Exploration sheet

Activity 1

You might have seen this puzzle before, there is an interactive version on the NRICH website.

There are five brown frogs and five green frogs sitting on their lily pads, as shown:

You have to swap the green frogs with the brown frogs, but - there are some rules.

***Only move one square at a time.**

***You can jump only over one frog of a different colour.**

***All frogs can only move forwards from the direction they start in.**

Find the minimum number of moves to swap 100 brown and 100 green frogs

No. of brown frogs	No. of green frogs	Minimum no. of moves to swap
2	2	
3	3	
4	4	
5	5	
n	n	

Activity 2:

Find a pattern to find the number of diagonals in a polygon. How does this relate to the number of handshakes in a group of people?

Polygon	No. of sides	No. of diagonals
Triangle	3	
Quadrilateral	4	
Pentagon	5	
Hexagon	6	
Heptagon	7	
Octagon	8	
n-gon	n	

Activity 3:

Imagine a large cube made up from 27 small red cubes.

Imagine dipping the large cube into a pot of yellow paint so the whole outer surface is covered, and then breaking the cube up into its small cubes.

How many of the small cubes will have yellow paint on their faces?
Will they all look the same?

Extend this idea, and create a formula to find the number of painted face for a cube of dimension n x n x n

Dimension of the cube	No. of small cubes with 3 face painted yellow	No. of small cubes with 2 face painted yellow	No. of small cubes with 1 face painted yellow	No. of small cubes with 0 face painted yellow
2x2x2				
3x3x3				
4x4x4				
5x5x5				
nxnxn				

Figurate numbers

Figurate numbers have been studied from ancient times in various civilizations.

The book by Elena Deza and Michel Marie Deza called ‘FIGURATE NUMBERS’, a 440 page book discusses the many theorems and results of plane figurate numbers, space figurate numbers, multidimensional figurate numbers and the theorems which I have listed below to pique your curiosity.

Pierre de Fermat studied the figurate numbers, conjecturing the Fermat polygonal number theorem in 1636. It was published in Fermat’s edition of Diaphantus’s book after his death in 1670. In his theorem, Fermat proposed that for any $k \geq 3$, every whole number can be expressed as the sum of at most k, k-gonal numbers. Even though he claimed to have proved the theorem, no one has ever found his proof.

After him Euler studied the topic and was unable to prove Fermat's polygonal theorem, but he left partial results which were subsequently used by Lagrange.

Lagrange would use Euler's ideas to complete the proof of the polygonal number theorem when $k=4$ and this case is now called Lagrange's four-square theorem.

In 1796, Gauss proved the case $k=3$, i.e., that every natural number is a sum of at most 3 triangular numbers. According to wikipedia, this is sometimes called the Eureka Theorem because Gauss wrote “EUREKA! $n = \Delta + \Delta + \Delta$” in his diary the day he proved it. It is also an immediate consequence of another theorem that was published a year later: Legendre's three-square theorem, which is incidentally equivalent to Fermat's claim. Even though Legendre was the first to publish a proof, the case is credited to Gauss since the diary entry is dated earlier

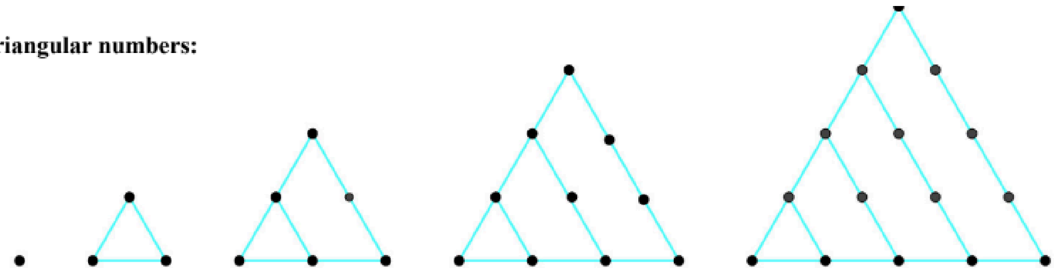
Augustin-Louis Cauchy, published in 1813 the first proof of the Polygonal Number Theorem in its entirety. Thus the theorem is sometimes called Fermat’s Polygonal Number Theorem, and sometimes called Cauchy’s Polygonal Number Theorem.

Many other mathematical formulations have deep roots in polygonal numbers and several famous theorems are based on these numbers. In particular, such natural numbers as perfect numbers, Mersenne numbers, Fermat numbers, Fibonacci and Lucas numbers, etc. are related to polygonal numbers. Furthermore, a modern application of polygonal numbers is seen in Pascal’s triangle and the binomial theorem

So let’s see what these figurate numbers mean, also known as a figural number, is a number that can be represented by a regular geometrical arrangement of equally spaced points along the sides. If the arrangement forms a regular polygon, the number is called a polygonal number.

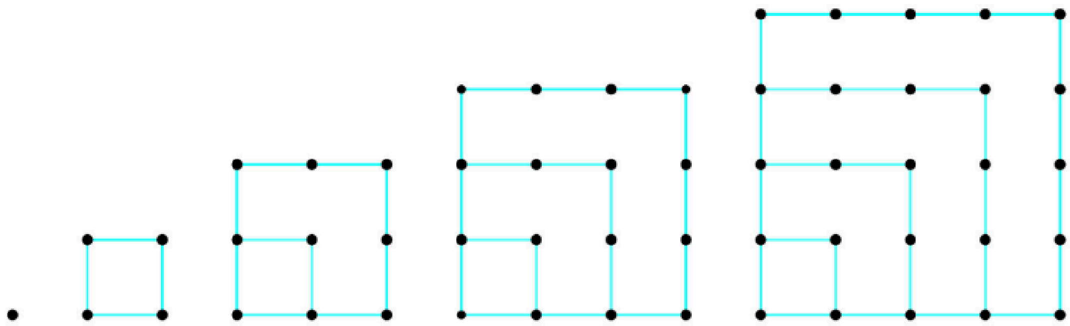
3-sided polygonal numbers are termed as triangular numbers,
4-sided polygonal numbers are termed as square numbers,
5-sided polygonal numbers are termed as pentagonal numbers,
6-sided polygonal numbers are termed as hexagonal numbers, ...
Let’s see polygonal numbers.

Triangular numbers:



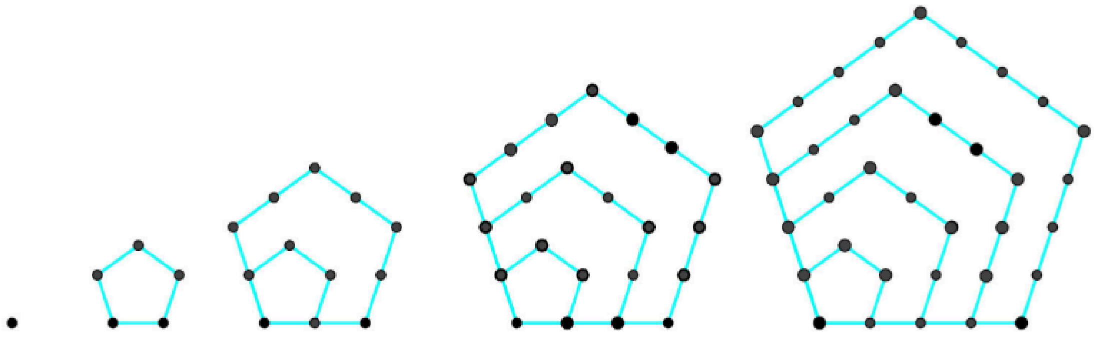
1st triangular number = 1, 2nd triangular number = 3, 3rd triangular number = 6
4th triangular number = 10, 5th triangular number = 15, …

Square numbers:



1st square number = 1, 2nd square number = 4, 3rd square number = 9,
4th square number = 16, 5th square number = 25, …

Pentagonal numbers:



1st pentagonal number = 1, 2nd pentagonal number = 5, 3rd pentagonal number = 12, 4th pentagonal number = 22, 5th pentagonal number = 35, …

Q. 1: Enlist first five hexagonal numbers (6-sided polygonal numbers).
Q. 2: Enlist first five septagonal numbers (7-sided polygonal numbers).
Q. 3: Enlist first five octagonal numbers (8-sided polygonal numbers).
Q. 4: Enlist first five nanagonal numbers (9-sided polygonal numbers).
Q. 5: Enlist first five decagonal numbers (10-sided polygonal numbers).
Q. 6: Give a formula for the nth triangular number.
Q. 7: Give a formula for the nth square number.
Q. 8: Give a formula for the nth pentagonal number.
Q. 9: Give a formula for the nth hexagonal number.
Q. 10: Give a formula for the nth septagonal number.
Q. 11: Give a formula for the nth octagonal number.
Q. 12: Give a formula for the nth nonagonal number.
Q. 13: Give a formula for the nth decagonal number.
Q. 14: What are the first five r-gonal numbers? Write them.
Q. 15: In general, give a formula for the nth r-gonal number.
Q. 16: Prove that every hexagonal number is a triangular number
Q. 17: Is there a relation between octagonal number and triangular number?
Q. 18: Derive the equation for finding the square triangular numbers
Q. 19: Derive the equation for finding the pentagonal triangular number
Q. 20: Derive the equation for finding the pentagonal square number