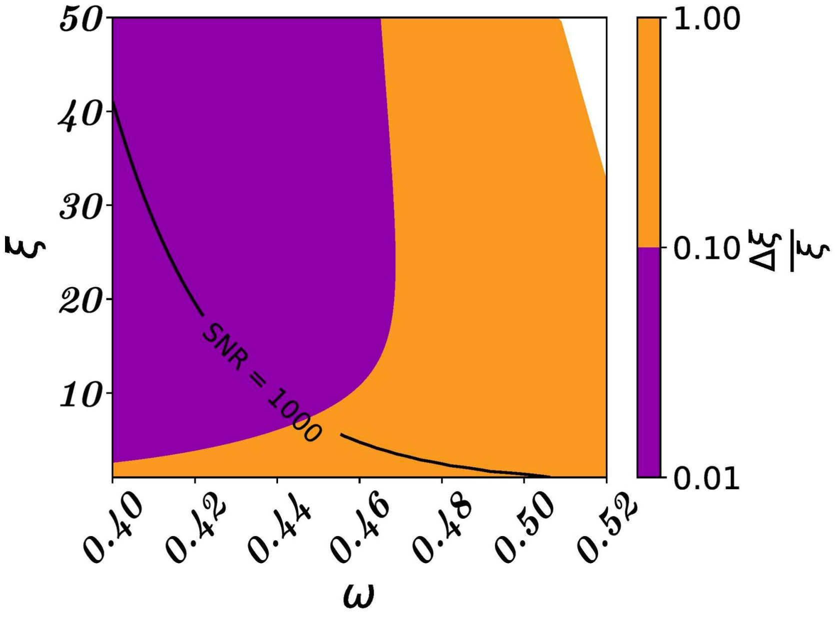




For ET

Fisher matrix analysis





For LISA

Markov chain Monte Carlo analysis

Likelihood function for GW signal is defined as

$$\mathscr{L}(\theta) = \prod_{b=1}^{N_b} \sqrt{\frac{n_b}{2\pi\Omega_{\rm n}(f_b)^2}} \exp\left(-\frac{n_b\left(\Omega_{\rm sig}(f_b,\theta) - \Omega_{fid}(f_b)\right)^2}{\Omega_{\rm n}(f_b)^2}\right)$$

Chi-squared distribution

$$\mathcal{L}(\theta) \equiv ln\left(\mathscr{L}(\theta)\right)$$

Parameters: $\{\omega, \xi, n_T\}$

Parameter	*Prior*	
	BBO	DECIGO *and* ET
ω	Flat, 0.45 $\to$ 0.70	Flat, 0.4 $\to$ 0.60
ξ	Flat, 1 $\to$ 30	Flat, 1 $\to$ 30
n_T	Flat, -0.2 $\to$ 0.1	Flat, -0.2 $\to$ 0.15

Likelihood function

- What we want to know: $\mathcal{P}(\theta | d)$
- What we know: $\mathcal{P}(d|\theta)$

$$\mathcal{P}(\theta | d) \times \mathcal{P}(d) = \mathcal{P}(d|\theta) \times \mathcal{P}(\theta)$$

Posterior distribution

Considered to be 1

Likelihood function

Prior

Likelihood function

- What we want to know: $\mathcal{P}(\theta | d)$
- What we know: $\mathcal{P}(d|\theta)$

$$\mathcal{P}(\theta | d) \text{ x } \mathcal{P}(d) = \mathcal{P}(\theta | d) \text{ x } \mathcal{P}(\theta)$$

$$\ln \mathscr{L}(\theta) = \ln \mathscr{L}(\theta_{\mathrm{ML}}) + \frac{\partial \ln \mathscr{L}(\theta)}{\partial \theta}\bigg|_{\theta_{\mathrm{ML}}} (\theta - \theta_{\mathrm{ML}}) + \frac{1}{2} \frac{\partial^2 \ln \mathscr{L}(\theta)}{\partial \theta^2}\bigg|_{\theta_{\mathrm{ML}}} (\theta - \theta_{\mathrm{ML}})^2 + \dots$$