Many reviews:
- Dixon 1310.5353
- Elvang, Huang 1308.1697

Conventions: 2005.07129

THE AMPLITUDE APPROACH TO EFTs

1. EFTs via on-shell amplitudes:
   - Spinor helicity formalism
   - "Building block" amplitudes
   - Higher-point amplitudes and factorization
2. Renormalization of the EFT:
   - RGE of Wilson coefficients via on-shell methods at one-loop
   - Selection rules
   - Natural zeros

$\boxed{1}$ EFT defined as a Taylor expansion:

$$\mathcal{L}\left(\frac{\phi}{\Lambda}, \frac{D_\mu}{\Lambda}\right) \simeq \mathcal{L}_4 + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \dots$$

BUILDING BLOCKS:

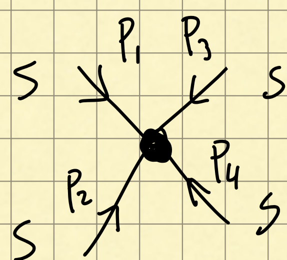
$$\boxed{\mathcal{O}_i \longrightarrow \mathcal{A}_i}$$

↳ fields and derivatives

↳ states defined by: $\vec{p}$, helicity, quantum number

MASSLESS!

e.g. $\underbrace{\phi^2 (\partial^\mu \phi)^2 + \dots}_{\text{need of field redefinitions to eliminate redundancies}} \longrightarrow$



by crossing $\begin{cases} p_1 \leftrightarrow p_2 : t \leftrightarrow u \\ p_1 \leftrightarrow p_3 : s \leftrightarrow t \end{cases}$

$$\mathcal{A} = s + t + u = 0$$

$$\overbrace{(\partial^\mu \phi)^4 + \dots} \longrightarrow \mathcal{A} = s^2 + t^2 + u^2$$

only one independent!

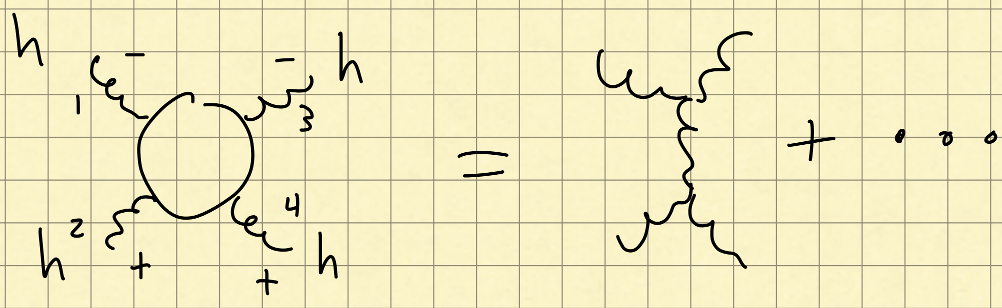
$$S = \mathbb{1} + i\,T$$

$$\langle \text{OUT} | T | \text{IN} \rangle = (2\pi)^4 \, \delta^{(4)}\left(\sum_i p_i^{\text{OUT}} - \sum_i p_i^{\text{IN}}\right) \mathcal{A}$$

↳ multi-particle state defined by $\vec{p}$ and helicities (s_z)

other examples more dramatic when massless particles have spin

Graviton scattering : After learning GR :



but $$\mathcal{A} = G_N \frac{\langle 13\rangle^4 [24]^4}{s \cdot t \cdot u}$$

Amazing simplicity !

to be understood later

IDEA: Theory defined by particle content and "building block" amplitudes (and global symmetries)

$$\mathcal{L}\left(\frac{\phi}{\Lambda}, \frac{D_\mu}{\Lambda}\right) \simeq \mathcal{L}_4 + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

Expansion of P/Λ: $\boxed{A^a_{\Lambda^0}}$ $\boxed{A^b_{1/\Lambda}}$ $\boxed{A^c_{1/\Lambda^2}}$

SPINOR-HELICITY FORMALISM

Lorentz Group: *

$$SO(1,3) \approx SL(2,\mathbb{C})$$ { 2×2 COMPLEX MATRICES OF DET = 1

$\Lambda^{\mu}{}_{\nu}$	M
$p^{\mu} \to \Lambda^{\mu}{}_{\nu} p^{\nu}$	Spinor ψ_{α} : $\psi \to \underbrace{M\psi}_{M_{\alpha}{}^{\beta}\psi_{\beta}}$
	$\alpha = 1,2$

Complex spinor rep $\bar{\psi}_{\dot{\alpha}}$: $\bar{\psi}_{\dot{\alpha}} \to (M^*)_{\dot{\alpha}}{}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}$

Product rep : $\psi_{\alpha} \otimes \bar{\psi}_{\dot{\alpha}} \equiv P_{\alpha\dot{\alpha}}$

$$\boxed{P \to M P M^{\dagger}}$$

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* As in rotations:

$$SO(3) \approx SU(2) \qquad 2 \otimes \bar{2} = 3 + 1$$

$\mathbb{X}_{2\times 2}$ = traceless

$$\vec{x} \to R_{3\times 3}\,\vec{x} \qquad \mathbb{X}_{2\times 2} = \vec{\sigma}\cdot\vec{x} \to U\,\mathbb{X}\,U^{\dagger}$$

$$|\vec{x}| = \mathrm{Det}\,\mathbb{X} \to \mathrm{Det}\,\mathbb{X}$$ { transformation that leaves $|\vec{x}|$ invariant

$$\boxed{P_\mu} \Longleftrightarrow \boxed{P_{\alpha\dot\alpha}} = \sigma^\mu_{\alpha\dot\alpha} P_\mu = \begin{pmatrix} P_0+P_3 & P_1-iP_2 \\ P_1+iP_2 & P_0-P_3 \end{pmatrix}$$

$$\sigma^\mu_{\alpha\dot\alpha} = \{\mathbb{1}, \vec{\sigma}\}$$

$$\boxed{\mathrm{Det}\,P = P_0^2 - P_1^2 - P_2^2 - P_3^2 = P_\mu P^\mu}$$

Since $\mathrm{Det}\,M = 1$

$$\boxed{\mathrm{Det}\,P \xrightarrow{\text{LORENTZ}} \mathrm{Det}\,P}$$

transformation that leaves

invariant $P_\mu P^\mu$

$$\parallel$$

Equivalent to $P_\mu \to \Lambda_\mu{}^\nu P_\nu$

WE WILL CONSIDER MASSLESS PARTICLES:

$$\mathrm{Det}\, P = 0 \qquad P_{\alpha\dot{\alpha}} = \psi_\alpha \cdot \psi_{\dot{\alpha}}$$

Usual notation:

$$1 \xrightarrow[p_1]{} \qquad \boxed{(P_1)_{\alpha\dot{\alpha}} = |1\rangle_\alpha \, [1|_{\dot{\alpha}}}$$

Important step:

$\boxed{\text{We will work with } P \text{ complex}}$:

$$P \neq P^\dagger \Rightarrow [1|_{\dot{\alpha}} \neq \left(|1\rangle_\alpha\right)^*$$

transforming under different $SL(2,\mathbb{C})$:

$$SL(2,\mathbb{C})_L \otimes SL(2,\mathbb{C})_R$$

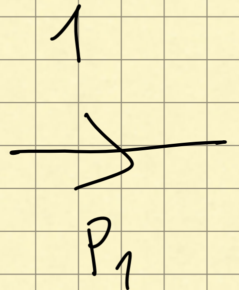
SUMMARY SO FAR:

$$p^\mu \qquad \Big| \qquad \sigma^\mu p_\mu = \begin{pmatrix} p_0+p_3 & p_1-ip_2 \\ p_1+ip_2 & p_0-p_3 \end{pmatrix} \equiv \underline{p}$$

$$p^\mu \to \Lambda^\mu{}_\nu p^\nu \qquad \Big| \qquad \sigma^\mu p_\mu \to \sigma_\mu p^{\mu\prime} \equiv M \sigma_\mu p^\mu M^\dagger$$

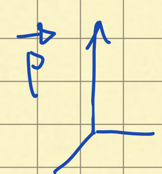
If massless particles:

$$\mathrm{Det}\, \underline{p}_1 = 0 \qquad \boxed{\underline{p}_1 = |1\rangle_\alpha [1|_{\dot\alpha}}$$



$$\underline{p}_1 = \begin{pmatrix} |1\rangle_1 [1|_{\dot 1} & |1\rangle_2 [1|_{\dot 2} \\ |1\rangle_2 [1|_{\dot 1} & \langle 1|_2 [1|_{\dot 2} \end{pmatrix}$$

Example:



$$p_\mu = (E, 0, 0, E)$$

$$\underline{p}_1 = \begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sqrt{2E}$$

$$[1| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sqrt{2E}$$

$$p^\mu \in \mathbb{R} \Rightarrow (\sigma^\mu p_\mu)^\dagger = \sigma^{\mu\dagger} p_\mu^* = \sigma^\mu \cdot p_\mu$$

$$\underline{p}^\dagger = \underline{p} \Rightarrow |1\rangle_\alpha = [1|^*_{\dot\alpha}$$

the amplitude must be build from

Lorentz invariants:

$$\mathcal{A}(p_1 \cdot p_2, p_i^2, \dots) \rightarrow \mathcal{A}(\langle 12 \rangle, \dots)$$

"Angle" Brackets $\quad \underbrace{|1\rangle_\alpha \, \varepsilon^{\alpha\beta}}_{\langle 1|^\beta} \, |2\rangle_\beta \equiv \langle 12 \rangle$

"Square" Brackets $\quad [1|_{\dot\alpha} \, \underbrace{\varepsilon^{\dot\alpha\dot\beta} \, [2|_{\dot\beta}}_{|2]^{\dot\alpha}} \equiv [12]$

Inv. since $\varepsilon^{\alpha\beta} M_\alpha{}^\gamma M_\beta{}^\delta = \varepsilon^{\gamma\delta} \underbrace{\mathrm{Det}M}_{= 1}$

$$\varepsilon_{\alpha\beta}\,\varepsilon^{\beta\gamma} = \delta_\alpha^\gamma \qquad \varepsilon_{\dot\alpha\dot\beta}\,\varepsilon^{\dot\beta\dot\gamma} = \delta_{\dot\alpha}^{\dot\gamma}$$

$$\varepsilon^{\gamma\beta}\,\varepsilon_{\beta\alpha} = \delta_\alpha^\gamma \qquad \varepsilon^{\dot\gamma\dot\beta}\,\varepsilon_{\dot\beta\dot\alpha} = \delta_{\dot\alpha}^{\dot\gamma}$$

$$\sigma^\mu{}_{\alpha\dot\alpha} = \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\,\bar\sigma^{\mu\,\dot\beta\beta}$$

$$\langle 1|^{\alpha} (\sigma^{\mu})_{\alpha\dot{\alpha}} |2]^{\dot{\alpha}} \equiv \langle 1|\sigma^{\mu}|2]$$

$$\hookrightarrow \boxed{p_1^{\mu} = \frac{1}{2}\langle 1|\sigma^{\mu}|1]}$$

Fierz contraction: $\sigma^{\mu}_{\alpha\dot{\alpha}} (\sigma_{\mu})_{\beta\dot{\beta}} = 2\, \epsilon_{\alpha\beta}\, \epsilon_{\dot{\alpha}\dot{\beta}}$

$$p_1^{\mu} \cdot p_{2\mu} = \frac{1}{4} \langle 1|\sigma^{\mu}|1] \langle 2|\sigma_{\mu}|2]$$

$$= \frac{1}{2} \langle 12\rangle [21]$$

$$\boxed{2 p_i^{\mu} \cdot p_{j\mu} = \langle ij\rangle [ji]}$$

$$p^{\mu} \in \mathbb{R} \Rightarrow \underline{P} = \underline{P}^{\dagger}$$

$$\Rightarrow \langle ij\rangle \sim [ij]^{*} \sim \sqrt{p_i \cdot p_j}$$

Properties:

$$\left.\begin{array}{l}\langle 12\rangle = -\langle 21\rangle \\ [12] = -[21]\end{array}\right\} \Rightarrow \langle ii\rangle = [ii] = 0$$

$$p_1|1] = |1\rangle[11] = 0 \; ; \quad \langle 1|p_1 = \langle 11\rangle[1| = 0$$

Momentum conservation: $\displaystyle\sum_j \langle i\, j\rangle[j k] = \langle i|\overset{0}{\overset{\|}{\sum p_j}}|k\rangle = 0$

Shouten:

$$\langle ij\rangle\langle kl\rangle - \langle ik\rangle\langle jl\rangle = \langle il\rangle\langle kj\rangle$$

$$\langle 12\rangle\langle 34\rangle = \langle 13\rangle\langle 24\rangle - \langle 14\rangle\langle 32\rangle$$

Fierz : $\langle 1|\sigma^\mu|2]\langle 3|\sigma_\mu|4] = -2\langle 13\rangle[24]$

$$|-p\rangle = i|p\rangle \; ; \; |-p] = i|p]$$

such that $|-p\rangle[-p| = -|p\rangle[p|$

And

$$\begin{cases} \varepsilon_\mu^+ = \dfrac{\langle q|\sigma_\mu|p]}{\sqrt{2}\,\langle qp\rangle} \\[2ex] \varepsilon_\mu^- = -\dfrac{\langle p|\sigma_\mu|q]}{\sqrt{2}\,[qp]} \end{cases}$$

$\xrightarrow[p]{}$

q reference momentum

Little group transformation (rotation around $\vec{p}_1$ axis)

$$\left\{ \begin{array}{ll} |1\rangle \to e^{\overbrace{-i\theta/2}^{\omega \in \mathbb{C} \text{ when we complexify}}} |1\rangle & \lambda = -1/2 \\ |1] \to e^{+i\theta/2} |1] & \lambda = +1/2 \end{array} \right.$$

- on state: $|p_1, \lambda\rangle \to e^{i\theta\lambda} |p_1, \lambda\rangle$

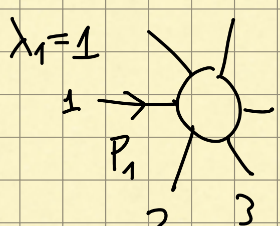
and then on the amplitude:

$$A \to e^{i\theta\lambda_1} A \qquad \lambda_1 = \text{helicity of state } 1$$

Important constraint:

$$\boxed{A\left(e^{-i\frac{\theta}{2}}|1\rangle, e^{i\frac{\theta}{2}}|1], \ldots\right) = e^{i\theta\lambda_1} A\left(|1\rangle, |1], \ldots\right)}$$

and similarly for particle 2, 3, ...

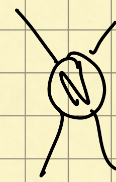


$$\propto \left\{ \begin{array}{l} \langle 1i\rangle\langle 1j\rangle \\ \langle 1i\rangle / [1j] \\ 1/[1i][1j] \end{array} \right.$$

Dimensions:

$$[A] = E^{4-n_{\text{states}}}$$

3pt  $\sim E$

4pt  $\sim E^{0}$

$\vdots$ $\quad$ $\vdots$

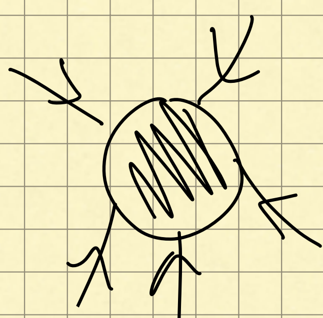
$\langle \vec{p}|\vec{q}\rangle = 2E\,(2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{q}) \Rightarrow [|\vec{p}\rangle] = [E]^{-1}$

$[s] = [T] = [E]^{0}$

$$[p] = E \Rightarrow [|i\rangle, [i|] = E^{1/2}$$

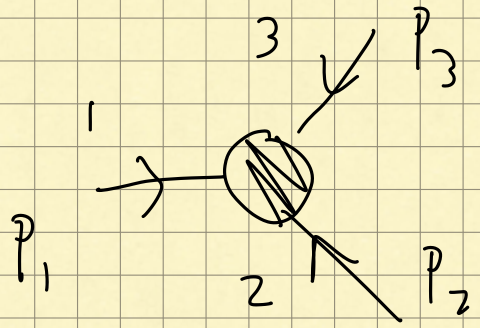
$$\big[[ij], \langle ij\rangle\big] = E \quad ; \quad [\Lambda] = E$$

We will take that states are always incoming:



$$\boxed{\text{3pt}}$$

$p_i^2 = 0$



$p_1 = p_2 + p_3$

$p_2^2 = 0 = 2p_2 p_3$

$\Downarrow$

All $\boxed{p_i \cdot p_j = 0}$

Possible to achive a nonzero A_{3pt} if momenta is complex $\left(\langle ij \rangle \neq [ij]^* \right)$

$$p_i \cdot p_j = \frac{1}{2} \langle ij \rangle [ji] = 0$$

$\hookrightarrow$ by either $\langle ij \rangle = 0$ or $[ij] = 0$

either all $\langle ij \rangle = 0$ or all $[ij] = 0$:

PROOF:

- $p_1 p_2 = \frac{1}{2} \langle 12 \rangle [21]$ Lets take $[21] = 0$
- $\langle 1 | p_2 | 3] = \langle 1 | (-p_1 - p_3) | 3] = -\underbrace{\langle 1 | p_1 | 3]}_{\langle 11 \rangle [1 \overset{\shortparallel}{0}} - \underbrace{\langle 1 | p_3 | 3]}_{13\rangle [33] \overset{\shortparallel}{0}} = 0$

$\shortparallel$

$\langle 12 \rangle [23]$ $\rightarrow \Delta$

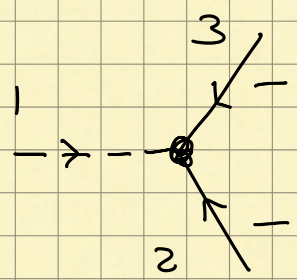
$[23] = 0$ and similarly $[13] = 0$

LEADING ORDER in EFTs:

$\left(\text{Expansion } \frac{\langle ij\rangle}{\Lambda}, \frac{[ij]}{\Lambda}\right)$

$$\boxed{A \propto \Lambda^{0}}$$

Yukawa Interaction:



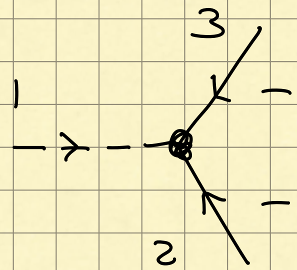
$\propto \langle 3| \text{ and } \langle 2| \text{ or } [3|^{-1} \text{ and } [2|^{-1}$

$\Longrightarrow$ Either $\langle 23 \rangle$ or $\dfrac{1}{[23]}$

Right dimension $= E$

We must fix $[ij]=0$

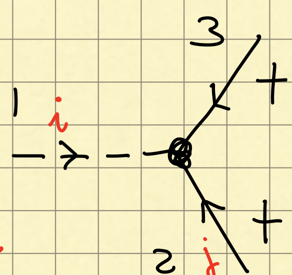
to guarantee $p_i \cdot p_j = 0$



$$A = Y\langle 23\rangle$$

If global symmetries, we can include invariant tensor under the symmetry

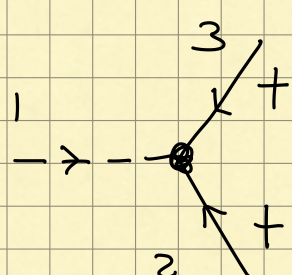
Example: 1, 2 states are doublets of $SU(2)_L$



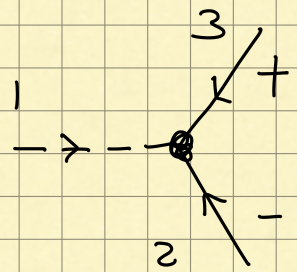
$i, j = 1, 2$

$$\boxed{A = Y\langle 23\rangle \varepsilon_{ij}}$$

Similarly:



$$\propto [23]$$



$$\propto \underbrace{\langle 3| \text{ and } \langle 2|^{-1}}\quad \left(\text{or } [2| \text{ and } [3|^{-1}\right)$$

$$\Downarrow$$

$$\frac{\langle 3\, i\rangle}{\langle 2\, i\rangle}$$

$\Downarrow$ only $i = 1$ nonzero

$$\frac{\langle 3 1\rangle}{\langle 2 1\rangle} \quad \text{but Dim} = 0$$

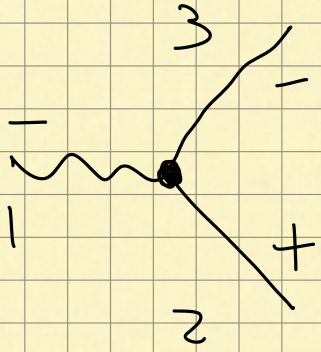
Not possible!

Show that $H\bar{\Psi}\Psi$ lead (on-shell) to this amplitude using:

$$u_{\mp} = P_{\mp}\begin{pmatrix}|p\rangle_\alpha \\ |p]^{\dot\alpha}\end{pmatrix} \; ; \; \bar{v}_{\mp} = \left(\langle p|^\alpha \quad [p|_{\dot\alpha}\right) P_{\pm} \qquad P_{\mp} = \frac{1}{2}\left(1 \pm \gamma_5\right)$$

upcoming fermion or antifermion

Gauge interactions:



$$\propto \underbrace{\langle 1|^{2},\ \langle 3|,\ \langle 2|^{-1}}_{\frac{\langle 13\rangle\langle 1i\rangle}{\langle 2i\rangle}}$$

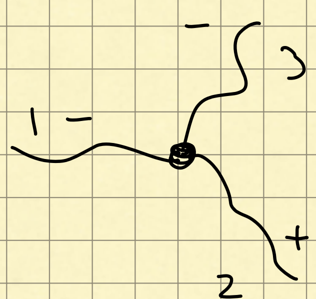
$\Downarrow$ only $i=3$ non zero

$$\boxed{\frac{\langle 13\rangle^{2}}{\langle 23\rangle}}$$

Dim $=1$

OK ✓

No other helicities!



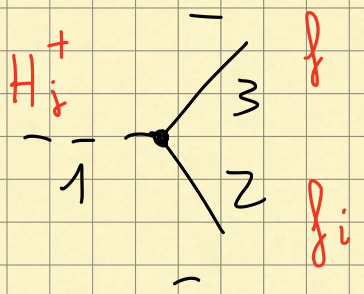
$$\propto \frac{\langle 13\rangle^{2}\langle ij\rangle}{\langle 2i\rangle\langle 2j\rangle} \xrightarrow[\text{NONZERO}]{\text{ONLY}} \frac{\langle 13\rangle^{3}}{\langle 21\rangle\langle 23\rangle}$$

Antisymmetric $1\leftrightarrow 2$ or $1\leftrightarrow 3$

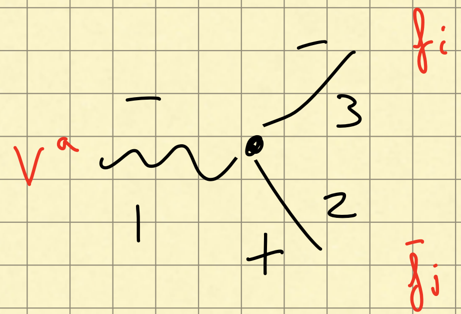
only possible for more than one

Leading order (Λ^0):

$\boxed{\text{SM}}$

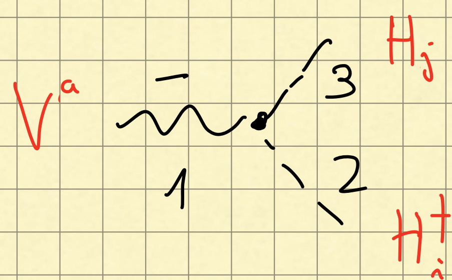


$$y \langle 32 \rangle \delta_{ij}$$

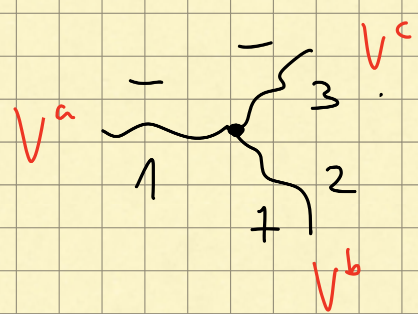


$$g \frac{\langle 13 \rangle^2}{\langle 32 \rangle} T^a_{ij}$$

AMPLITUDE WITH OTHER h NOT POSSIBLE



$$g \frac{\langle 31 \rangle \langle 21 \rangle}{\langle 23 \rangle} T^a_{ij}$$



$$g \frac{\langle 31 \rangle^3}{\langle 12 \rangle \langle 23 \rangle} f^{abc}$$

$+$ h.c.

later: need of being invariant tensor under a global symmetry

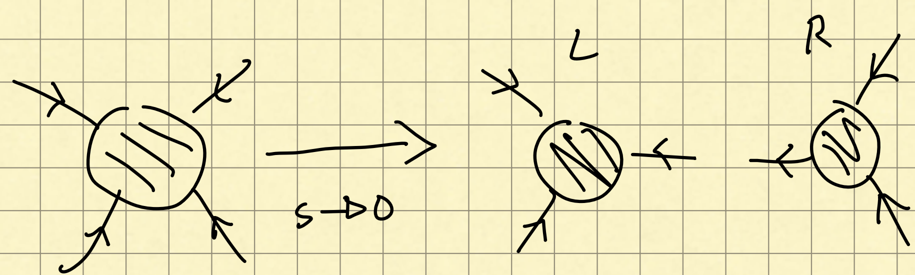
Higher-point amplitudes:

Same logic ⊕ Factorization

⇓

singularities must be simple poles arising from a particle exchange.

On the pole the 4pt must factorize in 3pts:



$$A_{4pt} \longrightarrow A^{L}_{3p}\,\frac{1}{s}\,A^{R}_{3pt}$$

→ If it's a fermion line

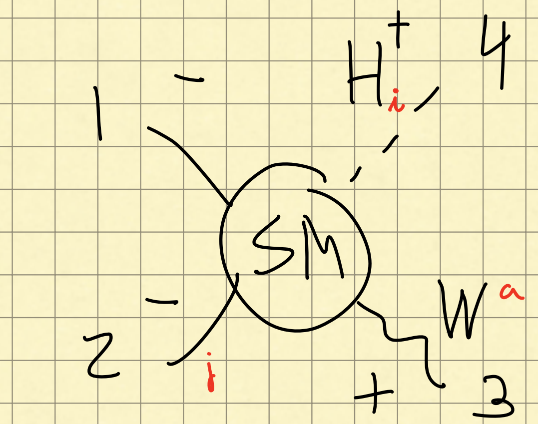
$$\boxed{\lim_{s\to 0} s\cdot A_{4p}(1,2,3,4) = i^{F}\left[i\,A_L(1,2,p)\right]\left[i\,A_R(-p,3,4)\right]}$$

Subtleties: 

must be inverted. We choose $\begin{cases} |-p\rangle = i\,|p\rangle \\ |-p] = i\,|p] \end{cases}$ Fullfills:

$|{-p}\rangle[-p| = -|p\rangle[p| = -P_{\alpha\dot{\alpha}}$

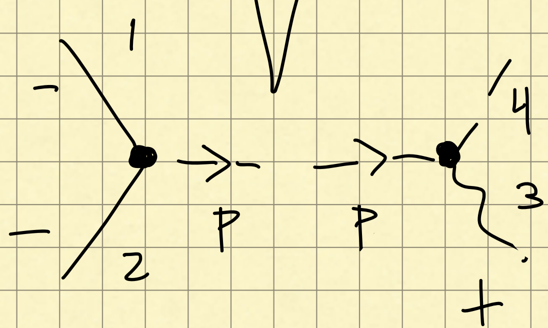
EXAMPLE: We'll see how global symmetries are forced upon us!



$$c\,\frac{\langle 12\rangle [3i][3j]\langle ij\rangle}{s\cdot t}$$

simple poles

$s \to 0$



$$Y_\psi \langle 12\rangle \cdot \frac{i^2}{s}\cdot \frac{[34][3p]}{[p4]}\, g T^a_{ij}$$

$$= \underbrace{g Y_\psi \cdot T^a_{ij}}_{c} \langle 12\rangle \cdot \frac{i^2}{s} \cdot \frac{[34][3\cancel{p}]\langle p2\rangle}{[\cancel{p}4]\,\langle p2\rangle}$$

$\cancel{p} \to -(\cancel{p_3}+p_4)$, $\quad \cancel{p} \to -(p_3+\cancel{p_4})$

$s = \langle 43\rangle[34]$

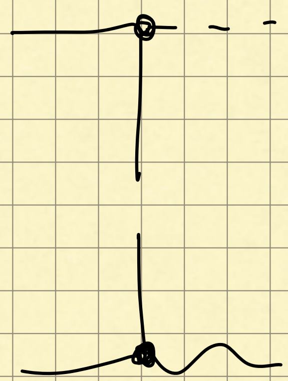
$$\frac{[34]\langle 42\rangle}{\langle 32\rangle}\;\frac{[23]}{[23]}$$

$$\boxed{\mathcal{M} = g Y_\psi T^a_{ij} \frac{\langle 12\rangle\langle 42\rangle}{\langle 43\rangle\langle 23\rangle}} \quad -\frac{1}{t}[34][32]\langle 42\rangle$$

$t = [32]\langle 23\rangle$

$i = 4$
$j = 2$

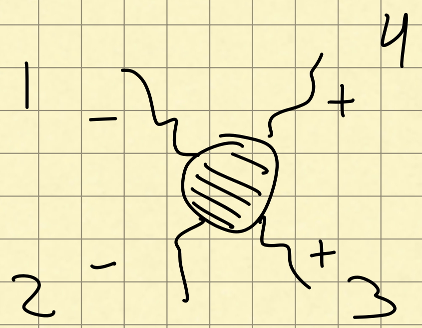
one has to check also proper factorization in the t channel



$\tilde{g}\, T^a$

tells us that $\tilde{g} = g$!!

Example 2: Graviton scattering

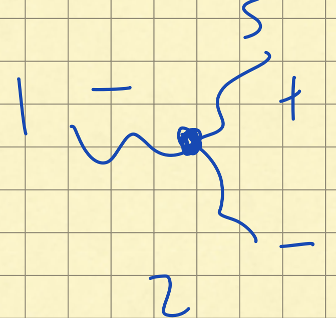


helicities

$$\propto G_N \frac{\langle 12\rangle^4 [34]^4}{s \cdot t \cdot u}$$

simple poles ⊕ crossing

Check factorization into 3pt:



$$= \sqrt{G_N}\left(\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}\right)^2$$

$\sqrt{G_N} = \frac{1}{M_P}$ Needed by dimensions

Not possibility to higher-spin since the denominator would have double poles

To get higher-point amplitude,
there is a recursion method:

BCFW method

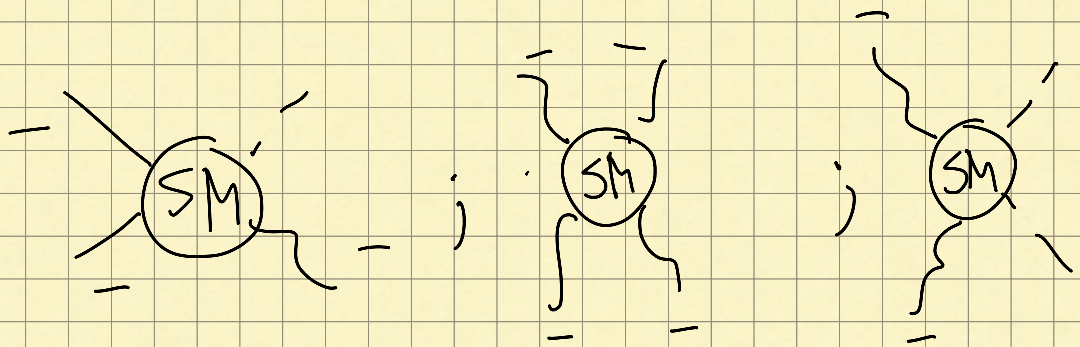
B → BRITTO

C → CACHAZO

F → FENG

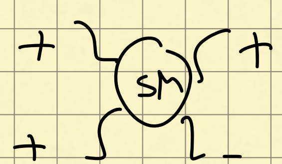
W → WITTEN

In the SM no(*) amplitude with all fields with positive (or negative) helicities:



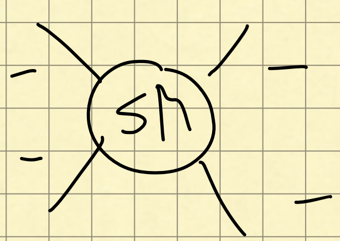
$= 0$

For gauge/gravity, zero also if one is different

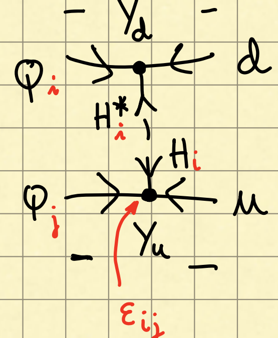


$$\boxed{\lambda^{SM}_{TOTAL} = 0}$$

(*) only one exception:



$=$



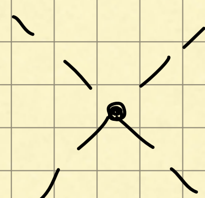
$\sim Y_d Y_u \dfrac{\langle 14\rangle\langle 23\rangle}{t}$

or Y_e

$\lambda_{TOTAL} = 2$

$$\boxed{\lambda_{TOTAL} = -2}$$

there can be 4pt that do not factorize to 3pt. In the SM:

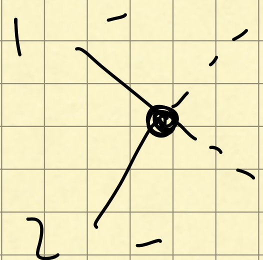
 $\mathcal{A} = \lambda$

↳ New "building block"!

Taking the idea of EFT, there is a new scale Λ that suppress higher dim operators

$\Rightarrow$ Here, it means new "building blocks"

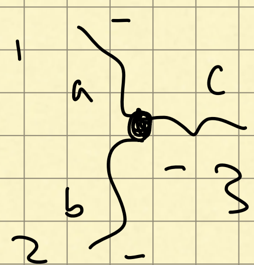
$$\boxed{A \propto \frac{1}{\Lambda}} \quad \longleftrightarrow \quad \text{Dim 5 operators}$$



$$A \propto \frac{\langle 12 \rangle}{\Lambda} \longleftrightarrow \frac{1}{\Lambda}(H \cdot L)(H \cdot \bar{L}^c)$$

$$\boxed{\mathcal{A} \propto \frac{1}{\Lambda^2}} \longleftrightarrow \text{Dim 6 operators}$$

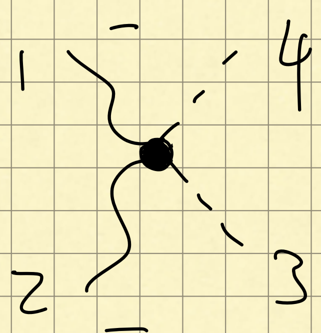
3pt:



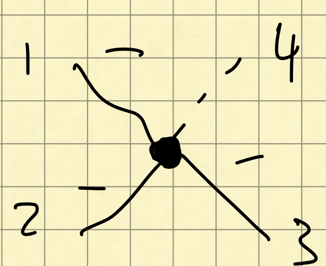
$$\frac{\langle 12\rangle\langle 23\rangle\langle 31\rangle}{\Lambda^2} f_{abc} \longleftrightarrow F^3$$

4pt:

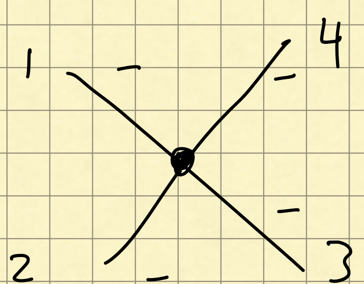
$\lambda_{TOTAL} = -2$:



$$\frac{\langle 12\rangle^2}{\Lambda^2} \longleftrightarrow |H|^2 F^2$$



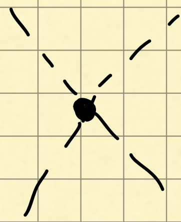
$$\frac{\langle 12\rangle\langle 13\rangle}{\Lambda^2} \longleftrightarrow H F^{\mu\nu} \bar{\psi} \sigma_{\mu\nu} \psi$$



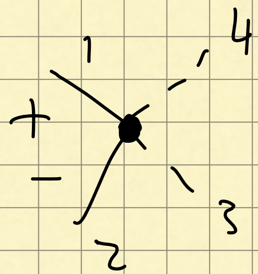
$$\frac{\langle 12\rangle\langle 34\rangle}{\Lambda^2}, \frac{\langle 13\rangle\langle 24\rangle}{\Lambda^2} \longleftrightarrow (\bar{\psi}\psi)^2$$

$$\langle 12\rangle\langle 34\rangle = \langle 13\rangle\langle 24\rangle - \langle 14\rangle\langle 32\rangle$$

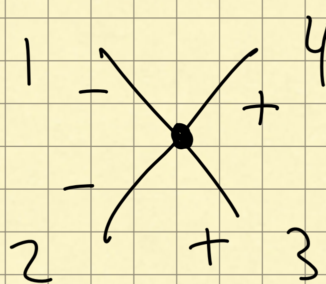
$\lambda_{TOTAL} = 0$:



$$\frac{\langle 12\rangle[12]}{\Lambda^2}, \quad \frac{\langle 13\rangle[13]}{\Lambda^2} \longleftrightarrow D^2 H^4$$

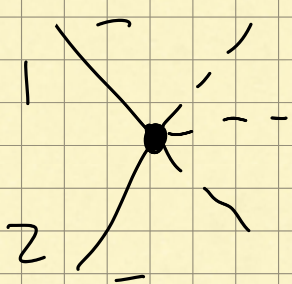


$$\frac{\langle 13\rangle[32]}{\Lambda^2} \longleftrightarrow \bar{\psi}\gamma^\mu\psi H^\dagger D_\mu H$$



$$\frac{\langle 12\rangle[34]}{\Lambda^2} \longleftrightarrow (\bar{\psi}\gamma_\mu\psi)^2$$

5 pt:



$$\frac{\langle 12\rangle}{\Lambda^2} \longleftrightarrow |H|^2 H\bar{\psi}\psi$$

6 pt:

$$\frac{1}{\Lambda^2} \longleftrightarrow |H|^6$$

Apart from complex-conjugated A : $\langle ij\rangle \longleftrightarrow [ij]$

Direct physics and clean selection rules for interference with the SM:

$$|A|^2 = \left|A_{SM} + \frac{1}{\Lambda^2} A_{BSM}\right|^2$$

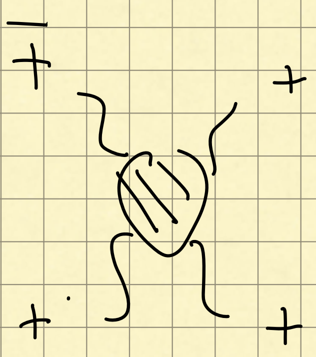
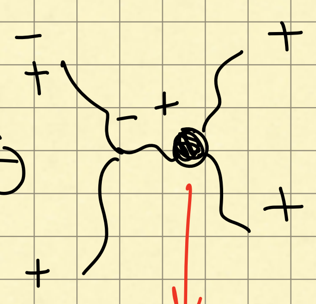
$$= |A_{SM}|^2 + A_{SM} A^*_{BSM} \frac{1}{\Lambda^2} + \text{h.c.}$$

$$+ \frac{1}{\Lambda^4} |A_{BSM}|^2$$

Different scaling

Dim 8 needed?

Example:

 = No SM $\oplus$ 

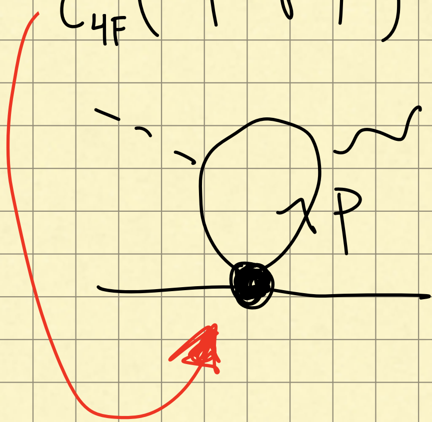
antisym.

Renormalization & EFT via on-shell method

Feynman approach:

At the one-loop we can have operator mixing:

$$C_{4F}(\bar{\psi}\gamma\psi)^2 \longrightarrow C_D\, H F\, \bar{\psi}\sigma_{\mu\nu}\psi$$



Can have divergent terms

$$\frac{C}{16\pi^2}\int\frac{d^4p}{p^4} \sim \frac{1}{\epsilon} + \ln\mu^2$$

Must be absorved in C_D:

$$C_D \to C_D(\mu)$$

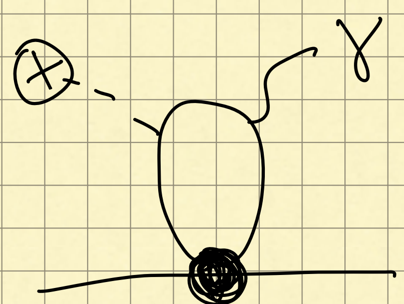
Anomalous dimension: $\gamma = \dfrac{dC_D}{d\ln\mu} \propto \dfrac{1}{16\pi^2}\, C_{4F}$

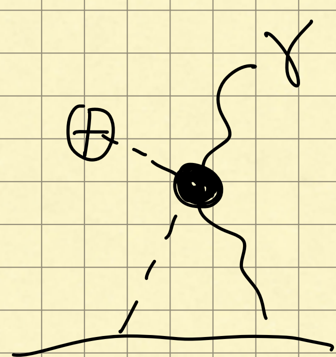
Very important for phenomenology:

Example:

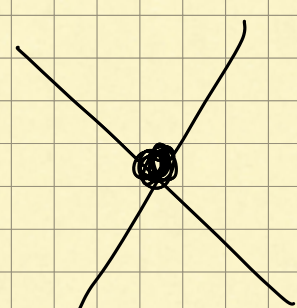
<u>Electric</u> dipole moment very well measured and extremely small in the SM

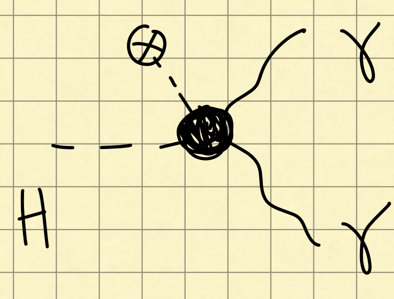
We can get a better bound to $(\bar{\psi}\gamma\psi)^2$ and $h\gamma\gamma$ if CP violating:





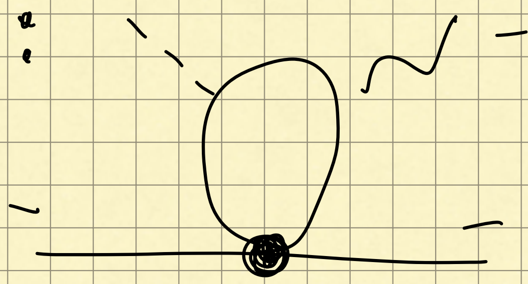
than from tree-level:



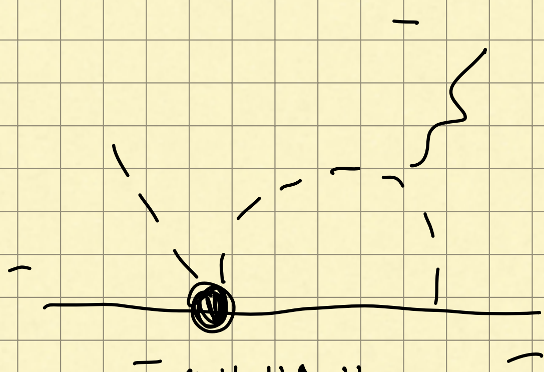


Emergence of selection rules hidden in Feynman approach:

CONTRIBUTIONS TO DIPOLES:

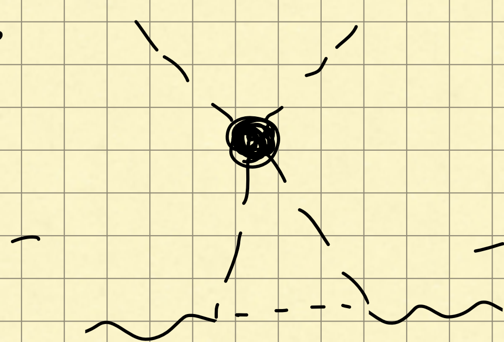


$(\bar{\psi}\gamma^{\mu}\psi)^2$



$\bar{\psi}\gamma^{\mu}\psi H D_{\mu} H$

CONTRIBUTIONS TO $|H|^2F^2$:



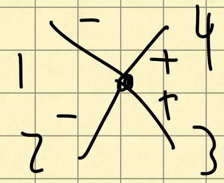
No div. part found from Feynman calc.

$\Downarrow$

Zero anomalous dimension

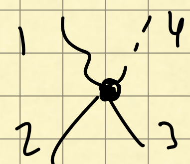
Using on-shell amplitude approach:

$$A_{4F} = \frac{C_{4F}}{\Lambda^2}\langle 12\rangle[34]$$



Loop? How to calculate?

$$A_{D} = \frac{C_{D}(\mu)}{\Lambda^2}\langle 12\rangle\langle 13\rangle$$



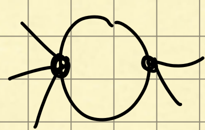
RGE of Wilson coefficients:

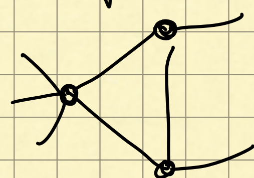
Generalized unitarity Methods
see e.g. Dixon 1310.5353

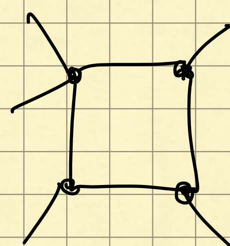
one-loop:

rational terms

$$\mathcal{M}_{loop} = \sum_a c_2^a I_2^{(a)} + \sum_b c_3^b I_3^{(b)} + \sum_c c_4^c I_4^{(c)} + R$$







Bubbles triangles Boxes

only UV divergent $\left(I_2 = \frac{1}{\epsilon} + \ln \mu^2 + \dots \right)$

$$I_m = (-1)^m \mu^{4-D} \int \frac{d^D\ell}{i(2\pi)^D} \frac{1}{\ell^2 (\ell - P_1)^2 (\ell - P_1 - P_2)^2 \cdots}$$

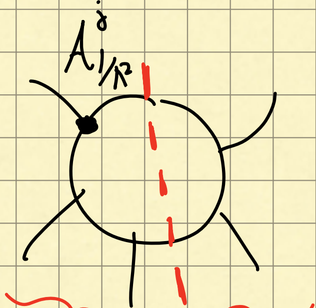
$$D = 4 - 2\epsilon$$

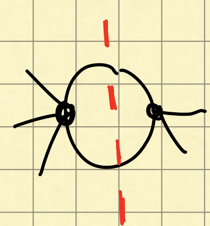
RGE of Wilson coefficients:

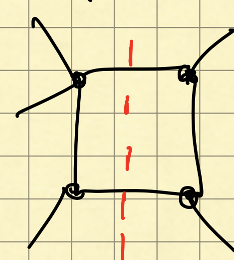
one-loop:

rational terms

$$\mathcal{M}_{loop} = \sum_a C_2^a I_2^{(a)} + \sum_b C_3^b I_3^{(b)} + \sum_c C_4^c I_4^{(c)} + R$$







A_L A_R

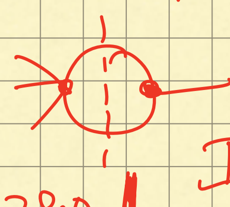
Bubbles triangles Boxes

zero contributions for IR safe amplitudes

$$\int A_L \cdot A_R \qquad \propto \sum_a C_2^a$$

$$\cong \gamma_{j\to i} A^i_{1/\Lambda^2}$$

bubble with one leg



$p^2=0$

$I_2(0)$

give zero!

$$\int dLIPS = \frac{\pi}{2}$$

$$\boxed{\gamma_{j\to i} A^{i}_{BSM} = - \frac{1}{4\pi^3} \frac{c_i}{c_j} \int dLIPS \sum_{\substack{\text{external legs}\\ \text{distribution}}} \sum_{\ell_1,\ell_2} \sigma_{\ell_1\ell_2} \hat{A}^{j}_{BSM}(\ell_1,\ell_2) \times A_{SM}(-\ell_2,-\ell_1)}$$

Amplitude containing A^{j}_{1/λ^2}

$\frac{1}{2}$: for identical particles

i^{n_F} $\quad n_F$: internal fermions

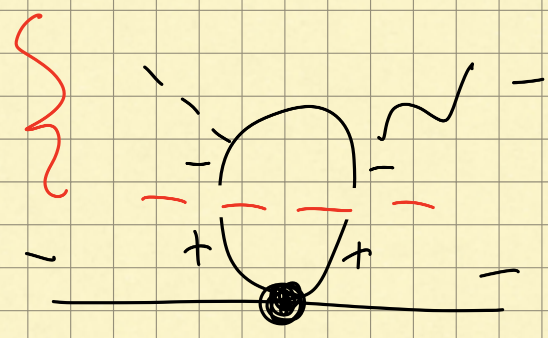
Caron-Huot and Wilhelm 1607.06448

Baratella, Fernandez, A.P. 2005.07129

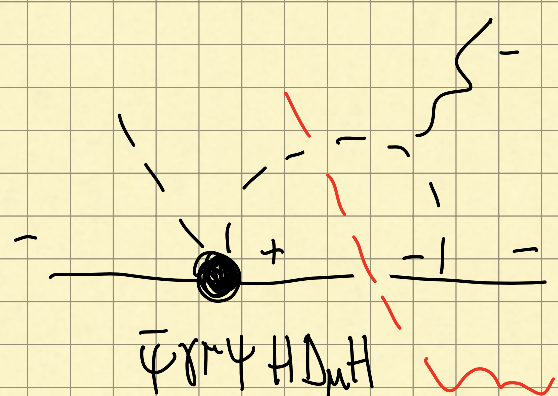
$$\underbrace{\int dLIPS}_{\text{Lorentz Invariant Phase Space}} \cdots = \int d^4\ell_1 \, d^4\ell_2 \, \delta^{(4)}(\ell_1+\ell_2-P) \, \delta^{+}(\ell_1^2) \, \delta^{+}(\ell_2^2)$$

Emergence of selection rules hidden in Feynman approach:

zero

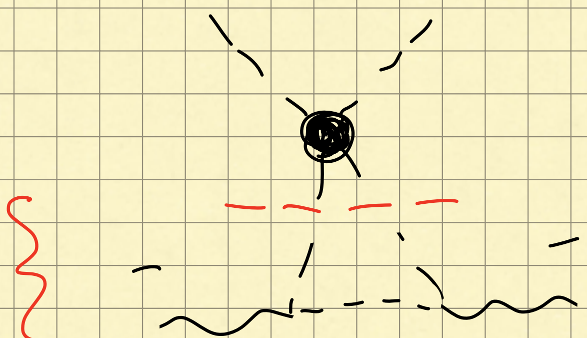


$(\bar{\psi}\gamma^\mu\psi)^2$



zero

zero



No div. part found from Feynman calc.

Rules from $\gamma_{j\to i}\, A_i \sim \int \hat{A}^L_j\, A^R_{SM}$:

$$\begin{cases} m_i = \hat{m}_j + m_{SM} - 4 \\ \lambda_i = \hat{\lambda}_j + \lambda_{SM} \end{cases} \qquad \begin{array}{c} \hat{m}_j \geq m_j \\ m_{SM} \geq 4 \\ \Downarrow \\ \boxed{\Delta m \geq 0} \end{array}$$

Defining $\Delta m \equiv m_i - m_j$

$\Delta\lambda = \lambda_i - \lambda_j$

4pt : $\lambda^{4pt}_{SM} = 0 \quad m_{SM} \geq |\lambda_{SM}| + 4$

$\Downarrow$ (*) apart from exception

$$\boxed{\Delta m \geq |\Delta\lambda|}$$

(*) Allows renormalization: $\psi^4 \leftrightarrow \psi^2\bar{\psi}^2$

$H^3\psi^2 \leftrightarrow H^3\bar{\psi}^2$

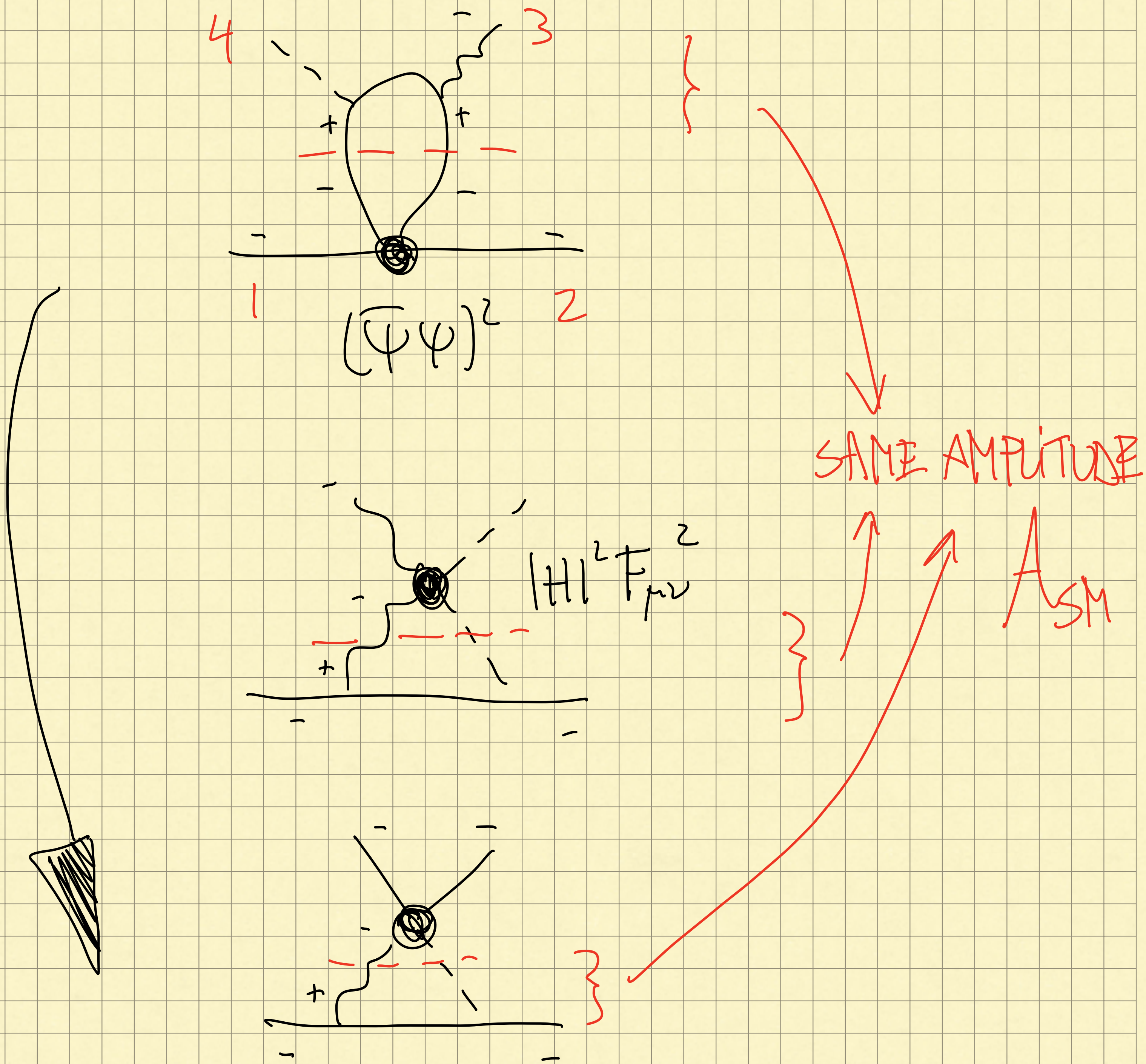
Cheung + Shen 1305.01844

λ \ m	3	4	5	6
0		$\bar{\psi}^2\psi^2$ D^2H^4 $DH^2\bar{\psi}\psi$		H^6
1			$H^3\bar{\psi}\psi$	
2		H^2F^2 ψ^4 $HF\psi\psi$		
	F^3			

EXCEPTION

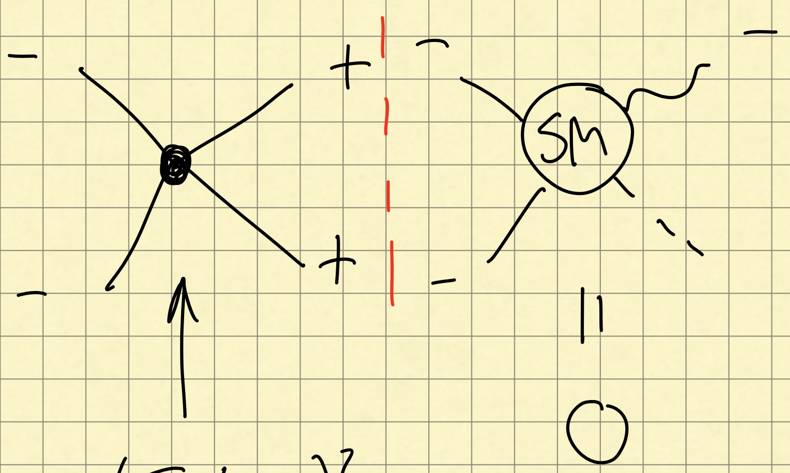
Renormalizin is possible

SIMPLICITY AND RECYCLING POWER



$$\gamma_{\text{dipole}}\frac{\langle 31\rangle\langle 32\rangle}{\Lambda^2} = -\frac{1}{4\pi^3}\int dLIPS\,\frac{\langle 23'\rangle\langle 4'1\rangle}{\Lambda^2}\,A_{SM}$$

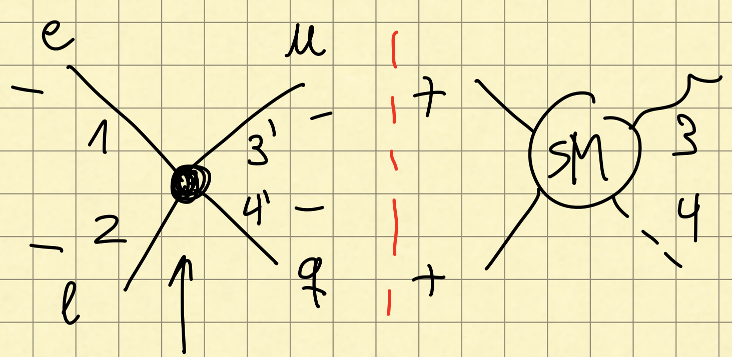
VIA AMPLITUDE METHODS



zero by helicity selection rules

$(\bar{\psi}\gamma_\mu\psi)^2$

J=1



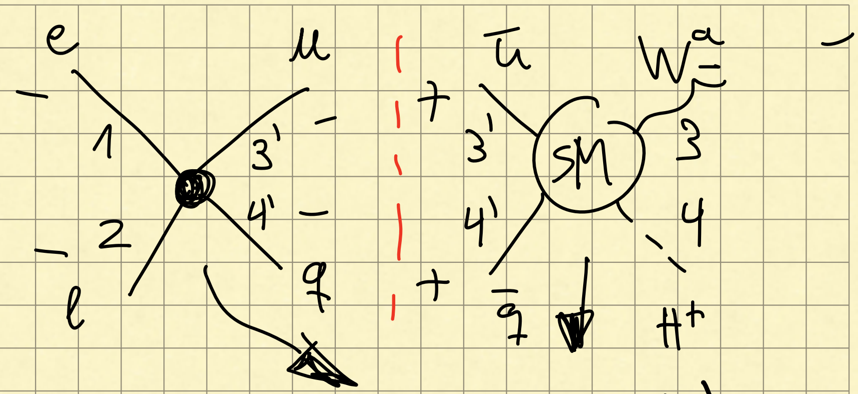
J=1

$(\bar{\psi}\psi)^2$

(a) $\langle 23'\rangle\langle 4'1\rangle$ J=1 e-l state

(b) $\langle 12\rangle\cancel{\langle 3'4'\rangle}$ J=0 e-l state

$$\gamma \frac{\langle 31\rangle\langle 32\rangle}{\Lambda^2} = -\frac{1}{4\pi^3}\int dLIPS\, \frac{\langle 23'\rangle\langle 4'1\rangle}{\Lambda^2} A_{SM}$$



$$\gamma \cdot \frac{\langle 31\rangle\langle 32\rangle}{\Lambda^2} = -\frac{Y_u g_2 N_c}{4\pi^3} C_{luqe} \int dLIPS \, \frac{\langle 23'\rangle\langle 4'1\rangle}{\Lambda^2} \cdot \frac{\langle 34\rangle\langle 33'\rangle}{\langle 43'\rangle\langle 3'4'\rangle}$$

$$\frac{1}{2}\int_0^{2\pi} d\phi \int_0^{\pi/2} s_\theta \, c_\theta \, d\theta$$

$$\begin{cases} |3'\rangle = c_\theta |3\rangle - s_\theta e^{i\phi} |4\rangle \\ |4'\rangle = s_\theta e^{-i\phi} |3\rangle + c_\theta |4\rangle \end{cases}$$

$$\boxed{\gamma} = \frac{Y_u g_2 N_c}{4\pi^2} C_{luqe} \int_0^{\pi/2} d\theta \, s_\theta^3 c_\theta =$$

$$= \boxed{\frac{Y_u \cdot g_2 N_c}{16\pi^2} C_{luqe}}$$

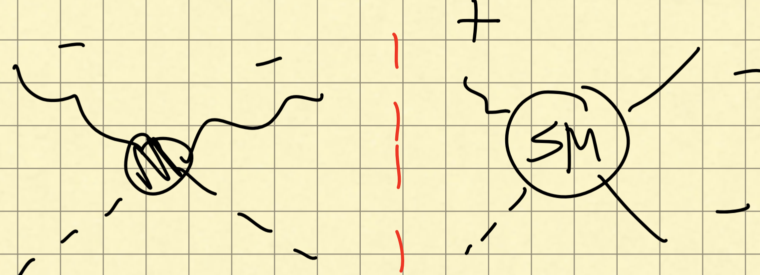
$$\gamma \frac{\langle 31\rangle\langle 32\rangle}{\Lambda^2} = -\frac{1}{4\pi^3}\int dLIPS\, \frac{\langle 23'\rangle\langle 4'1'\rangle}{\Lambda^2} A_{SM}$$

PARTIAL WAVE DECOMPOSITION

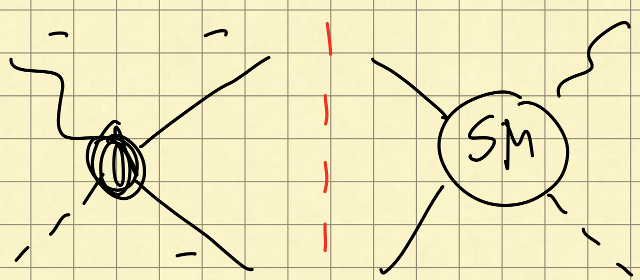
2010.13809

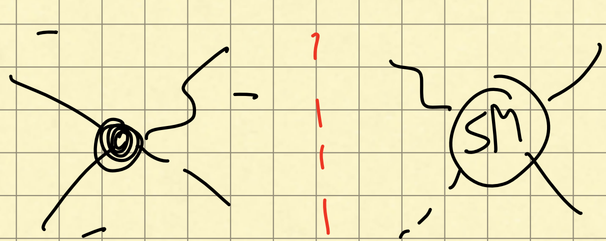
$$\boxed{\gamma \cdot a^{J=1}_{dipole} = -\frac{N_c}{8\pi^2}\, a^{J=1}_{44} \cdot a^{J=1}_{SM}}$$

ALSO:



BUT ALSO:





EVERYTHING IS RECYCLED

$\boxed{\text{IR Div:}}$ $\propto A_{tree}$

- From bubbles : Collinear → Contribute to γ (External legs)

- From triangles : Soft → Must be substracted