EFT

Schladming Lectures hep-ph/9606222
Les Houches Lectures 1804.05863

The basic idea is that you can calculate observables in a <u>systematic</u> expansion without knowing the exact underlying theory.

This is used all the time, since we do not know the underlying theory.

In HEP: Fermi theory of weak interactions — led to SM
S. Weinberg, PRL 17 (1966) 616 π scattering lengths
before QCD, quarks, ...

Cohen { operator analysis of weak decays and GUT: → SMEFT
Technicolor → HEFT

Neubert { HQET: introduce labels v

NRQED/NRQCD/NREFT: label changing interactions
modes

SCET

observable = calculation + error

(a) understand how to control error
(b) systematic expansion in δ (power counting parameter)

(c) can improve accuracy by working to higher order in δ

(d) Finite number of parameters at any given order in δ. Can eliminate these to get parameter-free predictions

(e) Work to δ^n: # of parameters increases with n.

(f) Almost trivial at classical level
- expand $f(\delta)$ in a Taylor series

(g) non-trivial at the quantum level (loop corrections)

(h) key ingredient is locality
- separation of scales into short distance Lagrangian coefficients and long distance matrix elements

(i) can include non-perturbative effects
eg. $\left(\frac{\Lambda_{QCD}}{m_b}\right)$ in HQET

$$-2\lambda_1 = \langle B | \bar{b}_v D_\perp^2 b_v | B \rangle$$

$$-12\lambda_2 = \langle B | \bar{b}_v g \sigma_{\alpha\beta} G^{\alpha\beta} b_v | B \rangle$$

$\lambda_1, \lambda_2 \sim O(\Lambda_{QCD}^2)$ enter masses decay spectra, form factors, etc.

- completely non-perturbative EFT in χPT

- mixture in QCD factorization into

$\hat{\sigma}(\alpha_s) \otimes$ PDF

pert in $\alpha_s(Q)$ non-perturbative

PDF's universal

(j) Allows you to organize your thinking and get estimates for quantities using locality and gauge invariance.

(k) scale separation: deal with only one scale at a time. greatly simplifies calculation otherwise $f(m_1/m_2)$ can be complicated

single scale: #

B decay: M_W, m_b, Λ_{QCD}

(l) symmetries manifest

χPT: spontaneously broken chiral symmetry

HQET: spin-flavor symmetry $b\uparrow$ $b\downarrow$ $c\uparrow$ $c\downarrow$

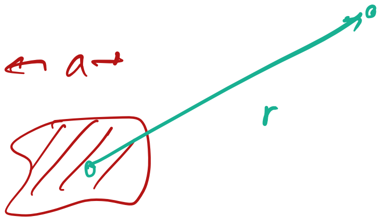
(m) sum large logs (RG improvement)

$$\left(\alpha_s \log \frac{M_W}{m_b}\right)^n$$

fixed order perturbation theory can break down

(n) efficient way to characterize new physics

Electrostatics



Scales $r \gg a$

$$V(r) = \frac{1}{r} \sum c_{lm} \left(\frac{a}{r}\right)^l Y_{lm}(\Omega)$$

(1) expansion parameter $\delta \sim \frac{a}{r}$

(2) Far away observer parameterizes $V(r)$ in terms of short distance coefficient $c_{lm} a^l$

$M \sim \frac{1}{a}$ $\qquad = \frac{c_{lm}}{M^l}$

(3) prediction: $Y_{lm}(\Omega) \propto \frac{1}{r^l}$

(4) dimensional analysis: $c_{lm} \sim 1$

or $\quad c_{lm} a^l \sim a^l$

don't know what a is, but can compare different l to get an estimate of a.

(5) underlying symmetry $\longrightarrow$ measurable predictions

cubic symmetry $\Rightarrow$ $c_{lm} = 0$ unless $m \equiv 0 \pmod 4$

(6) more multipoles
$\Rightarrow$ better description with smaller errors

(7) do not need to know a

$$(c_{lm} a^l) \times \frac{1}{r^l} Y_{lm}(\Omega)$$

$\llcorner$ unknown dimensionfull #

EFT: need to think. cannot apply rules blindly

HQET: $\frac{\lambda_1}{m_b^2}, \frac{\lambda_2}{m_b^2}$ but endpoint of spectra

non-perturbative shape function $S(k)$

eg. $B \to X e \nu$ endpoint $\to$ get shape function for $\frac{d\Gamma}{dE_e}$

m_b, m_c: expand in $\frac{m_Q}{M_W}$ for weak decays

also have $\frac{m_c}{m_b}$ & $\frac{\Lambda_{QCD}}{m_c}$ $\frac{\Lambda_{QCD}}{m_b}$

χPT: $\frac{m_s, \ m_u, m_d}{\Lambda_\chi}$ $SU(3)$ χPT

$\frac{m_u, m_d}{m_s}$ $SU(2)$ χPT integrate out K

expansion depends on what is being measured.

QFT

e.g. QCD, QED

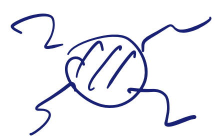
$$\mathcal{L} = \sum \bar{\psi}_r \, i \not{D} \, \psi_r - m_r \bar{\psi}_r \psi_r - \frac{1}{4} G_{\mu\nu}^2 + \frac{\theta g^2}{32\pi^2} G \tilde{G}$$

(1) already an EFT neglect weak interactions...

(2) parameters: masses $m_r(\mu)$

$\alpha_s(\mu)$

$\bar{\theta} < 10^{-10}$

What can we compute? 

Green's functions are gauge dependent

Can compute S-matrix elements

on-shell external particles

physical polarizations

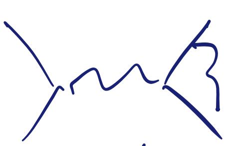
Fields are not particles

LSZ reduction formula $\langle 0 | \phi | p \rangle \neq 0$

field particle

can get S-matrix from ϕ correlation functions.

ϕ = any operator, e.g. ϕ, ϕ^3, $\phi(\partial\phi)^2$ ...

radiative corrections ∞ : 

need a regularization/renormalization scheme

part of the definition of the theory

Will use dim reg in $4-2\epsilon$ dimensions

Used for (almost) all analytic calculations

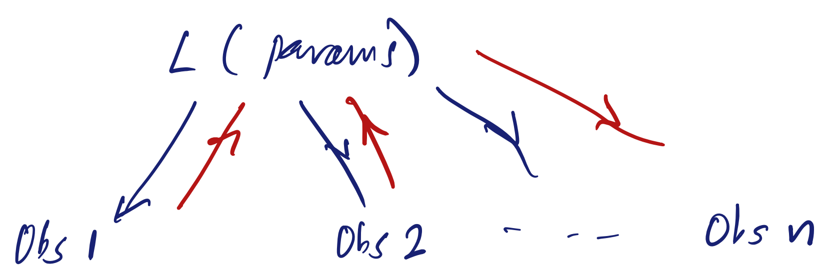
preserves gauge invariance and chiral symmetry

$$\mathcal{L} = \bar{\psi} i \not{D} \psi \ \ldots \ + \underbrace{c.t.}_{\text{small but } \infty \ \ \propto \frac{1}{\epsilon} \ldots}$$

$\mathcal{L}$ parameters are ∞

Lagrangian parameters m not observables

m_B = B meson mass
m_π = pion mass
} observable



get params from some observables to determine rest. Lagrangian parameters are intermediate steps because of how we do calculations.

Scattering amplitudes; avoid Lagrangian parameters

QED: Lagrangian $m_e(\mu)$ $\alpha(\mu)$ in $\overline{MS}$ scheme

Fix from m_e (pole mass) $\qquad \dfrac{\alpha}{r}$ = Coulomb potential

$\alpha = \dfrac{1}{137.036}$

$L_{QED} = L_{EFT}$ neglect strong, weak interactions.

(not in QCD)

Given L_{EFT} can compute using L_{EFT}

do not need to know QED is low energy limit of SM

do not need to consult an oracle

do not need to know "high scale" M_W (or m_π)

"renormalizable theory" vs "non-renormalizable theory"

old-fashioned terminology. not really very different.

QCD: $\mathcal{L} = \bar\psi i \not{D} \psi - m\bar\psi\psi - \frac{1}{4} G^2$

c.t. have the same structure (beware of gauge-fixing)

EFT $\mathcal{L} = \mathcal{L}_{ED} - \frac{4G_F}{\sqrt{2}} \underbrace{(\bar\psi\gamma^\mu P_L\psi)(\bar\psi\gamma_\mu P_L\psi)}_{\dim 6}$

$+ \cdots \dim 8 \cdots$

∞ terms with counterterms.

(a) but to finite order in p^2/M_W^2 only a finite number of coefficients.

(b) can compute without knowing where operators came from. weak decay computed before we knew SM.

An interacting QFT can be constructed from low-energy inputs.

"full theory" $\longrightarrow$ EFT

QCD χPT

SM $\longrightarrow$ Fermi theory

EFT and full theory not the same. They have a different divergence structure. ⟵ one of the reasons for constructing EFT.

not a minor change

"matching" construct EFT from full theory

in some cases, do not know full theory and only have EFT, or matching non-perturbative as in χPT.

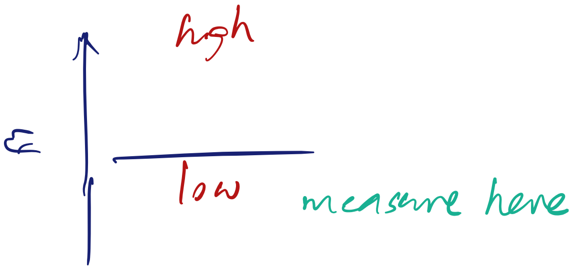
EFT and full theory need not have same d.o.f.

e.g. χPT : mesons, baryons QCD quarks, gluons.

1+1 dim : sine-Gordon Thirring duality
bosons fermions

even QCD with t A_μ, ψ not same
without t in two theories

low energy dynamics insensitive to high-energy quantities provided inputs are low energy



α_{Coulomb}, m_e, m_p

but if you use high-energy inputs $\alpha_s(\mu)$ at say $\mu \sim 500$ GeV

then results depend on parameters such as

m_t . $m_p(\alpha_s(\mu), m_t)$

high-energy $\rightarrow$ constraints on EFT from symmetries
eg P, T , etc.

NRQM: spin-statistics thm from QFT

EFT: often talk about "integrating out"
but in χPT or HQET what is being integrated out?

better: think about problem you want to solve
and what are the relevant scales/fields.
ie. what you want

Fields not always obvious.
NRQCD / SCET multiple gluon fields
soft, ultrasoft
collinear, ultrasoft.

Dimensional Analysis

$\int \mathcal{D}\phi \; e^{iS}$ $\qquad S = \int d^d x \, L$ = dimensionless

$[S] = 0 \qquad [\psi] = \frac{d-1}{2} \qquad [\phi] = \frac{d-2}{2} \qquad [A_\mu] = \frac{d-2}{2}$

$[gA] = 1 \quad \Rightarrow \quad [g] = \frac{4-d}{2} \qquad [D] = 1$

operators in $d = 4$ $\qquad [\phi] = 1 \qquad [\psi] = 3/2 \qquad [A] = 1$

$D = 0 \qquad 1 \longrightarrow \Lambda$

$D = 1 \qquad \phi$ shift field to remove

$D = 2 \qquad \phi^2$

total derivative

$D = 3 \qquad \phi^3, \; \bar\psi\psi, \; \partial^2\phi, \; \cancel{\phi\,\partial\phi}$ not Lorentz inv

$D = 4 \qquad \phi^4, \; \bar\psi\psi\phi, \; \bar\psi \not{D} \psi, \; (D\phi)^2, \; F_{\mu\nu}^2, \; \partial_\mu(\phi\,\partial\phi) \to 0$

total derivatives $= 0 \Leftarrow$ Integration by parts. (IBP)

$\int \partial^2\phi = 0 \qquad \int \phi\, D^2 \phi = -\int (D\phi)^2$, etc.

Remove some terms in Lagrangian – simplifies calculations. ~ factor of 2 reduction.

① Integration by parts (IBP)

② Field redefinitions:

$$\phi \longrightarrow \phi + \frac{\phi^3}{M^2} + \cdots$$

(a) can change Green's functions but not S-matrix elements.

(b) remove some (redundant) operators

(c) Green's functions can be ∞, but S-matrix is finite cdl eg. diagonalize kinetic energy $\bar{\Psi}_i \, i\not{D} \Psi_j \; K_{ij} \to \delta_{ij}$

fermions : In 4d, use Weyl spinors

$\psi_{L,R} = P_{L,R} \psi \qquad P_{L,R} = \frac{1 \mp \gamma_5}{2}$

$\bar{\psi}_L \psi_R \qquad \bar{\psi}_R \psi_L \qquad \bar{\psi}_L \not{D} \psi_L \qquad \bar{\psi}_R \not{D} \psi_R$

use e^-_L and e^-_R or e^-_L and $e^+_L \sim C(e^-_R)^\dagger$

$$\mathcal{L}_{EFT} = \mathcal{L}_{d \le 4} + \frac{\mathcal{L}_5}{M} + \frac{\mathcal{L}_6}{M^2} + \cdots$$

$$\mathcal{L}_{d \le 4} = (D_\mu \phi^\dagger D^\mu \phi) - m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

$\mathcal{L}_{EFT} = \infty$ number of terms eg $(\phi^\dagger \phi)^3$ at dim 6

$M =$ scale of "new physics" like $1/a$ in multipole example

IMPORTANT :

$\phi^\dagger \phi =$ dim 2 $\qquad \phi^3, \bar{\psi}\psi$: dim 3

EFT: dynamics of light particles with masses $\ll M$.

$$\therefore \quad \mathcal{L} \sim - m^2 \phi^\dagger \phi - g \phi^3 - m_f \bar{\psi} \psi$$

$g \sim O(m)$

$$= - \left(\frac{m}{M}\right)^2 M^2 \phi^\dagger \phi - \left(\frac{g}{M}\right) M \phi^3 - \left(\frac{m_f}{M}\right) M \bar{\psi} \psi$$

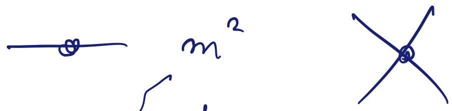
$\therefore$ dim 2, 3 operators are counted like dim 4 because of the extra $(m/M)^2$ or (m/M) suppression.

scattering amplitude has an expansion parameter

$$\delta = \frac{p}{M} \qquad p = \text{momentum or mass } (p^2 = m^2)$$
$$= \text{scale in EFT (ie. low scale)}$$

$$A_n = p^{4 - E_\phi - \frac{3}{2}E_\psi} \qquad \left.\begin{array}{l} E_\phi = \text{external scalars} \\ \frac{3}{2}E_\psi : \text{external fermions} . \times \; \psi \sim \sqrt{E} \end{array}\right\}$$

so E for ϕ, ψ



m^2 $\qquad$ λ

low scale

dim 5 : $\left(\frac{p}{M}\right)$ $\qquad$ dim 6 : $\left(\frac{p}{M}\right)^2 = (\dim 5)^2$ etc.

relative to $\mathcal{L}_{d \leq 4}$ amplitude

ie. $A \sim \left(\frac{p}{M}\right)^n p^{4-E}$ $\qquad$ $n = \sum n_i$ vertices

$$= \sum (D_i - d) = \sum (D_i - 4)$$
$$= D - d$$

count powers of M in the denominator

Weinberg χPT : count powers of p in the numerator

$\langle O \rangle = p^{D - E_\phi - \frac{3}{2}E_\psi}$ $\qquad$ dim D operator with ϕ, ψ external lines.

IR : $p \to 0$ $\qquad$ lowest dim operators dominate

in interacting theory : naive + anomalous dimension

Proton Decay

SM B, L are accidental symmetries : terms with $D \leq 4$ conserve them. First B violating term

$$\mathcal{L} \sim \frac{qqq\ell}{M_G^2} \quad \text{dim } 6$$

proton decay : $\Gamma \sim \frac{m_p}{16\pi} \left(\frac{m_p}{M_G}\right)^4$

$$\tau = \frac{1}{\Gamma} \sim 10^{30}\,\text{yr} \left(\frac{M_G}{10^{15}\,\text{GeV}}\right)^4$$

If dim 5 then only $\left(\frac{m_p}{M_G}\right)^2$ enhanced by $10^{30} \to$ so 1 yr

$n - \bar{n}$ oscillations : $\Delta B = 2$ $\quad \frac{(udd)^2}{M^5}$ dim 9 operator

$\left(\frac{M_n}{M}\right)^5$ suppression in amplitude.

dim 6 $\quad \frac{(H^\dagger H)^3}{M^2}$ could be at a few TeV

$M_G \gg M$ possible because B is a symmetry

scale at which $\cancel{B}$ is M_G

similarly for other symmetries : $L, P, C, CP, T, \ldots$

ν masses

$$\mathcal{L} = \frac{(HL)(HL)}{M} \quad \text{dim 5 operator}$$

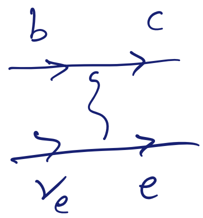
$\Delta L = 2$ and leads to $\mathcal{L} = \frac{v^2}{M} \nu\nu$

Majorana mass $\quad m_\nu \sim \frac{v^2}{M}$

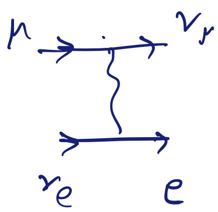
$v \sim 246$ GeV $\quad m_\nu \sim 10^{-2}$ eV $\Rightarrow M \sim 6\ 10^{15}$ GeV

Important to check experimentally if ν masses violate lepton number.

Low-energy weak interactions



or



spinors

$$A \sim \left(\frac{-ig}{\sqrt{2}}\right)^2 V_{cb} \left(\bar{c}\gamma^\mu P_L b\right)\left(\bar{e}\gamma_\mu P_L \nu_e\right)$$

$$\times \left(\frac{-i}{k^2 - M_W^2}\right)$$

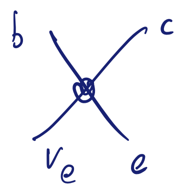
$k^2 \ll M_W^2$

$$\frac{1}{k^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{k^2}{M_W^2} + \cdots\right)$$

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} V_{cb}\left(\bar{c}\gamma^\mu P_L b\right)\left(\bar{e}\gamma_\mu P_L \nu_e\right)$$

$$\frac{4G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2} \qquad M_W = \frac{gv}{2} \Rightarrow \frac{4G_F}{\sqrt{2}} = \frac{2}{v^2}$$

determines v, but not M_W.



$$\Gamma = \frac{|V_{cb}|^2 G_F^2 m_b^5}{192\pi^3} f(\rho) \qquad \rho = \frac{m_c^2}{m_b^2}$$

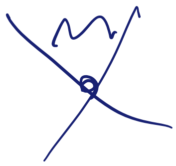
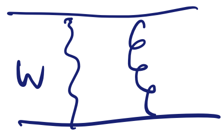
$$f(\rho) = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho$$

① expanded out $\frac{m_c}{m_W}$, $\frac{m_b}{m_W}$ ($\frac{k^2}{M_W^2}$ terms)

② keep full dependence on $\frac{m_c}{m_b}$ = low energy scales.

③ Advantages at higher orders and including radiative corrections

α_s corrections

 vs 

$L \sim \log \frac{M_W}{m_b}$

radiative corrections

$$1 + \alpha_s L + (\alpha_s L)^2 \ \ldots \qquad \text{LL series}$$

$$\alpha_s + \alpha_s (\alpha_s L)^2 \ \ldots \qquad \text{NLL series}$$

LO NLO

RG improvement : factor ~ 2

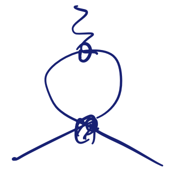
EFT valid until $K^2 \sim M_W^2$

G_F doesn't give M_W.

dim 8 operator $-\frac{4G_F}{\sqrt{2}} \frac{V_{cb}}{M_W^2} \partial_\alpha(\bar{b}\gamma^\mu P_L c)\, \partial^\alpha(\bar{e}\gamma_\mu P_L \nu_e)$

Loops

dim 6 :



$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \sim \Lambda^2$$

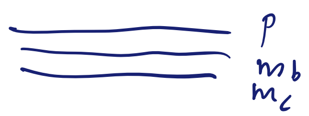
cutoff

dim 8 : $$\int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^2-m^2} \sim \Lambda^4$$

and expansion naively breaks down

$$1 + \underbrace{\frac{1}{M^2}\Lambda^2}_{\text{dim 6}} + \underbrace{\frac{1}{M^4}\Lambda^4}_{\text{dim 8}} + \cdots$$

$\underline{\Lambda \approx M_{EW}}$



non-analytic structure

branch cuts, logs, etc. in

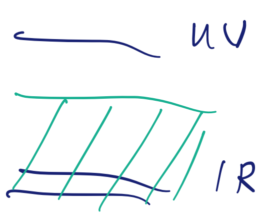
low energy scales

expand out $\frac{\text{low energy}}{\text{high energy}}$

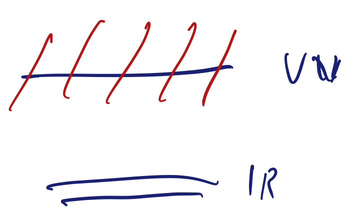
EFT integral is not the "correct one"

may not even know what the correct one is

if we don't know UV theory.



IR part of integral is correct because EFT gives low energy amplitude (by construction)



UV need not be correct.

different UV → local operators.

matching : adjust EFT coefficients to give correct loop amplitude <u>if</u> UV theory known.

power counting $\frac{p}{M_W}$ part of defn of EFT

tells you how to organize expansion.

want a way to do loops that maintains power counting

Dim reg: $\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{(k^2)^a}{(k^2-m^2)^b}$

$$= \frac{i}{(4\pi)^{d/2}} \mu^{2\epsilon} \frac{\Gamma(a+d/2)\,\Gamma(b-a-d/2)}{\Gamma(d/2)\,\Gamma(b)} (-1)^{a-b} \left(m^2\right)^{d/2-a+b}$$

$d = 4-2\epsilon$.

eg. $\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2-M^2)^2} = \frac{i}{(4\pi)^{2-\epsilon}} \mu^{2\epsilon} \frac{\Gamma(2-\epsilon)\,\Gamma(\epsilon)}{\Gamma(2-\epsilon)\,\Gamma(2)} (m^2)^{-\epsilon}$

$$\bar{\mu}^2 = 4\pi\mu^2 e^{-\gamma_E}$$

$$= \frac{i}{16\pi^2} \left[\frac{1}{\epsilon} + \log \frac{\mu^2}{m^2} + \cdots \right]$$

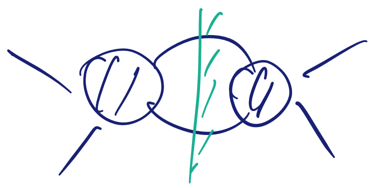
1. $\bar{\mu}$ only appears as $\log \bar{\mu}$. No powers of $\bar{\mu}$
2. scaleless integrals vanish $\int \frac{d^4 k}{(2\pi)^d} = 0$
3. no power divergences. Only log divergences

$$\int \frac{d^d k}{k^2-m^2} \sim m^2 \log \frac{\mu^2}{m^2} \qquad \int \frac{d^d k}{k^2} = 0$$

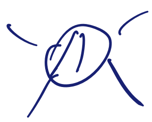
$$\int \frac{d^d k}{k^2 - m^2} \cdot k^2 \sim m^4 \log \frac{\mu^2}{m^2}$$

only get powers of IR scale m in numerator.
No high scale enters the integral. No cutoffs.

similar to scattering amplitude:



unitary cuts → construct loops



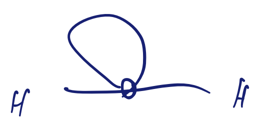
possible polynomial in momentum can be added
only knows about low-energy scales.

polynomial → would have to introduce a random scale Λ not in the theory to have Λ^2, Λ^4 terms.

Dim reg: only scales that enter integral are those in the propagator denominators $\frac{1}{k^2 - m^2}$

~ like complex integration by residues.
- singularities fixed by low scale.

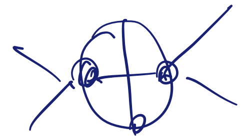
SM:



$$\sim m_H^2 \log \frac{m_H^2}{\mu^2}$$

do not get $\Lambda^2 \log \frac{m_H^2}{\mu^2}$

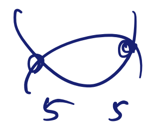
power counting amplitude $\dfrac{O_D}{M^{D-d}}$



$$D - d = \sum_i (D_i - d_i) \quad \text{even for loops}$$

since loop integrals do not generate M.

$$\mathcal{L} = \mathcal{L}_{\leq 4} + \frac{\mathcal{L}_5}{M}$$



$$\sim \frac{1}{M^2}\left[\frac{1}{\epsilon} + \cdots\right]$$

need a dim 6 counterterm.

$$\mathcal{L}_{EFT} = \mathcal{L}_{\leq 4} + \frac{\mathcal{L}_5}{M} + \frac{\mathcal{L}_6}{M^2} + \cdots - ,$$

have the <u>entire</u> series to absorb all the divergences. Cannot ignore them.

$D = 5 \Rightarrow D - d = 1 \geq 0 \Rightarrow$ generate all higher order terms.

$D \leq 4 \Rightarrow D - d \leq 0$ <u>do not generate all terms.</u>

$\frac{1}{\epsilon}$ terms lead to RGE.

$$\mu \frac{d}{d\mu} (\dim 5) \propto \dim 5$$

$$(\dim 6) \propto \dim 6 + (\dim 5)^2$$

$$\vdots$$