

# Many-body localisation: new results for weak interactions

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Phys. Rev. Lett. **133**, 116502 (2024) + **WIP**

[Jeanne Colbois](#) (Toulouse → Singapore → Grenoble)

[Nicolas Laflorencie](#) (Toulouse)

[Asmi Haldar](#) (Toulouse)



[Pdf of the talk](#)  
[Pdf of paper](#)  
[List of References](#)



SCAN ME



FONDATION  
SIMONE ET CINO  
**DEL DUCA**  
INSTITUT DE FRANCE

Indo-French Workshop in classical and quantum  
dynamics in out of equilibrium systems, ICTS  
Bangalore, December 2024

# Plan of the talk

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Quick Recap on Many-Body Localization (MBL) : motivations & hallmarks

Status of MBL : Controversies, and current views

Introduction

The rest of the talk: Stepping away from the « strong » interaction regime

Results

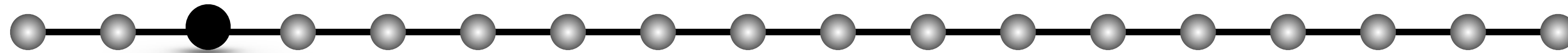
Related : Correlations as markers of route to ergodicity

Nicolas' talk  
Friday

# From single to many: Anderson to Many-Body Localization

Single particle in a  
random potential

$$\mathcal{H} = \sum_i \left[ t \left( c_i^\dagger c_{i+1} + \text{hc} \right) + \epsilon_i n_i \right]$$



# From single to many: Anderson to Many-Body Localization

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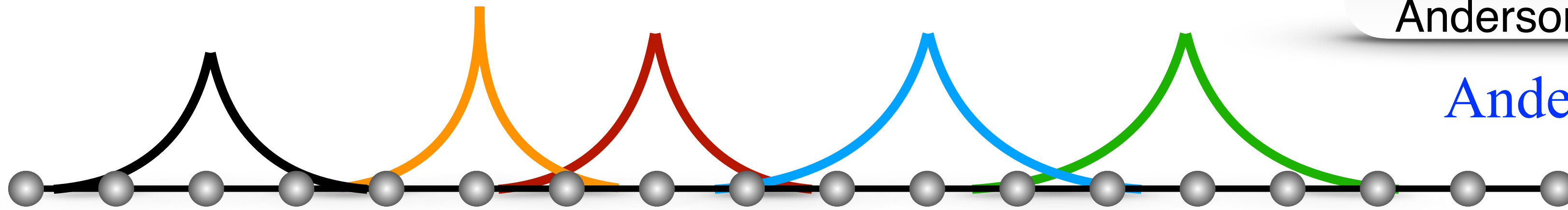
$$\mathcal{H} = \sum_i \left[ t \left( c_i^\dagger c_{i+1} + \text{hc} \right) + \epsilon_i n_i \right]$$
$$= \sum_m \tilde{\epsilon}_m b_m^\dagger b_m$$

$$b_m = \sum_i \phi_i^m c_i$$

$$|\phi_i^{(m)}|^2 \sim \exp\left(-\frac{|i - i_0^{(m)}|}{\xi_m}\right)$$

Anderson localized orbitals

Anderson localization



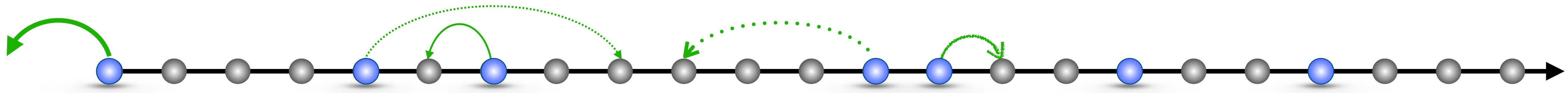


# From single to many: Anderson to Many-Body Localization

Many interacting particles in a random potential

$$\mathcal{H} = \sum_i \left[ t \left( c_i^\dagger c_{i+1} + \text{hc} \right) + \epsilon_i n_i + V n_i n_{i+1} \right]$$

$$= \sum_m \tilde{\epsilon}_m b_m^\dagger b_m + \sum_{j,k,l,m} V_{jklm} b_j^\dagger b_k^\dagger b_l b_m$$



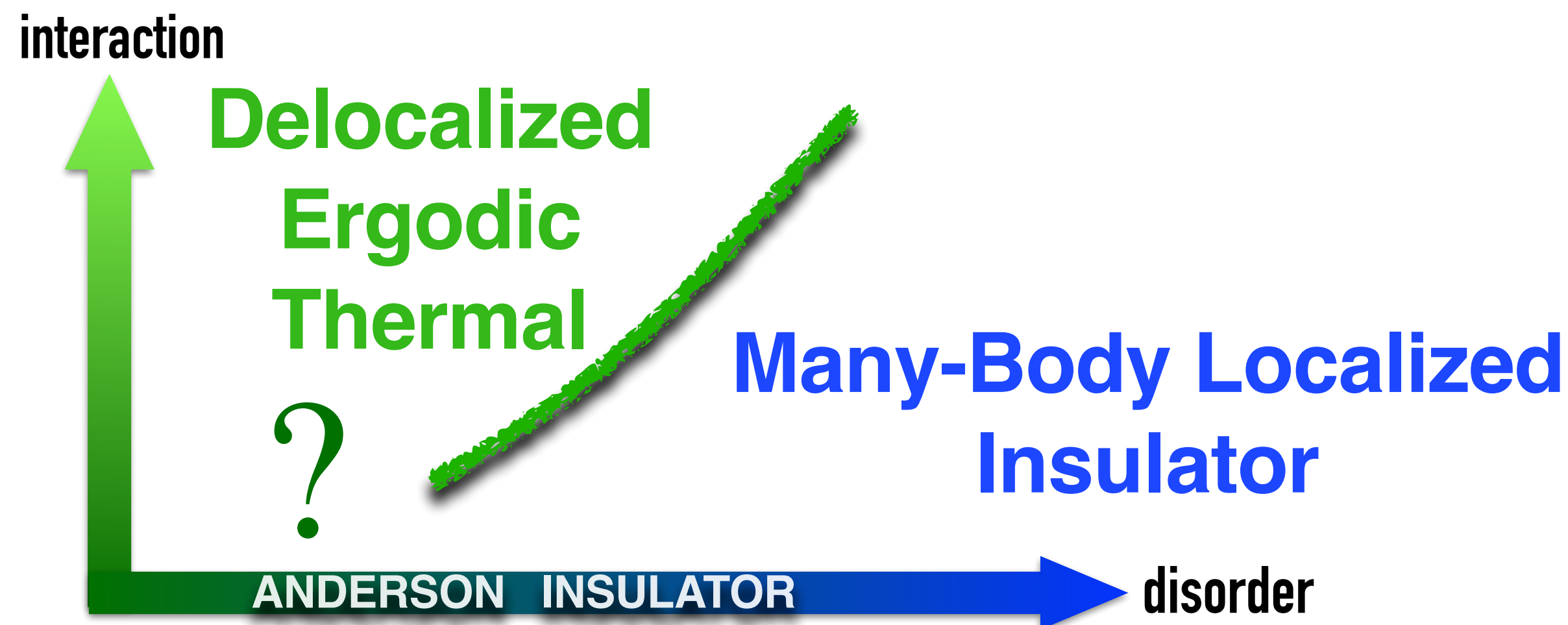
Intuition: Interactions favour delocalisation / thermalization

Localization can survive interactions :  
**Many-Body Localized (MBL) phase & Ergodic to MBL phase transition**

Gornyi, Mirlin, Polyakov (2005)

Basko, Aleiner, Altshuler (2006)

Pal, Huse (2010)

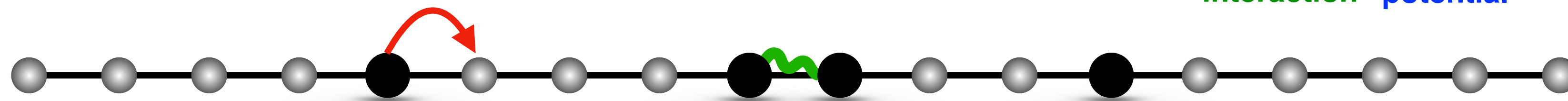


# Minimal Spin-chain model for MBL

Interacting electrons  
in 1d

$$H = \sum_i \left[ t \left( c_i^\dagger c_{i+1} + hc \right) + V n_i n_{i+1} + \epsilon_i n_i \right]$$

quantum tunneling  
short-range interaction    random potential



Only existing symmetry  
(neither needed nor impeding for  
MBL):

Particle number conservation

$$\Delta = V$$

Eq. : XXZ spin chain  
in random field

$$H = \sum_i \left[ \frac{t}{2} \left( S_i^+ S_{i+1}^- + hc \right) + \Delta S_i^z S_{i+1}^z + h_i S_i^z \right] + C$$

spin-flip    Ising interaction    random magnetic field    Total magnetization  $S^z$  conservation

Special case : Random-field Heisenberg spin chain     $\Delta = 1$

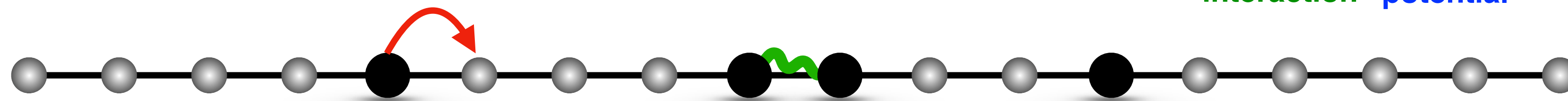
$$h_i \in [-h, h]$$

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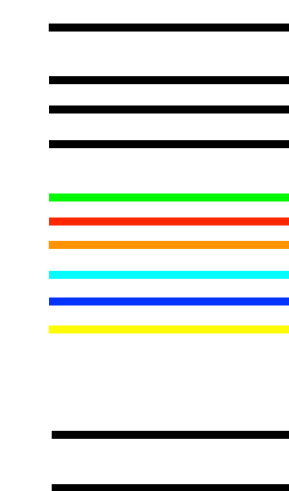
- Expect two family of eigenstates

Ergodic, Thermal states



Eigenstates look all the same (~ Random Matrix Theory)

Many-Body Localized states



Eigenstates all different (do not “speak” to each other), behave as ground-states

Disorder  $h$

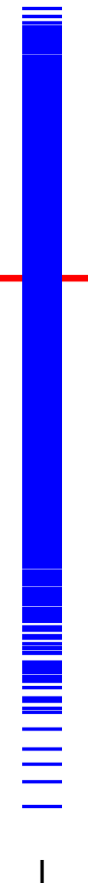


# Specs of Many-Body Localization

## 1. Spectral statistics

$$\epsilon = (E - E_{\min}) / (E_{\max} - E_{\min})$$

Many-body spectrum



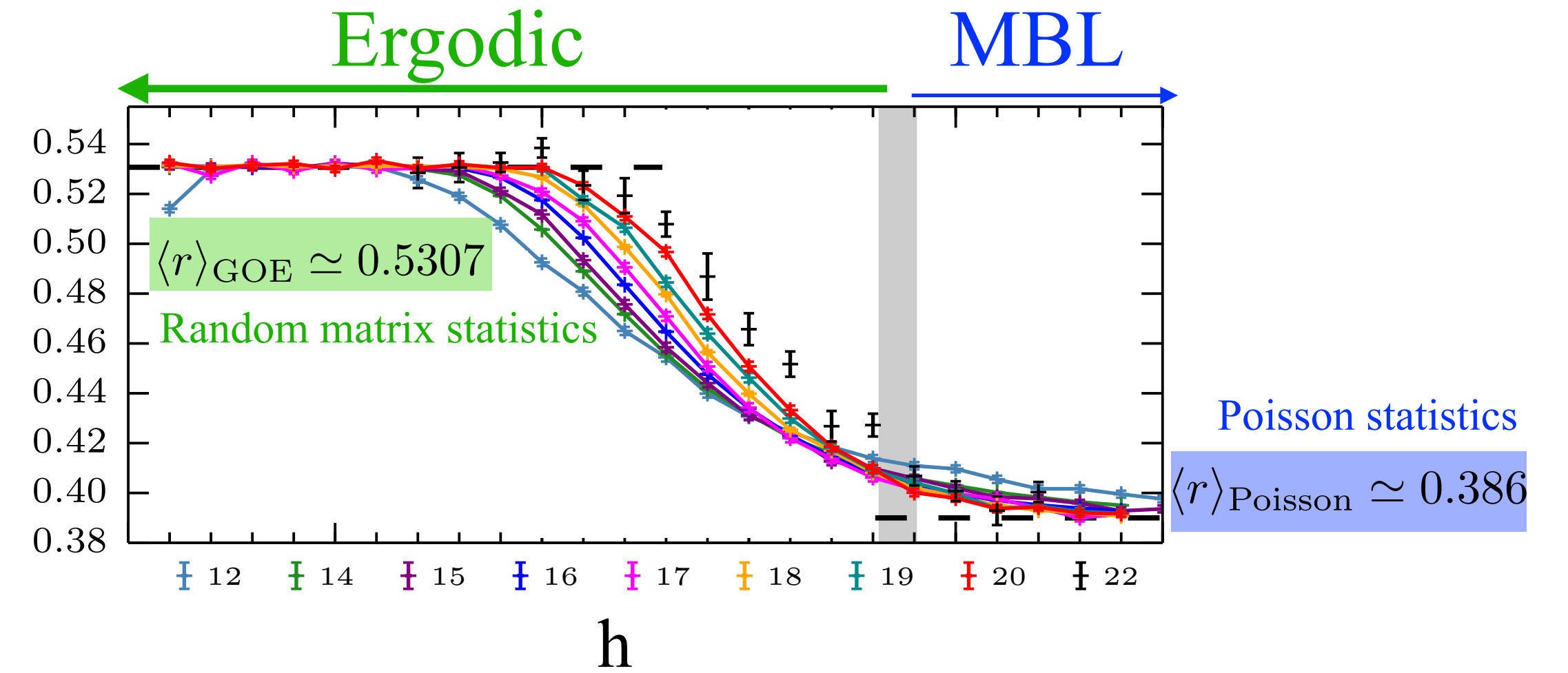
Level spacings  $s_n = E_n - E_{n-1}$

Ratio of consecutive gaps

$$r = \min(s_n, s_{n+1}) / \max(s_n, s_{n+1})$$

Luitz *et al.* (2015)

$\langle r \rangle$





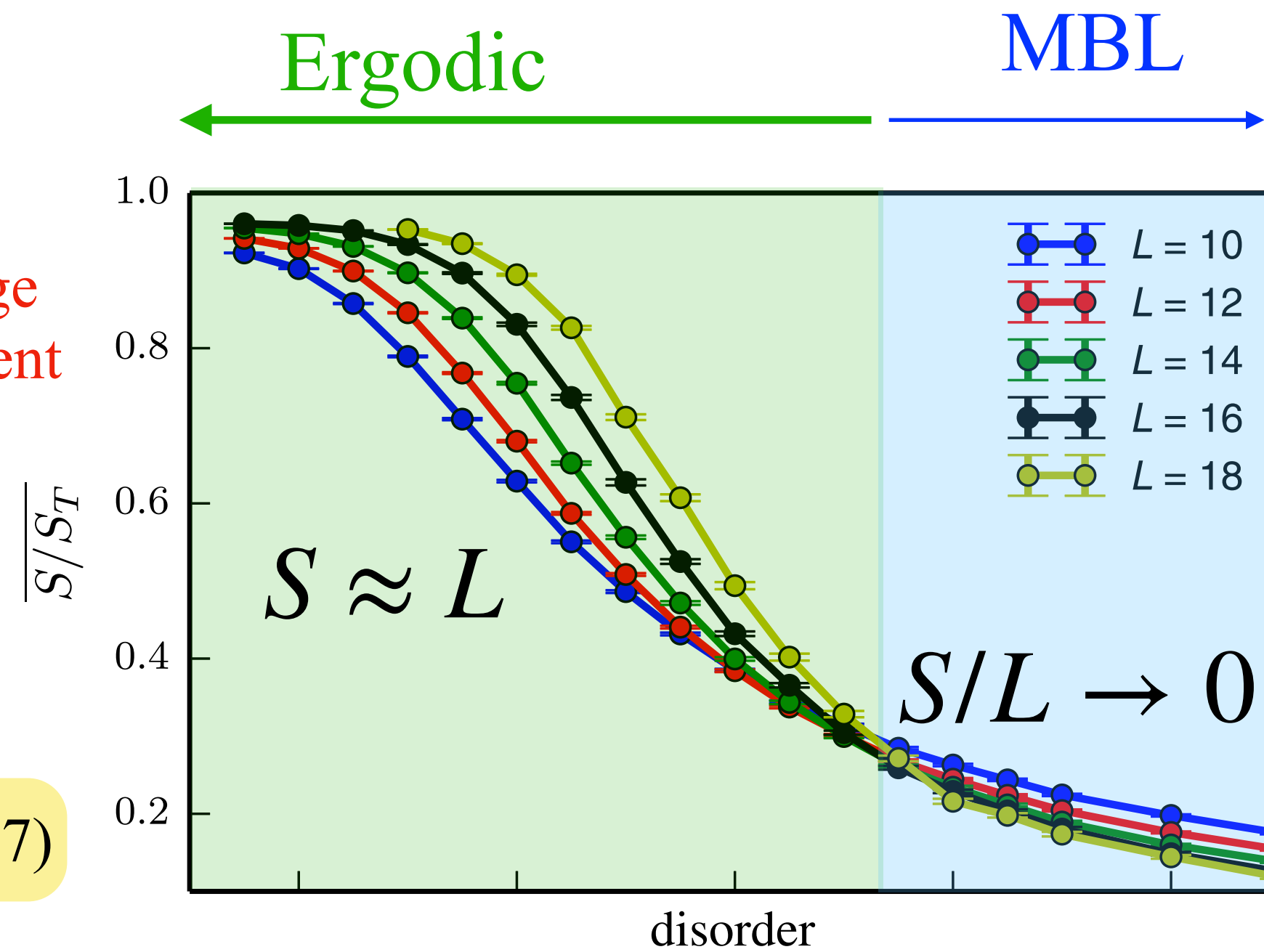
# Specs of Many-Body Localization

## 1. Spectral statistics

## 2. Entanglement : Volume vs. Area law

Eigenstates have large  
(volume) entanglement

Khemani *et al.* (2017)



MBL eigenstates have low entanglement  
(~ close to product states)

# Specs of Many-Body Localization

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**1. Spectral statistics**

**2. Entanglement : Volume vs. Area law**

**3. Out of equilibrium dynamics**

**Dynamics after quench :**  $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$

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### Memory of initial state

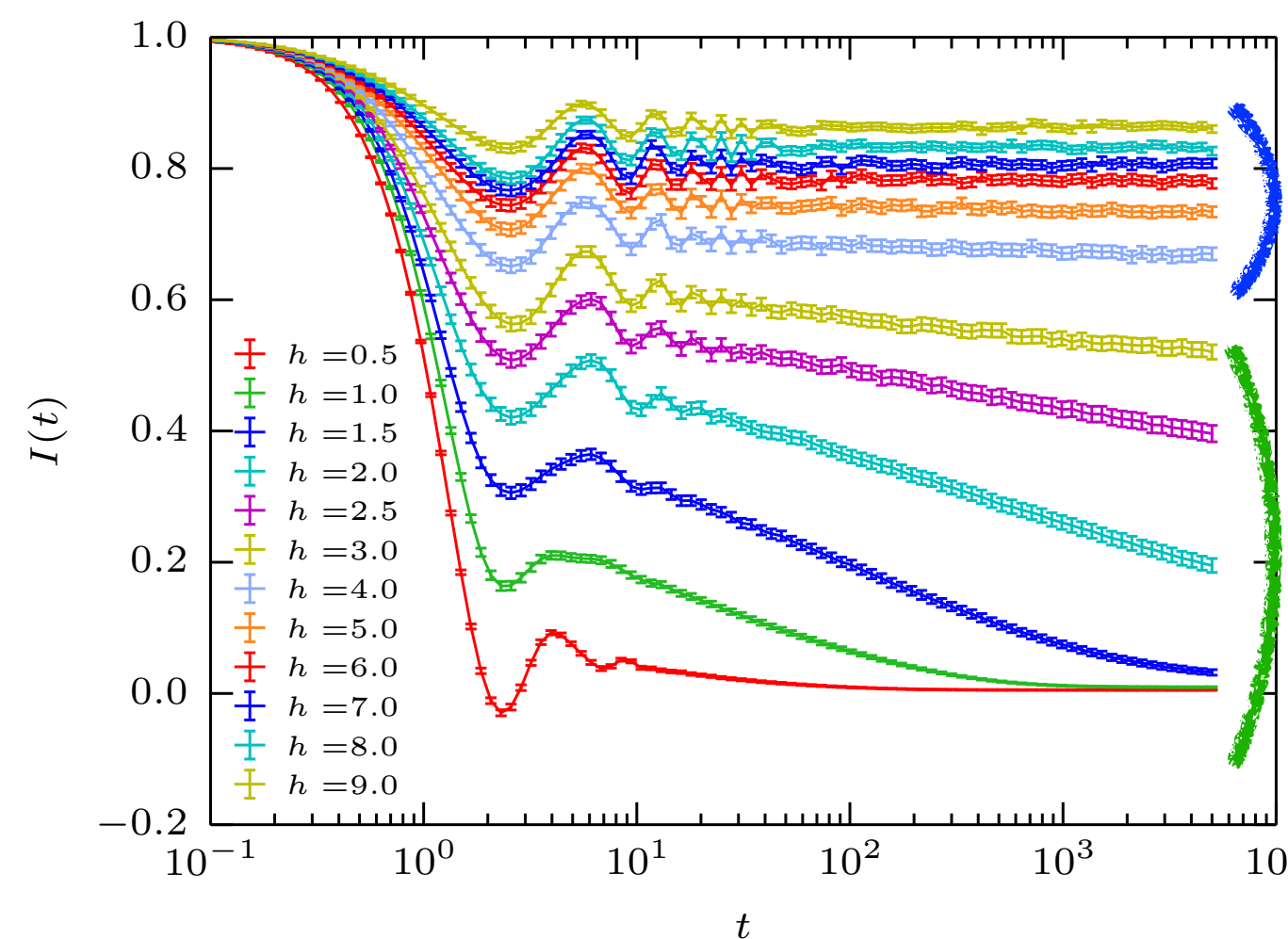
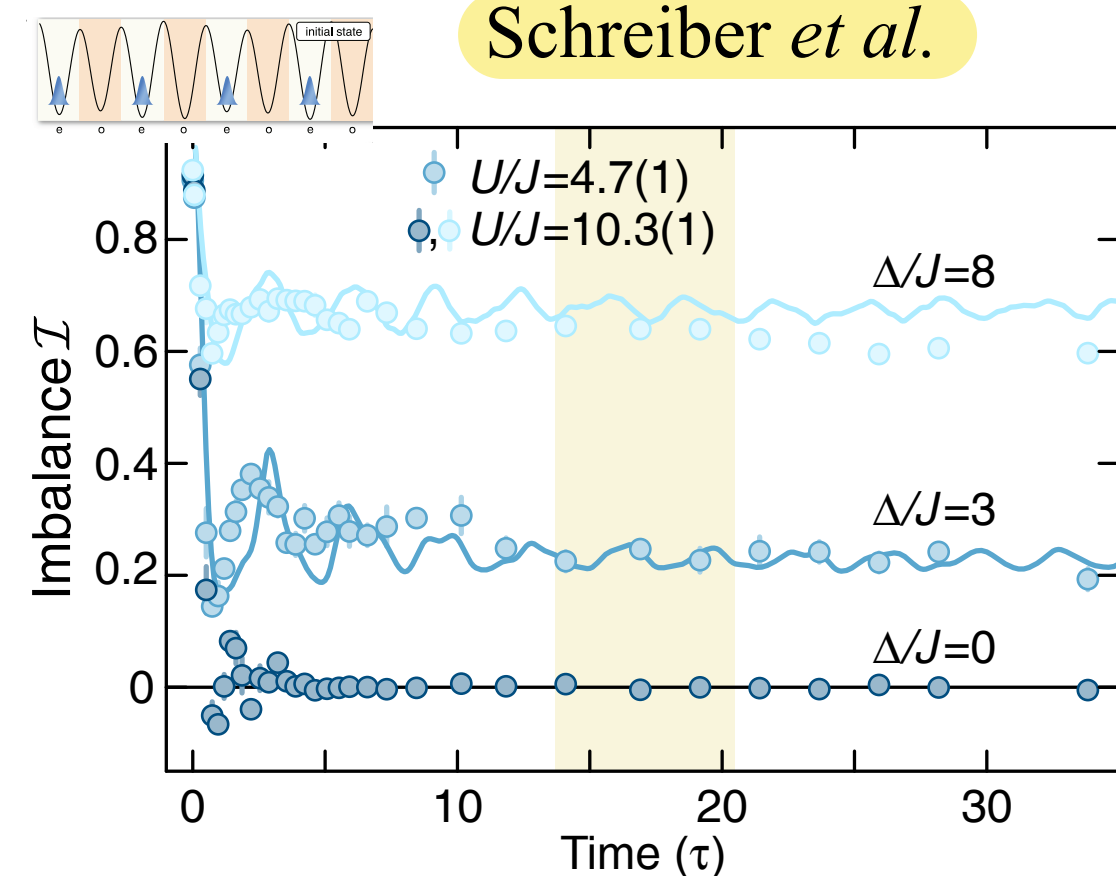
#### ► Experiments

#### ► Numerics:

Schreiber *et al.*

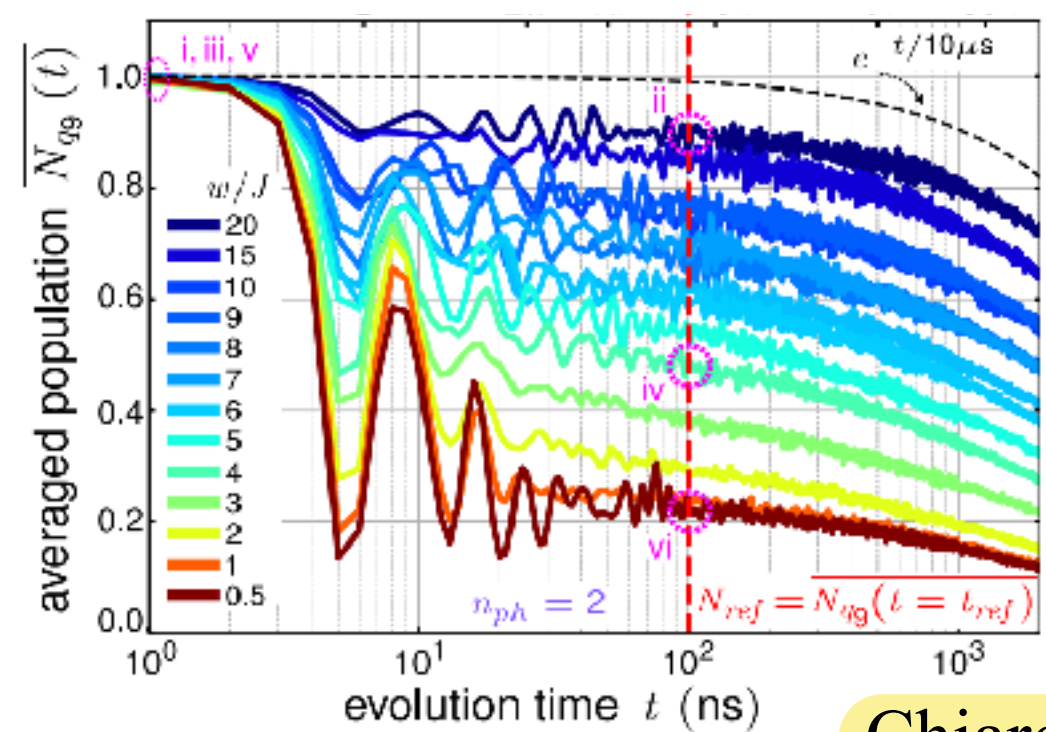
$$|\Psi_0\rangle = |\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\cdots\rangle$$

$$I(t) = \frac{4}{L} \sum_i \langle \Psi_0 | S_i^z S_i^z(t) | \Psi_0 \rangle$$



**Strong disorder: MBL**  
Memory of the initial state  
even at infinite time

**Weak disorder: Ergodic**  
No Memory  
converges to thermal ensemble



Chiaro *et al.*

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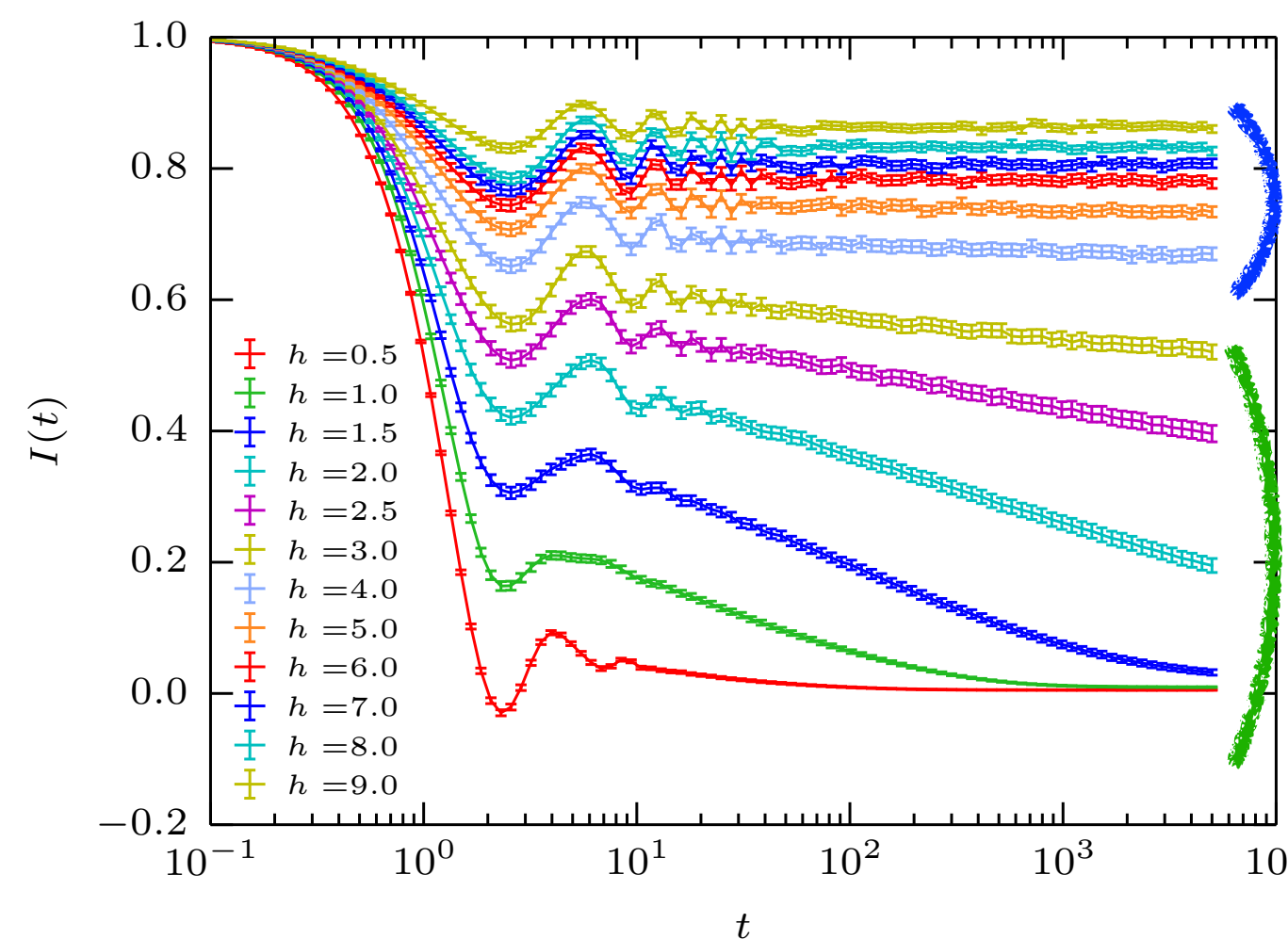
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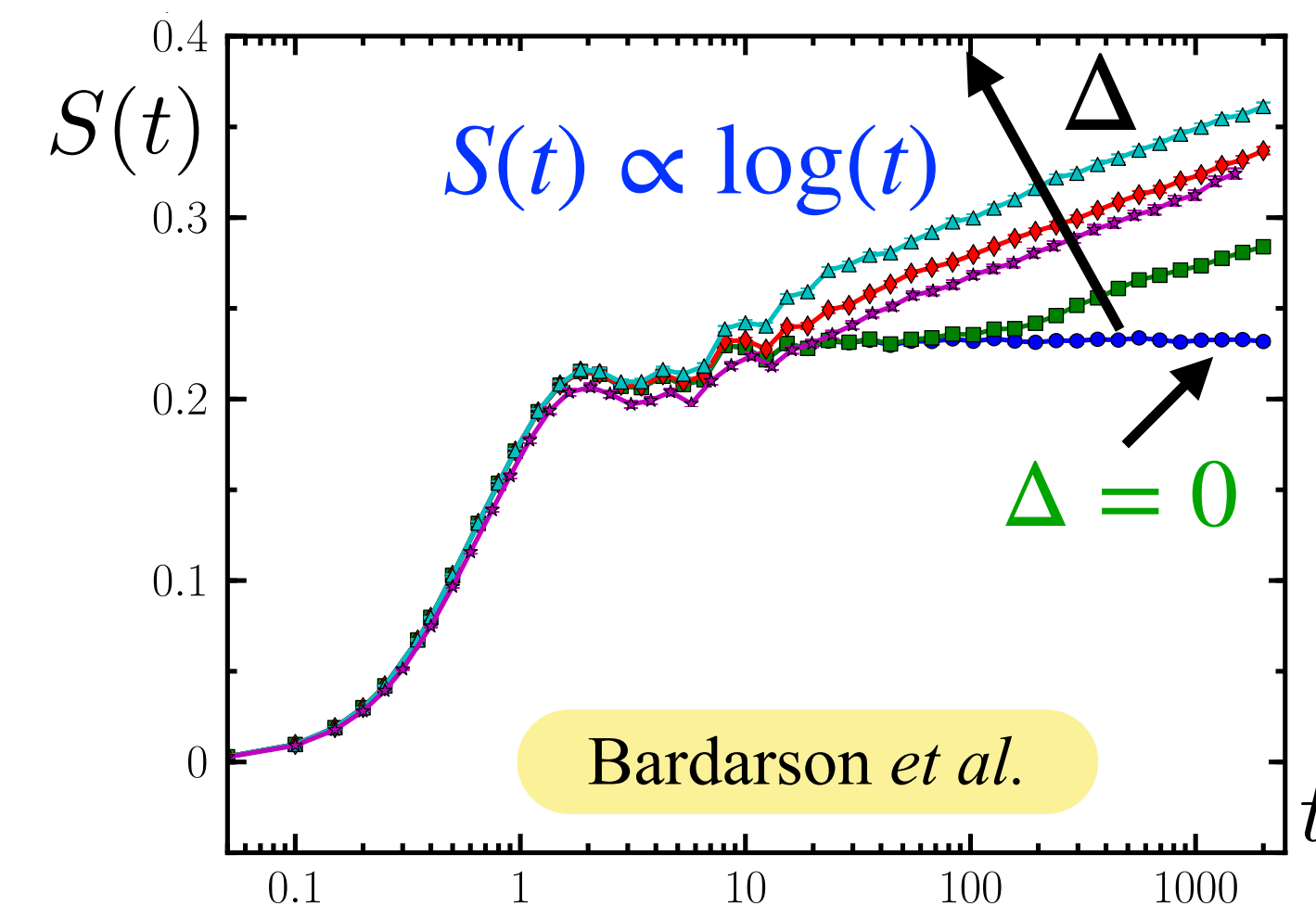


**Strong disorder: MBL**  
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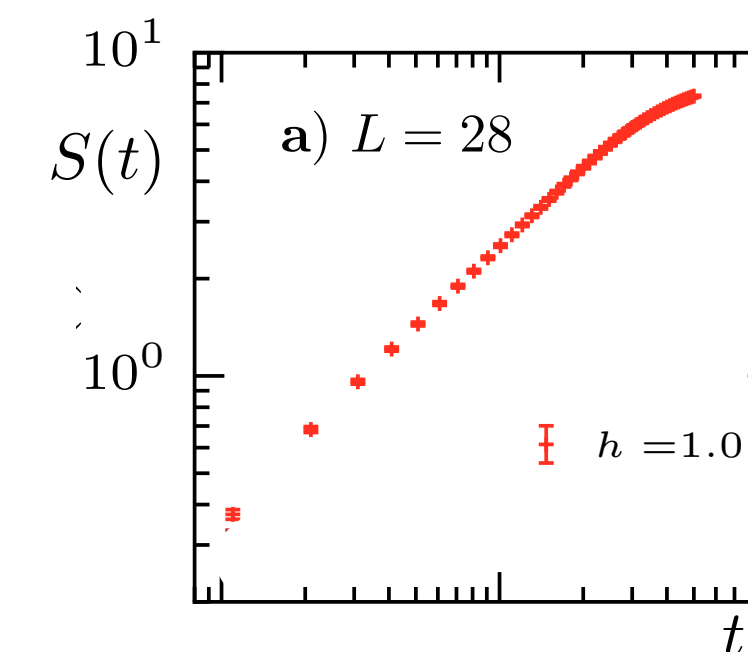
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### Slow (log) growth of information

#### MBL: Logarithmic spread

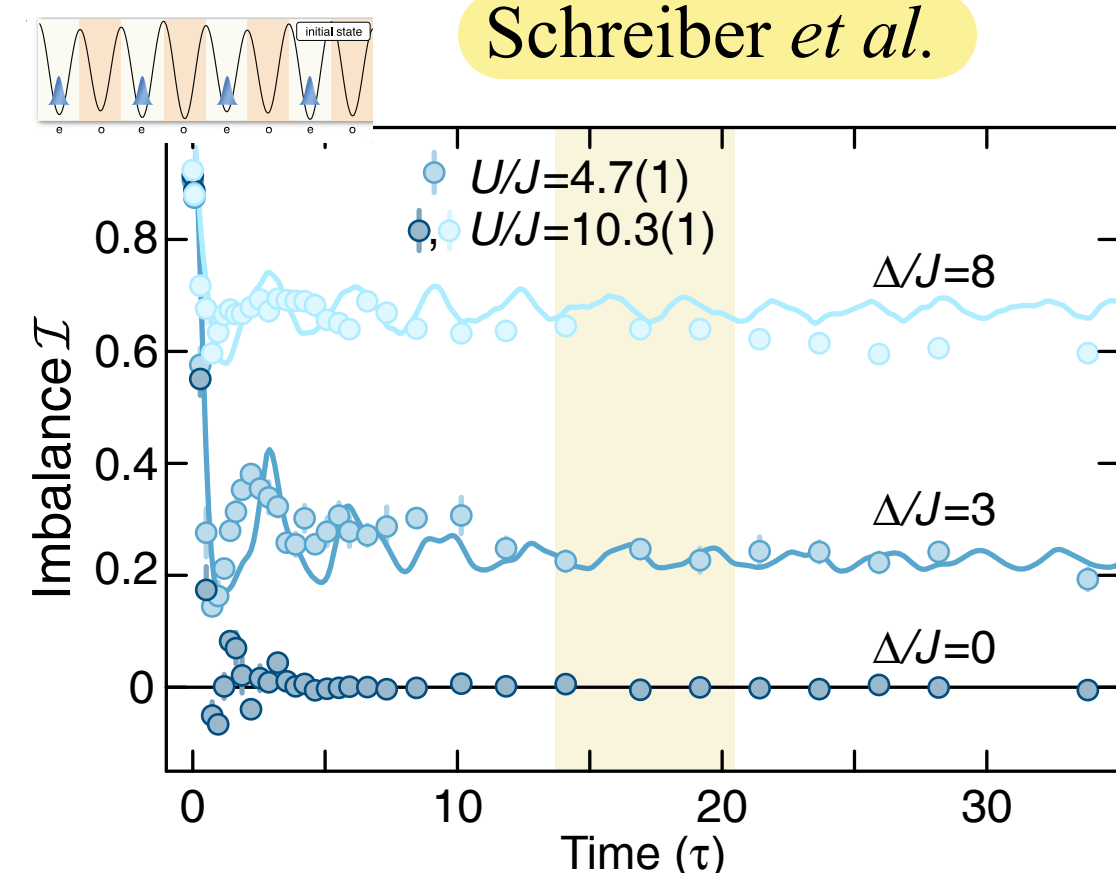


Bardarson *et al.*

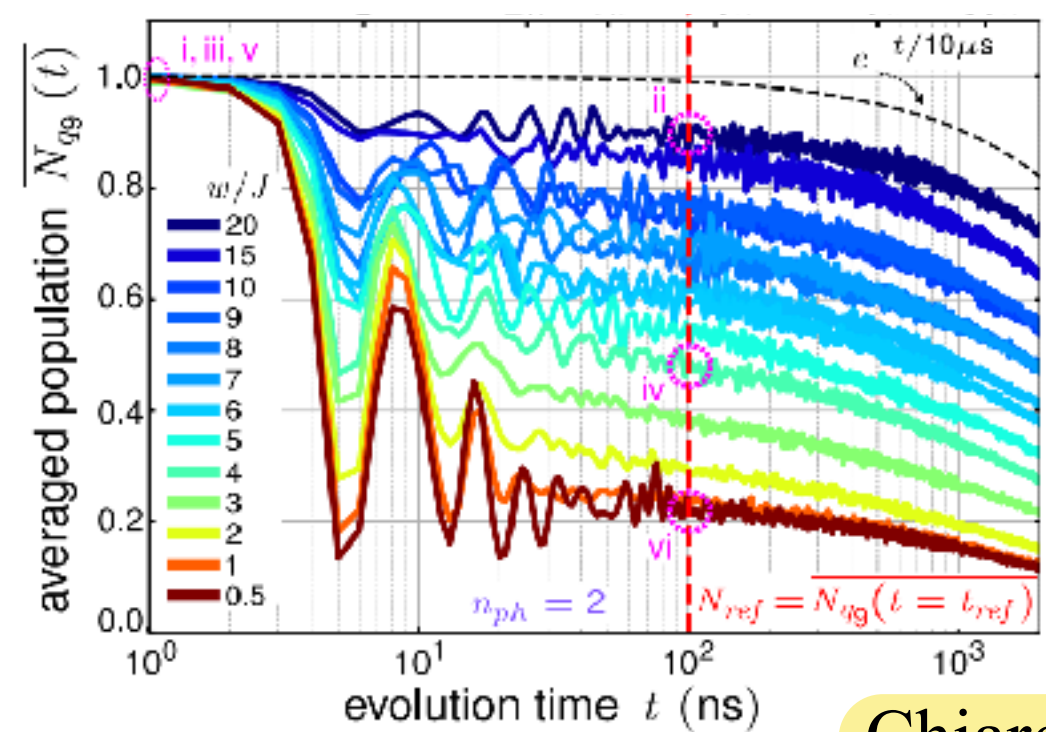


$S(t) \propto t$

**Ergodic**  
**Ballistic spread**



Schreiber *et al.*



Chiaro *et al.*



# Many-Body Localization ( $\sim$ 2008-2018)

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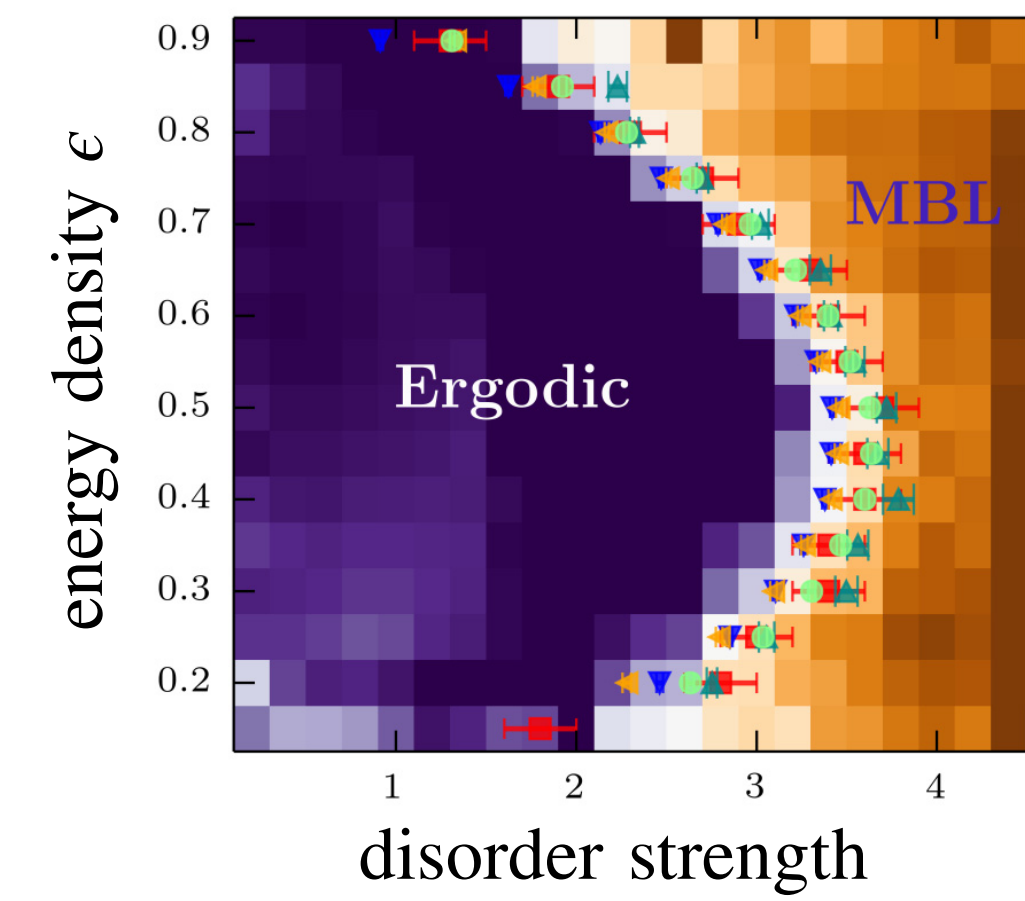
# Many-Body Localization (~2008-2018)

Numerics

After a lot of work the  
MBL problem seemed  
well understood (\*) (\*\*)

(\*) at least in 1d

(\*\*) albeit **not** the Ergodic /  
MBL phase transition



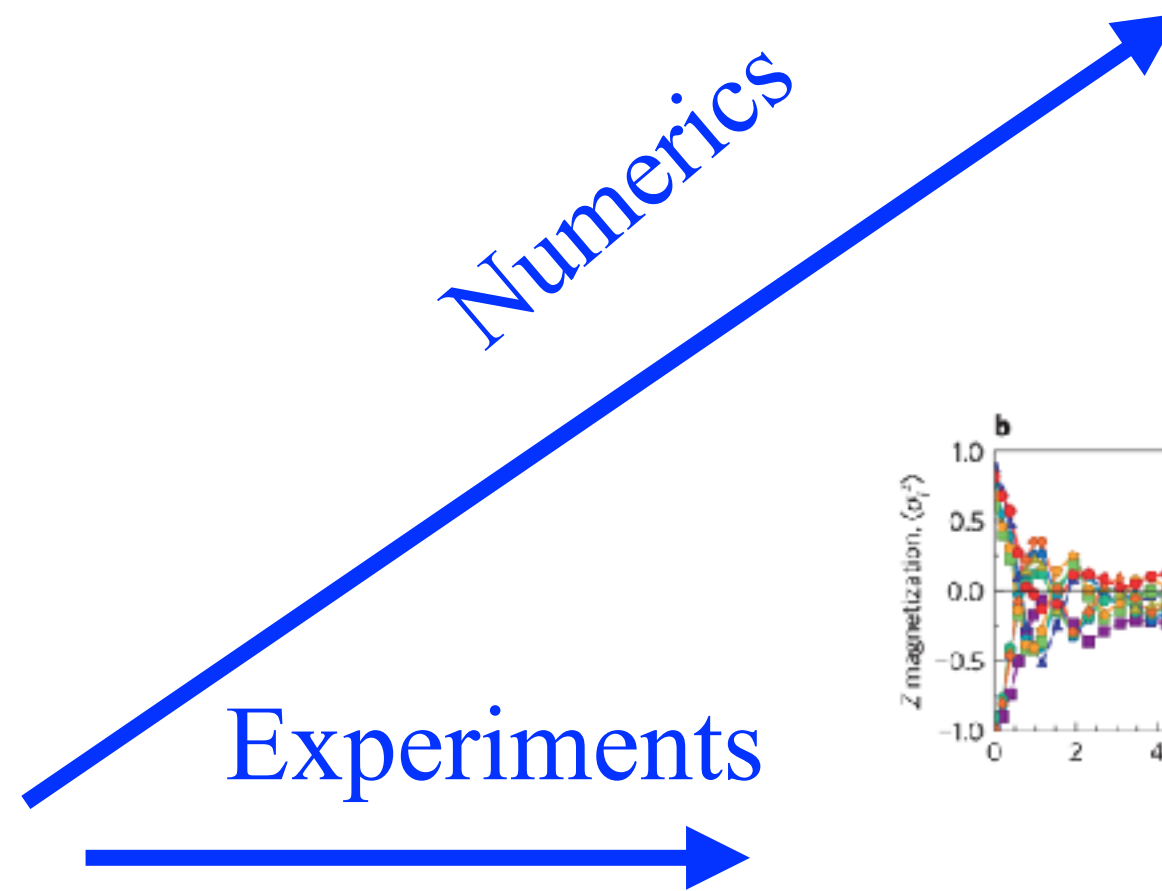
Luitz *et al.* (2015)

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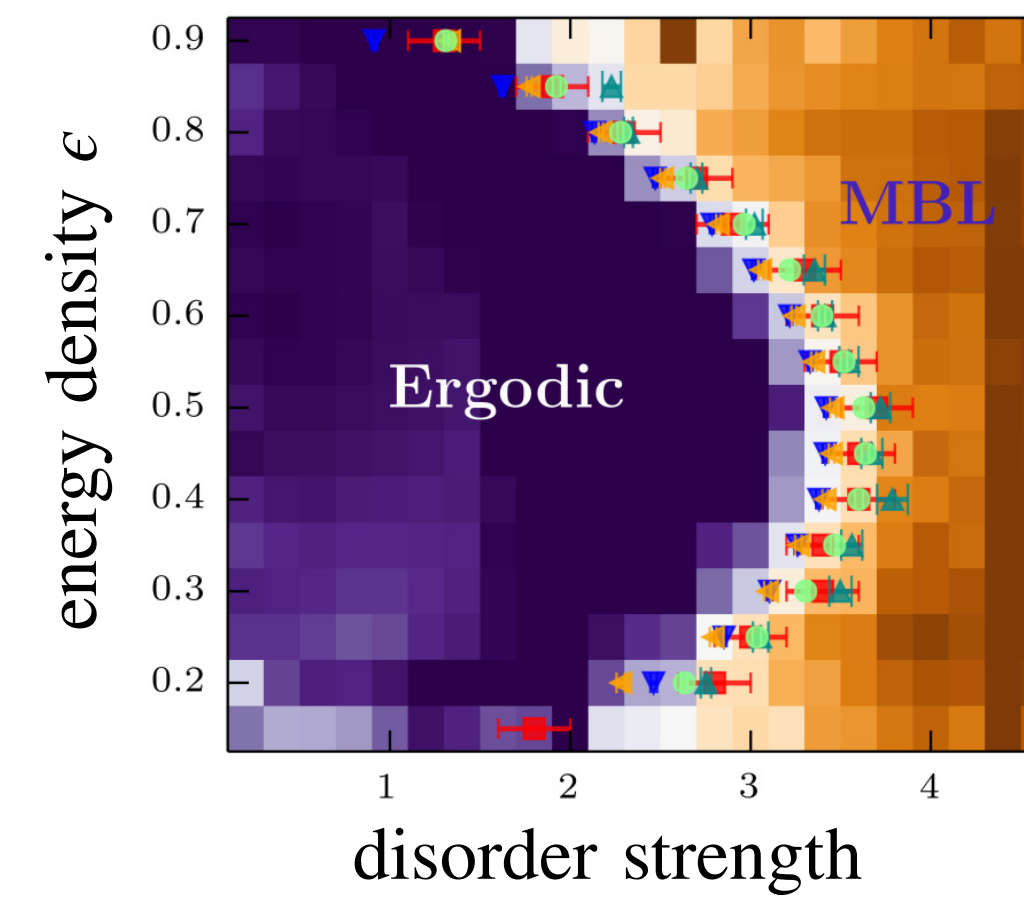
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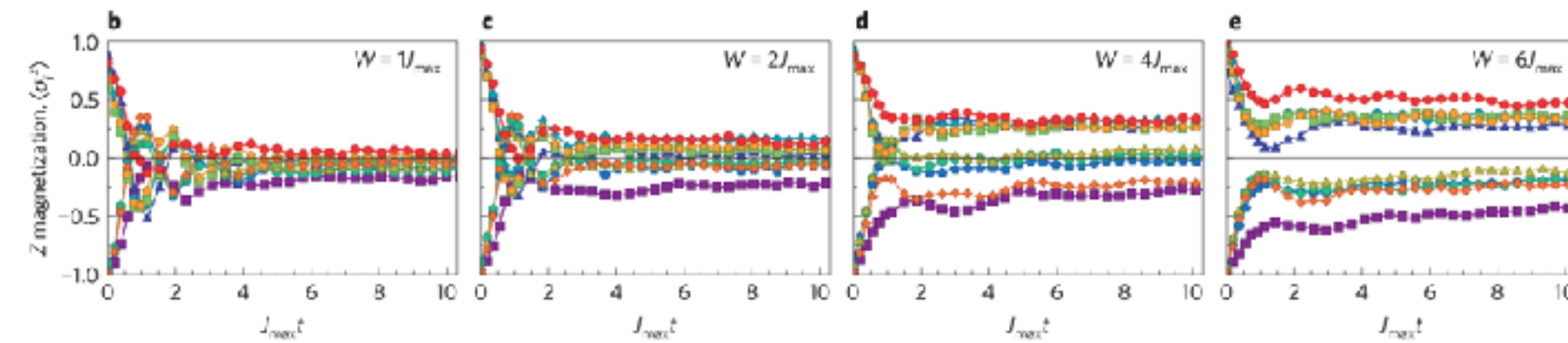
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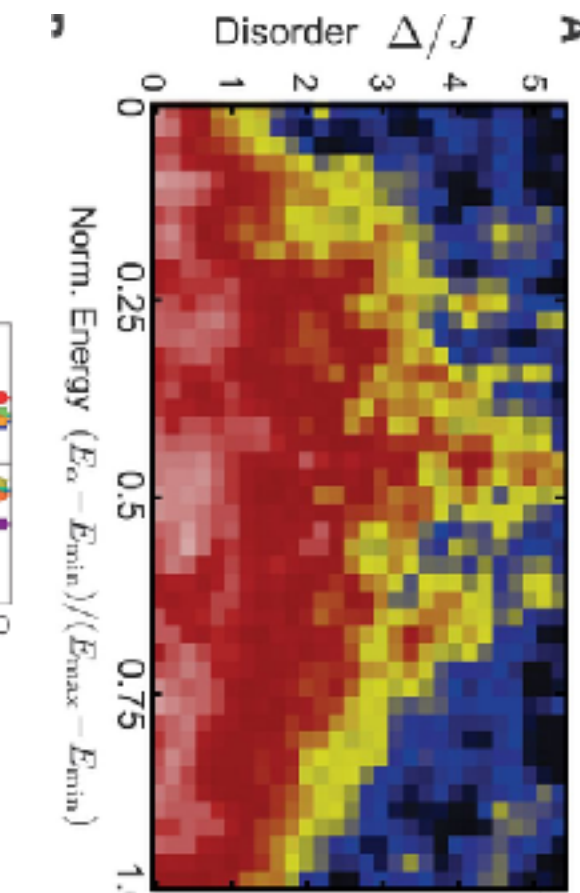
Cold-atom, trapped ions, superconducting qubits ...



Luitz *et al.* (2015)



Smith *et al.* (2017)



Roushan *et al.* (2017)

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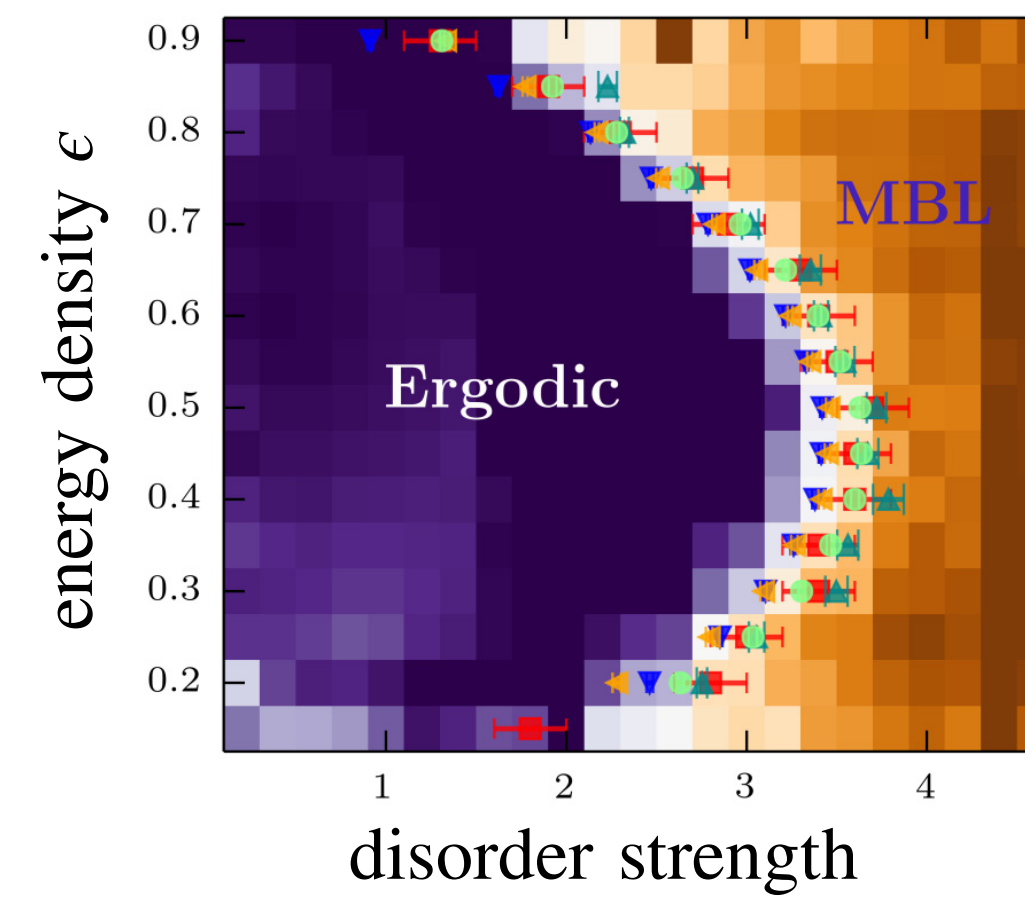
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Experiments

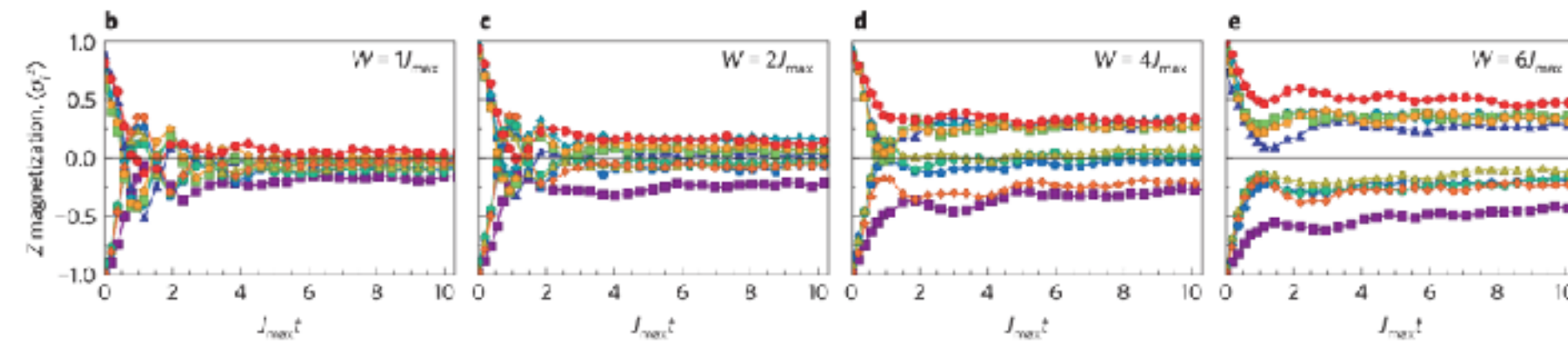
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Phenomenology

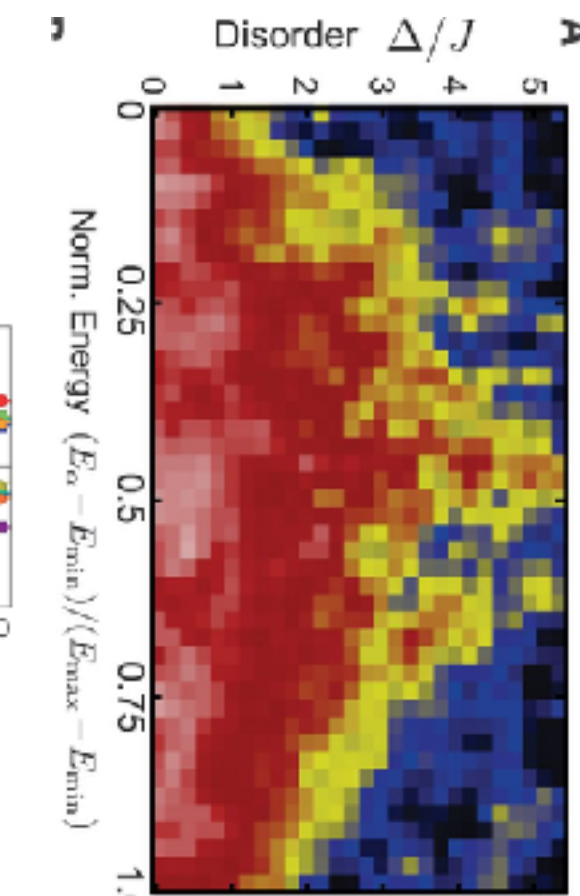
Oganesyan, Huse, Serbyn, Abanin, Papić



Luitz *et al.* (2015)



Smith *et al.* (2017)



Roushan *et al.* (2017)

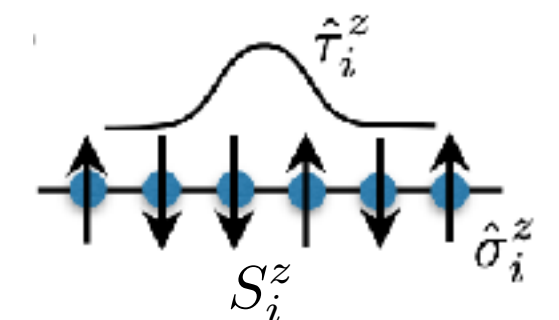
## Local integral of motions (liom's)

$$H_{\text{MBL}} = - \sum_i h_i \tau_i^z + \sum_{i,j} J_{ij} \tau_i^z \tau_j^z + \sum_{i,j,k} J_{ijk} \tau_i^z \tau_j^z \tau_k^z + \dots$$

“Fixed point”  
MBL Hamiltonian

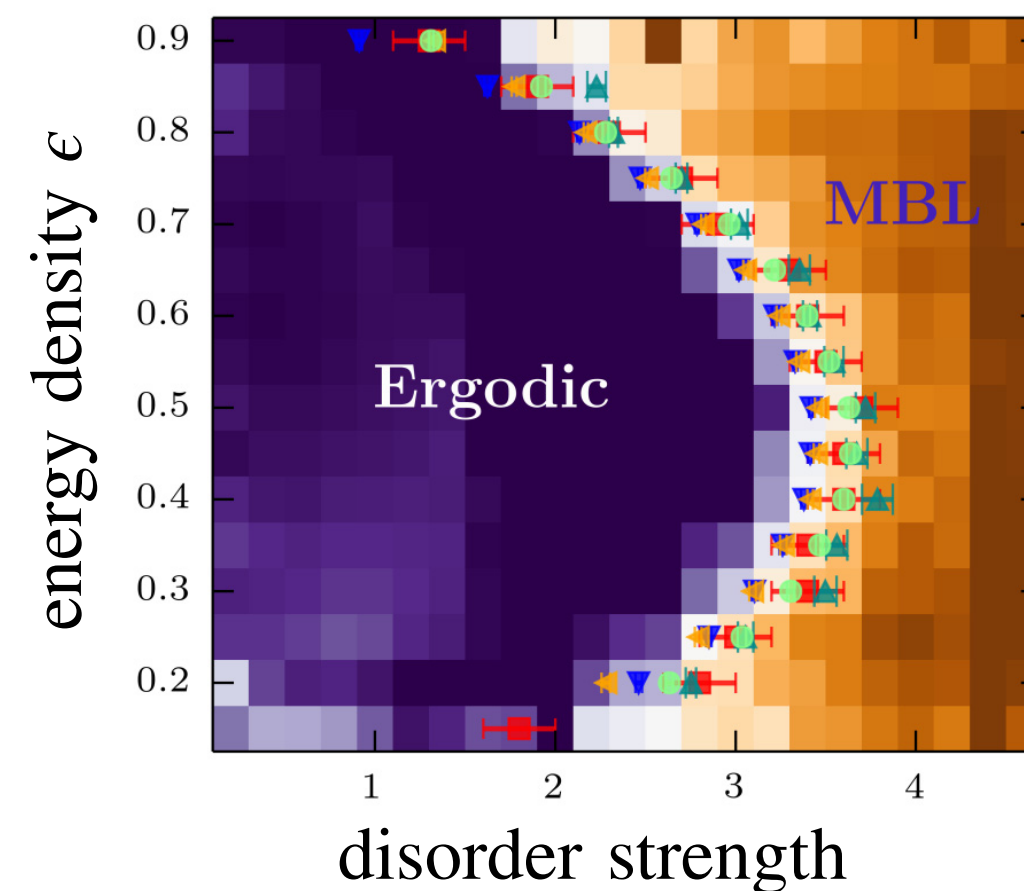
$$[H, \tau_i^z] = 0$$

$$\tau_i^z = \tilde{Z} S_i^z + \sum_{r,\alpha} \exp(-r/\xi) S_{i+r}^\alpha$$

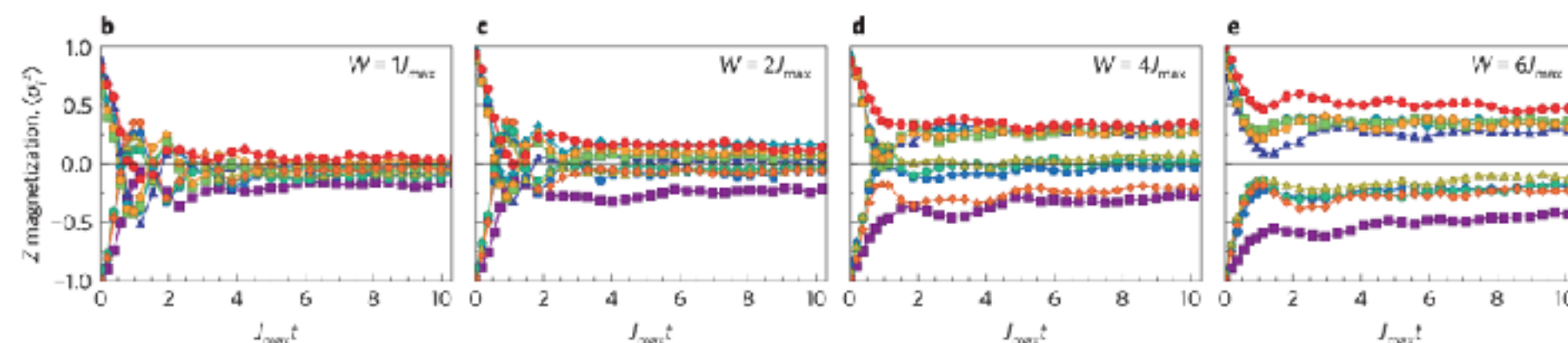




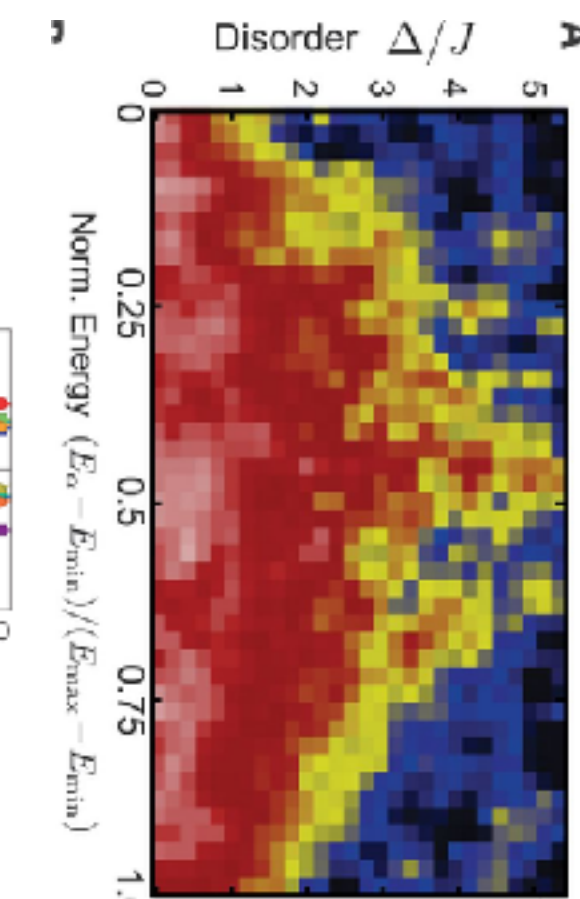
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Numerics

Analytics

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Oganesyan, Huse, Serbyn, Abanin, Papić

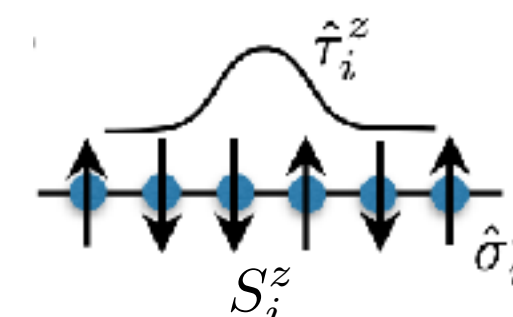
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MBL Hamiltonian

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Imbrie, De Roeck *et al.* Proof (?) for one model

# MBL : really ? (2019-)

EPL, 128 (2019) 67003  
doi: 10.1209/0295-5075/128/67003

## Can we study the many-body localisation transition?

R. K. PANDA<sup>1,2</sup>, A. SCARDICCHIO<sup>1,3</sup>, M. SCHULZ<sup>1</sup>, S. R. TAYLOR<sup>1(a)</sup> and M. ŽNIDARIČ<sup>4</sup>

PHYSICAL REVIEW E 104, 054105 (2021)

## Dynamical obstruction to localization in a disordered spin chain

Dries Sels<sup>1,2</sup> and Anatoli Polkovnikov<sup>3</sup>

PHYSICAL REVIEW E 102, 062144 (2020)

## Quantum chaos challenges many-body localization

Jan Šuntajs<sup>1</sup>, Janez Bonča<sup>2,1</sup>, Tomaž Prosen<sup>2</sup> and Lev Vidmar<sup>1,2</sup>

PHYSICAL REVIEW B 105, 224203 (2022)

## Challenges to observation of many-body localization

Piotr Sierant<sup>1,2</sup> and Jakub Zakrzewski<sup>2,3,\*</sup>

PHYSICAL REVIEW B 105, 174205 (2022)

Editors' Suggestion

## Avalanches and many-body resonances in many-body localized systems

Alan Morningstar<sup>1</sup>, Luis Colmenarez<sup>2</sup>, Vedika Khemani<sup>3</sup>, David J. Luitz<sup>4,2</sup> and David A. Huse<sup>1,5</sup>



Annals of Physics  
Volume 427, April 2021, 168415



## Distinguishing localization from chaos: Challenges in finite-size systems

D.A. Abanin<sup>a,1</sup>, J.H. Bardarson<sup>b,1</sup>, G. De Tomasi<sup>c,1</sup>, S. Gopalakrishnan<sup>d,e,1</sup>, V. Khemani<sup>f,1</sup>, S.A. Parameswaran<sup>g,1</sup>, F. Pollmann<sup>h,1,1</sup>, A.C. Potter<sup>j,1</sup>, M. Serbyn<sup>k,1</sup>, R. Vasseur<sup>l,1</sup>

1. Finite-size effects in numerics

2. Anomalously weak relaxation but not MBL?

3. Avalanches & Resonances

## The internal clock of many-body delocalization

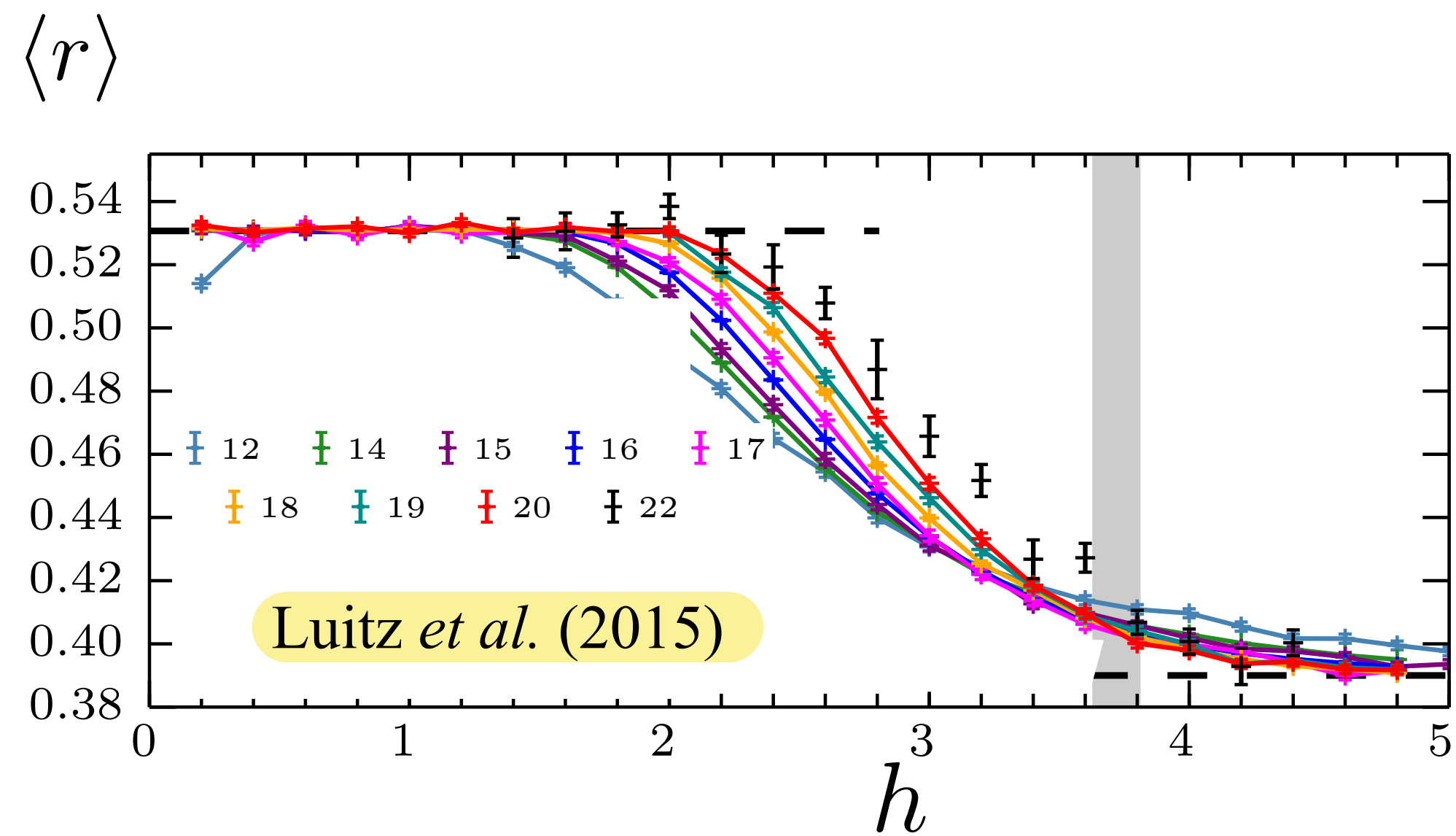
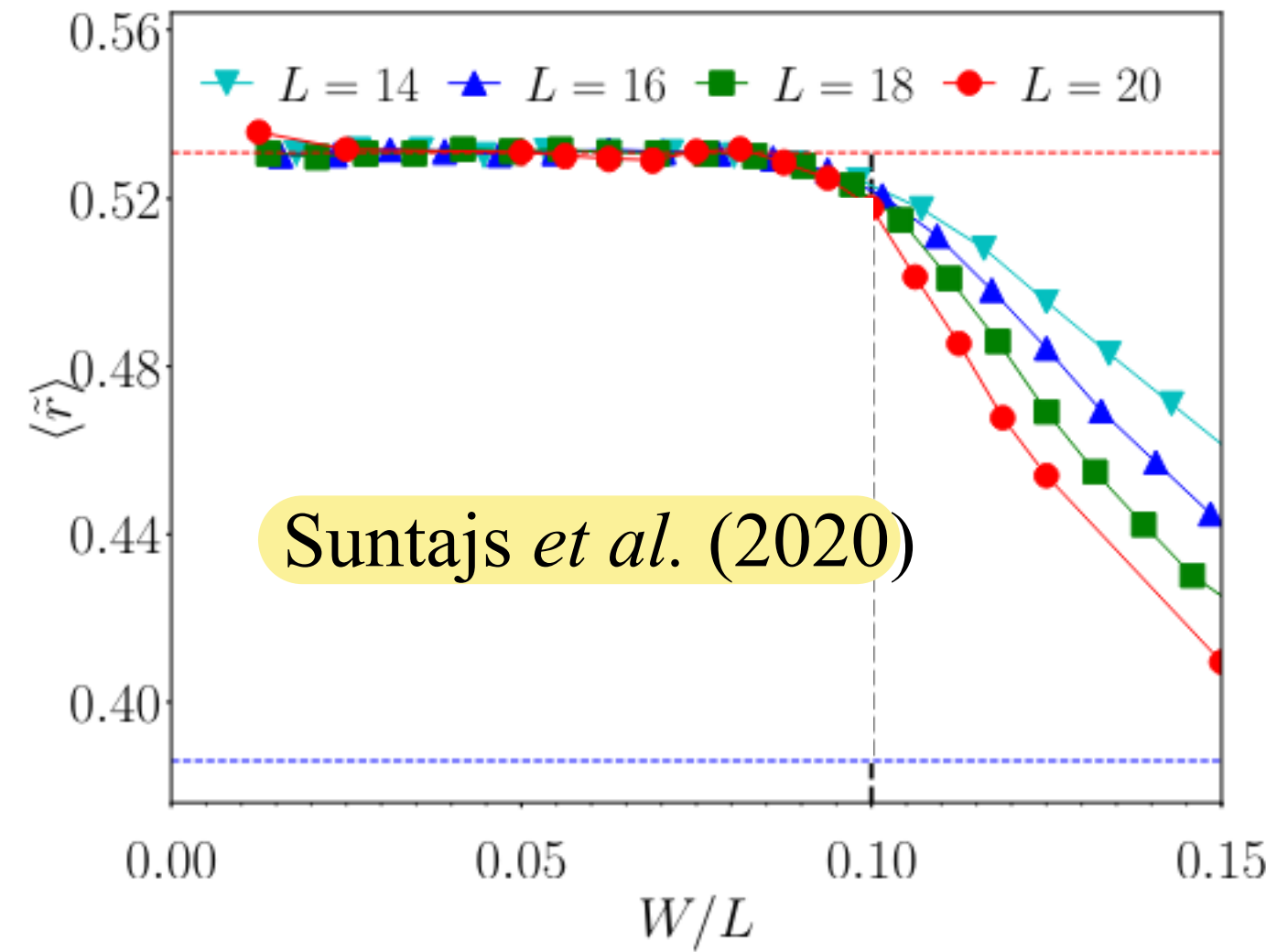
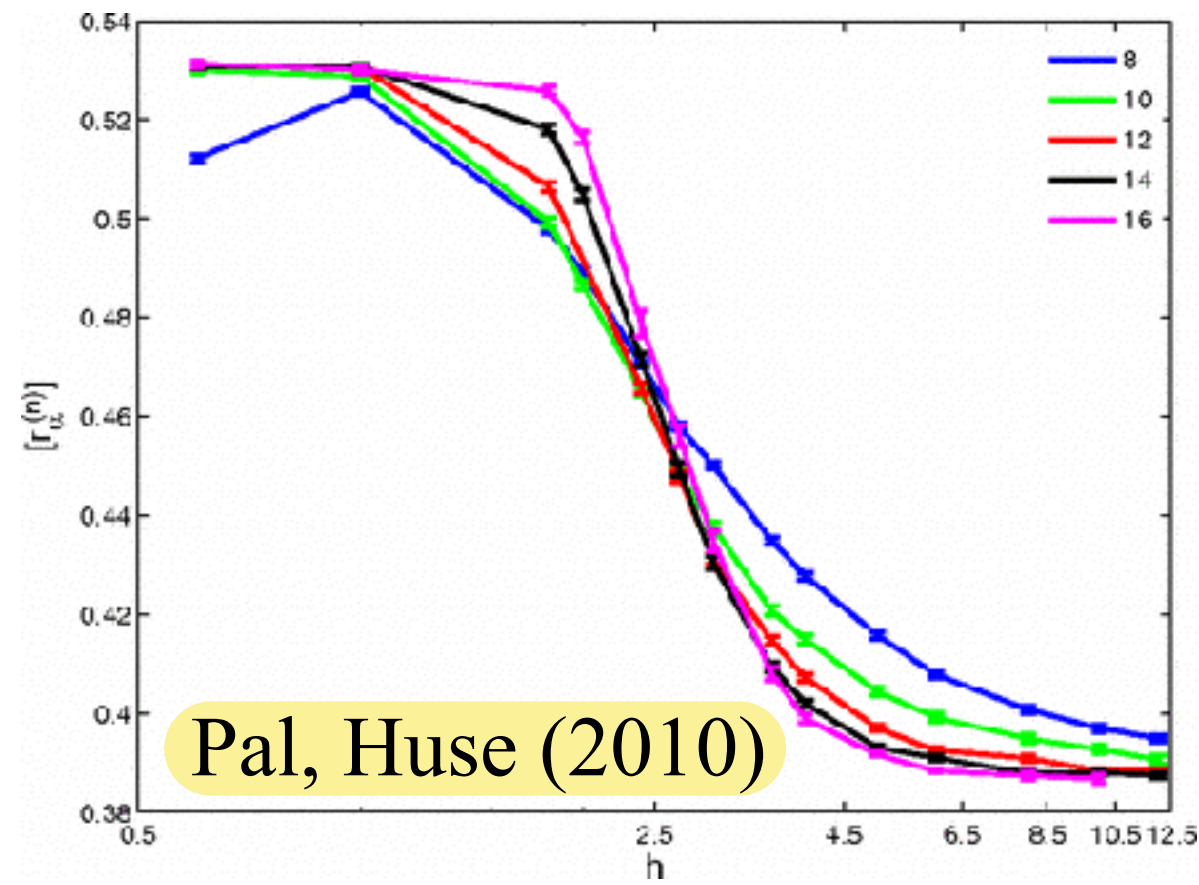
Ferdinand Evers, Ishita Modak, Soumya Bera

After a decade of many claims to the opposite, there now is a growing consensus that generic disordered quantum wires, e.g., the XXZ-Heisenberg chain, do not exhibit many-body localization

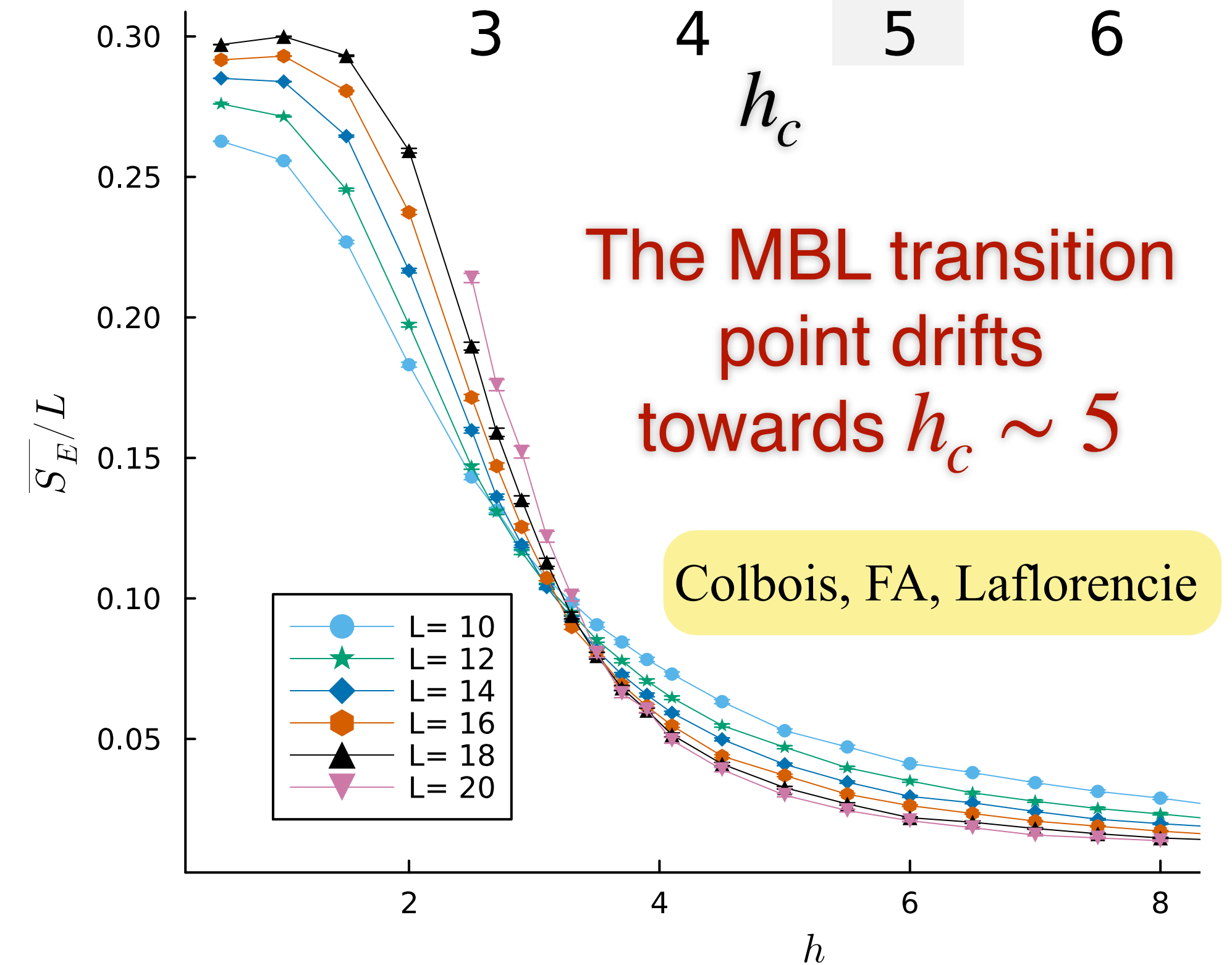
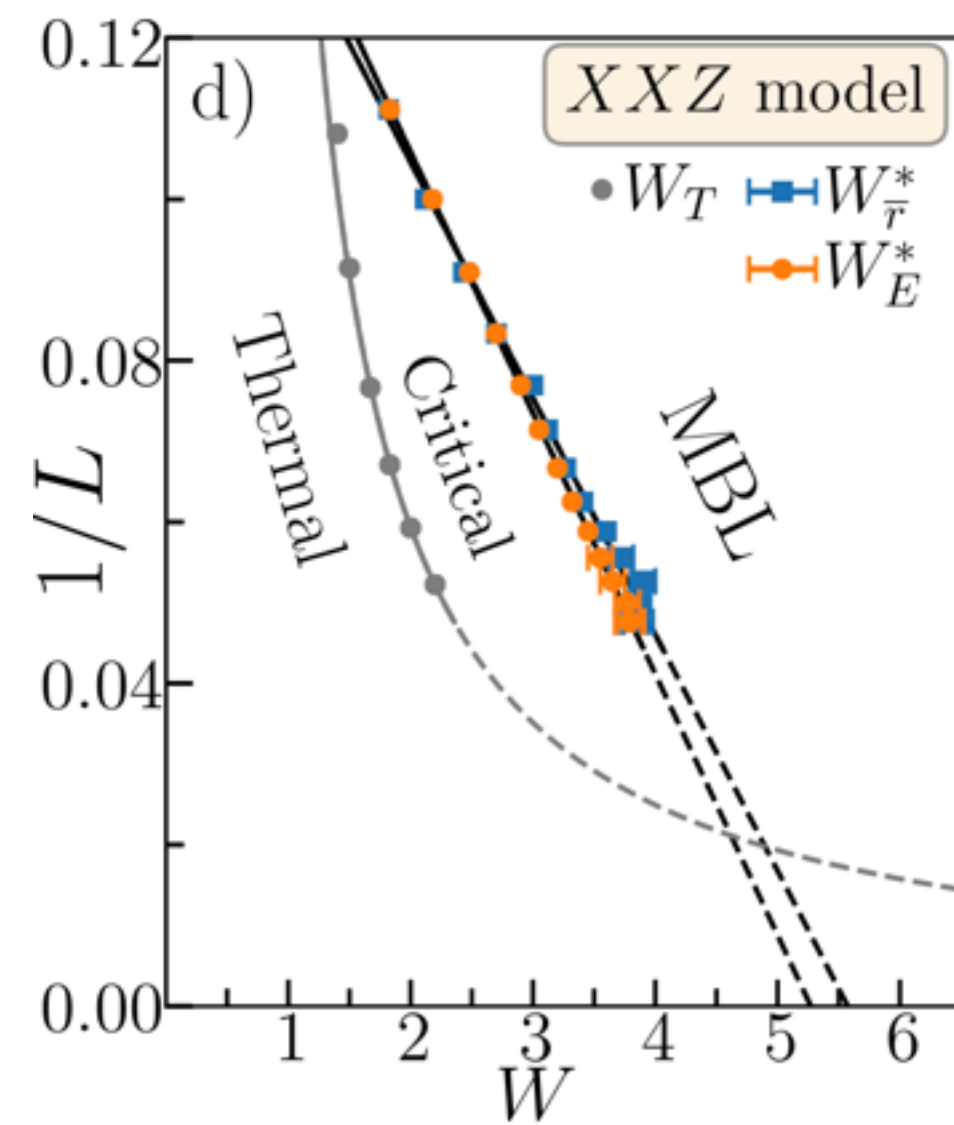
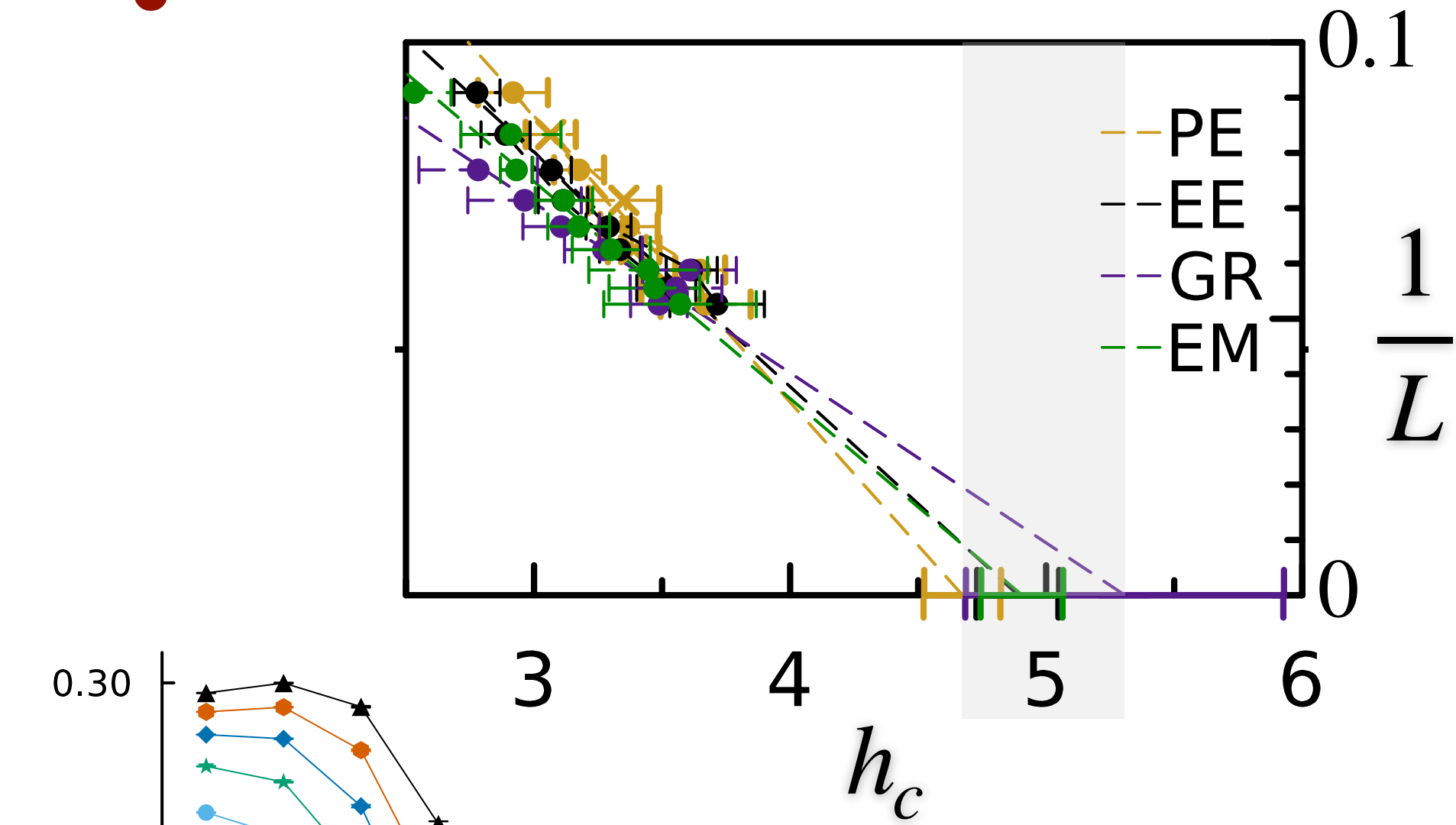


# Finite-size effects & their interpretations : some examples ...

## ► Gap Ratio



## ► Entanglement



The MBL transition point drifts towards  $h_c \sim 5$

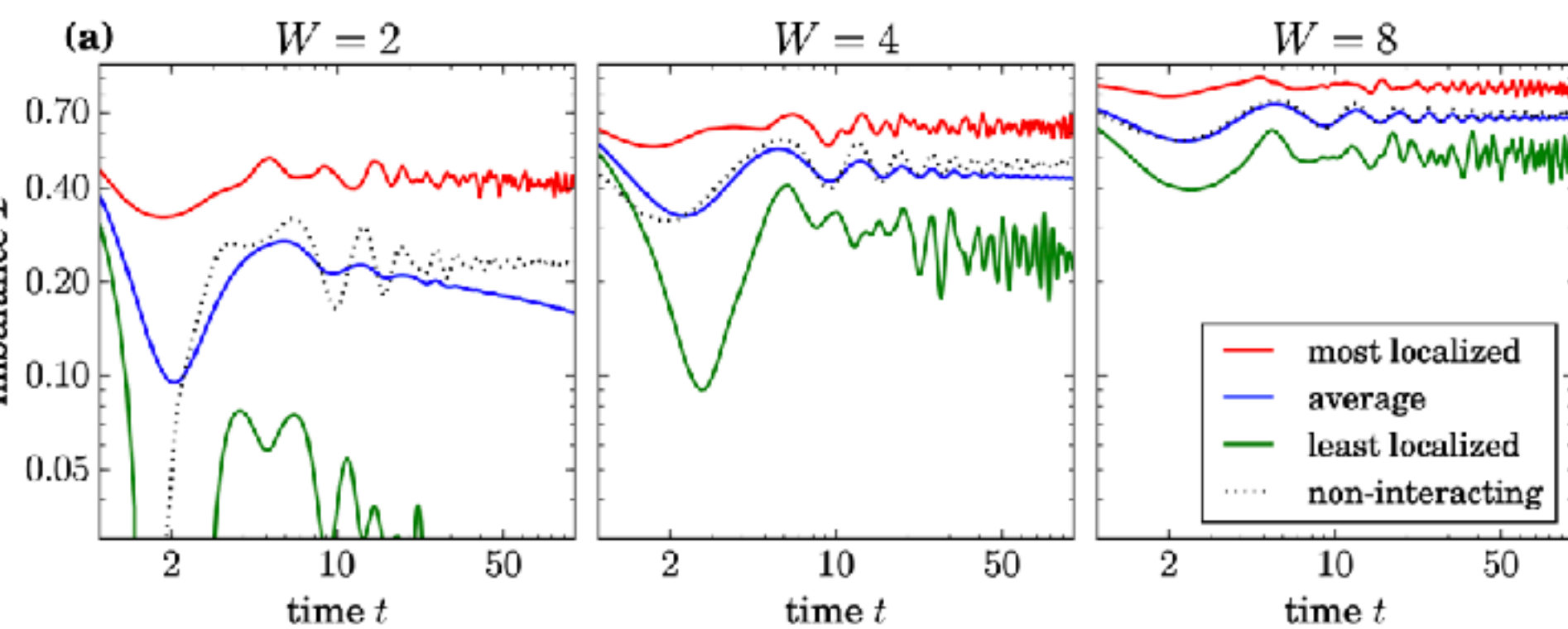
Sierant et al. (2020)

# Very slow dynamics inside the MBL phase

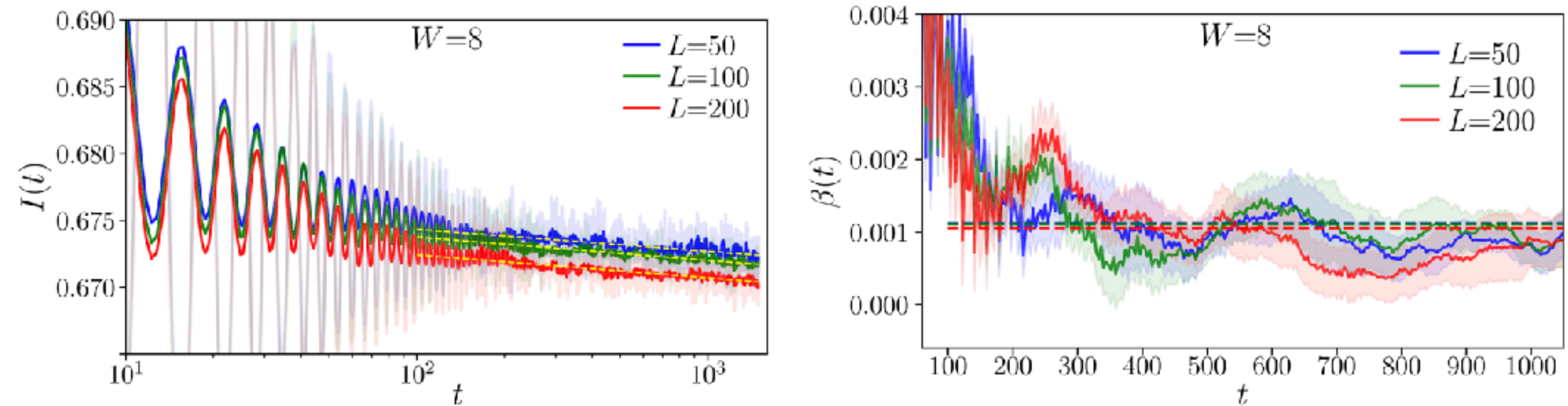
**Dynamics after quench :**  $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$

**Larger-scale dynamics of Imbalance decay** using Matrix-Product-States methods (t-DMRG, TEBD, TDVP)

Doggen *et al.* (2018)

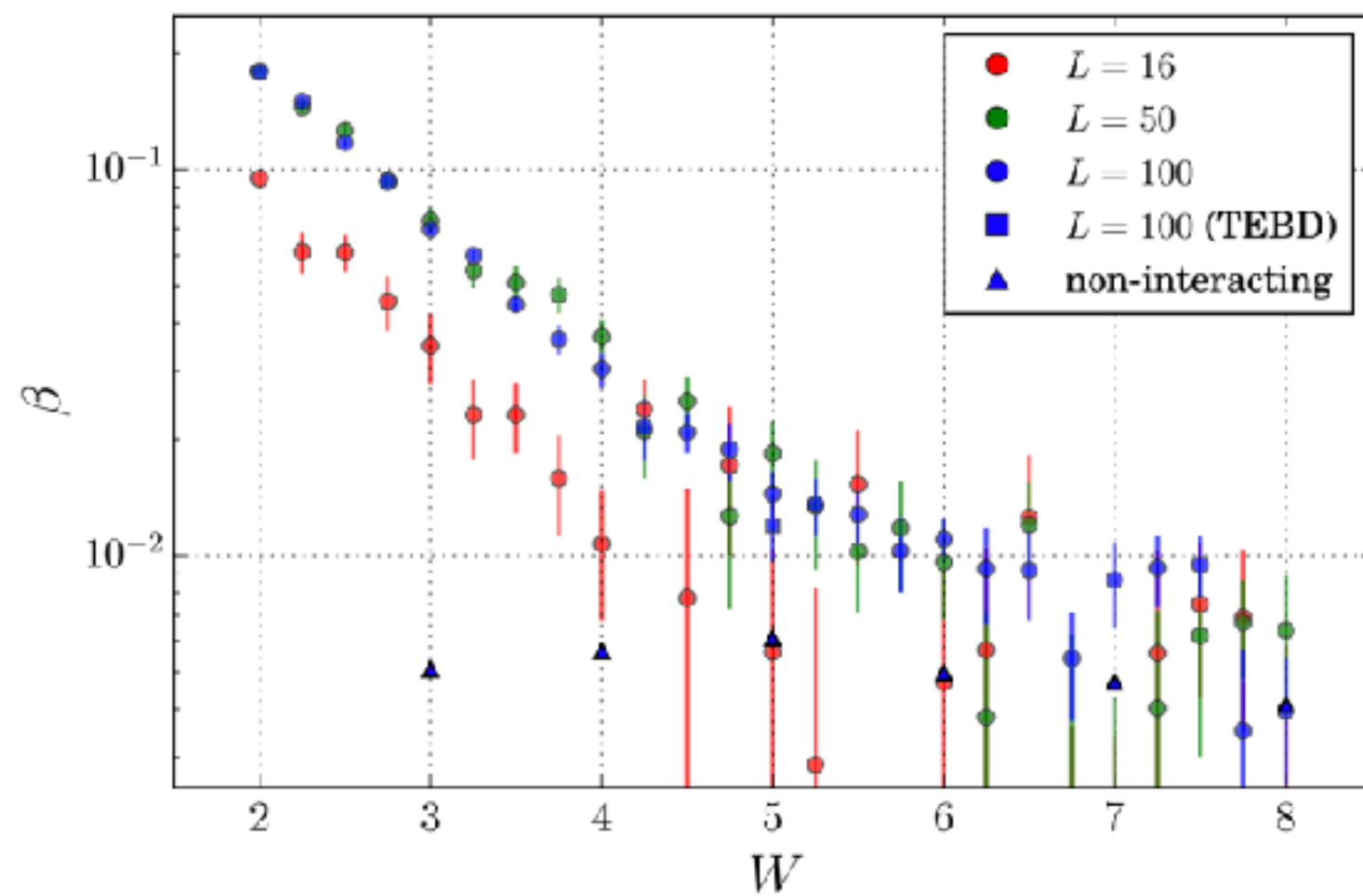


Sierant, Zakrzewski (2022)

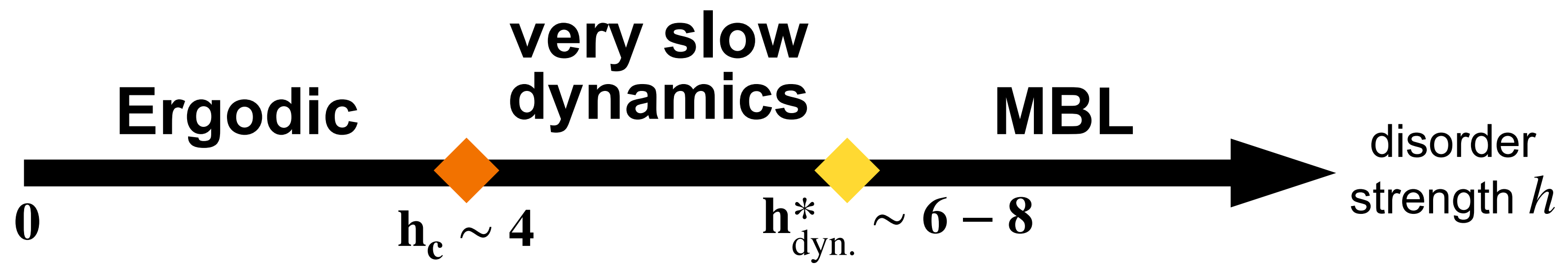


**Very slow decay of the imbalance**

$$I(t) \sim t^{-\beta}$$



**Suggested dynamical phase diagram**





# Searching for the invisible : Avalanche and resonances

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- Idea: **the liom picture cannot explain the transition to the ergodic phase.** Other phenomenological approaches emerged:

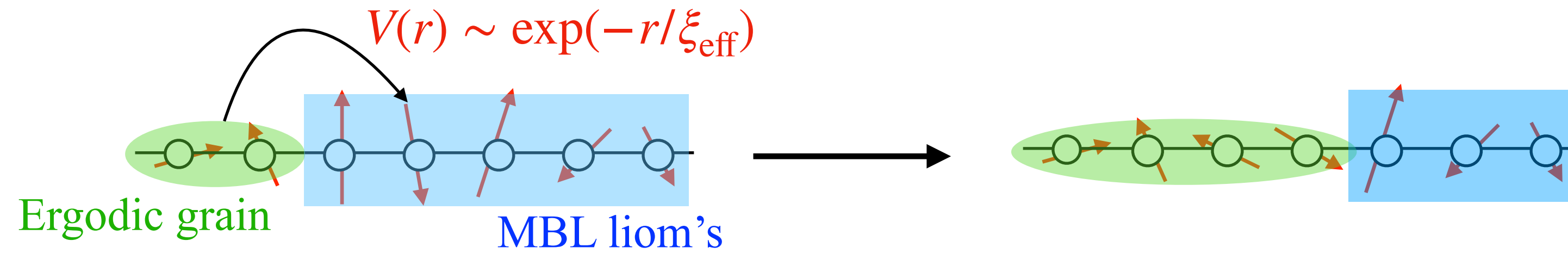
# Searching for the invisible : Avalanche and resonances

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## Avalanche instability

De Roeck, Huveneers

An ergodic grain can thermalise its neighbourhood, which is then included in the grain, thermalize again its neighbourhood .. and eventually leads to full thermalization



- The ergodic grain (initial size  $n_0$ ) grows as  $n_0 \rightarrow n_0 + r$  if the coupling  $V(r)$  is larger than effective level spacing  $2^{-(n_0+r)}$
- For large enough initial “localization” length  $\xi_{\text{eff}} > \xi^* = 2/\ln 2$ , this leads to an **instability towards thermalised phase : avalanche**

→ Avalanches never observed so far on systems with realistic disorder...

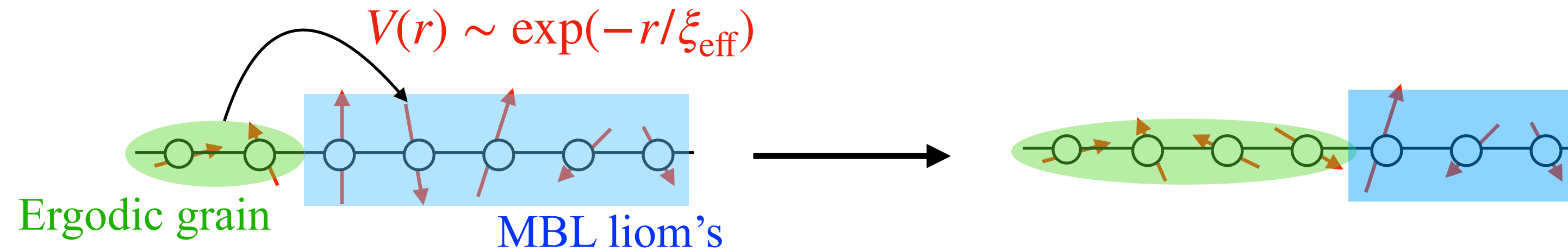
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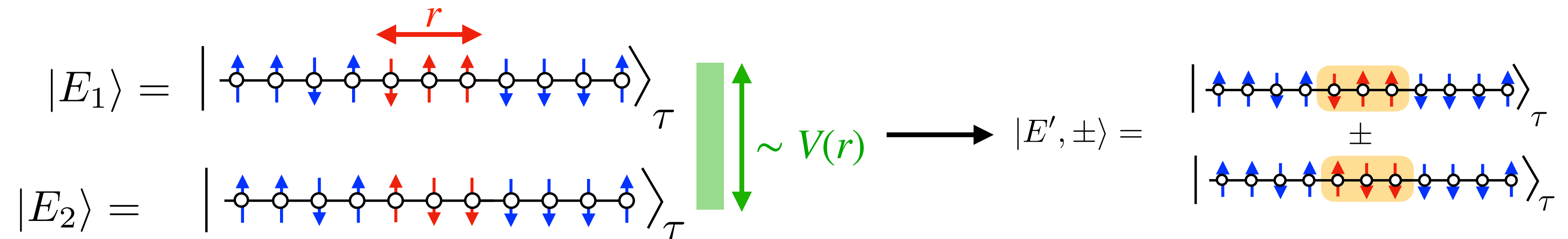


- The ergodic grain (initial size  $n_0$ ) grows as  $n_0 \rightarrow n_0 + r$  if the coupling  $V(r)$  is larger than effective level spacing  $2^{-(n_0+r)}$
- For large enough initial “localization” length  $\xi_{\text{eff}} > \xi^* = 2/\ln 2$ , this leads to an **instability towards thermalised phase : avalanche**

→ Avalanches never observed so far on systems with realistic disorder...

## Many-body resonances

More « direct » hybridisation between liom states by local operators



→ Hints of such many-body resonances in some (mostly phenomenological, some microscopic) models

Crowley, Chandran (2020-22)

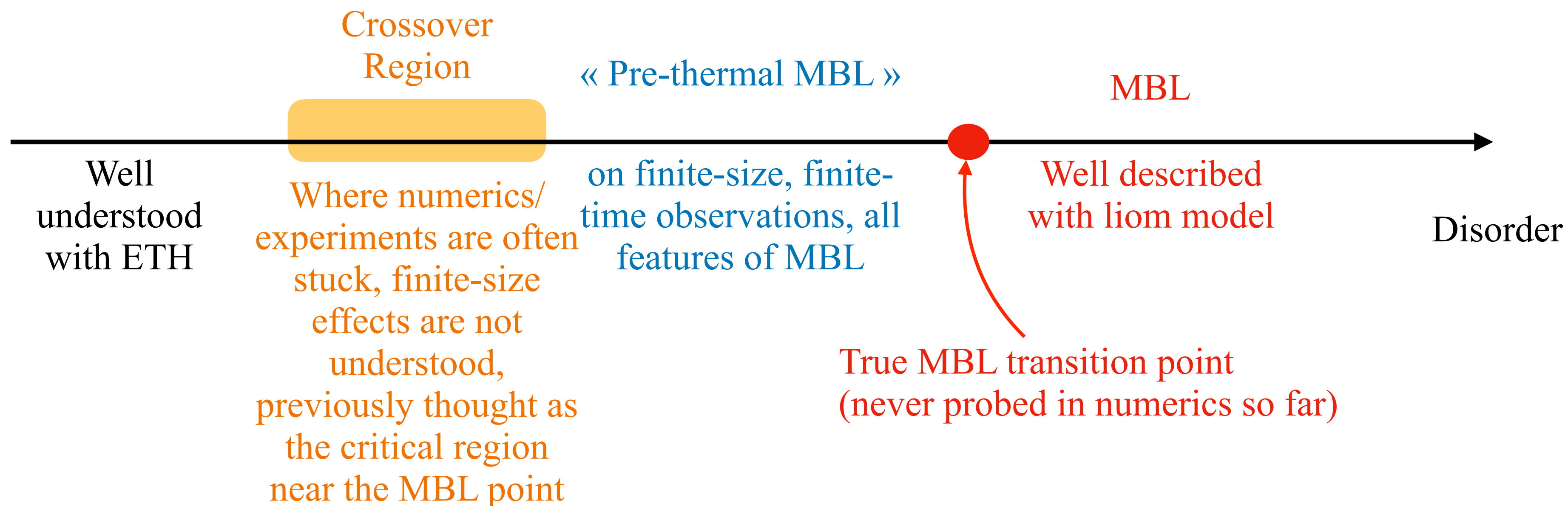
Garratt, Roy, Chalker (2021-22)

Morningstar *et al.* (2022)

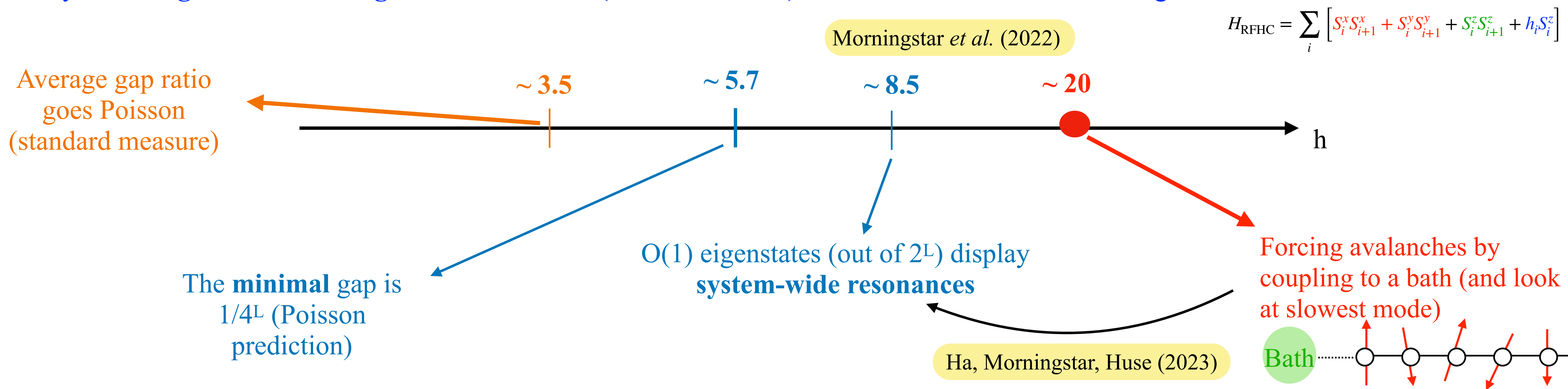
# Sketch of the new phenomenology

Morningstar *et al.* (2022)

Long *et al.* (2023)



- By « forcing » or « searching » for rare events (extreme values) for the random field Heisenberg chain





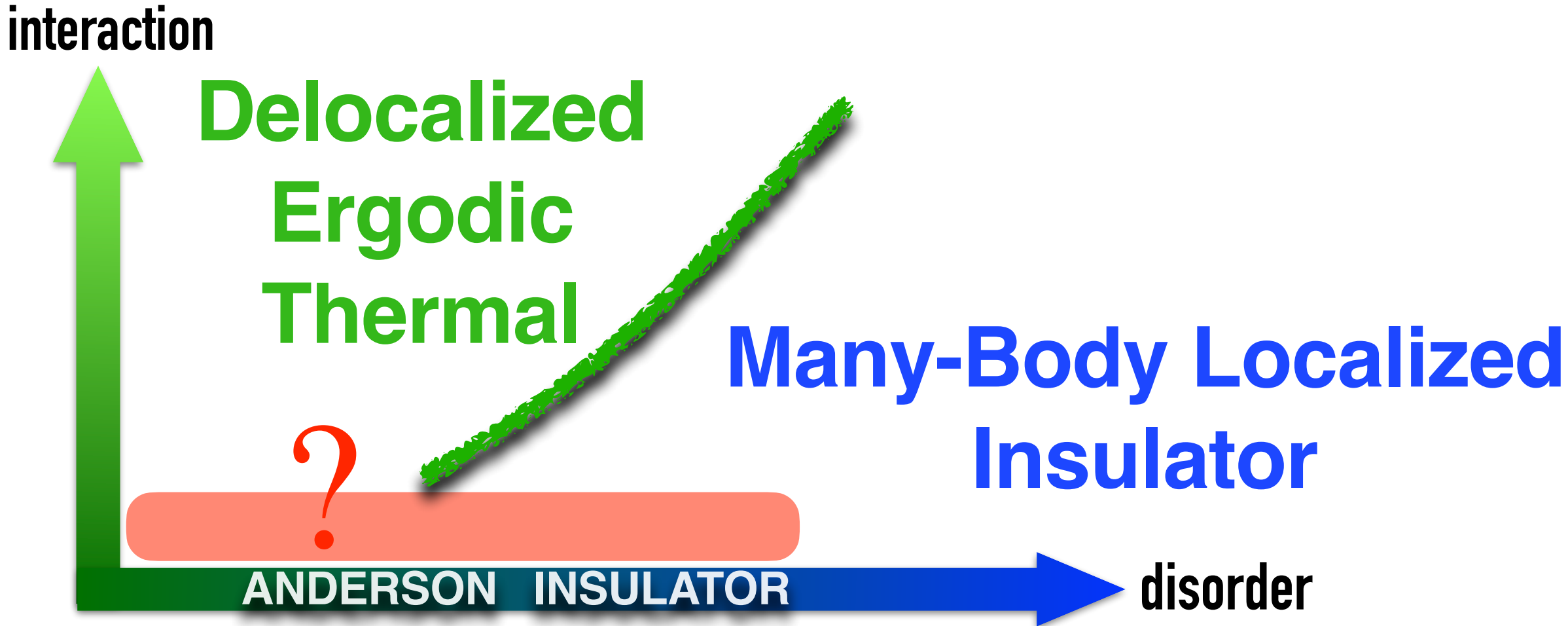
# Stepping away from the « strong » interaction regime

J. Colbois, FA, N. Laflorencie Phys. Rev. Lett. **133**, 116502 (2024)

Original motivation: Can localization survive interaction?

What happens at weak  $\Delta$  ?

Most studies focused on the « strongly interacting » Heisenberg chain  $\Delta = 1$



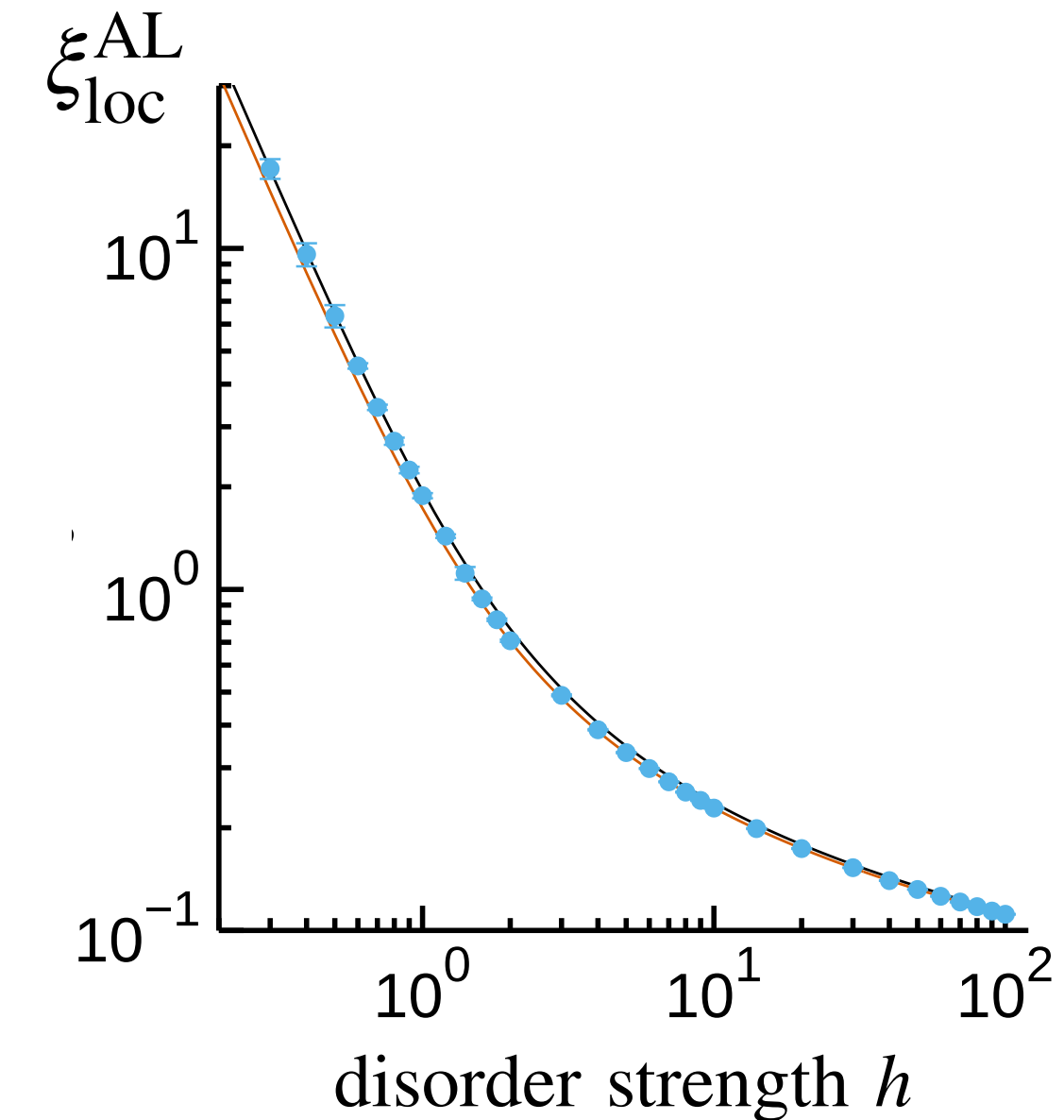
# Stepping away from the « strong » interaction regime

- Consider the weak interaction limit  $\Delta \rightarrow 0$ :  $H_{\text{XXZ}} = \sum_i \left[ S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z + h_i S_i^z \right] \longrightarrow H_{\text{MBAL}} = \sum_i \left[ S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + h_i S_i^z \right]$   
Many-body Anderson insulator

- Idea:** Any reasonable definition of the many-body localization length should converge to the (non-interacting) Anderson localization length  $\xi_{\text{loc}}^{\text{AL}}$

$$\xi_{\text{loc}}^{\text{AL}} = 1/\ln \left[ 1 + (h/h_0)^2 \right] \sim \begin{cases} h^{-2} & \text{(weak disorder)} \\ \frac{1}{\ln h} & \text{(strong disorder)} \end{cases}$$

Colbois, Laflorencie (2023)



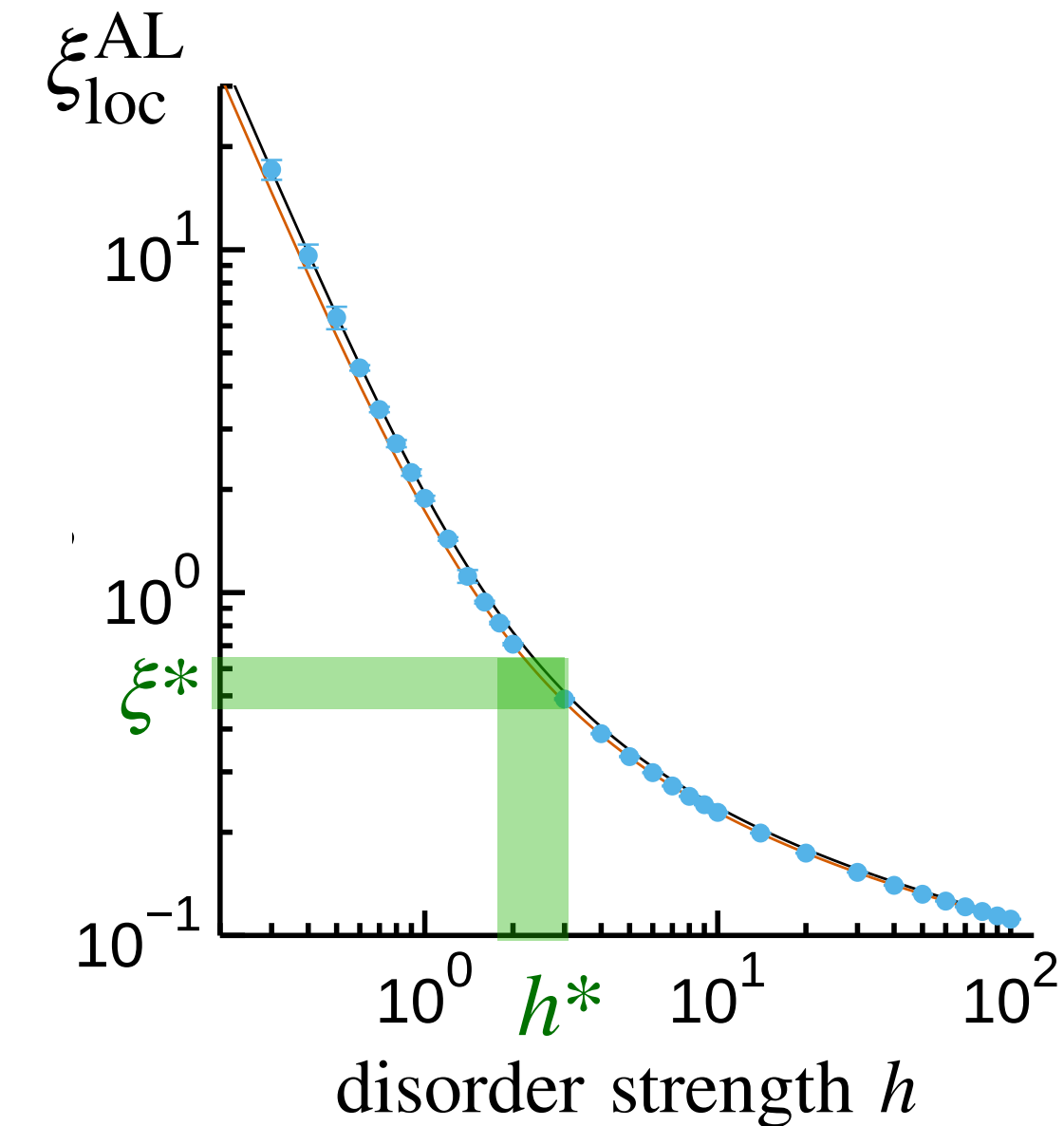
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Colbois, Laflorencie (2023)



- Naive / Literal application of the avalanche criterion  $\xi_{\text{eff}} > \xi^*$  for infinitesimal interactions :

There is a critical disorder  $h^*$  below which MBL is not stable

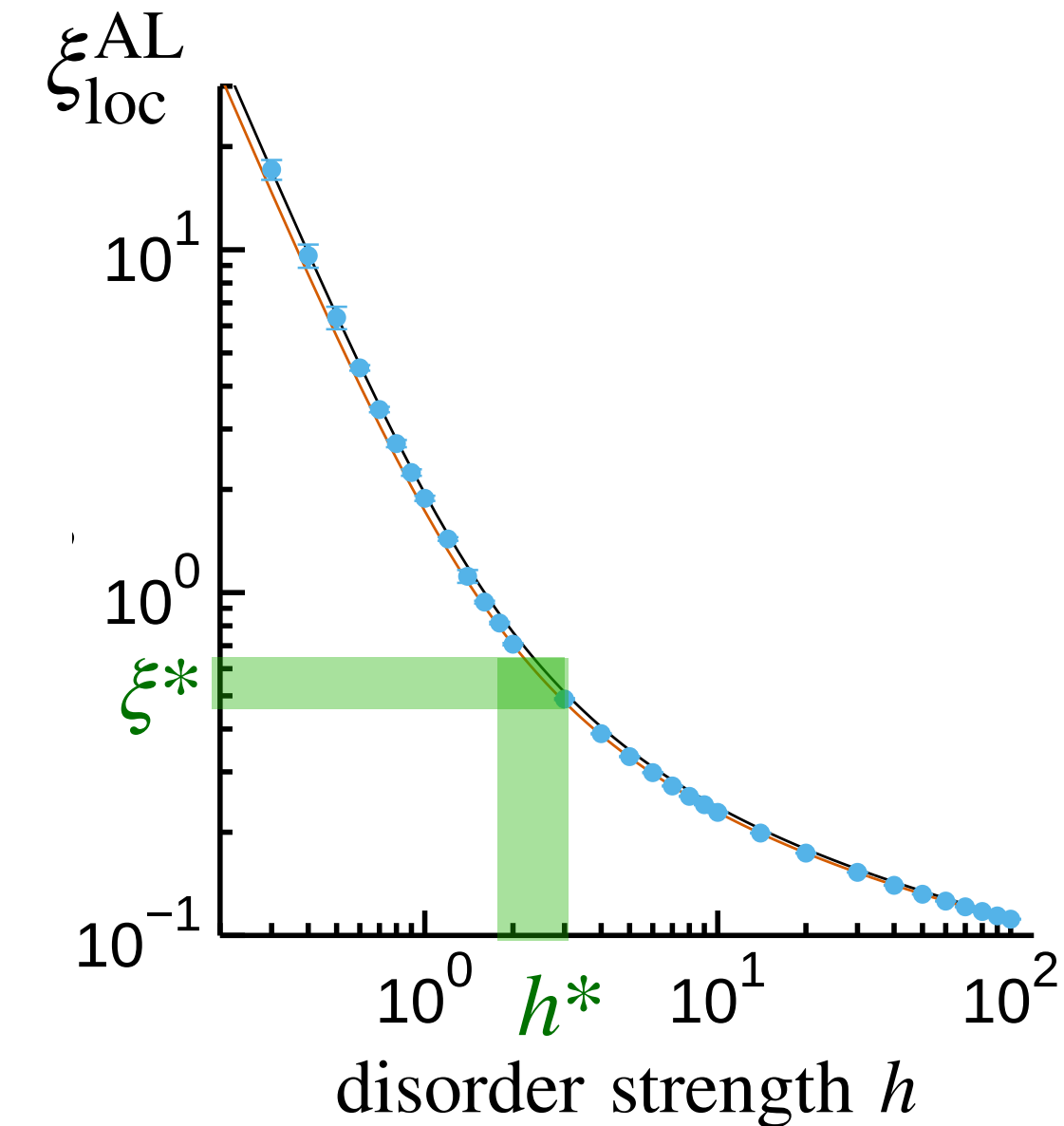
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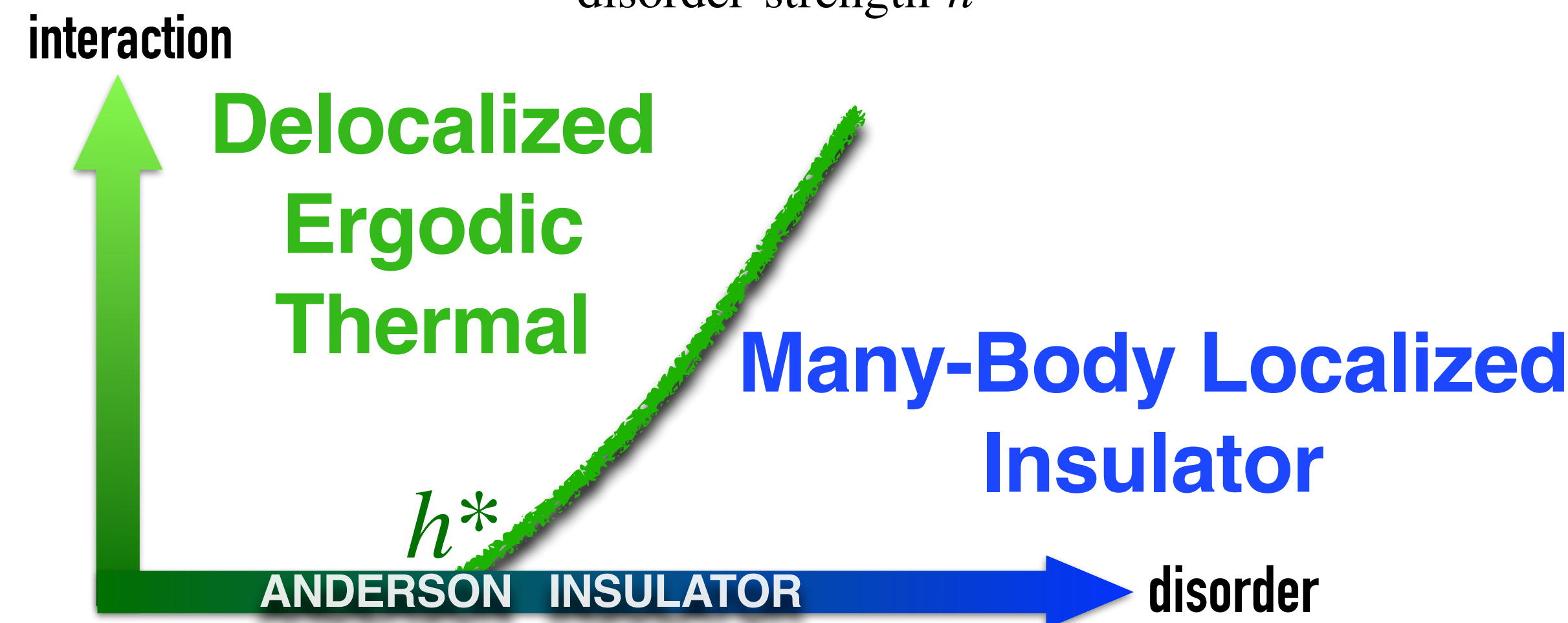
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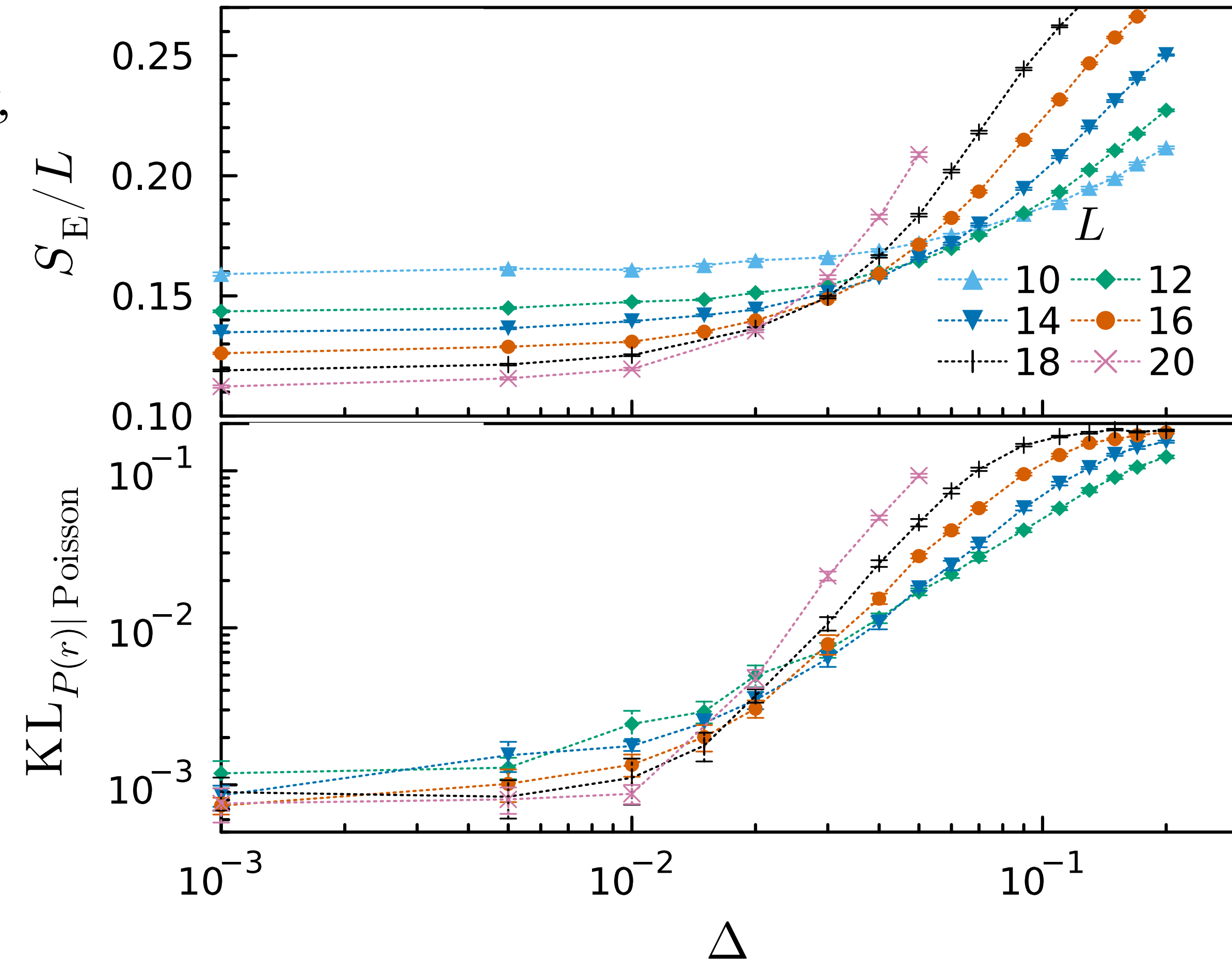


# Numerical for small $\Delta \ll 1$

- Shift-invert computations of mid-spectrum eigenstates for  $L \in [10 - 21]$  using **standard estimates** (entanglement entropy, spectral statistics, multifractal analysis and local observables)

Working at fixed (small) disorder

$$h = 1 \quad (\xi_{\text{loc}}^{\text{AL}} \approx 2)$$

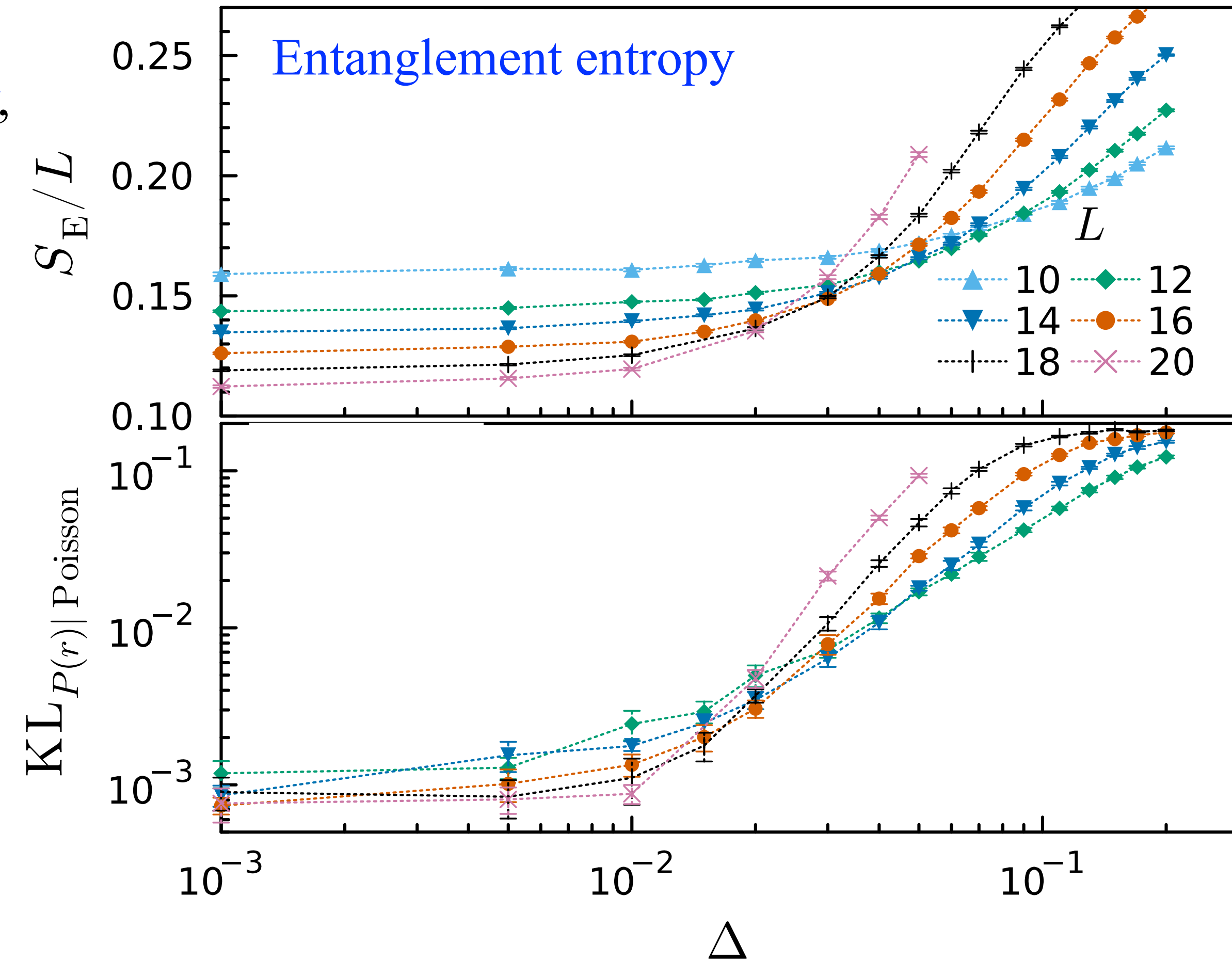


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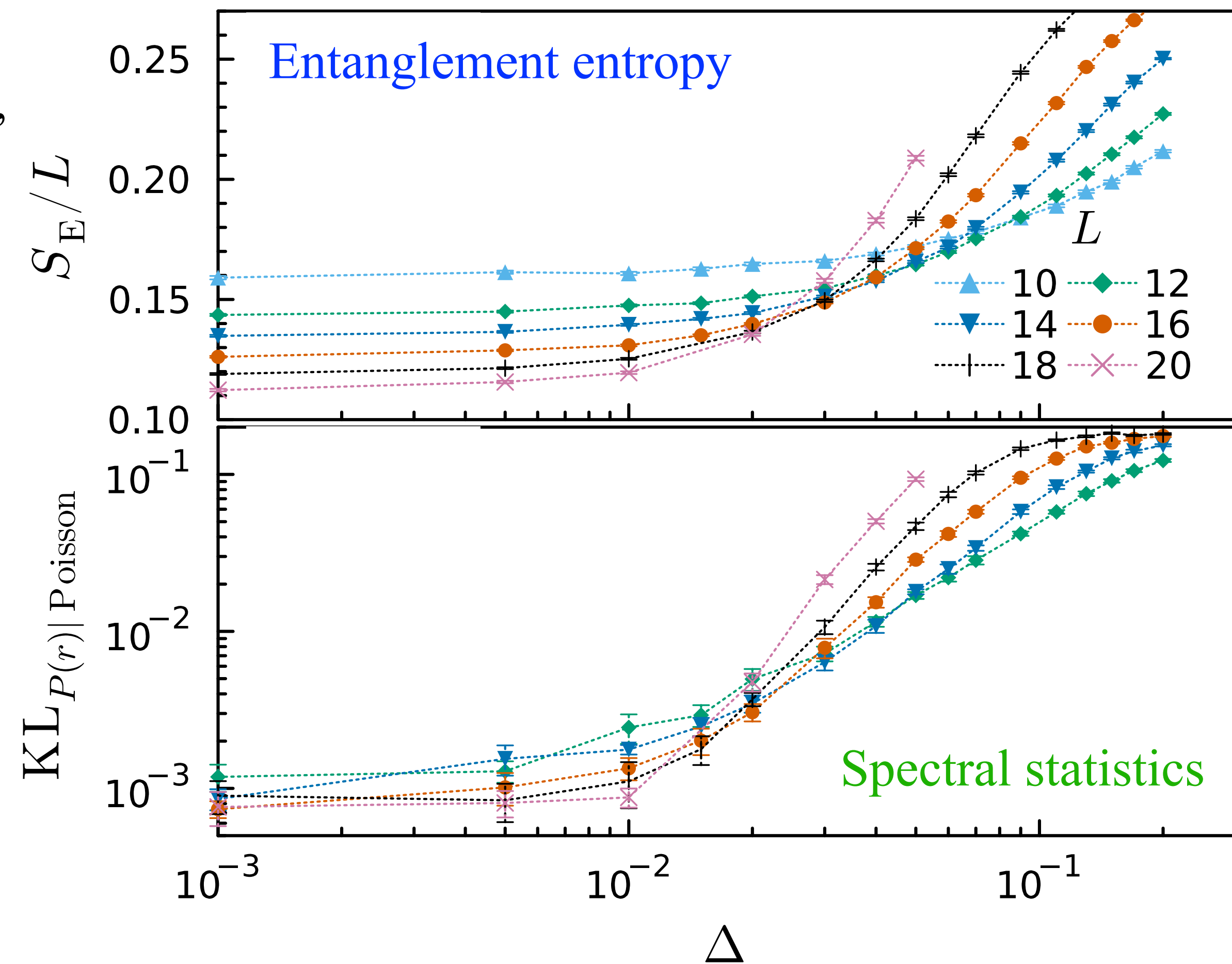


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$$\text{KL}_{P(r)|\text{Poisson}} = \int_0^1 P(r) \ln \left[ P(r) \frac{(1+r)^2}{2} \right] dr$$

$$\xrightarrow{L \gg 1} \begin{cases} 0 & \text{if Poisson} \\ \approx 0.19 & \text{GOE} \end{cases}$$



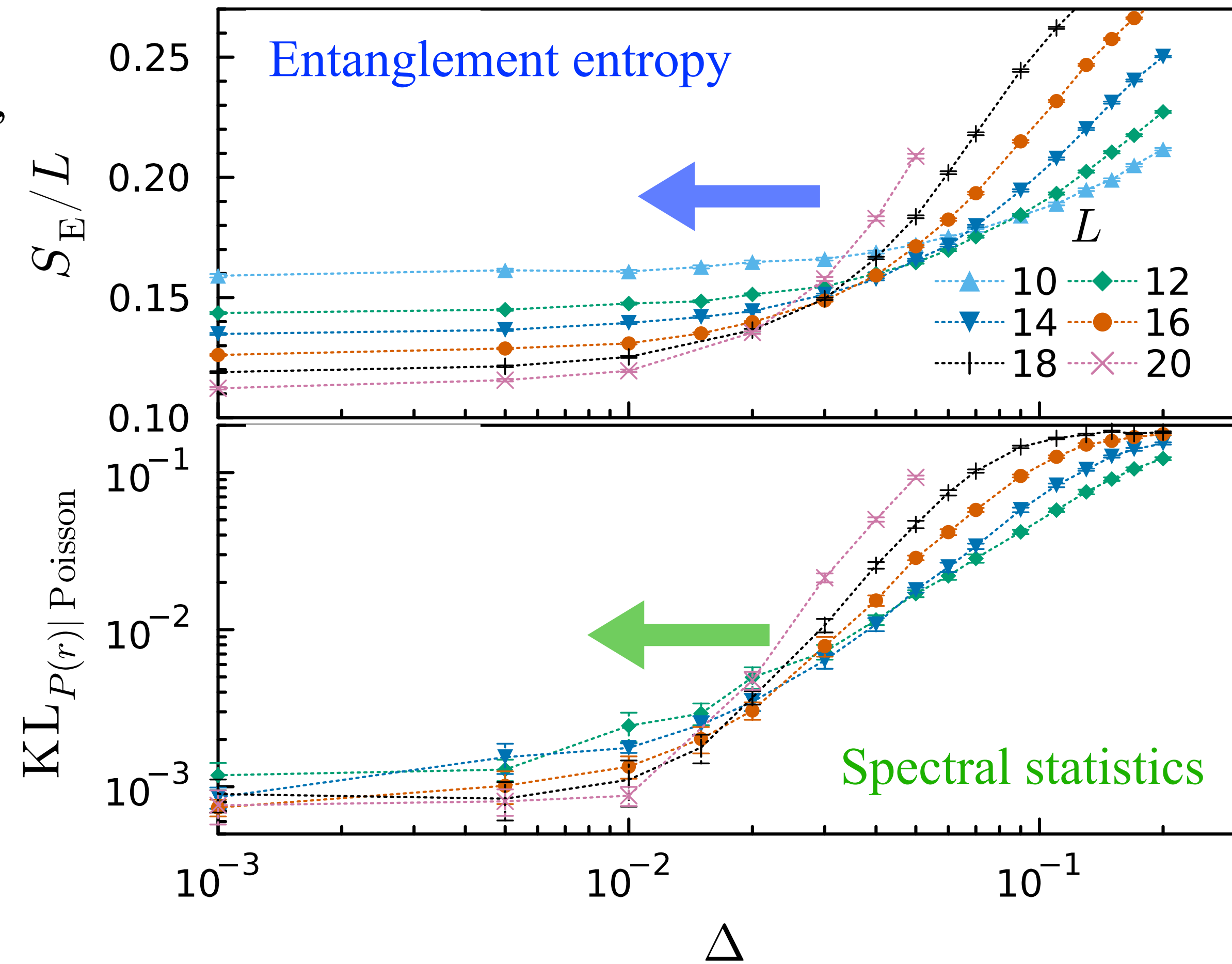
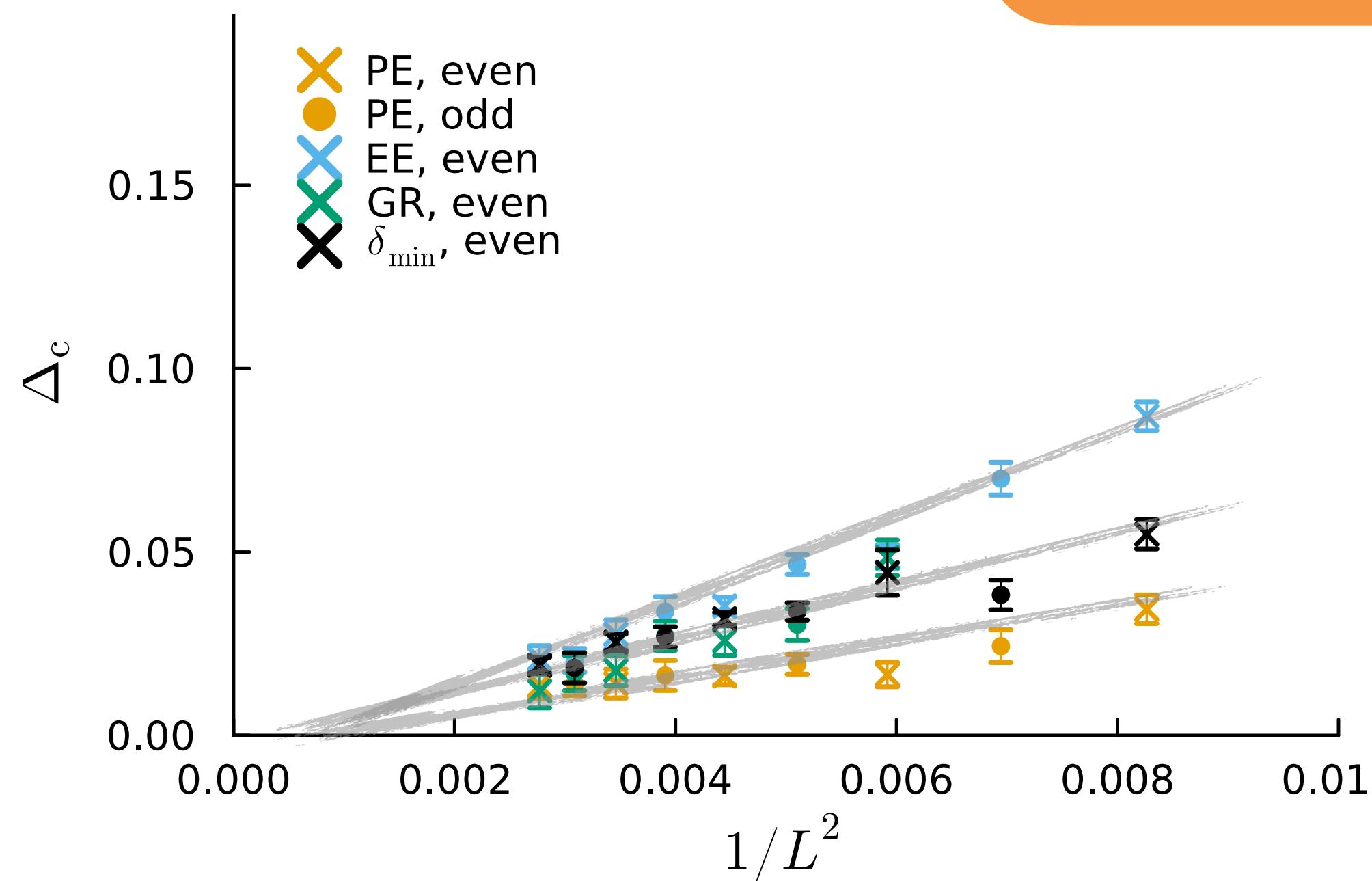
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Drifts of crossing points with L



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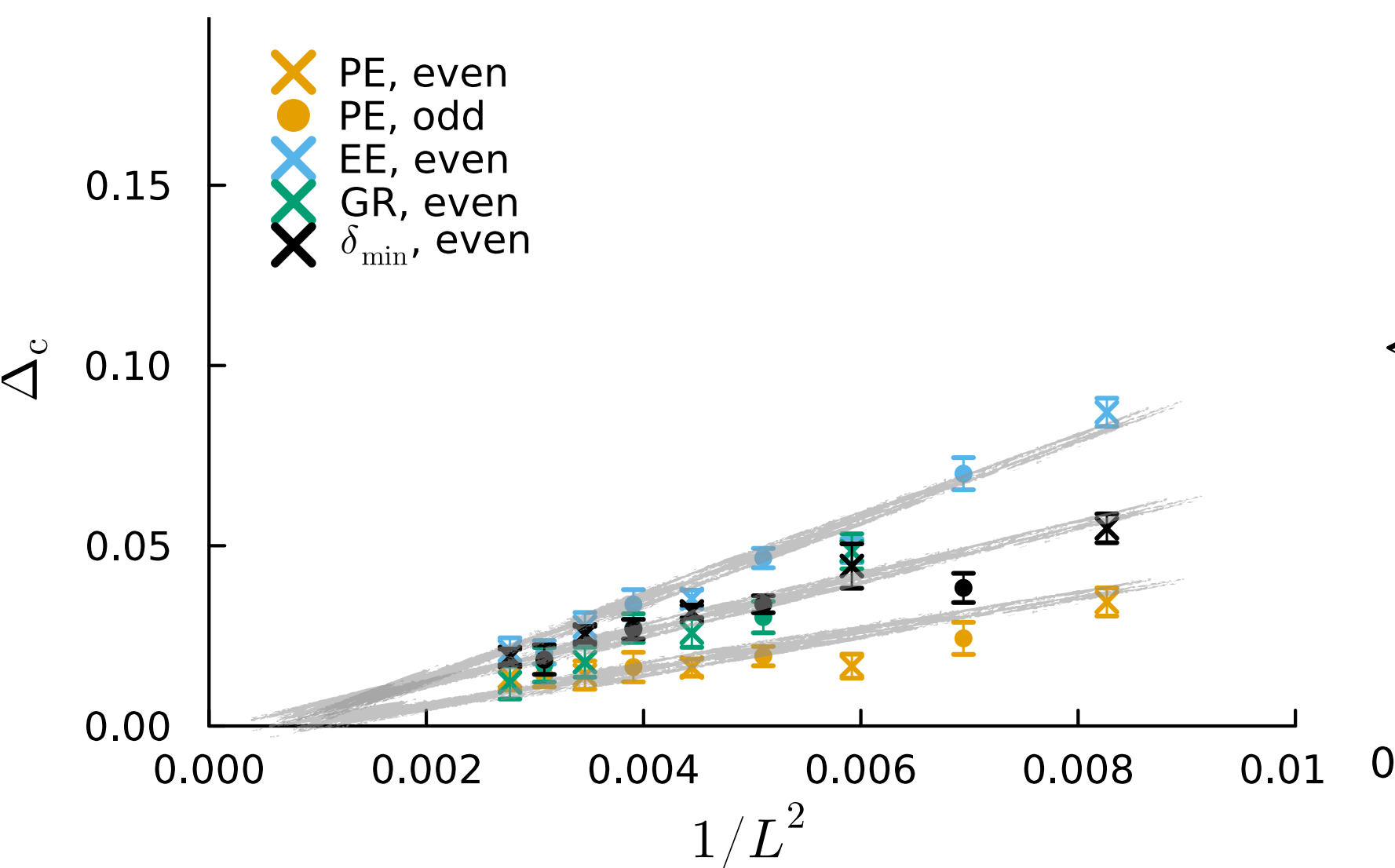
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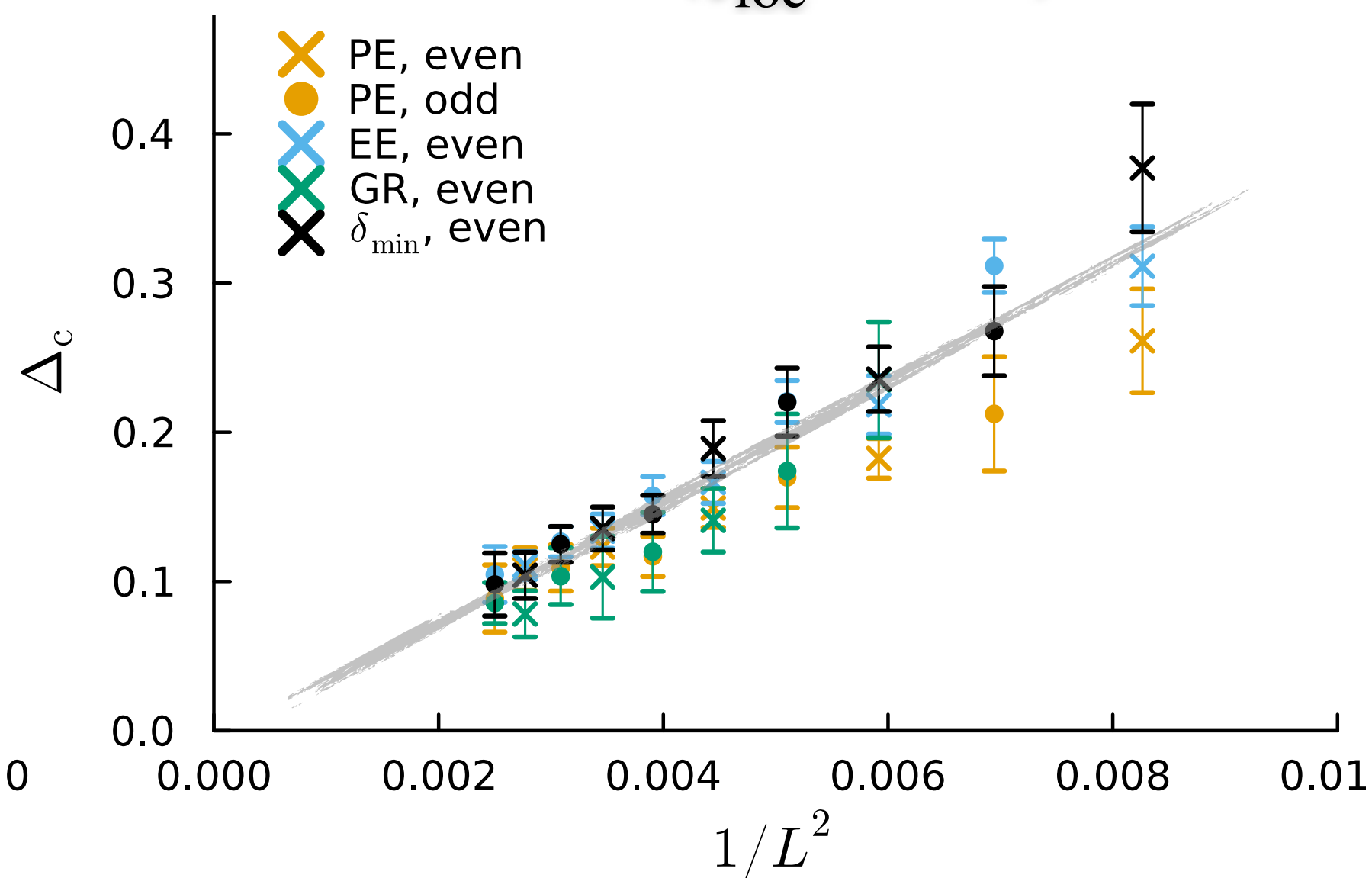
Working at fixed (small) disorder

Drifts of  
crossing points  
with  $L$

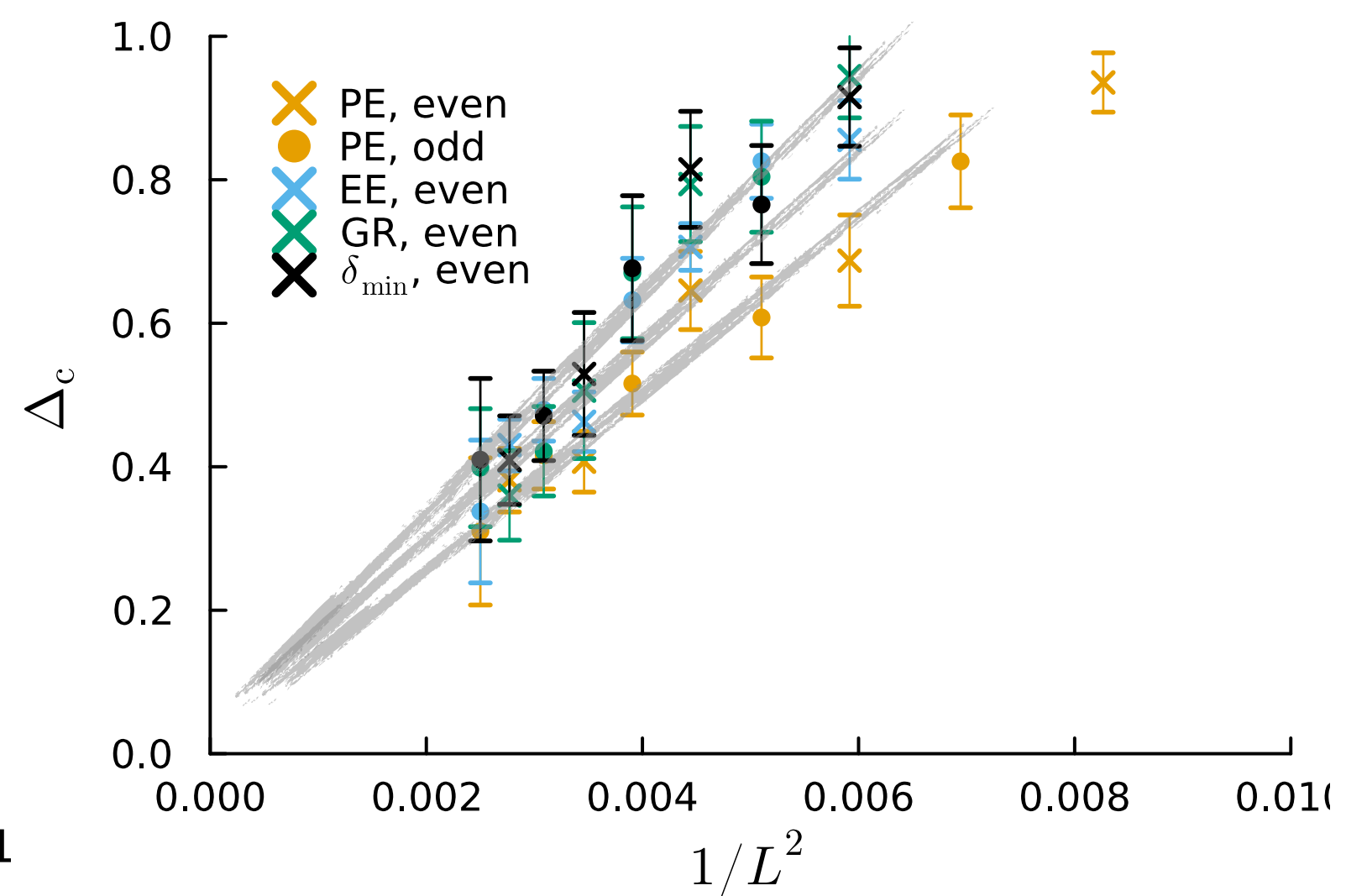
$h = 1$  ( $\xi_{\text{loc}}^{\text{AL}} \approx 2$ )



$h = 2$  ( $\xi_{\text{loc}}^{\text{AL}} \approx 0.7$ )



$h = 3$  ( $\xi_{\text{loc}}^{\text{AL}} \approx 0.5$ )



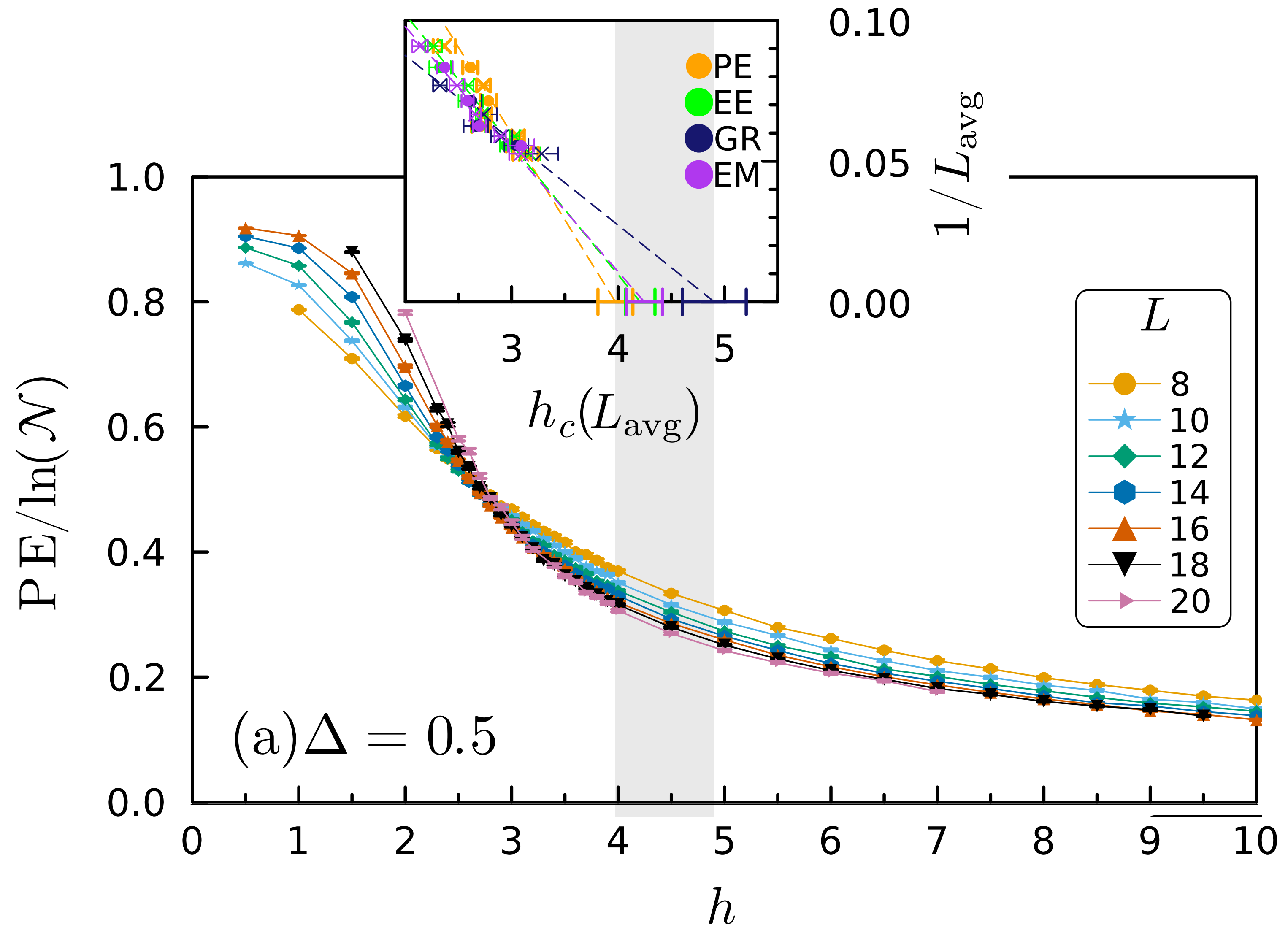
# Numerics at intermediate $\Delta < 1$

Working at fixed interaction  $\Delta$

Participation entropy (PE)

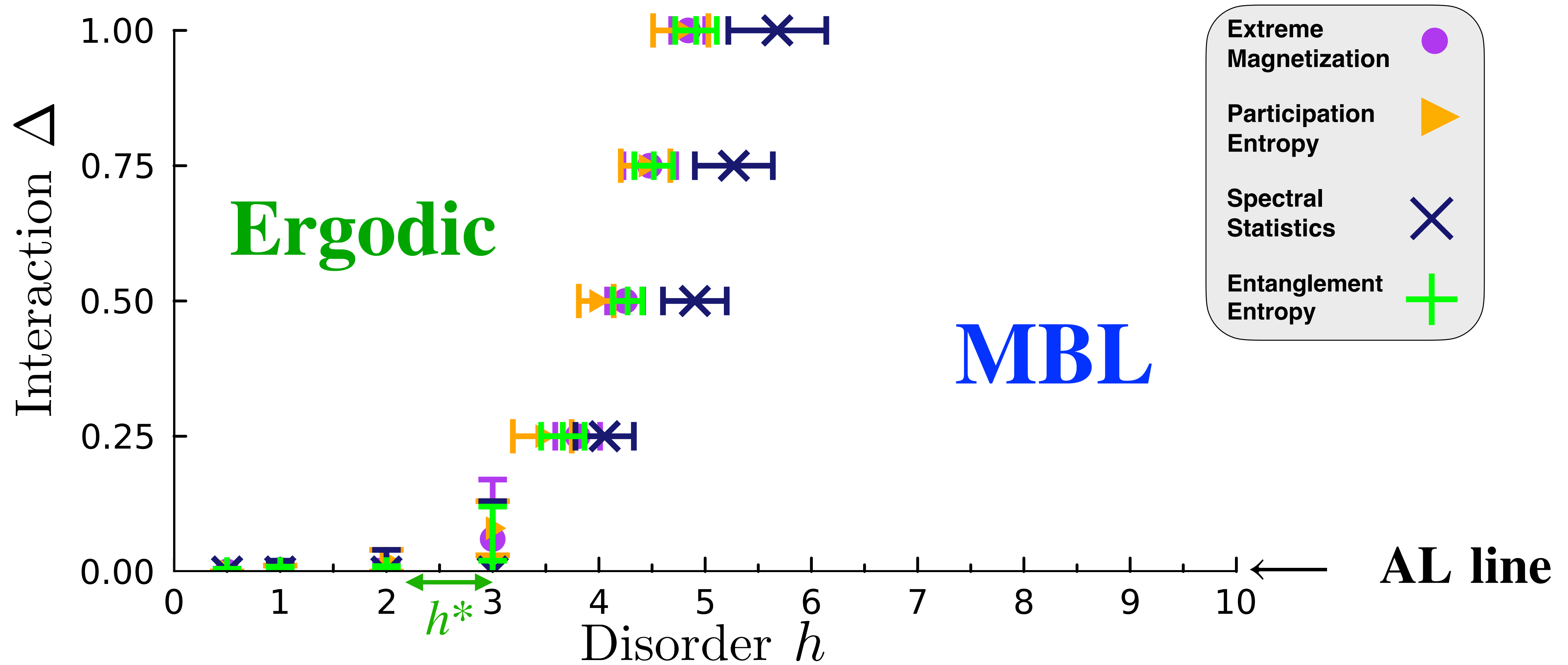
$$S_1^p = - \sum_i p_i \ln(p_i) \quad p_i = |\langle n|i \rangle|^2$$

Characterizes localization of eigenstates in the computational basis  $|n\rangle = \sum_i n_i |i\rangle$



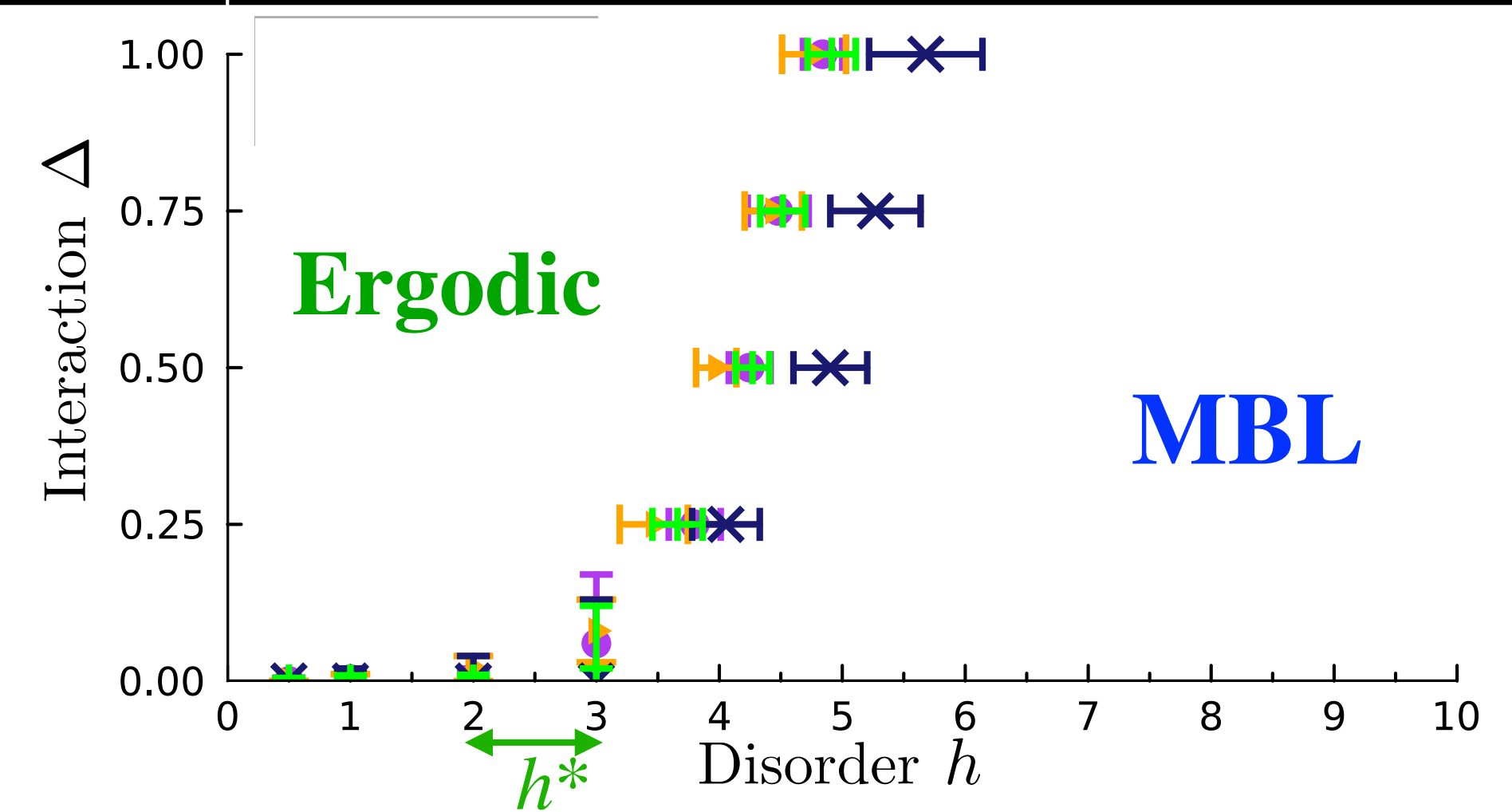
# Phase diagram

Phys. Rev. Lett. **133**, 116502 (2024)





- Q:** Is the critical field  $h^*$  the one from avalanche theory?  
**A:** It doesn't have to be,  $h_{\text{avl}}^*$  is a lower bound
- Q:** Could larger systems change the value of  $h^*$  ?  
**A:** Perhaps, but  $h^*$  can only **increase**



1. **Q:** Is the critical field  $h^*$  the one from avalanche theory?

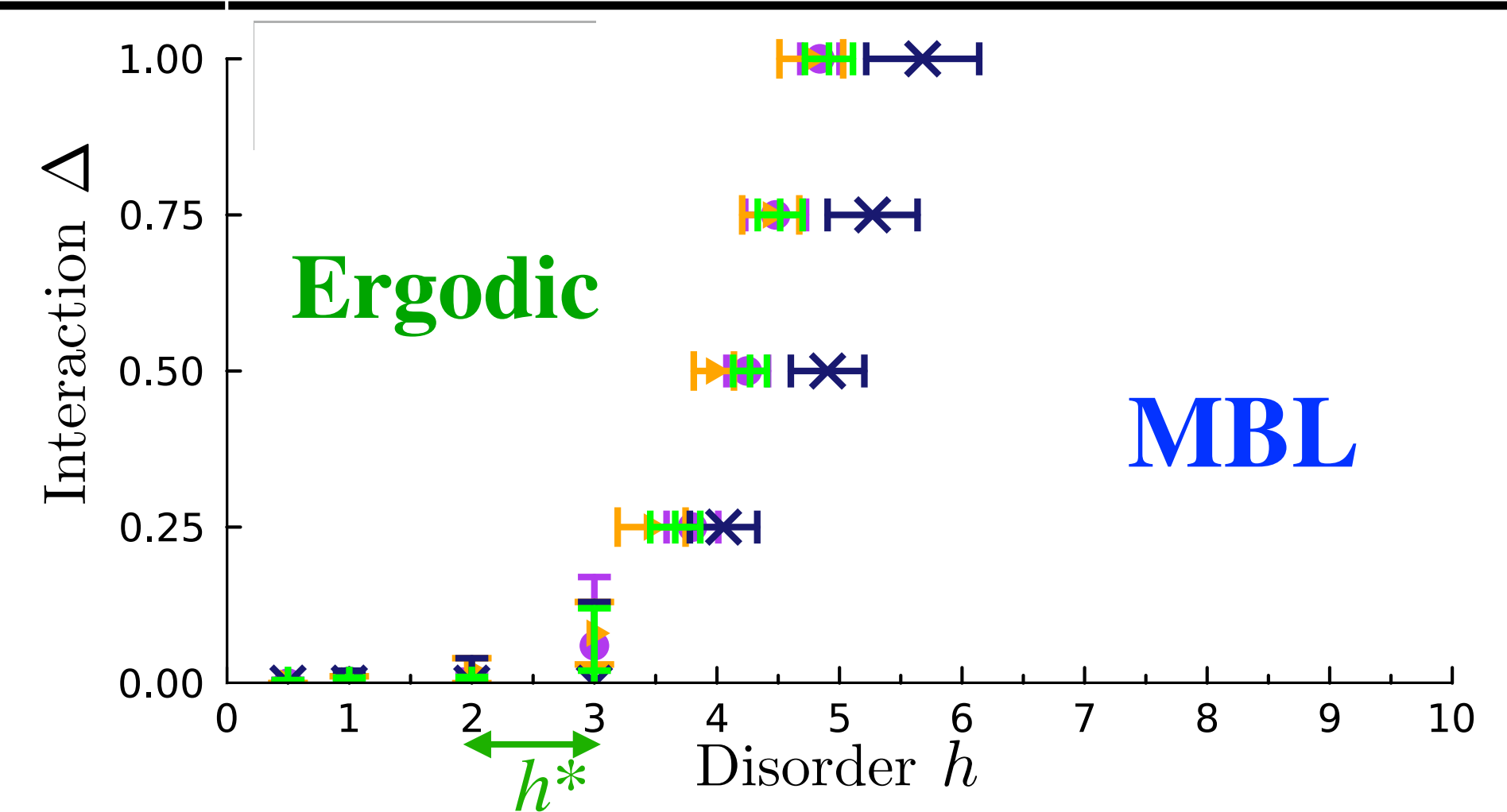
**A:** It doesn't have to be,  $h_{\text{avl}}^*$  is a lower bound

2. **Q:** Could larger systems change the value of  $h^*$  ?

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3. **Q:** Are there dynamical signatures of this weak-interaction behavior?

**A:** **YES ! Work in progress**



**Imbalance after a quench**

1. **Q:** Is the critical field  $h^*$  the one from avalanche theory?

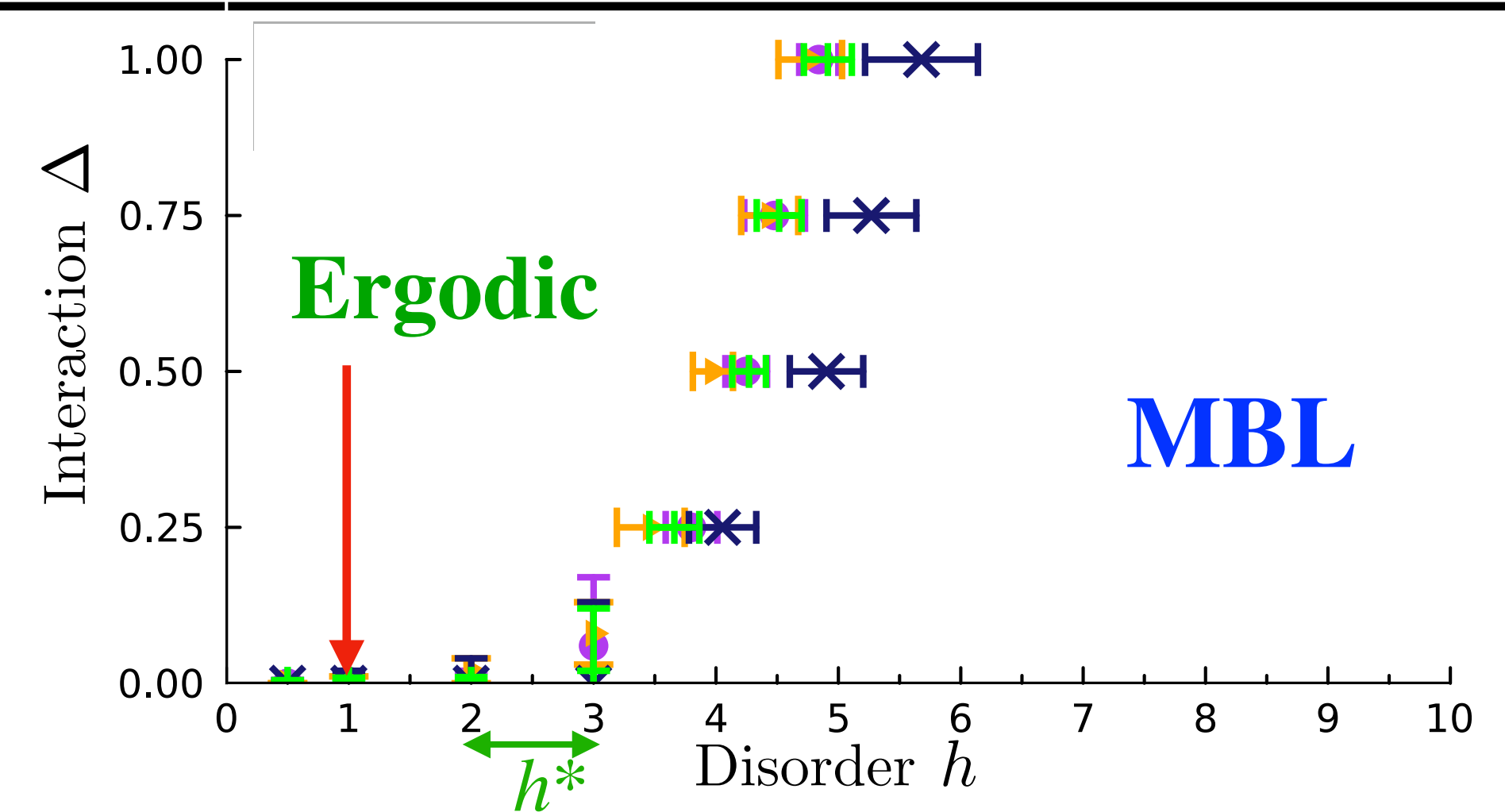
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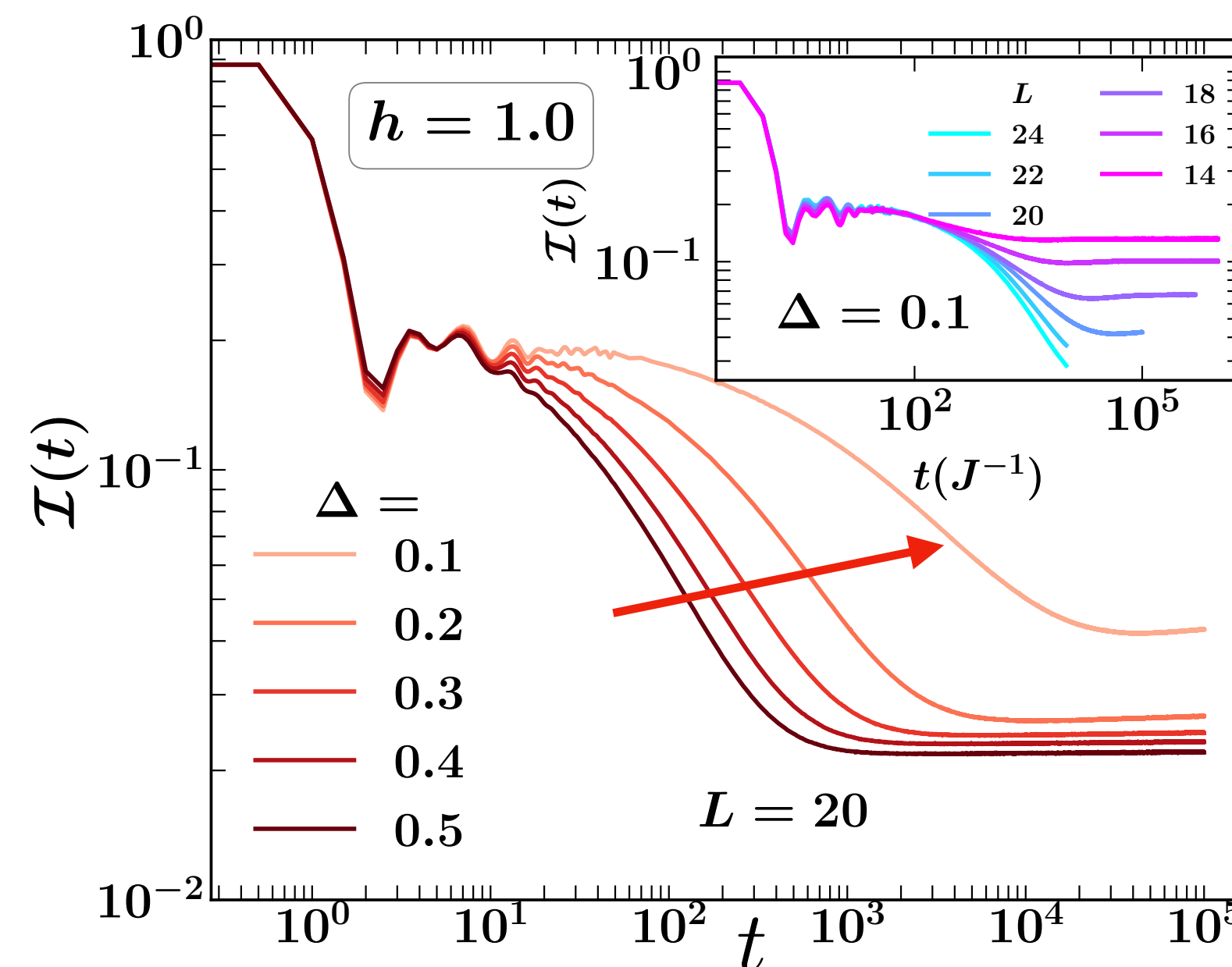
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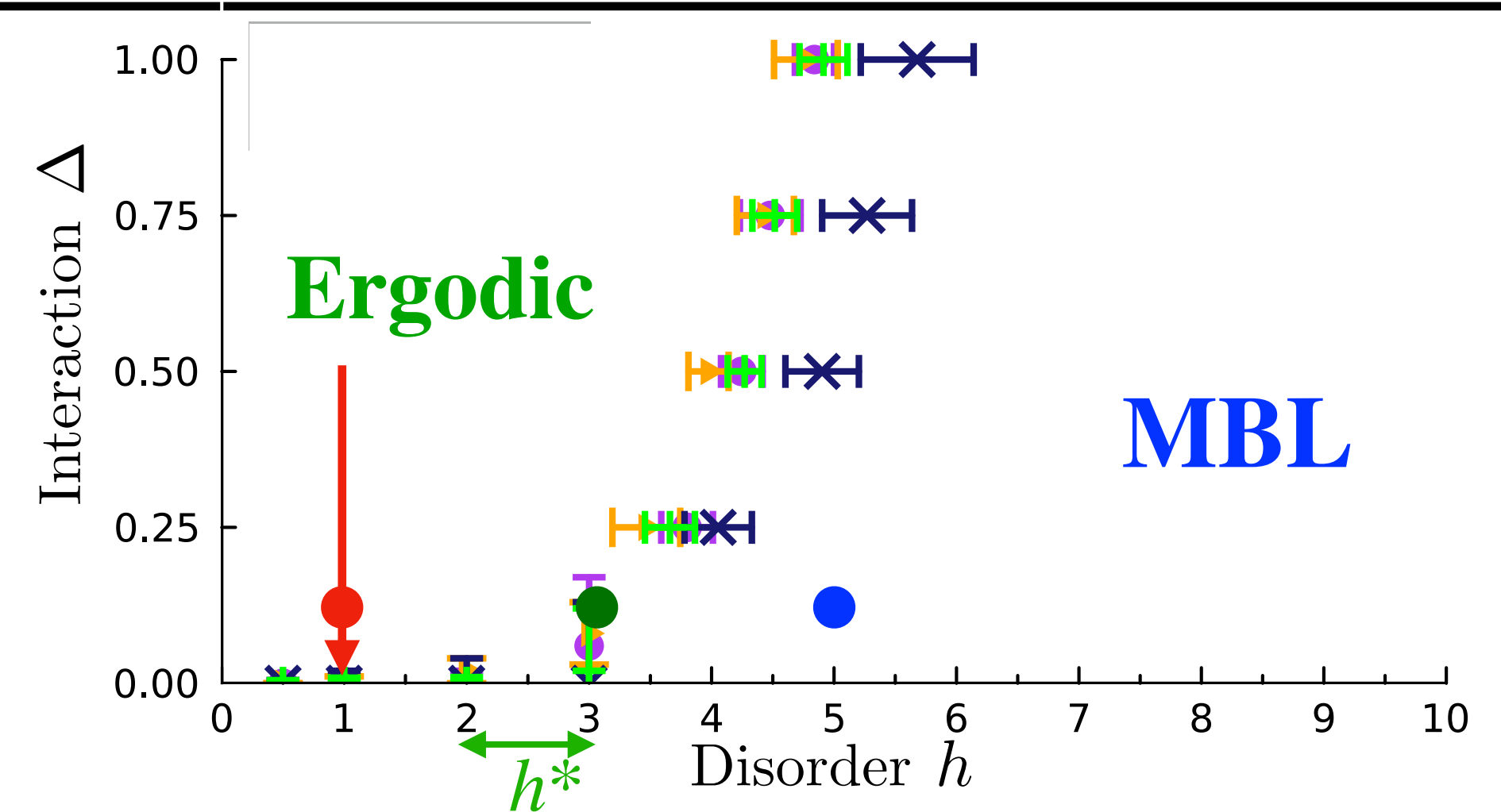
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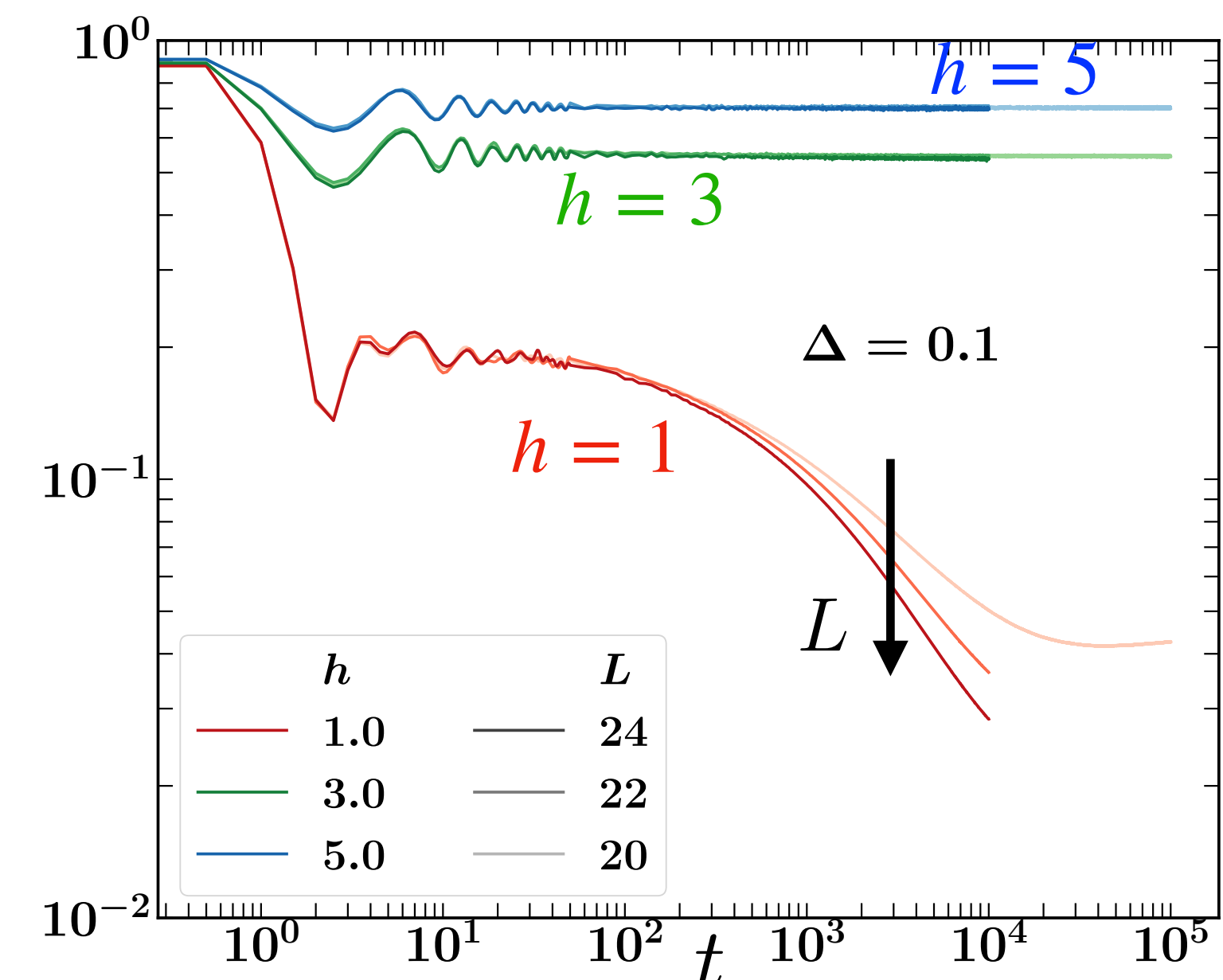
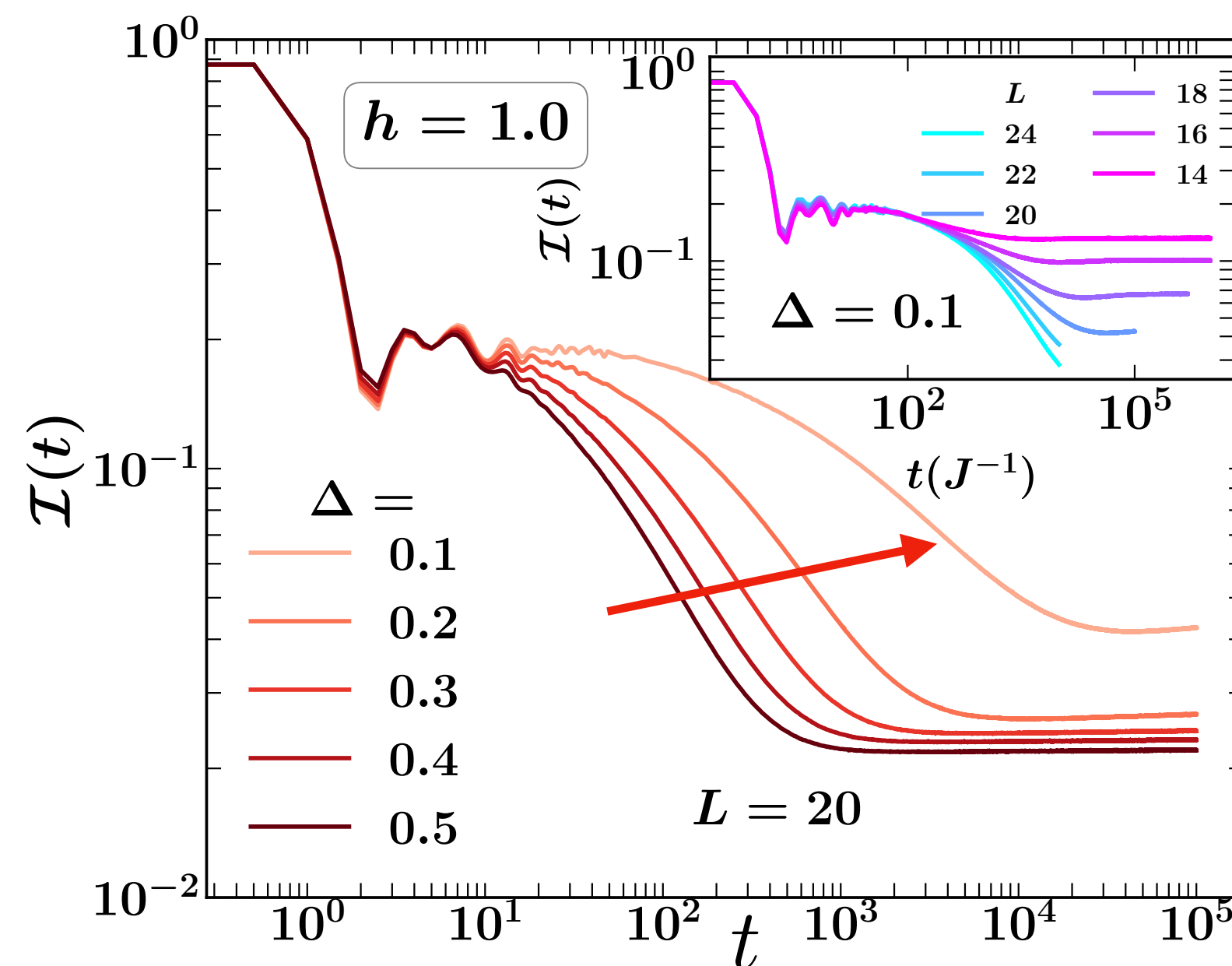
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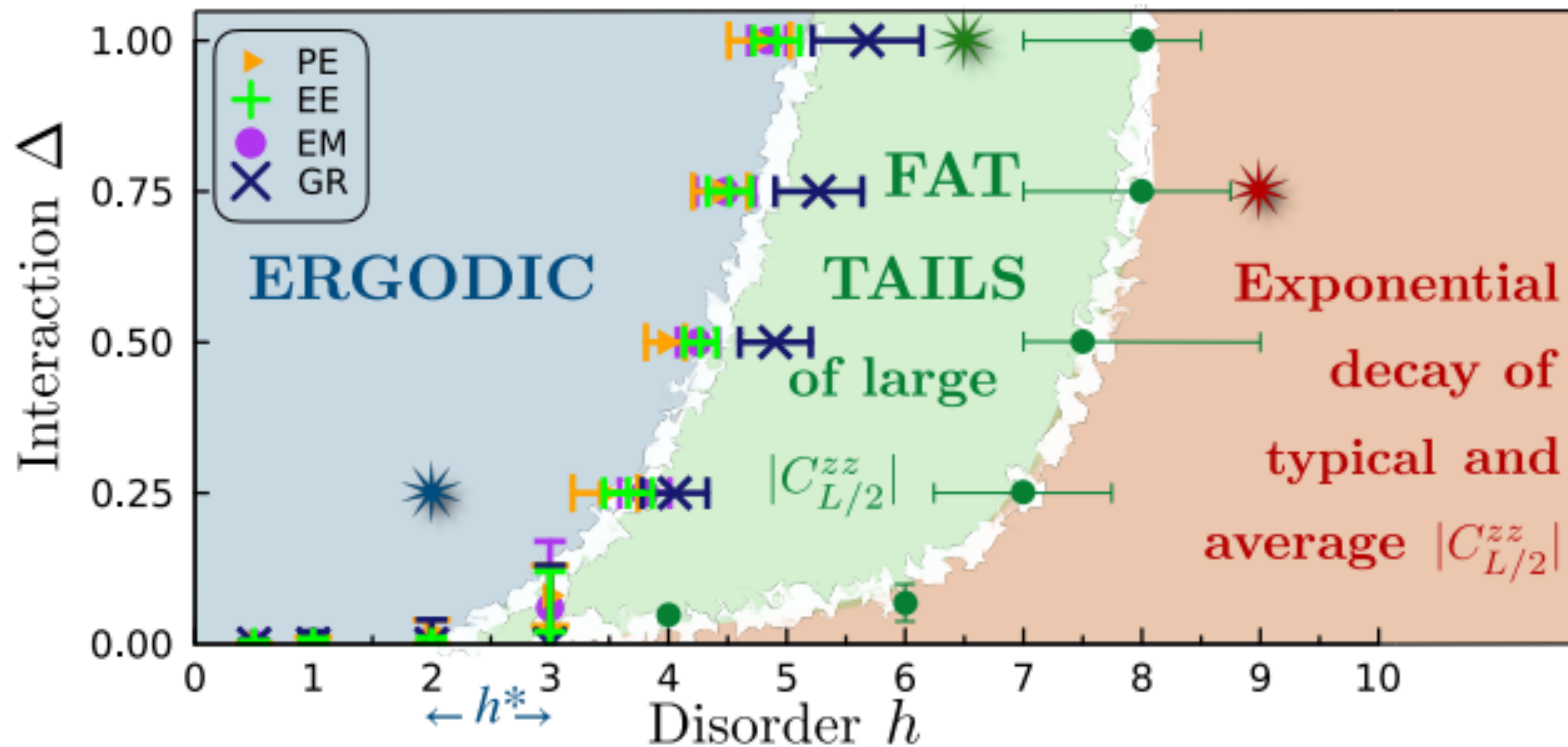
Imbalance after a quench





# For more colors ... see Nicolas' talk on Friday

arXiv:2410.10325, (PRB in press)







# Extremal Magnetization

Extremal Magnetization (EM) = which spin in the chain is most polarised?  $\delta_{\min} = \min_i (1/2 - |\langle S_i^z \rangle|)$

Laflorencie, Lemarié, Macé (2020)

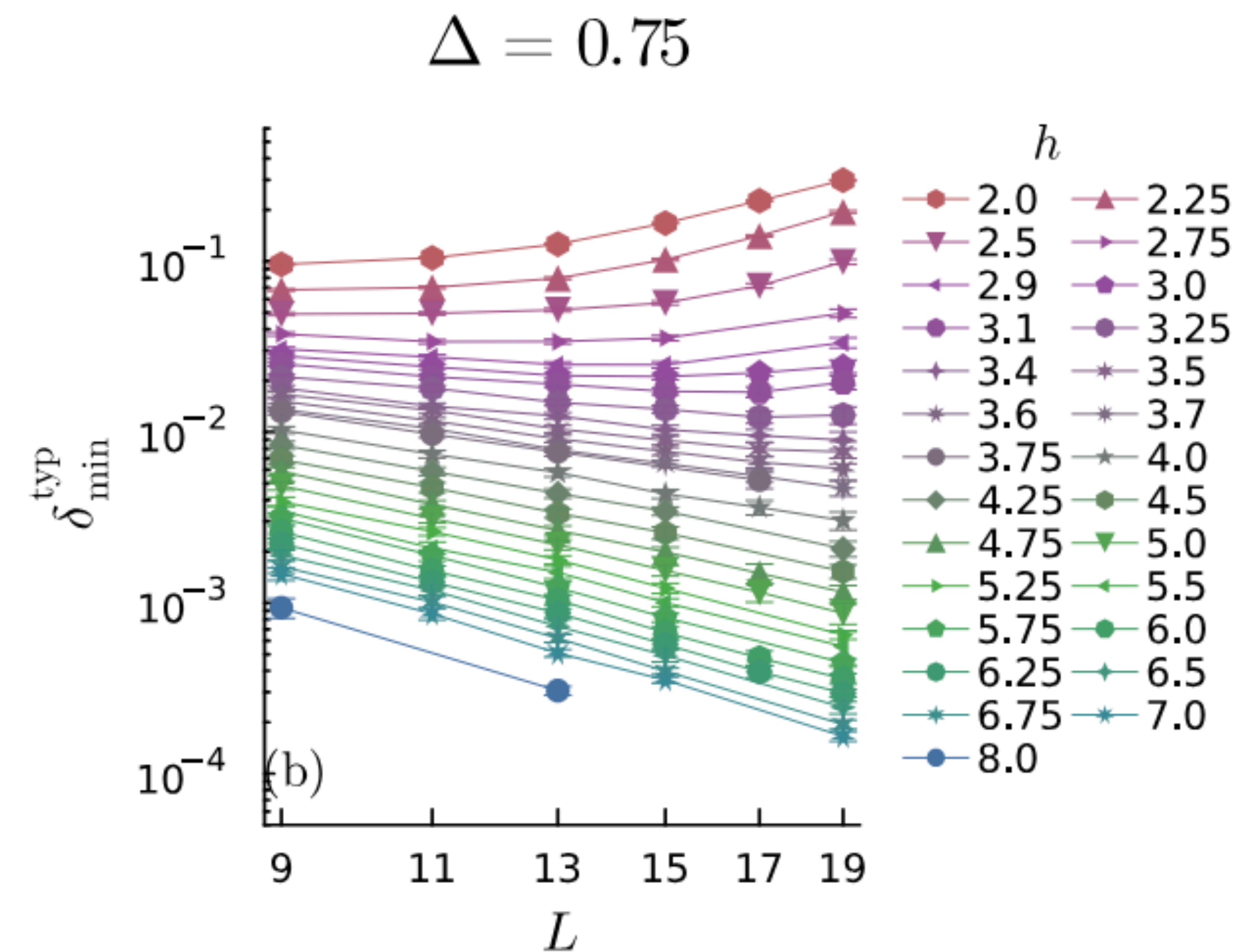
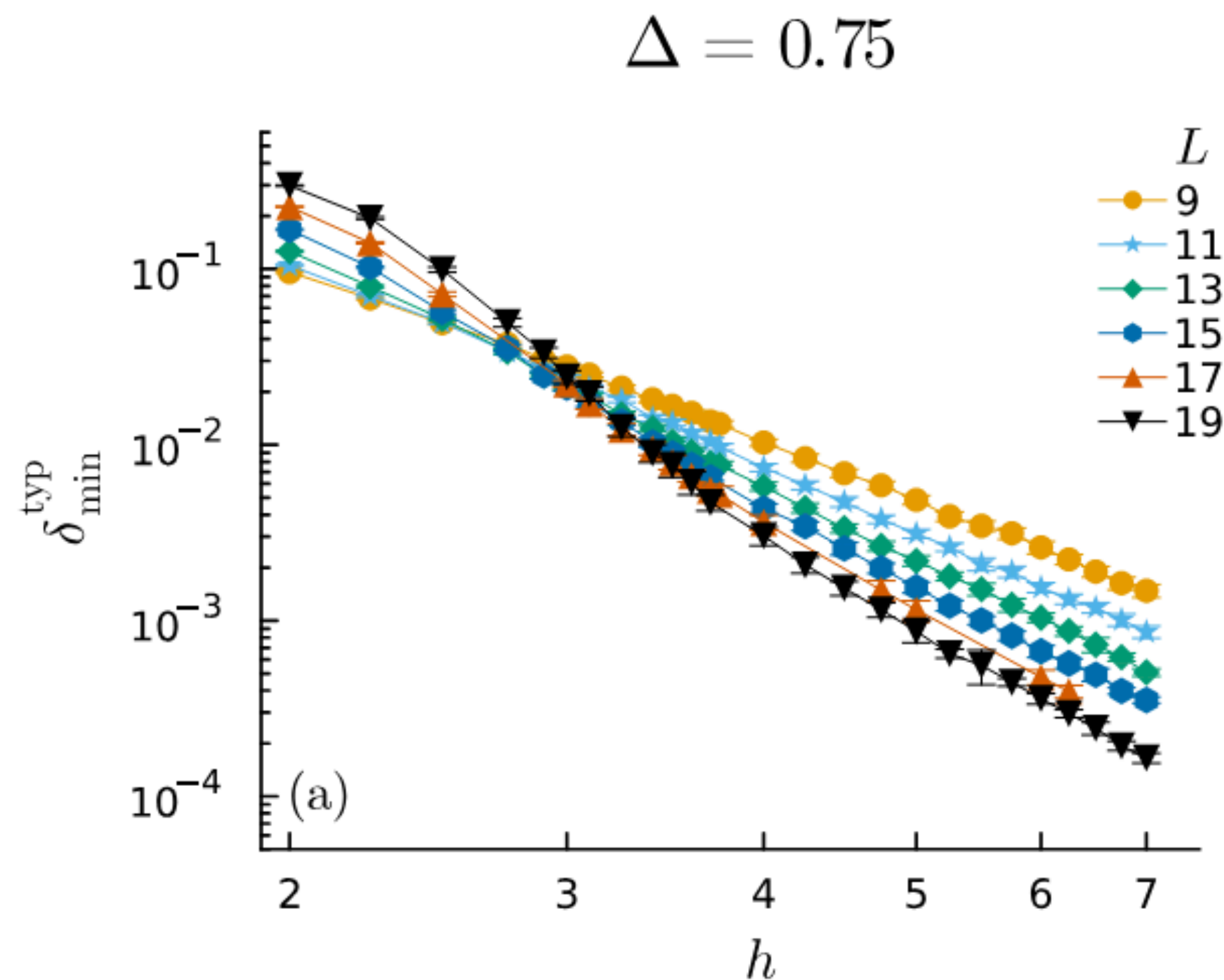
Ergodic

$$\delta_{\min}^{\text{typ}} \rightarrow 1/2$$

Localized regimes (AL, MBL)

$$\delta_{\min}^{\text{typ}} = L^{-\gamma(h)}$$

$$\gamma_{\text{AL}} = \frac{1}{2\xi_{\text{AL}} \ln(2)}$$





# System-wide resonances (?)

$$|E', \pm\rangle = \frac{1}{\sqrt{2}} \left( \left| \begin{array}{cccccccccccc} \downarrow & \uparrow & \downarrow & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{orange} & & & & & & & & & & & \text{orange} \end{array} \right\rangle \pm \left| \begin{array}{cccccccccccc} \uparrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \uparrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{orange} & & & & & & & & & & & \text{orange} \end{array} \right\rangle \right)$$

The diagram illustrates two states,  $|E', +\rangle$  and  $|E', -\rangle$ , in a 12-site chain. The top state,  $|E', +\rangle$ , features a red double-headed arrow pointing down at the first and last sites, which are highlighted with orange rounded rectangles. The remaining sites have blue arrows pointing up or down in an alternating pattern: up at sites 2, 4, 5, and 7; down at sites 3, 6, 8, 9, 10, 11, and 12. The bottom state,  $|E', -\rangle$ , features a red double-headed arrow pointing up at the first and last sites, also highlighted with orange rounded rectangles. The remaining sites have blue arrows pointing up or down in an alternating pattern: up at sites 2, 3, 4, 6, 7, and 8; down at sites 5, 9, 10, 11, and 12. The two states are separated by a plus-minus sign ( $\pm$ ).