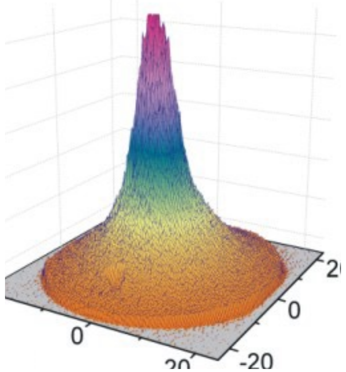
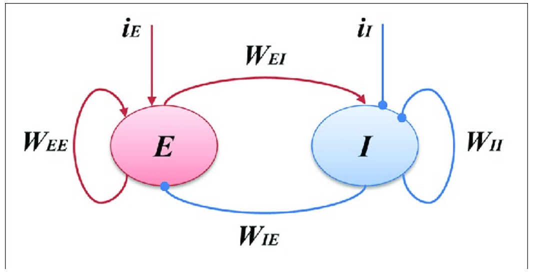
Physics without Newton's third law: phase transitions in non-reciprocal matter

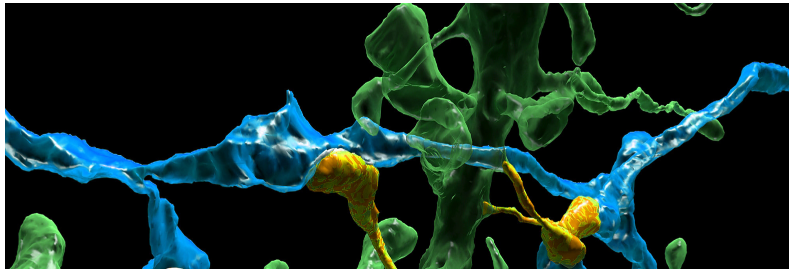
E2DQM 24 Bengaluru, July 2024

Alex Edelman , Michel Fruchart , Ryo Hanai, Kyle Kawagoe, Sergei Shmakov, Cheyne Weis, Shuoguang Liu, Xioayuan Huang, Peter Littlewood and Vincenzo Vitelli





Shmakov, PBL Physical Review E 109, 024220 (2024)
Weis et al arXiv:2207.11667
Fruchart, Hanai, PBL, VV, Nature 592, 363-369 (2021)
Hanai, PBL, Physical Review Research 2, 033018 (2020)
Hanai, Edelman, Ohashi, PBL, Phys. Rev. Lett. 122, 185301 (2019)



U.S. DEPARTMENT OF ENERGY Office of Science
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A Bose condensate viewed as a collection of synchronized oscillators


EST. 1971
Taktschläge
Largo
Larghetto
Adagio
Andante
Moderato
Allegro
Presto
Prestissimo

Christiaan Huygens 1629-95

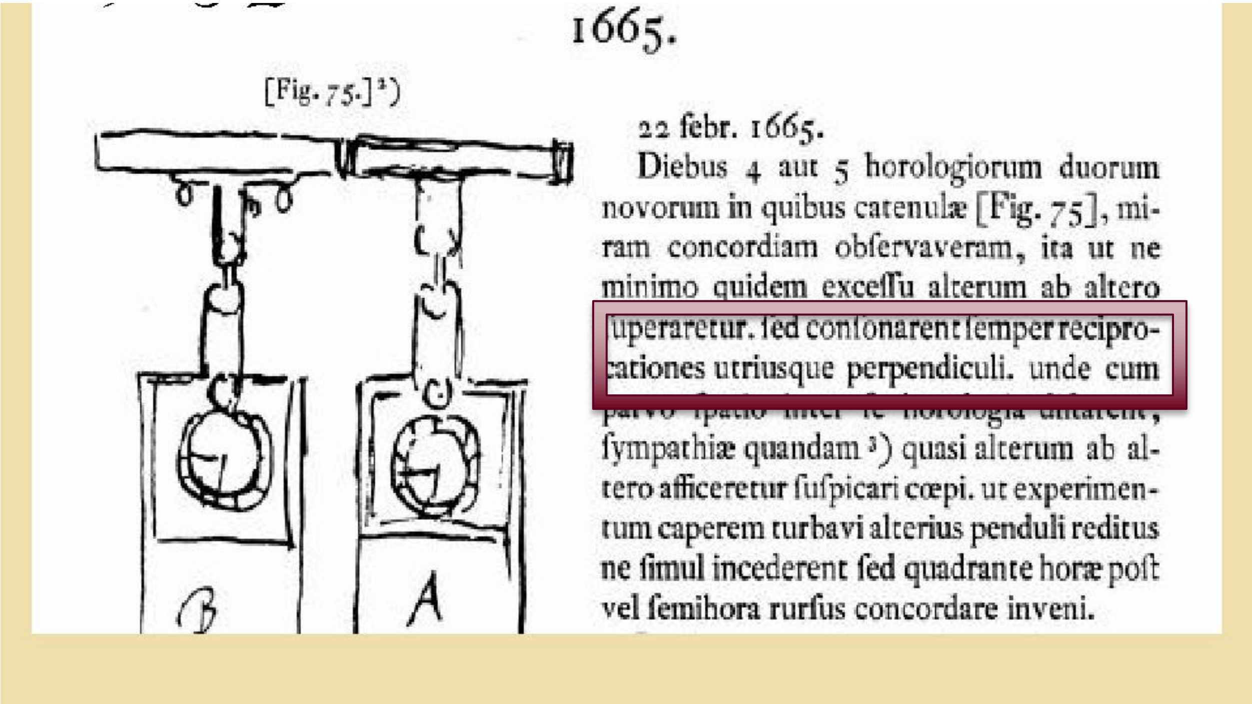
1656 – Patented the pendulum clock

1663 – Elected to Royal Society

1662-5 With Alexander Bruce, and sponsored by the Royal Society, constructed maritime pendulum clocks – periodically communicating by letter

Huygens Clocks

In early 1665, Huygens discovered ``..an odd kind of sympathy perceived by him in these watches [two pendulum clocks] suspended by the side of each other."



1665.

[Fig. 75.][2])

22 febr. 1665.

Diebus 4 aut 5 horologiorum duorum novorum in quibus catenulæ [Fig. 75], miram concordiam obſervaveram, ita ut ne minimo quidem exceſſu alterum ab altero ſuperaretur. ſed conſonarent ſemper reciprocationes utriusque perpendiculi. unde cum parvo ſpatio inter ſe horologia diſtarent, ſympathiæ quandam [3]) quasi alterum ab altero afficeretur ſuſpicari cœpi. ut experimentum caperem turbavi alterius penduli reditus ne ſimul incederent ſed quadrante horæ poſt vel ſemihora rurſus concordare inveni.

Out of phase!

He deduced that effect came from “imperceptible movements” of the common frame supporting the clocks

"Huygens' Clocks", M. Bennett et al, Proc. Roy. Soc. A **458**, 563 (2002)

Huygens clocks: the first experimental nonlinear dynamical system ?

Oscillators are non-linear (amplitude of oscillation not small)

Non-equilibrium – steady state established between balance of driving and dissipation

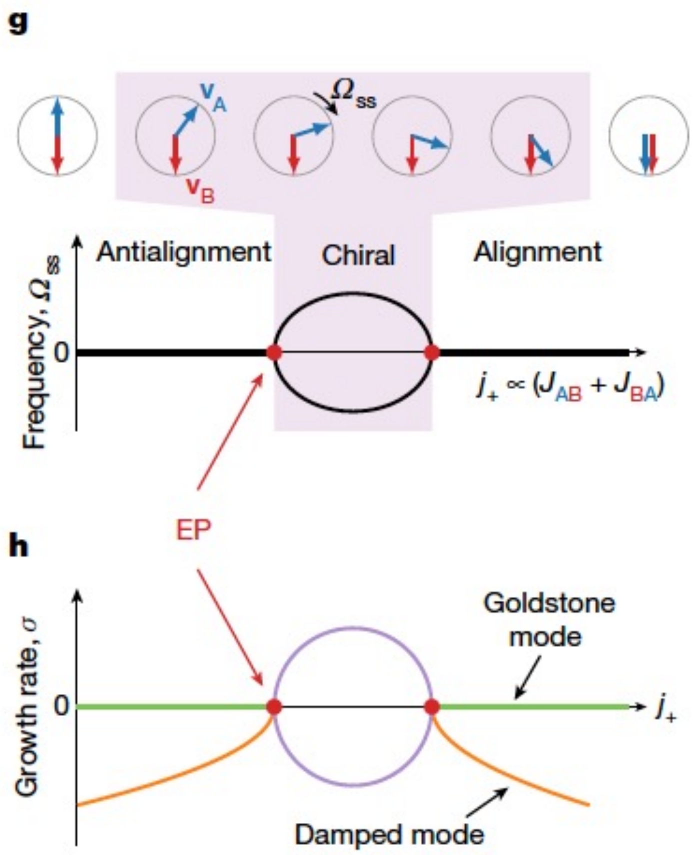
There exist periodic steady states that are stable to small external perturbation ("noise")

By tuning parameters in the model (e.g. elastic vs. inertial coupling) one can make abrupt transitions from one state to another (e.g. in-phase to out-of-phase)

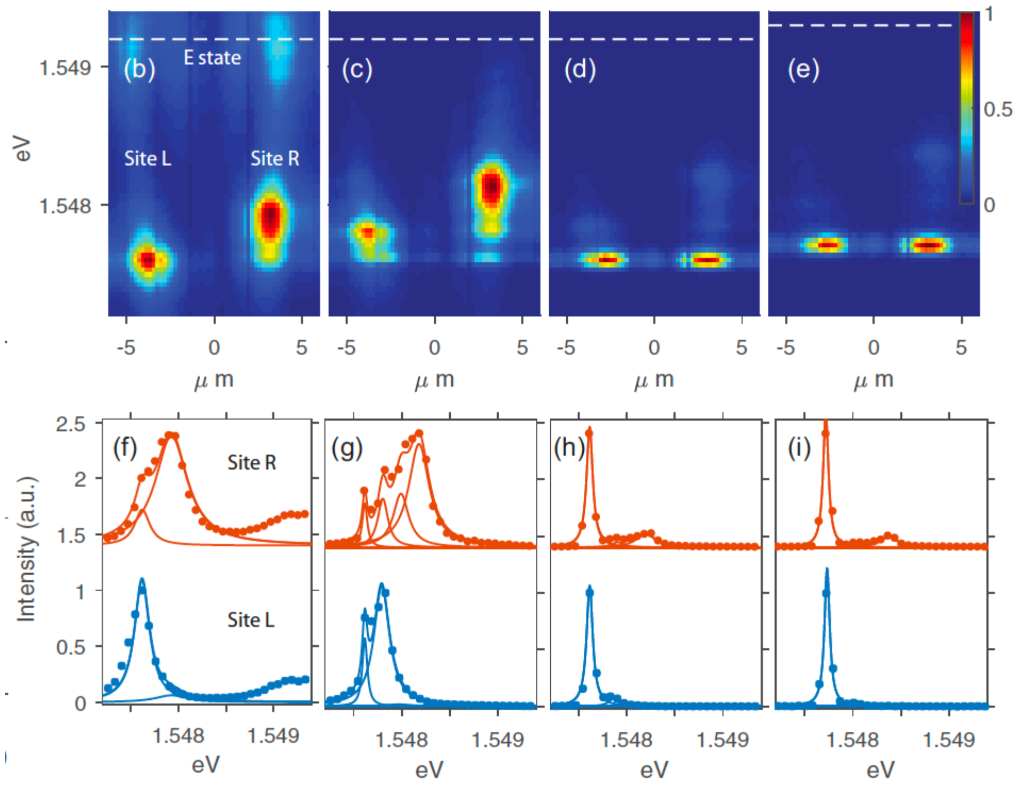
How does this transition happen?

- Directly? viz. FM -> AF
- Via disordered state? viz. FM -> PM -> AFM
- Or something else?

Generically the transition from synchronized to antisynchronized occurs via an intermediate chiral state at an exceptional point.



Fruchart et al. Nature 592, 363-369 (2021)



Kim et al PRB 101.085302 (2020)

Three topics of this talk

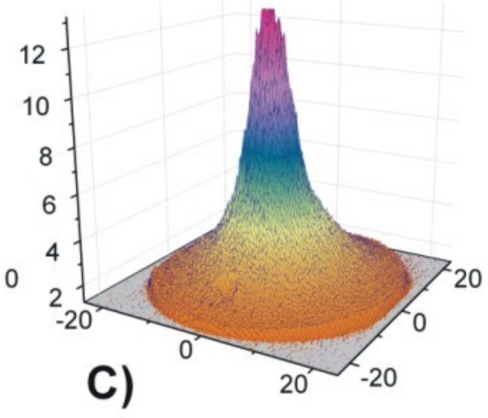
- Are there ‘phase transitions’ between different kinds of synchronized states and how does one describe those transitions?
- Is there a synchronization equivalent of a critical point in a many-body system, and does it come decorated with ‘universal’ fluctuation dynamics?
- Is there a connection between synchronized quantum systems (e.g. a laser) and synchronization in classical models (e.g. flocks, Kuramoto models) ?

Polaritons

$$|\text{pol}\rangle = c_1|\text{exc}\rangle + c_2|\text{ph}\rangle$$

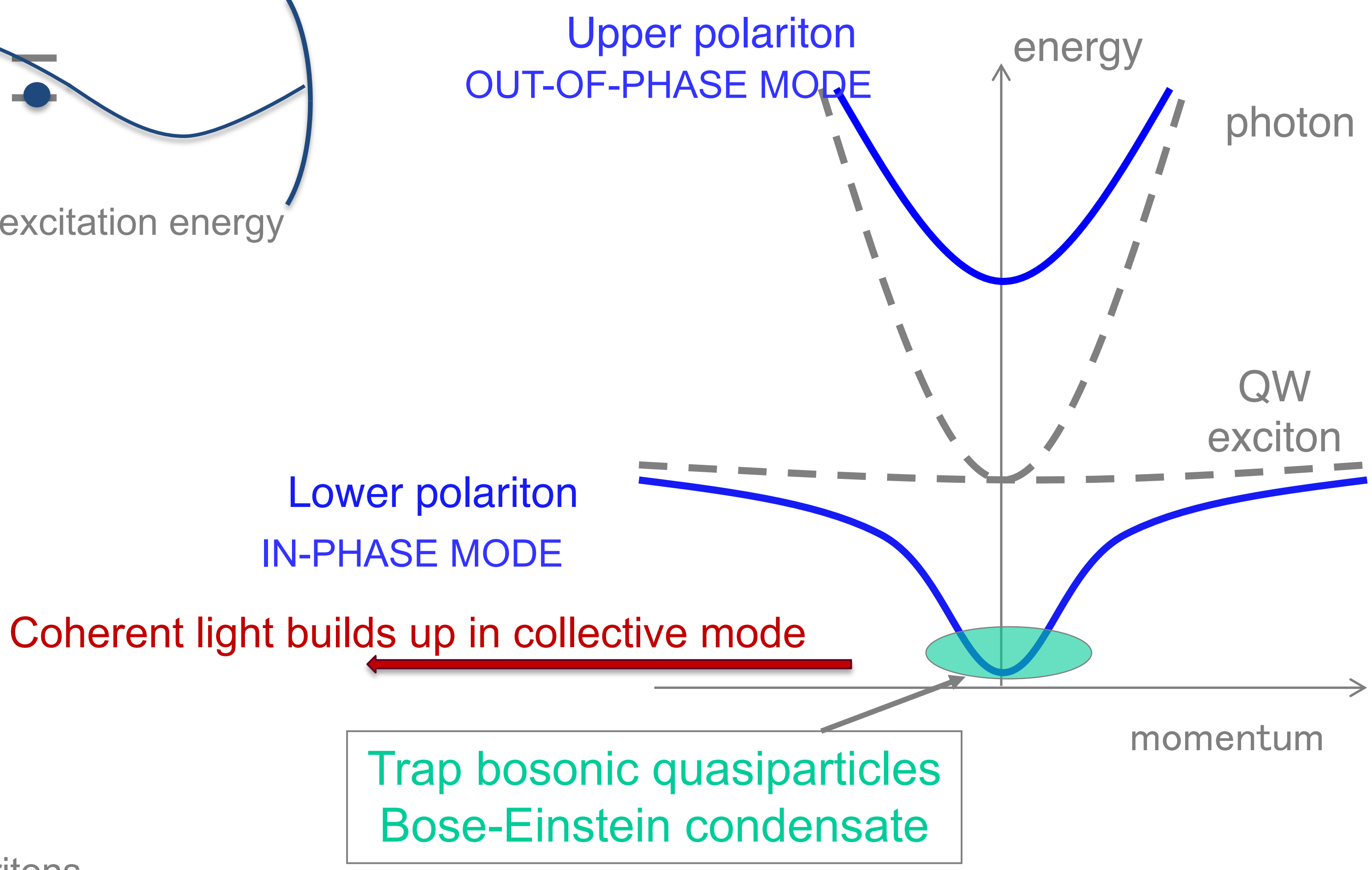
Atoms in a (large) optical cavity

Tune the lowest (q=0) mode to the excitation energy

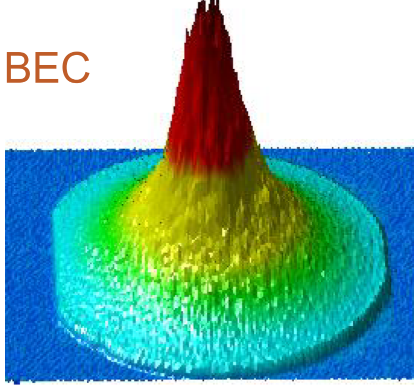


Momentum distribution of polaritons

Kasprzak et al Nature 2006

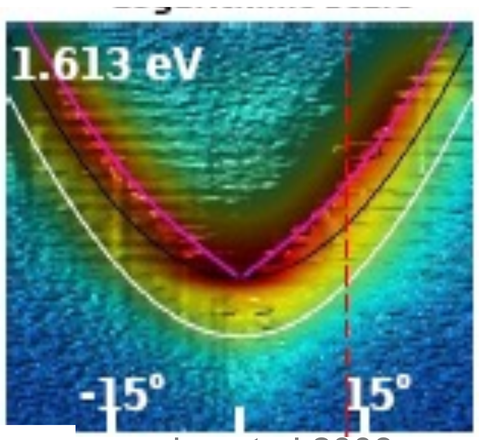


BEC



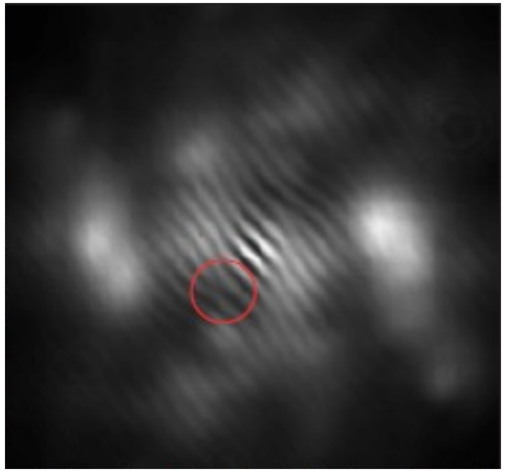
Kasprzak et al 2006

Bogoliubov spectrum



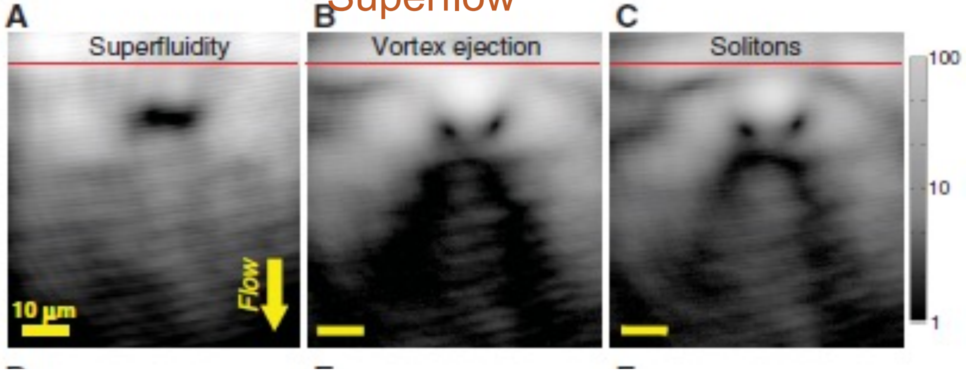
unomiya et al,2008

Vortices



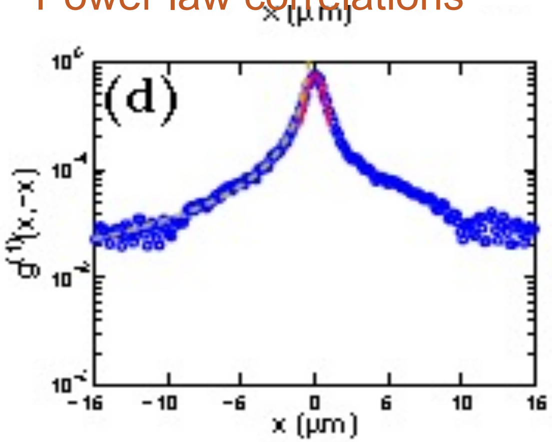
Lagoudakis et al 2008

Superflow



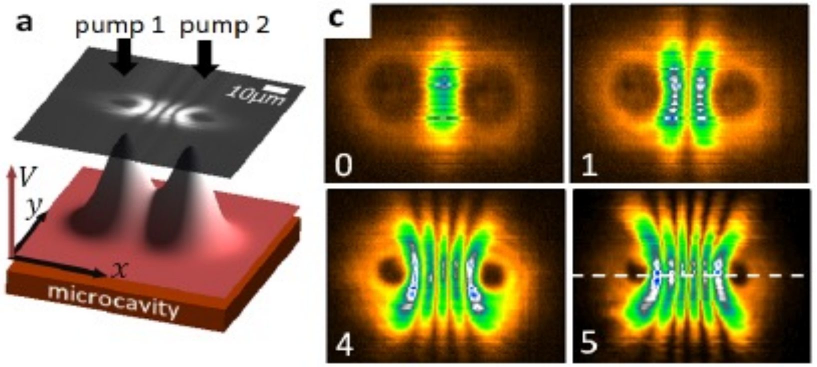
Amo et al, 2011

Power law correlations



Roumpos et al 2012

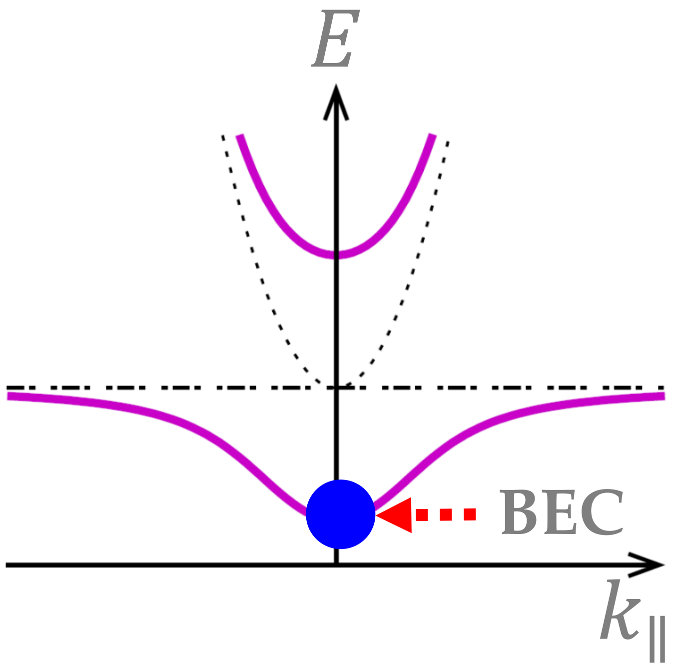
Coupled condensates



Tosi et al 2012

Two component order parameter: mixed light and matter

Non-equilibrium condensate is established as a steady-state solution with coherent oscillation (a.k.a. laser) – balancing excitation and decay

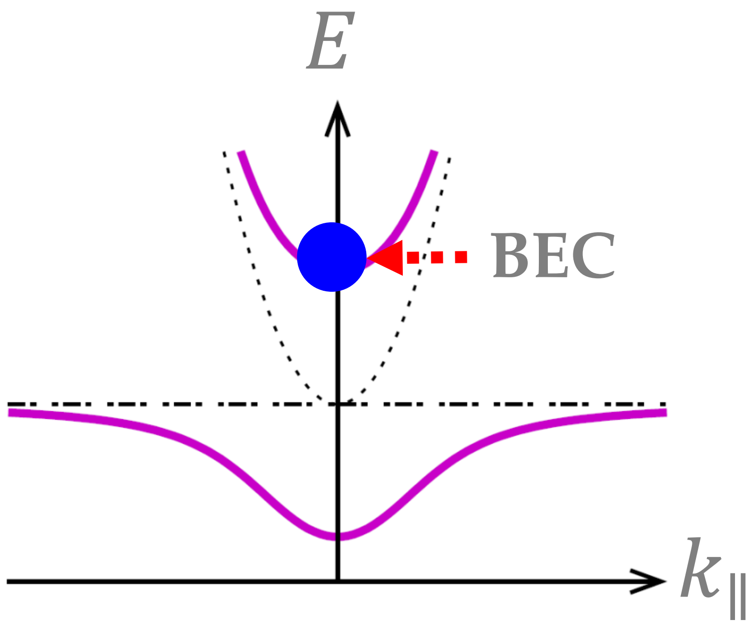


Conventional BEC in “in-phase” mode
Solution of non-linear 2x2 non-linear Schrodinger equation with eigen-frequency at (renormalized) lower polariton frequency

Hanai, Edelman, Ohashi, PBL, Phys. Rev. Lett. 122, 185301 (2019)

Two component order parameter: mixed light and matter

Non-equilibrium condensate is established as a steady-state solution with coherent oscillation (a.k.a. laser) – balancing excitation and decay



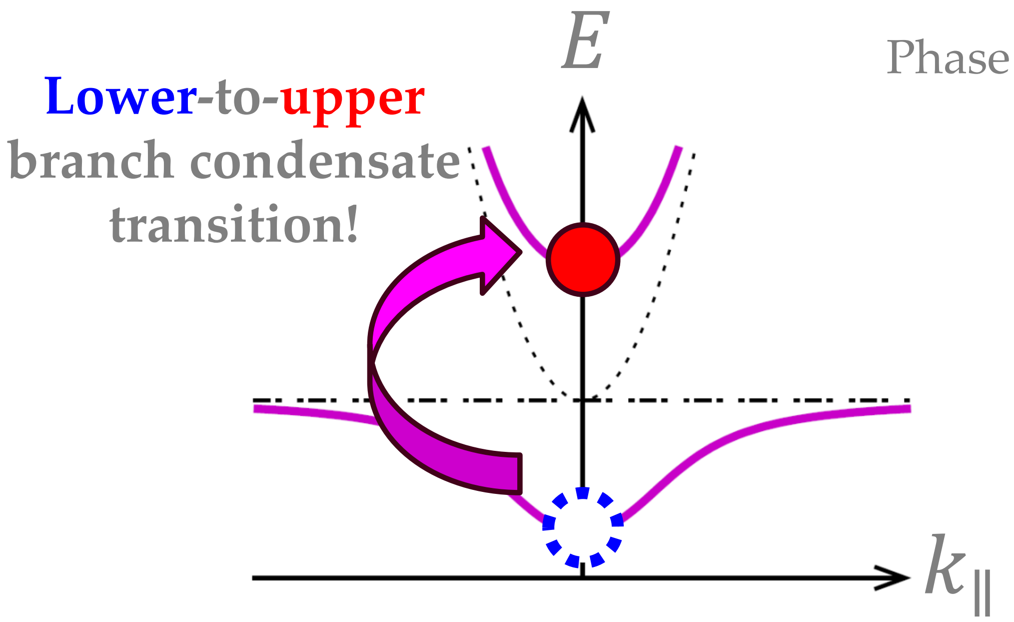
Non-equilibrium BEC in "out-of-phase" mode
Solution of non-linear 2x2 non-linear Schrodinger equation with eigen-frequency at (renormalized) upper polariton frequency

Hanai, Edelman, Ohashi, PBL, Phys. Rev. Lett. 122, 185301 (2019)

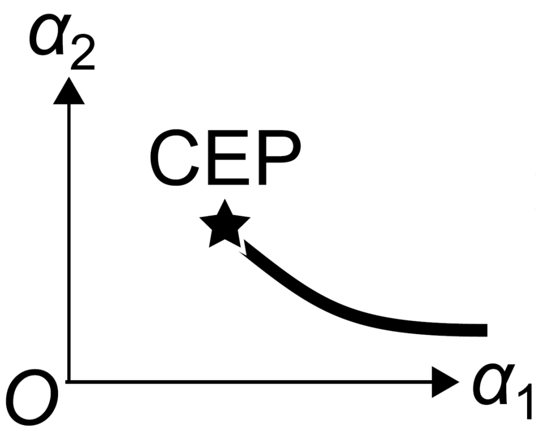
No symmetry distinction between upper and lower condensates

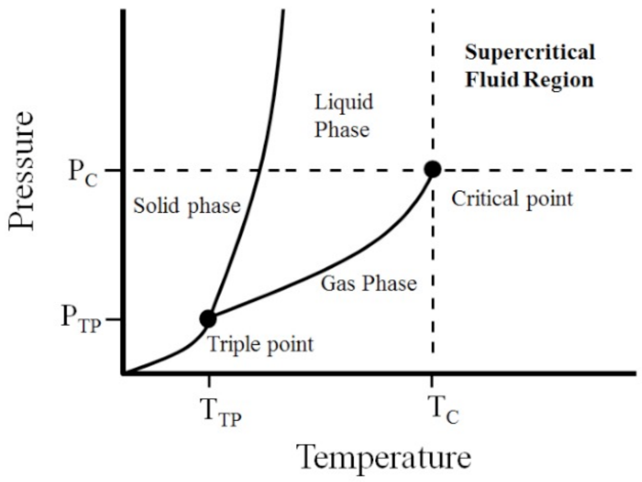
The transition can jump (first order-like) but can also coalescence if the parameters are such that the modes become degenerate: an exceptional point

Can only occur if the Hamiltonian is non-Hermitian



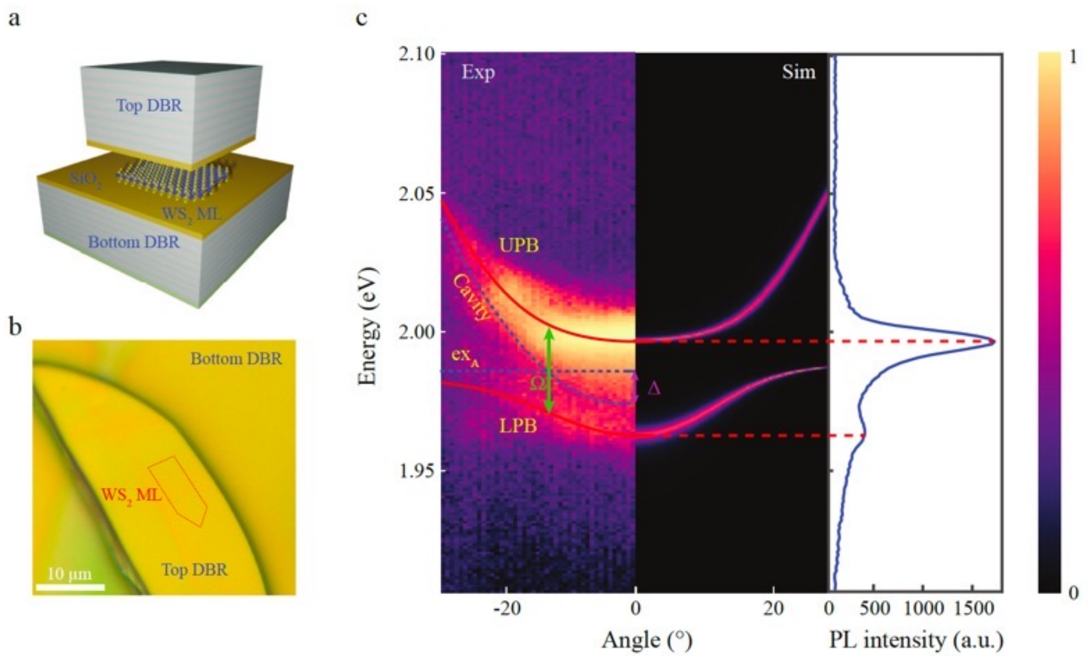
Phase diagram with two tuning parameters: e.g. pumping and loss



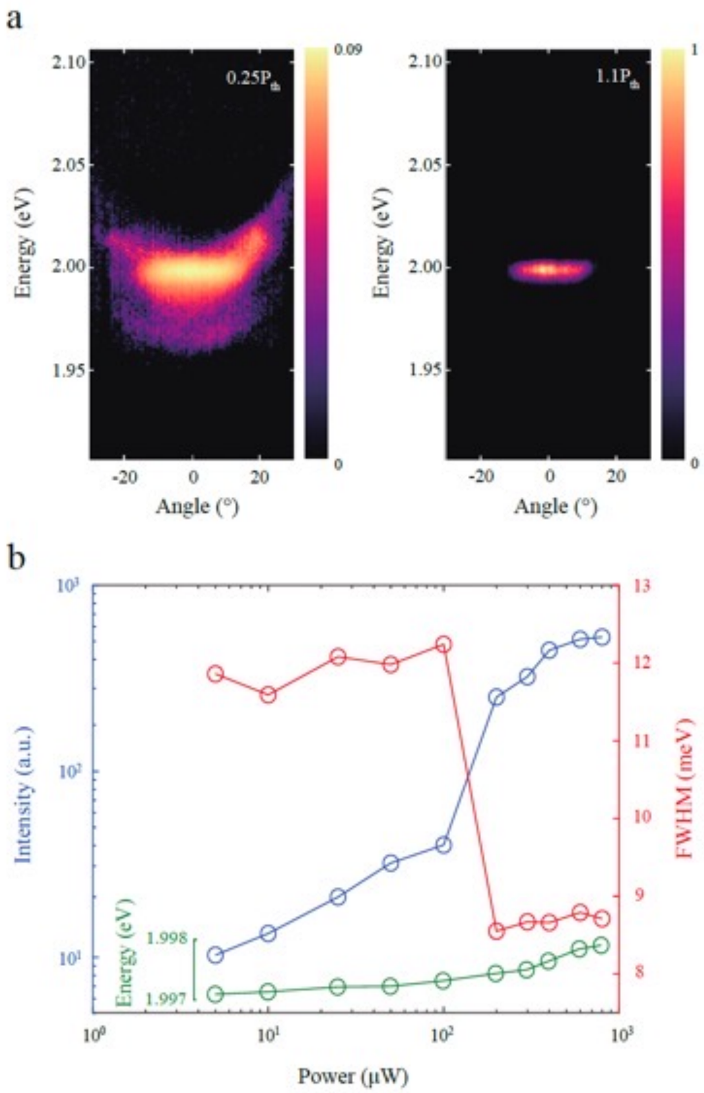


At the "critical exceptional point" CEP, "in-phase" and "out-of-phase" solutions are degenerate
Analogous to conventional critical point (e.g. liquid-gas) but there are no massive modes – very different critical dynamics

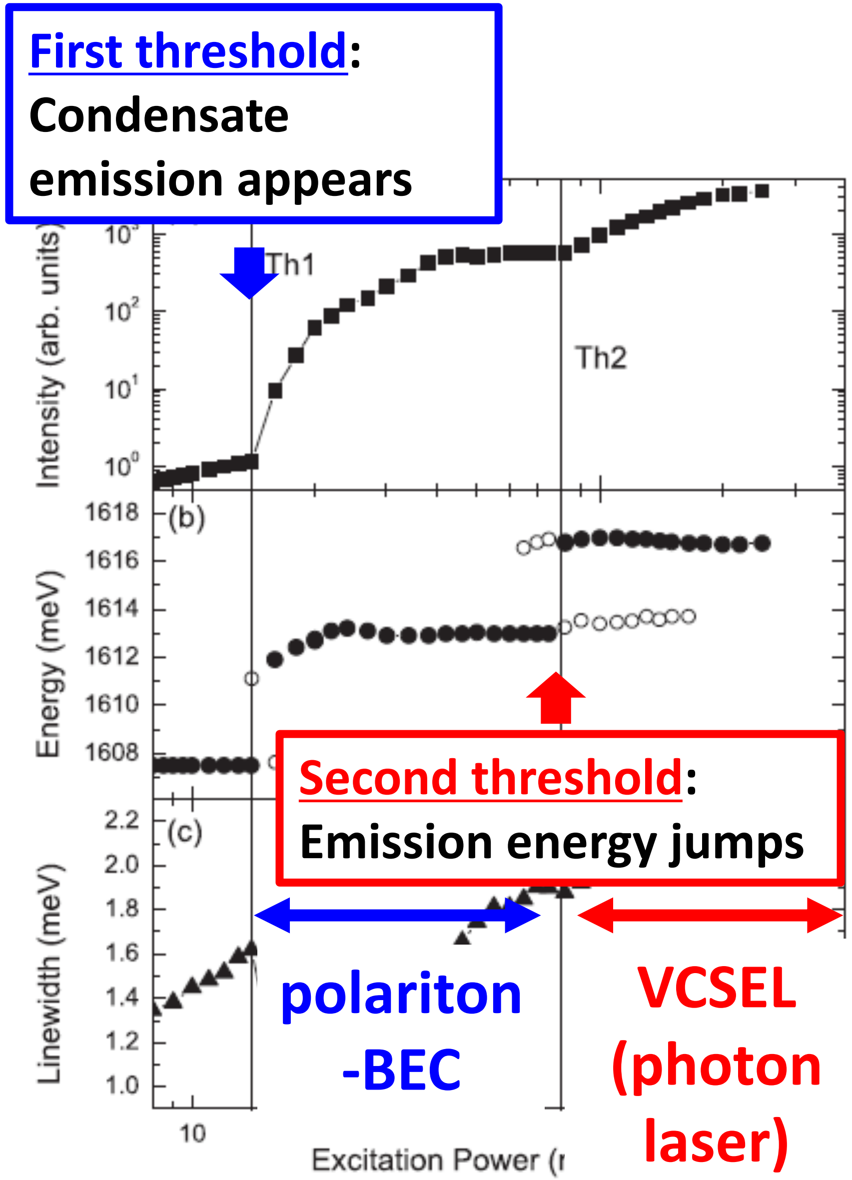
Upper polariton condensate in WS_2 monolayer cavity



Chen et al. Nano Letters 23 (20), 9538-9546



Two-threshold behavior



J. Tempel et.al, PRB **85**, 075318 (2012).

“ **Second threshold** = **strong-to-weak coupling transition** ”

Normal = $U(1)$-*symmetric*

Polariton-BEC = $U(1)$-*broken*

Photon laser = $U(1)$-*broken*

… no new broken symmetries!

Why a transition?

Minimal model* - two-component damped driven Gross-Pitaevskii equation with noise

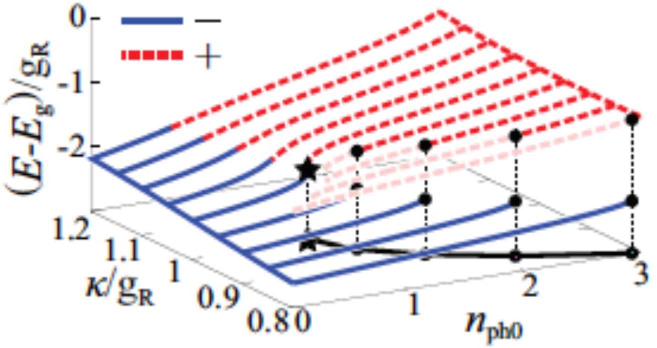
$$i\partial_t \Psi_\alpha(\boldsymbol{r},t) = \sum_{\beta=1,\mathrm{g}} [A_{\mathrm{GP}}]_{\alpha\beta}(\nabla^2)\Psi_\beta(\boldsymbol{r},t) + \eta_\alpha(\boldsymbol{r},t),$$

Photon field with dissipation

$$A_{\mathrm{GP}}(\nabla^2) = \begin{pmatrix} \omega_{\mathrm{l}} - i\kappa - K_{\mathrm{l}}\nabla^2 & g \\ g & \omega_{\mathrm{g}} + iP - \tilde{K}_{\mathrm{g}}\nabla^2 + \tilde{U}_{\mathrm{g}}|\Psi_{\mathrm{g}}|^2 \end{pmatrix},$$

Exciton field with pumping and nonlinearity

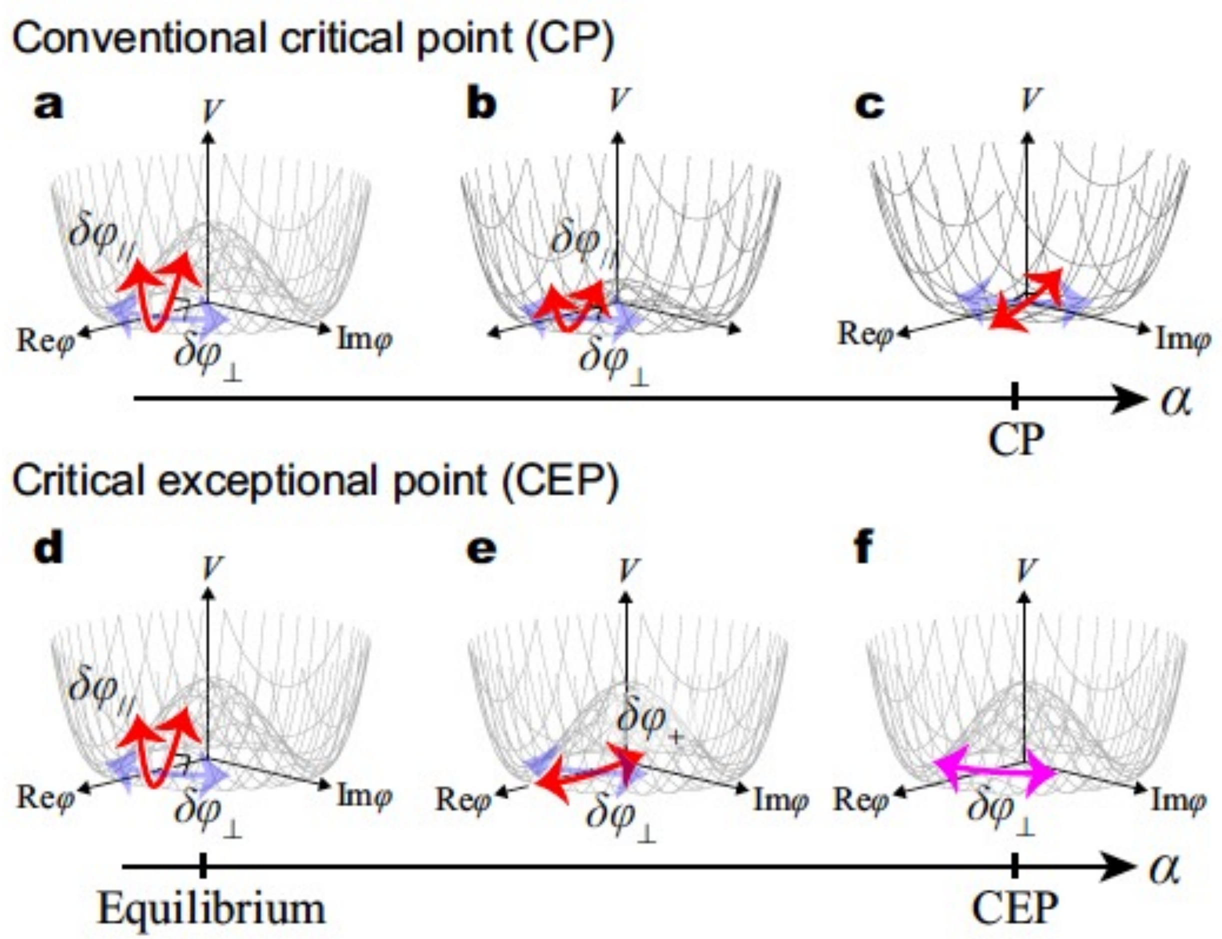
Nontrivial stationary solutions have to be oscillating at a fixed frequency: either E_- or E_+
There is a single point in parameter space where both solutions exist – a so-called exceptional point



Non-Hermitian matrices do not generally have real eigenvalues;
in our case the balance of dissipation and decay self-tunes to a stationary state

* M. Wouters and I. Carusotto, PRL 99, 140402 (2007).

Near a critical exceptional point, the two modes coalesce: all fluctuations project onto a massless mode



Conventionally at a critical point a massive mode becomes gapless
But is always orthogonal to transverse (Goldstone) mode

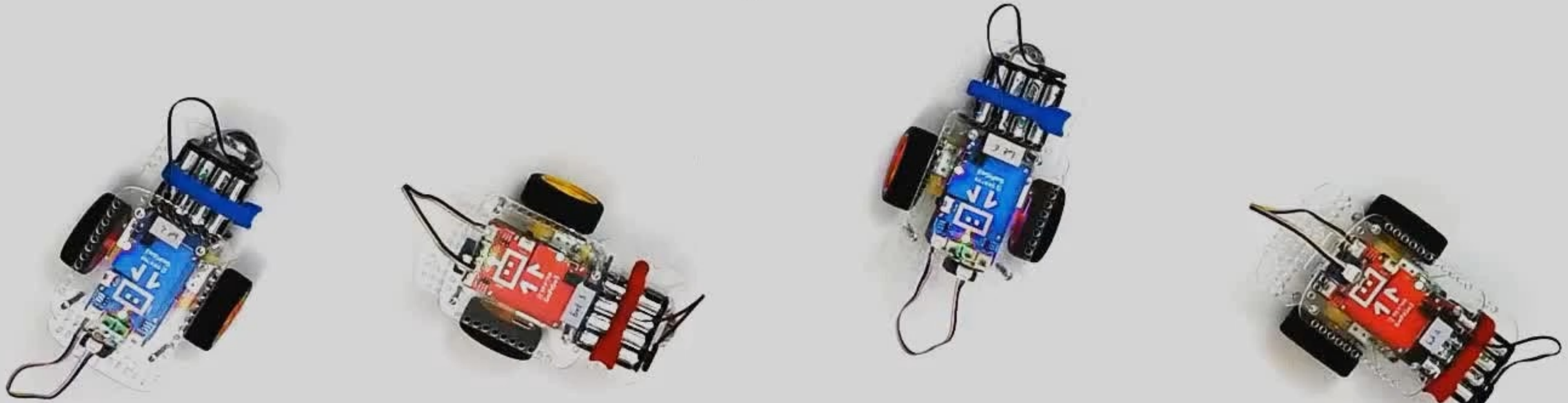
At a critical exceptional point the two modes coalesce and are indistinguishable
Much stronger fluctuations: no long-range order below 4 dimensions (?)
Upper critical dimension 8 dimensions (?)

Hanai & PBL , Physical Review Research 2, 033018 (2020)

Toward a general theory

Non-reciprocal phase transitions

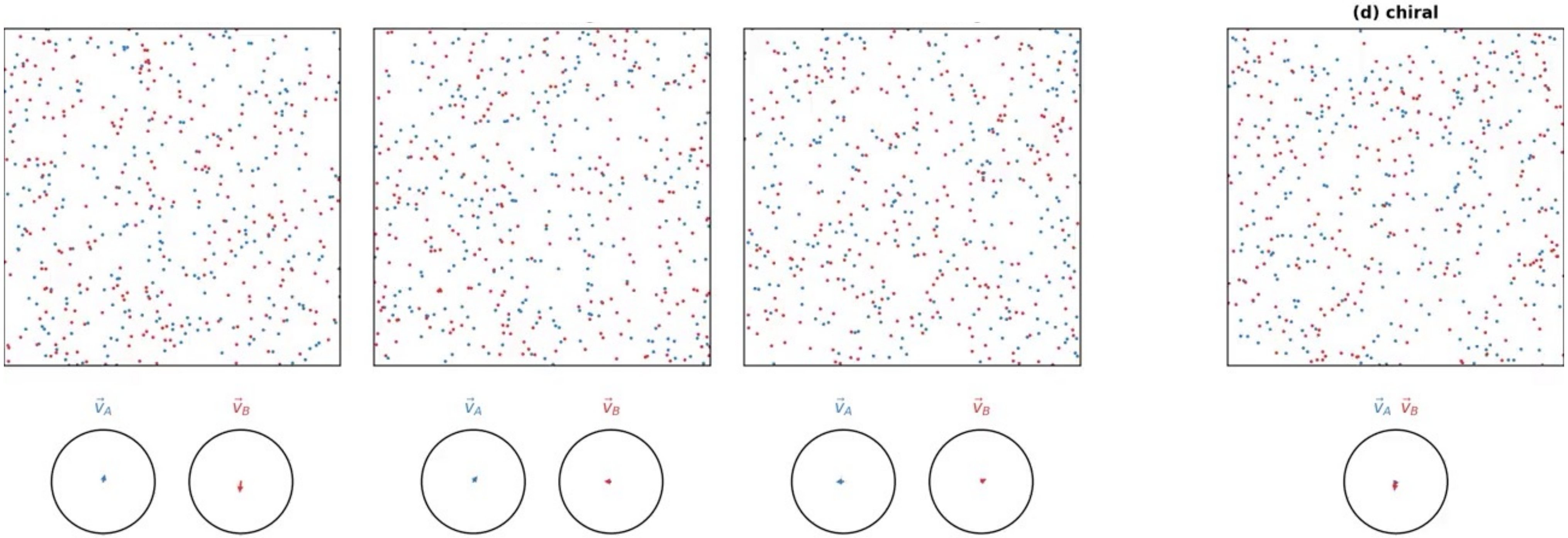
Michel Fruchart *, Ryo Hanai *,
Peter B. Littlewood, Vincenzo Vitelli



Nature (2021), arXiv:2003.13176

* These authors contributed equally to this work

Non-reciprocal Phase Transition : hawks and doves



High noise

Disordered phase

Low noise

Aligned phase

Low noise

Anti-aligned phase

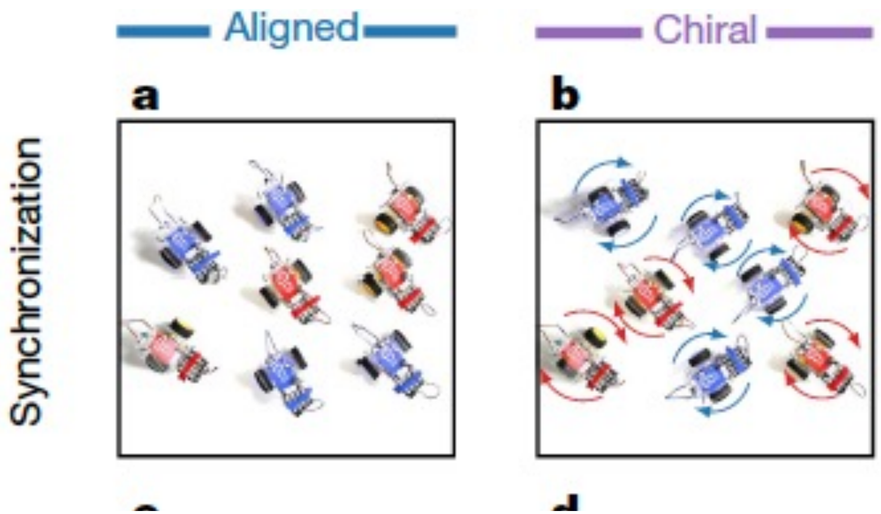
Reciprocity dominated

Low noise

Chiral phase

Non-Reciprocity induced!

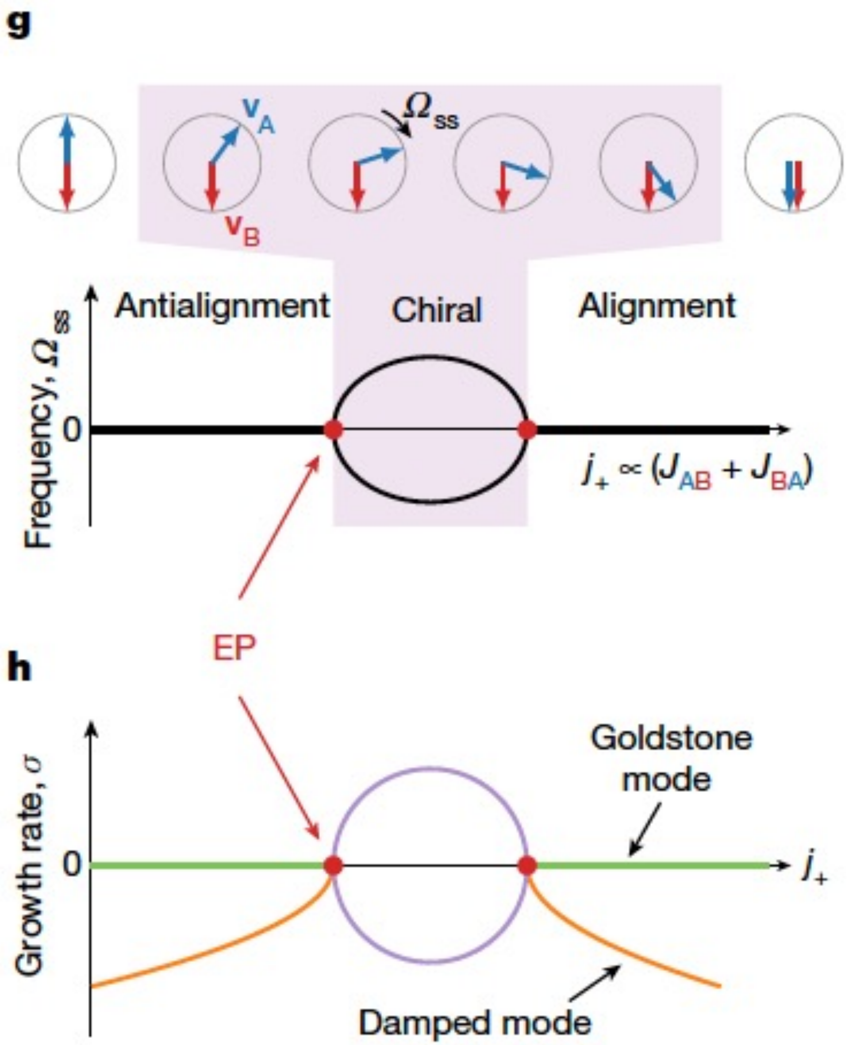
Transition from aligned/chiral/antialigned occurs when linear response in ordered phase reaches an exceptional point



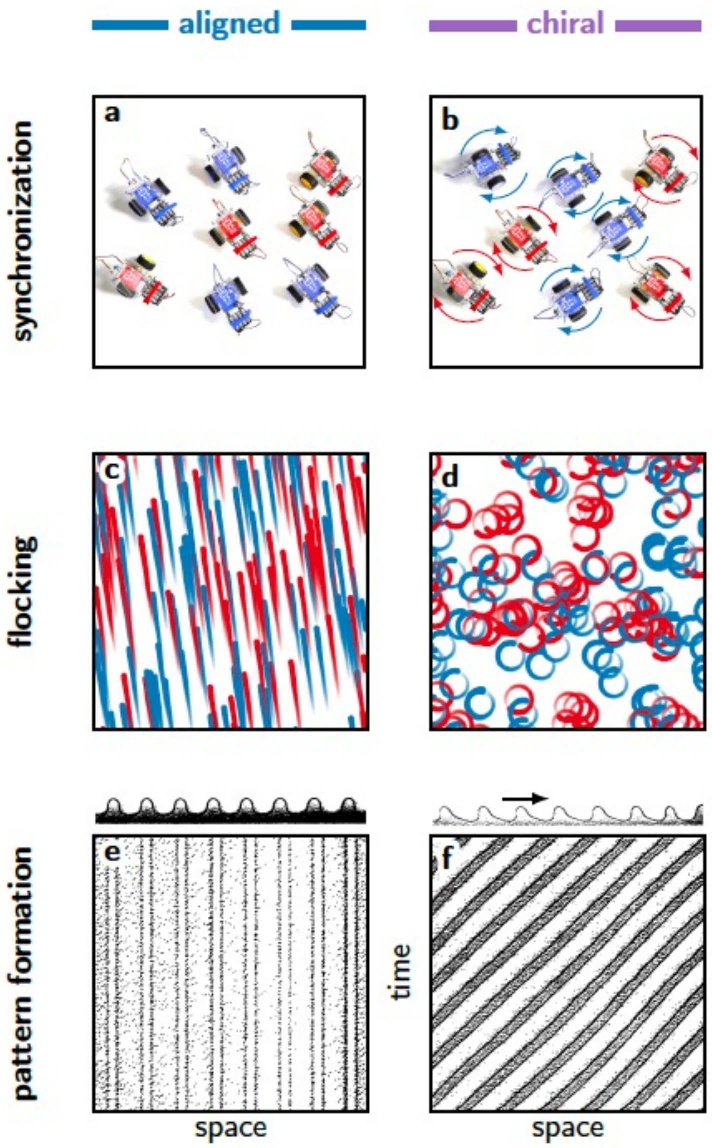
A Hamiltonian system is hermitian/symmetric
Eigenvalues are real, and eigenvectors are orthogonal

A non-Hermitian/non-symmetric system:
Eigenvalues can be complex, and eigenvectors are not orthogonal

At an exceptional point, two eigenvalues are degenerate, and corresponding eigenvectors merge. System is non-invertible



Models with continuous symmetry : U(1), O(2),

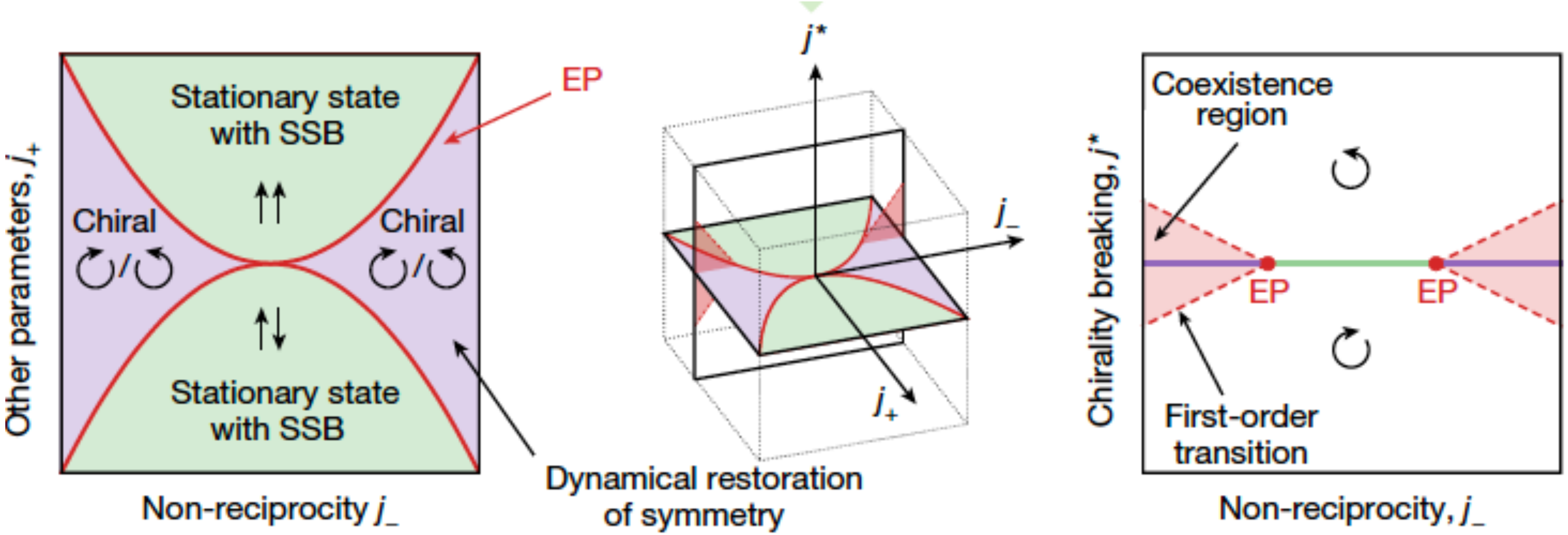


contrarian Kuramoto models

multi-component flocking

Swift-Hohenberg

General phase diagram of 2 - component non-reciprocal condensates



"PT" symmetric model
Spontaneous chiral symmetry breaking
EP are lines (co-dimension 1)

Flocking model (Vicsek)
Interfacial travelling wave
Wilson-Cowan model of neurons

"PT" non-symmetric model
Chiral symmetry explicitly broken
EP's are points (co-dimension 2)

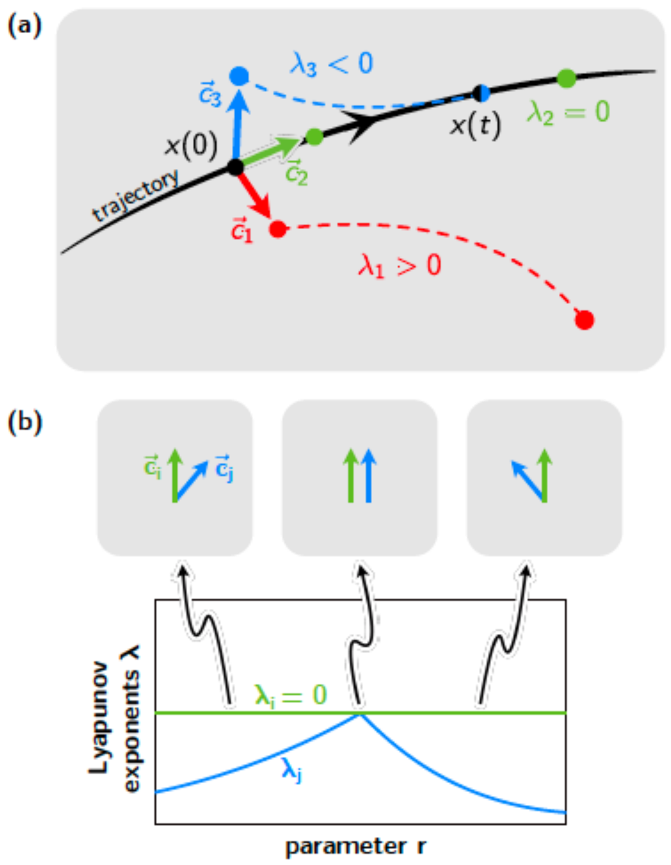
Polariton condensate
Kuramoto model

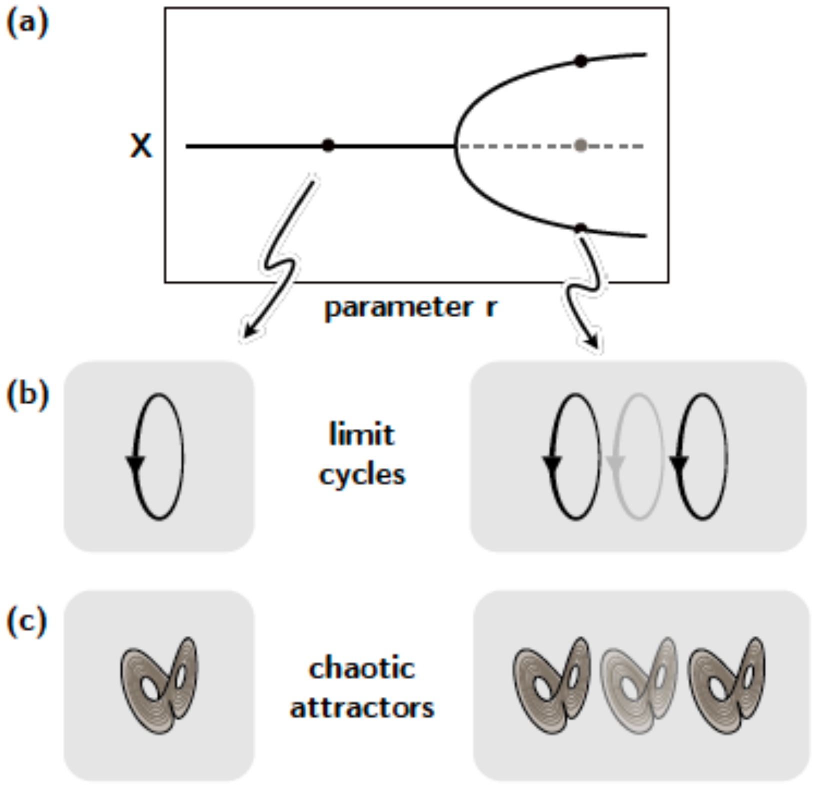
Exceptional points can describe merging of limit cycles

$$\delta\dot{X} = J\delta X.$$

perturbations along the eigenvector c_i of J with eigenvalue $\lambda_i + i\omega$ grows or decays as $e^{\lambda_i t}$.

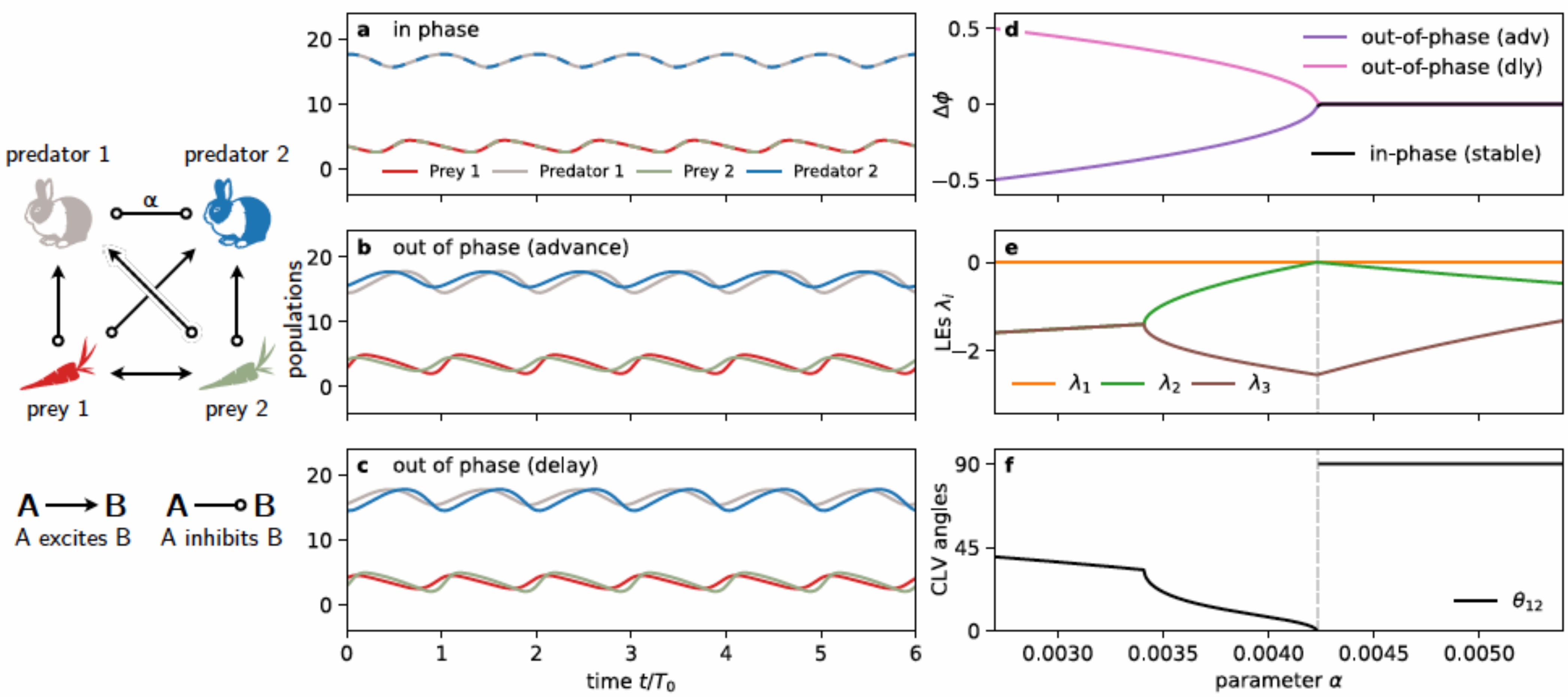
Merger of Lyapunov eigenvectors





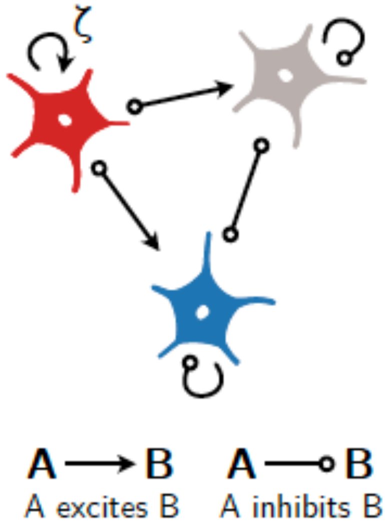
C Weis, et al., arXiv:2207.11667, 2022

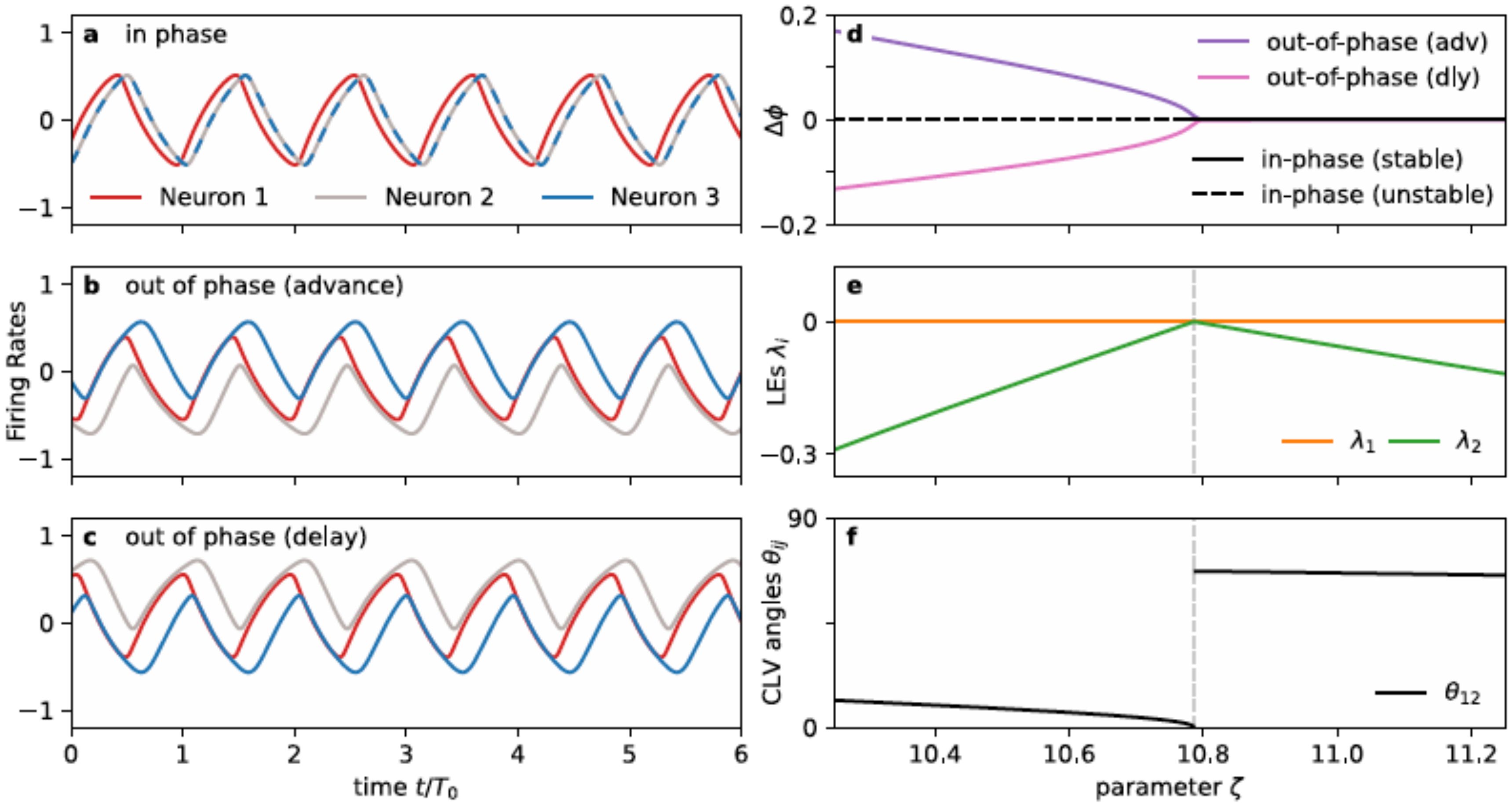
Rosenszweig-McArthur predator-prey model



C Weis, et al., arXiv:2207.11667, 2022

Wilson-Cowan model of excitatory/inhibitory neurons

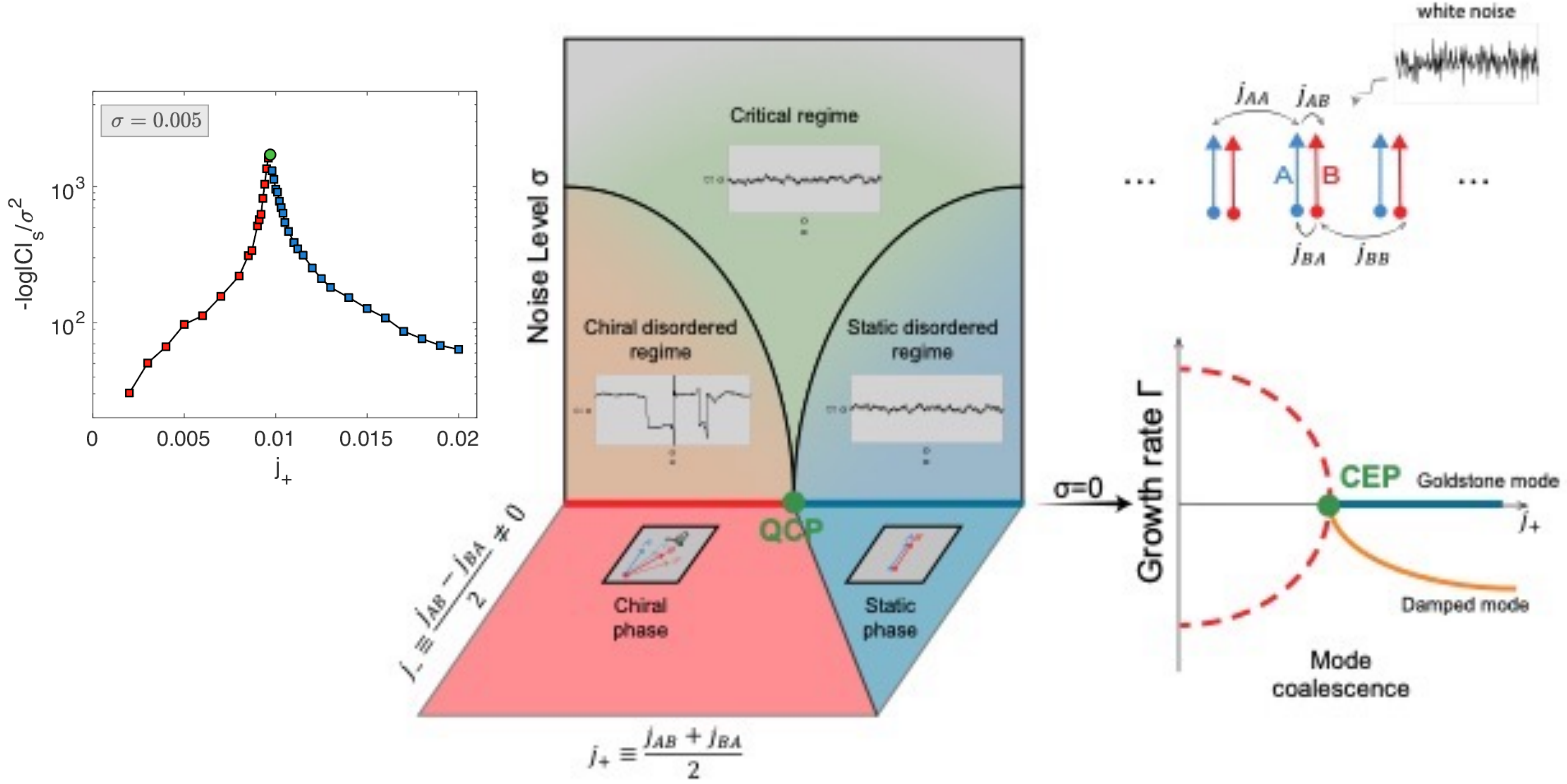




C Weis, et al., arXiv:2207.11667, 2022

Beyond mean field: dynamics, roughening, and pattern formation

Adding noise : Analogy to a quantum critical point?



Liu, Hanai, PBL unpublished

1D non-reciprocal XY model with noise: roughening transition

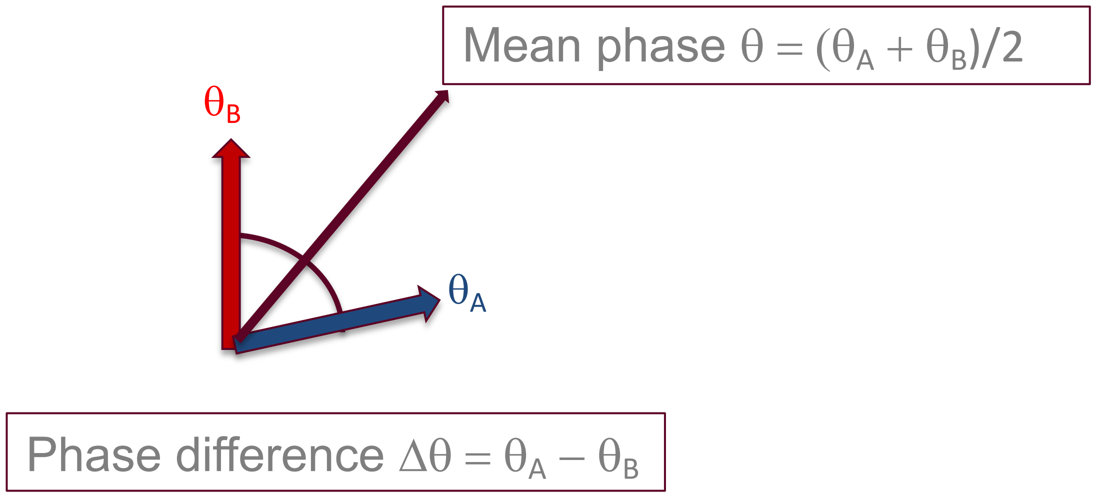
Model described by non-reciprocal spins

U(1) symmetry for average phase

Z_2 (Ising like) for phase difference

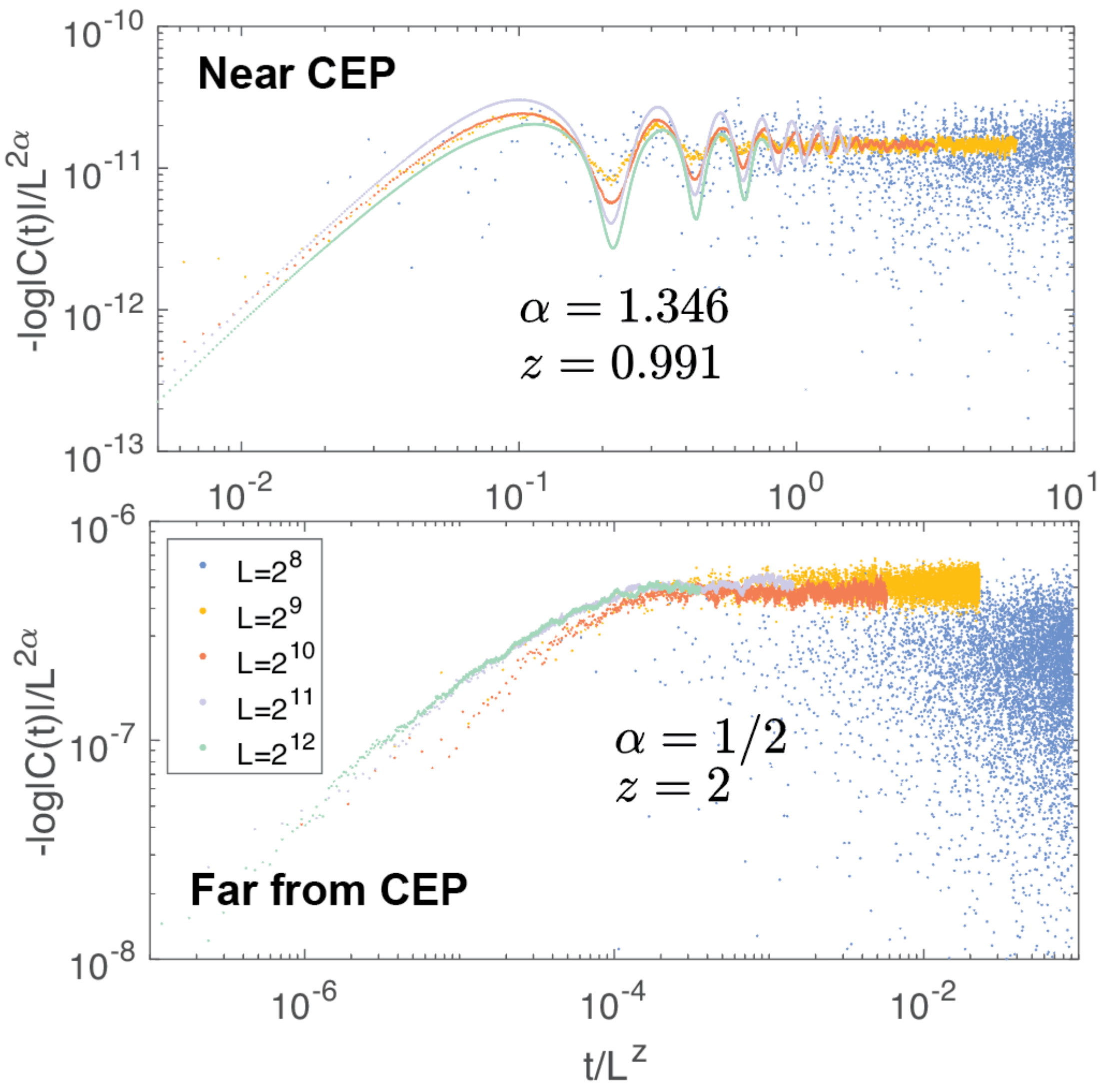
The effect of noise should destroy the ordered phase(s) and create a roughening transition.

[Note: Kardar-Parisi-Zhang term is forbidden by symmetry]



$$C(t, t+\Delta t) = \frac{\langle \overline{\Psi^*(t+\Delta t, x)\Psi(t,x)} \rangle}{\langle \overline{|\Psi(t)|^2} \rangle} \sim e^{-(\Delta t/\tau)^{2\beta}},$$

Finite size scaling near CEP vs. Far from CEP



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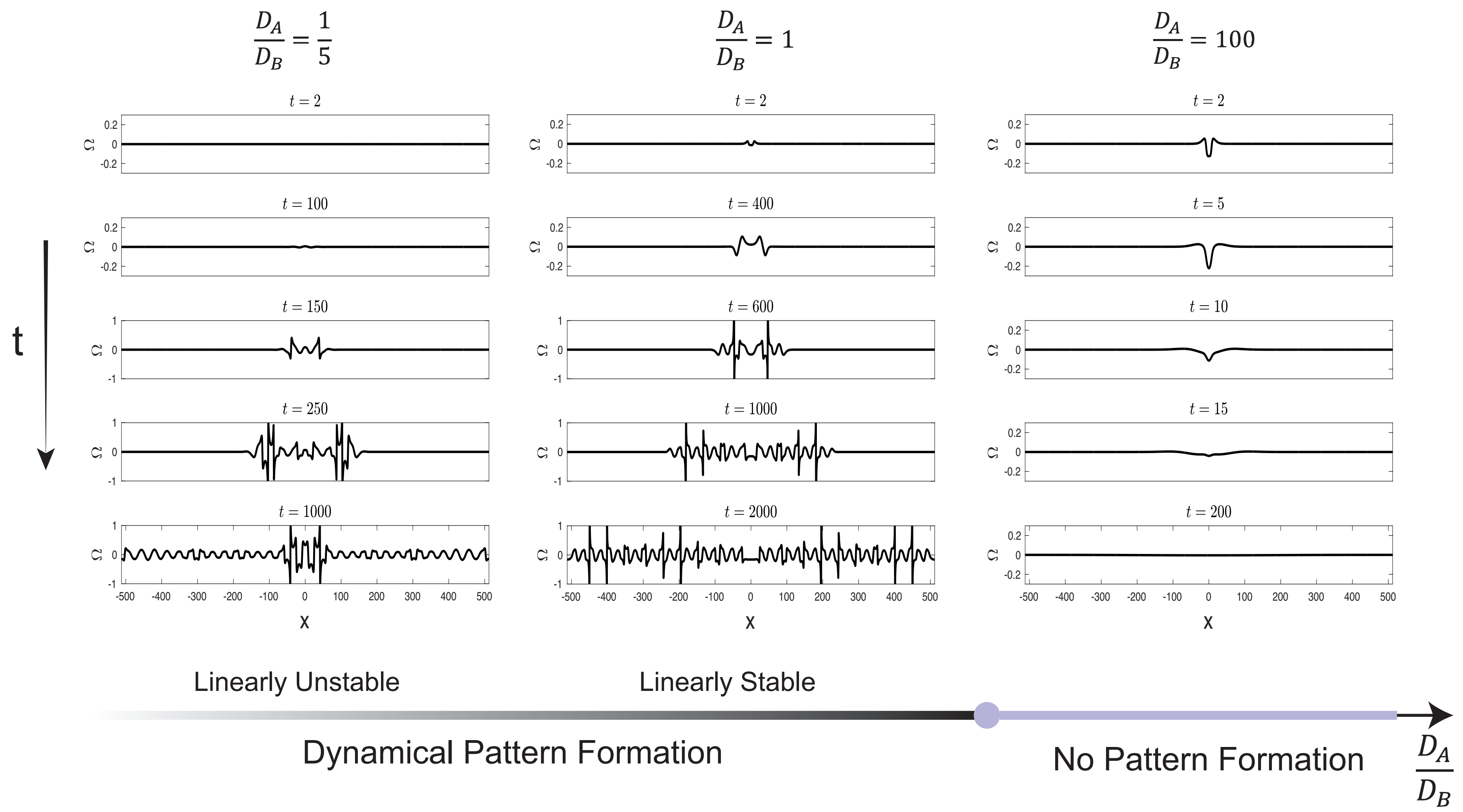
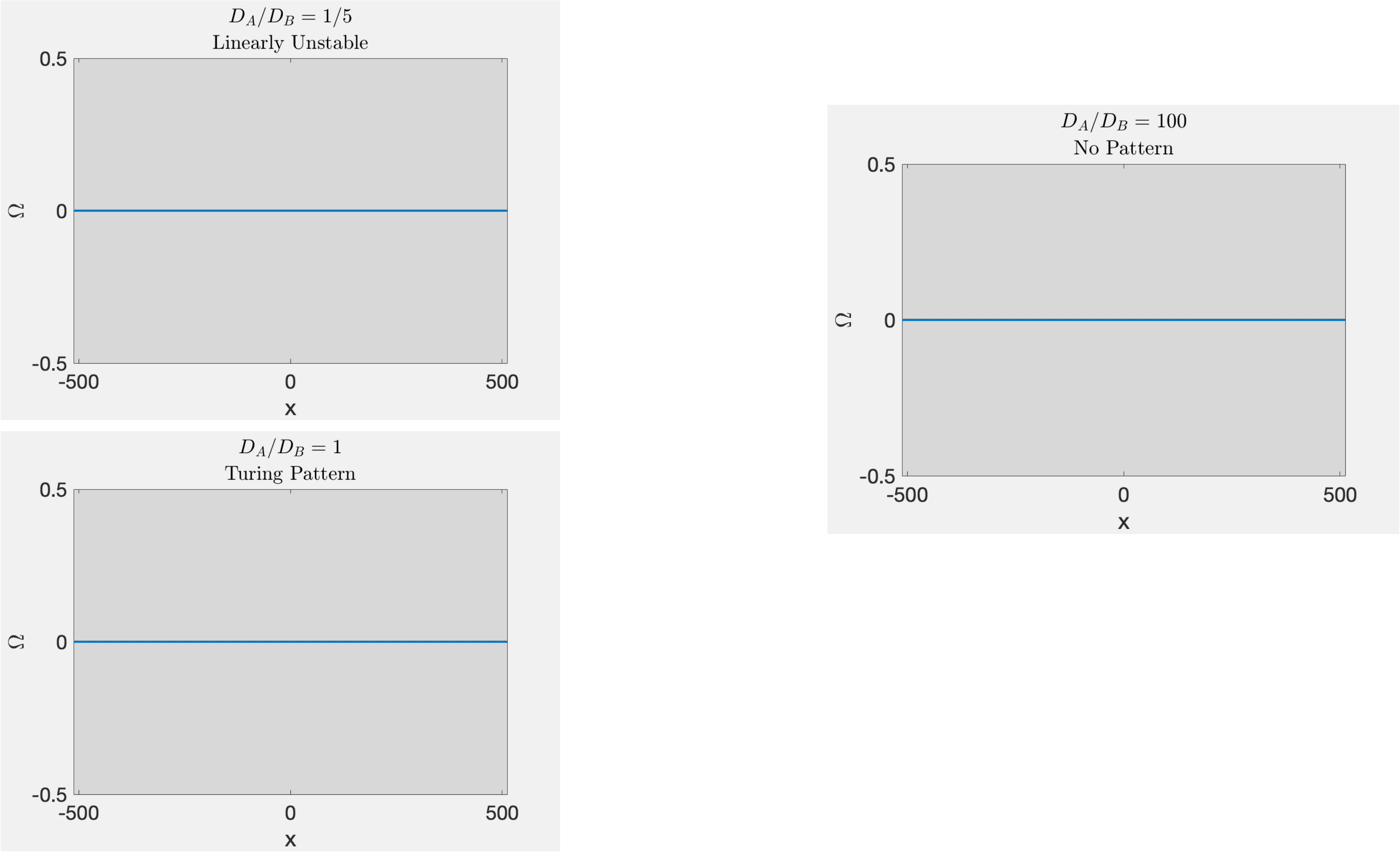
Enhanced fluctuations near CEP
Underdamped sound mode

Crossover to Edwards-Wilkinson
scaling away from CEP

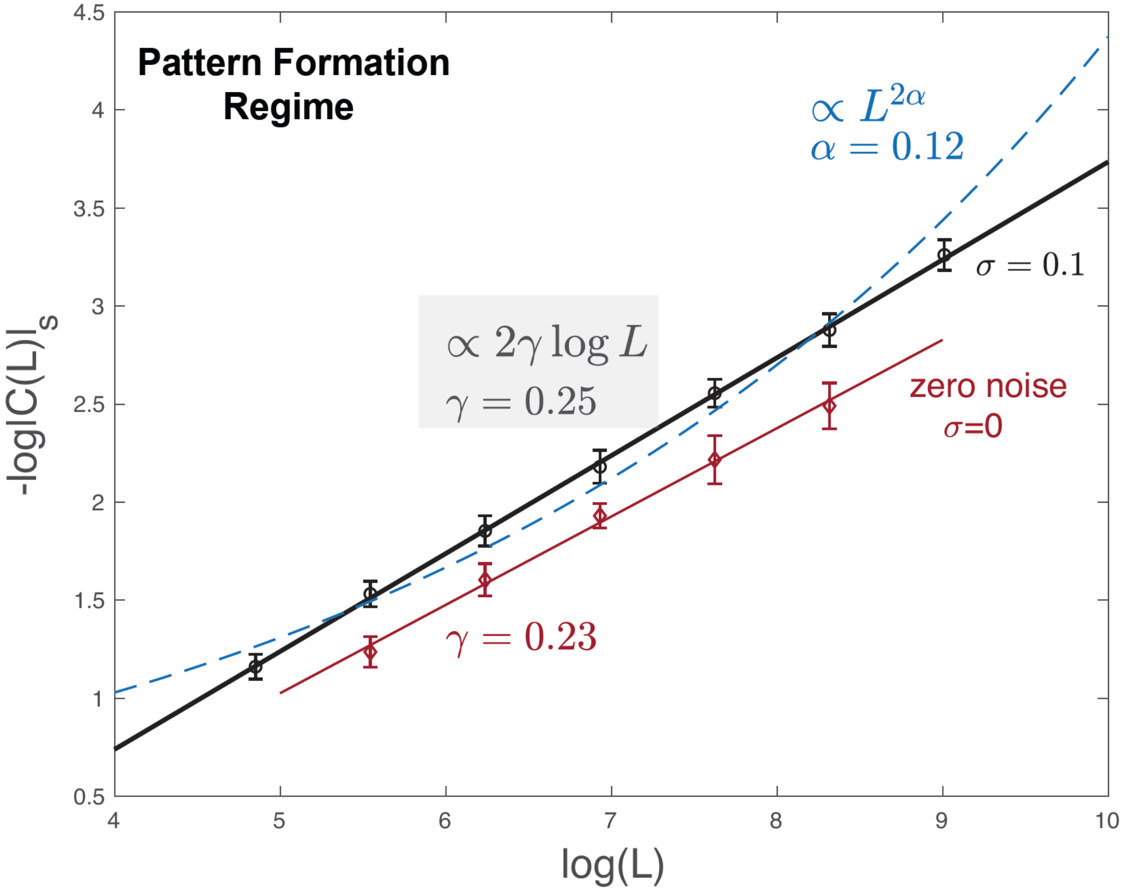
Diffusion-Driven Dynamical Pattern Formation - Turing-Hopf bifurcation

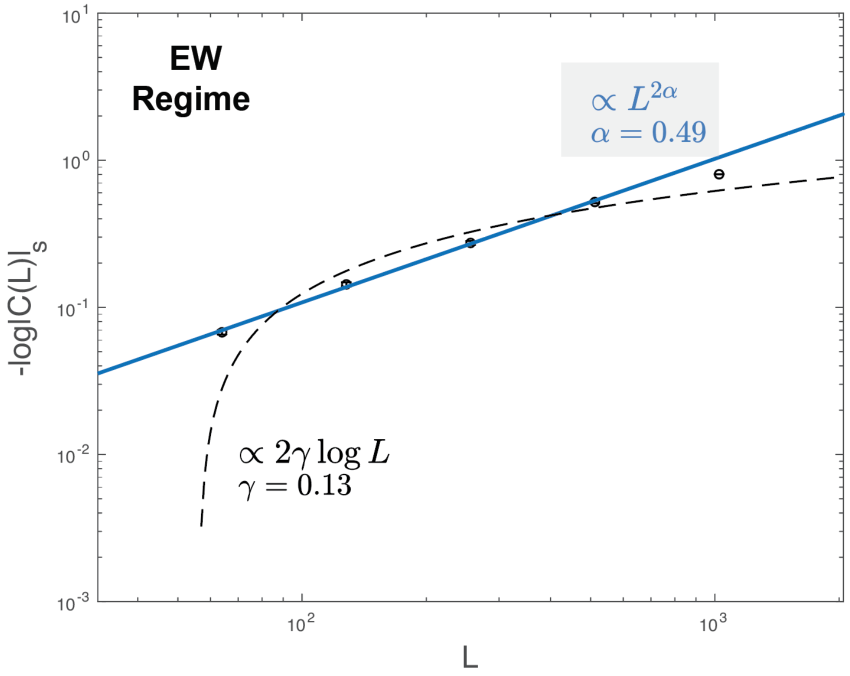




$D_A/D_B = 1/5$
Linearly Unstable
0.5
0
-0.5
-500
500
x
Ω
$D_A/D_B = 1$
Turing Pattern
0.5
0
-0.5
-500
500
x
Ω
$D_A/D_B = 100$
No Pattern
0.5
0
-0.5
-500
500
x
Ω

Deterministic Quasi-Long Range Order





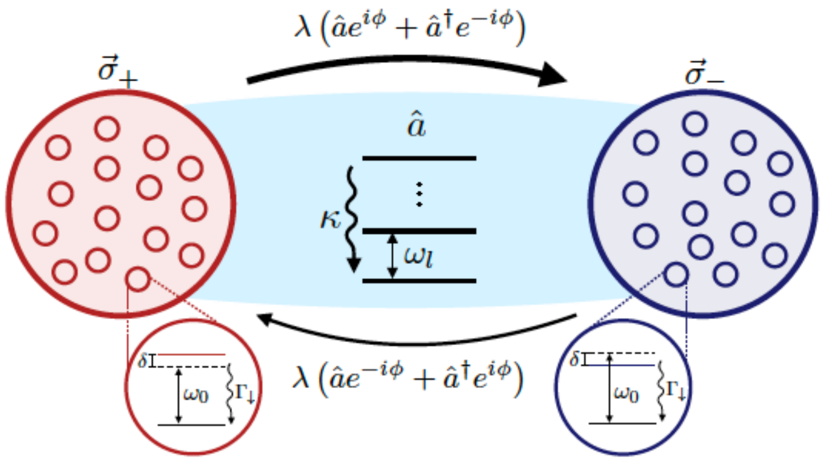
Can we construct “non-reciprocal quantum matter” ?

Some proposals

The non-reciprocal Dicke model

Ezequiel I. Rodríguez Chiacchio,[1] Andreas Nunnenkamp,[2,*] and Matteo Brunelli[3,†]

Physical Review Letters 131 (11), 113602

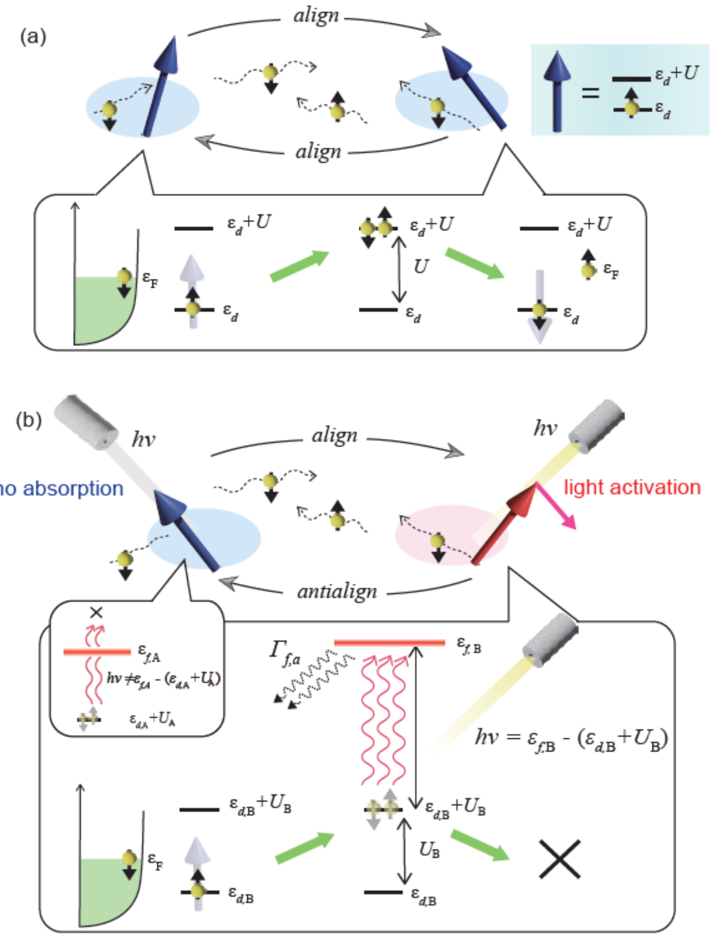


$$\hat H = \hat H_0 + \frac{\lambda}{2\sqrt{N}}\sum_{j=1}^{N}\sum_{m=\pm}\left(e^{-im\phi}\hat a + e^{im\phi}\hat a^\dagger\right)\hat\sigma^x_{j,m}$$

Photoinduced non-reciprocal magnetism

Ryo Hanai,[1,*] Daiki Ootsuki,[2,†] and Rina Tazai[1,‡]

arXiv:2406.05957



$$\dot{S}_a = \sum_{b(\neq a)} J_{ab} S_a \times S_b - \alpha_a S_a \times \dot{S}_a \quad -\gamma_a n S_a - \sum_{b(\neq a)} \Omega_{ab} S_b,$$

Conclusion

There are classes of dynamical many-body phase transition between different "stationary" states of non-Hermitian (or non-reciprocal) systems that are marked by Critical Exceptional Points where fluctuations merge with a Goldstone mode and become strongly enhanced

Nearby there are unusual states – "time crystals"

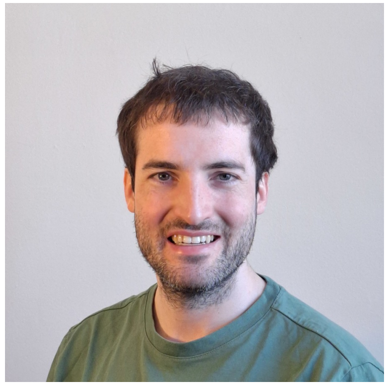
CEP's in extended systems are the dynamical system analog of a critical end point in a thermodynamic system

"Universality classes" (if they exist) are different from thermodynamic phase transitions. No consensus yet on systems beyond mean field

Easy to make in biology … possible in microscopic driven quantum systems?

The Team

Stefan Ihle

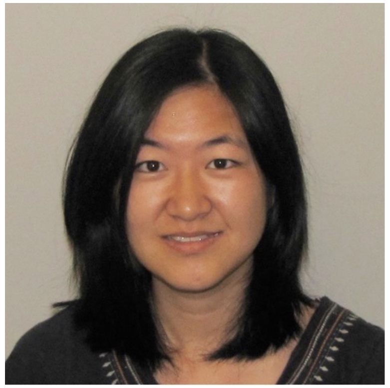


Ryo Hanai

Michel Fruchart

Alex Edelman

Shuoguang Liu



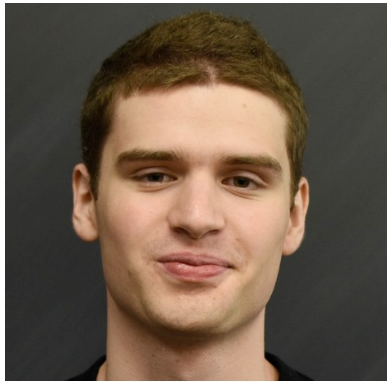
Vincenzo Vitelli

Kyle Kawagoe

Cheyne Weis

Xiaoyuan Huang

Sergei Shmakov



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